Erratum: $\pi\gamma \rightarrow \pi\pi$ transition and the ρ radiative decay width from lattice QCD **[Phys. Rev. D **98**, 074502 (2018)]**

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I. $\pi\gamma \rightarrow \pi\pi$ CROSS SECTION

The $\pi\gamma \rightarrow \pi\pi$ cross section was erroneously evaluated using the amplitude for a two-pion final state with definite isospin, $I = 1$. The final state observable in experiment is the unsymmetrized $|\pi^+\pi^0\rangle$ state. Equation (16) should therefore be replaced by

$$\sigma(\pi^+\gamma \rightarrow \pi^+\pi^0; s, q^2) = \frac{e^2}{16\pi} k |\vec{p}_\pi| \frac{4|\mathcal{V}_{\pi\gamma \rightarrow \pi\pi}^{(u)}(q^2, s)|^2}{m_\pi^2} \quad (\text{E1})$$

with the unsymmetrized transition amplitude

$$\mathcal{V}_{\pi\gamma \rightarrow \pi\pi}^{(u)} = \frac{1}{\sqrt{2}} \mathcal{V}_{\pi\gamma \rightarrow \pi\pi}. \quad (\text{E2})$$

The plot in Fig. 11 in our original publication should be replaced by the plot in Fig. 1 of this erratum for the unsymmetrized final state, corresponding to a cross section that is smaller by a factor of 2.

II. DECAY WIDTH AND PHOTO COUPLING

Equation (17) for the $\rho \rightarrow \pi\gamma$ decay width was missing a factor of 2 and should read

$$\Gamma(\rho \rightarrow \pi\gamma) = \frac{4}{3} \alpha \left(\frac{m_\rho^2 - m_\pi^2}{2m_\rho} \right)^3 \frac{|G_{\rho\pi\gamma}|^2}{m_\pi^2}. \quad (\text{E3})$$

Consequently, our lattice-QCD prediction for this decay width in Eq. (74) should be corrected to

$$\Gamma(\rho \rightarrow \pi\gamma) = 168(13)(8) \text{ keV}. \quad (\text{E4})$$

FIG. 1. The $\pi^+\gamma \rightarrow \pi^+\pi^0$ cross section.

FIG. 2. Correction version of the plot in Fig. 12.

Using the corrected Eq. (E3) and the experimental decay width $\Gamma(\rho \rightarrow \pi\gamma)_{\text{exp}} = 68(7)$ keV, the numerical value of the experiment-based coupling is corrected to

$$|G_{\rho\pi\gamma}|_{\text{exp}} = 0.0508(26). \quad (\text{E5})$$

Our lattice result for this coupling, $|G_{\rho\pi\gamma}| = 0.0802(32)(20)$, remains unchanged. A significant difference between the lattice result and the experimental value can be expected, given that we did not perform an extrapolation to zero lattice spacing and physical pion mass, and we did not estimate the resulting systematic errors.

III. SIGN OF THE IMAGINARY PART OF THE RESONANCE POLE LOCATION

In the paragraph above Eq. (7), in the paragraph above Eq. (13), in Eq. (13) itself, and in the sentence above Eq. (17), $m_R^2 + im_R\Gamma_R$ should be changed to $m_R^2 - im_R\Gamma_R$. This also has the consequence that the imaginary part of the form factor F changes sign, and the plot in Fig. 12 should be replaced by the plot shown in Fig. 2 of this erratum.

IV. CORRECTIONS OF TYPOGRAPHICAL ERRORS

Finally, we correct a number of typos in the manuscript that did not affect our results.

- (1) In Eq. (45), $\eta^{(t_y)}$ should be changed to $\xi^{(t_y)}$.
- (2) In Eq. (60), in the paragraph above Eq. (60), and in the first two sentences of Sec. VI.B, $|s, q^2; \vec{P}, \Lambda, r\rangle_{IV}$ should be changed to $|s, \vec{P}, \Lambda, r\rangle_{IV}$.
- (3) In the caption of Fig. 4, $\langle s, q^2; \vec{P}, \Lambda, r|$ should be changed to $\langle s, \vec{P}, \Lambda, r|$.
- (4) In the caption of Table V, $|n, \vec{P}, \Lambda, \rangle_{FV}$ should be changed to $|n, \vec{P}, \Lambda, r\rangle_{FV}$.
- (5) In the legend in the bottom-right panel of Fig. 3, square roots were missing. That is, the expressions in this panel should be replaced by

$$\sqrt{\frac{8\pi}{k_n^{\vec{P},\Lambda}} \frac{\partial \phi^{\vec{P},\Lambda}}{\partial \sqrt{s}}}$$

$$\sqrt{\frac{8\pi}{k_n^{\vec{P},\Lambda}} \left(\frac{\partial \delta_I}{\partial \sqrt{s}} + \frac{\partial \phi^{\vec{P},\Lambda}}{\partial \sqrt{s}} \right)}$$

$$\sqrt{\frac{8\pi}{k_n^{\vec{P},\Lambda}} \left(\frac{\partial \delta_{II}}{\partial \sqrt{s}} + \frac{\partial \phi^{\vec{P},\Lambda}}{\partial \sqrt{s}} \right)}.$$

Here, besides adding the missing square roots, we also simplified the equations by canceling common factors in numerator and denominator.

ACKNOWLEDGMENTS

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- [1] M. Niehus, M. Hoferichter, and B. Kubis, The $\gamma\pi \rightarrow \pi\pi$ anomaly from lattice QCD and dispersion relations, [J. High Energy Phys.](#) **12** (2021) 38.