Triply heavy baryon spectroscopy in the relativistic quark model

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Triply heavy baryons are investigated in the framework of the relativistic quark model based on the quark-diquark picture in the quasipotential approach in QCD. Masses of the ground and excited states of the Ω_{ccc} , Ω_{bbb} , Ω_{ccb} , and Ω_{cbb} baryons are calculated. Orbital and radial excitations between the diquark and quark as well as between quarks inside the diquark are considered. The diquark internal structure is consistently taken into account by the form factor of the diquark-gluon interaction expressed through the overlap integral of the diquark wave functions. A detailed comparison with previous calculations is given.

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I. INTRODUCTION

Recently, significant experimental progress has been achieved in studying hadrons with heavy quarks. Many new states of heavy mesons as well as of heavy baryons were observed, some of which have properties implying their exotic nature (for recent reviews, see, e.g., [1–3] and references therein). A special interest represents the long-awaited discoveries very recently made by the LHCb Collaboration of the doubly charmed baryon Ξ_{cc}^{++} [4]; the tetraquark $T_{cc\bar{c}\bar{c}}$, composed of two charm quarks and two charm antiquarks, X(6900) [5]; and the doubly charmed tetraquark, T_{cc}^+ , with a quark content $cc\bar{u}\bar{d}$ [6]. All these new states require the production of at least two charm quark-antiquark pairs. The next important step forward will be the discovery of the triply heavy baryons, composed only from heavy charm and/or bottom quarks and, thus, requiring the production of three heavy quark-antiquark pairs. The first observation of the simultaneous production of three J/ψ mesons in proton-proton collisions was very recently presented by the CMS Collaboration [7]. Estimates of the production cross section of triply heavy baryons in proton-proton [8] and heavy ion [9] collisions indicate that triply charmed Ω_{ccc} baryons have good chances to be observed at LHC.

In this paper, we apply the relativistic quark model based on the quasipotential approach in QCD to calculate the mass spectra of triply heavy baryons. These baryons contain only heavy quarks, and in the literature, they are usually treated as nonrelativistic systems. However, the

investigation of the heavy quark dynamics in heavy quarkonia shows that heavy quarks should be treated relativistically [10]. Indeed, estimates of the charm quark velocity v in charmonia show that it is about one half of the velocity of light c , while the bottom quark velocity in bottomonia is about one third of c . Our previous investigations of meson [10,11], baryon [12–15], and tetraquark [3,16] properties showed that relativistic effects play a very important role. Thus, we treat triply heavy baryons completely relativistically without application of the expansion in heavy quark velocity. To achieve this goal, we use the relativistic quark-diquark model which was previously developed and applied for the consideration of heavy [12,13], doubly heavy [14], and strange [15] baryons. Constructing the triply heavy baryon, we assume that two quarks of the same flavor form a doubly heavy diquark,¹ and the baryon is a relativistic bound system of this doubly heavy diquark and heavy quark. The masses and wave functions of diquarks are obtained by solving the relativistic quasipotential equation with the quark-quark interaction, which is one half of the quark-antiquark interaction in mesons. The diquark is considered to be a composite, not a pointlike object. The diquark internal structure is taken into account by calculating the diquark form factor, which enters the diquark-gluon interaction, and is expressed as the overlap integral of the diquark wave functions. The account of the diquark size softens the diquark-gluon interaction thus increasing the baryon mass. This effect allowed us to get the correct prediction for the doubly charmed baryon Ξ_{cc}^{++} mass long before its discovery [14]. It is important to point out that consistent treatment of

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¹This assumption was previously successfully tested in calculations of strange baryon spectra [15]. All observed ground and excited states of Λ , Σ , and Ξ baryons fit well in the picture where diquarks are composed from quarks of the same constituent mass.

the relativistic quark dynamics permitted us to get predictions for meson [10,11], baryon [12–15], and tetraquark [3] masses and decays in good agreement with experimental data using the universal set of model parameters, which we keep fixed in the present calculations of the triply baryon spectroscopy. Note that in most quark models the parameters for descriptions of meson and baryon properties are varied.

The paper is organized as follows. In Sec. II, we briefly describe our relativistic quark-diquark model. The quasipotential equation and quark-quark and quark-diquark interaction potentials are given. Doubly heavy diquarks are considered in Sec. III. Their masses are calculated up to the first radial excitation and second orbital excitation. The form factors entering the diquark-gluon interaction are evaluated, and their appropriate parametrization is given. In Sec. IV, we calculate the masses of the ground, orbitally, and radially excited states of triply heavy baryons and compare our results with previous calculations. Finally, Sec. V contains our conclusions.

II. RELATIVISTIC QUARK-DIQUARK MODEL

The relativistic quark-diquark model for the description of doubly heavy, heavy baryon, and hyperon spectroscopy based on the quasipotential approach and quark-diquark picture of baryons was developed and previously used in Refs. [12–15]. Here, we apply this model for the consideration of the triply heavy baryon spectroscopy. We use the same assumptions and model parameters. For completeness, we give its brief outline. In this approach, the complicated relativistic three-body problem is reduced to the solution of two more simple relativistic two-body problems. First, we introduce diquarks which are considered to be bound states of two quarks. It is assumed that quarks of the same flavor form a diquark. Second, the baryon is considered to be a bound system of a diquark and quark. In a quasipotential approach, interactions of two quarks in a diquark and of the quark and diquark in a baryon are described by the diquark wave function Ψ_d and by the baryon wave function Ψ_B , respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type [10]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,B}(\mathbf{q}), \quad (1)$$

with the relativistic reduced mass given by

$$\mu_R = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

and the center-of-mass system relative momentum squared on a mass shell defined by

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}, \quad (3)$$

where M is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of quarks (q_1 and q_2) which form the diquark or of the diquark (d) and quark (q) which form the baryon (B), and $\mathbf{p}$ is their relative momentum.

To construct the kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1), which is the quasipotential operator of the quark-quark or quark-diquark interactions, it is assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of long-range vector and scalar linear confining potentials [12,13]. The vector confining potentials contain additional effective Pauli terms, which introduce anomalous chromomagnetic moments of quarks and diquarks.

The quark-quark (qq) interaction quasipotential for the diquark is given by

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p) \bar{u}_2(-p) \mathcal{V}(\mathbf{p}, \mathbf{q}; M) u_1(q) u_2(-q), \quad (4)$$

with

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{1}{2} \left[\frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k}) \gamma_1^\mu \gamma_2^\nu + V_{\text{conf}}^V(\mathbf{k}) \Gamma_1^\mu(\mathbf{k}) \Gamma_{2;\mu}(-\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}) \right],$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, $\mathbf{k} = \mathbf{p} - \mathbf{q}$, γ_μ , and $u(p)$ are the Dirac matrices and spinors. The effective long-range vector vertex of the quark is defined by [10]

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_{\mu\nu} \tilde{k}^\nu, \quad \tilde{k} = (0, \mathbf{k}), \quad (5)$$

and κ is the anomalous chromomagnetic moment of quarks.

The quark-diquark (qd) interaction quasipotential in the baryon has the form

$$\begin{aligned} V(\mathbf{p}, \mathbf{q}; M) = & \frac{\langle d(P) | J_\mu | d(Q) \rangle}{2\sqrt{E_d(P)E_d(Q)}} \bar{u}_q(p) \frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k}) \gamma^\nu u_q(q) \\ & + \psi_d^*(P) \bar{u}_q(p) J_{d;\mu} \Gamma_q^\mu(\mathbf{k}) V_{\text{conf}}^V(\mathbf{k}) u_q(q) \psi_d(Q) \\ & + \psi_d^*(P) \bar{u}_q(p) V_{\text{conf}}^S(\mathbf{k}) u_q(q) \psi_d(Q), \end{aligned} \quad (6)$$

where $\langle d(P) | J_\mu | d(Q) \rangle$ is the vertex of the diquark-gluon interaction which takes into account the diquark internal structure, $J_{d;\mu}$ is the effective long-range vector vertex of the diquark, the diquark momenta are $P = (E_d(p), -\mathbf{p})$, $Q = (E_d(q), -\mathbf{q})$ with $E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2}$, and $\psi_d(P)$ is the diquark wave function [12].

The vector and scalar confining potentials in the non-relativistic limit in configuration space are the linear potentials with the mixing coefficient ε ,

$$V_{\text{conf}}^V(r) = (1 - \varepsilon)(Ar + B), \quad V_{\text{conf}}^S(r) = \varepsilon(Ar + B), \quad (7)$$

and their sum is just the usual static Cornell-like potential,

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + Ar + B, \quad (8)$$

with the freezing QCD coupling constant

$$\alpha_s(\mu^2) = \frac{4\pi}{(11 - \frac{2}{3}n_f) \ln \frac{\mu^2 + M_B^2}{\Lambda^2}}. \quad (9)$$

The scale $\mu = 2\mu_{NR}$ is chosen to be twice the nonrelativistic reduced mass $\mu_{NR} = m_1 m_2 / (m_1 + m_2)$. The background mass is taken $M_B = 0.95$ GeV, and the parameter $\Lambda = 413$ MeV was fixed from the light meson spectroscopy [11].

All parameters of the model are kept fixed from previous calculations of meson and baryon properties [10,12,14]. The constituent heavy quark masses are $m_c = 1.55$ GeV, $m_b = 4.88$ GeV. The parameters of the linear potential are $A = 0.18$ GeV² and $B = -0.3$ GeV. The value of the mixing coefficient of vector and scalar confining potentials is $\varepsilon = -1$, and the anomalous chromomagnetic quark moment is $\kappa = -1$. Note that the long-range chromomagnetic contribution to the potential, which is proportional to $(1 + \kappa)$, vanishes for the chosen value of $\kappa = -1$.

III. DOUBLY HEAVY DIQUARKS

As the first step, we consider the doubly heavy diquarks and calculate their masses and form factors. We assume that in triply heavy baryons two quarks of the same flavor form a doubly heavy diquark. We solve the quasipotential equation (1) numerically with the complete relativistic potential (4). Since a diquark is composed of heavy quarks of the same flavor, it is necessary to take into account the Pauli principle. The total baryon wave function must be antisymmetric. It is antisymmetric in color; thus, the rest of the wave function must be symmetric. The symmetric form of the flavor part of the considered doubly heavy diquark implies that the product of the spin and orbital parts is also symmetric. For the S and D states, which have orbitally symmetric wave functions, the diquark spin wave function must be also symmetric, and thus, the diquark spin is 1. For the P states with antisymmetric orbital wave function, the diquark spin wave function must be also antisymmetric, and the diquark spin is 0. The resulting value of the total momentum of the diquark is $j = 1$ for the S and P states, while values $j = 1, 2, 3$ are possible for the D states. Note that we do not consider higher orbital excitations of the quarks inside the diquark.

The calculated masses of the ground and excited states of diquarks are presented in Table I. We also give the values of the parameters ξ and ζ . They parametrize with high accuracy the r dependence of the form factor $F(r)$ in

TABLE I. Masses M and form factor parameters of the doubly heavy diquarks.

Quark content	State nl_j	M (MeV)	ξ (GeV)	ζ (GeV ²)
cc	$1s_1$	3226	1.30	0.42
	$1p_1$	3460	0.74	0.315
	$2s_1$	3535	0.67	0.19
	$1d_{1,2,3}$	3704	0.39	0.42
	$2p_1$	3712	0.60	0.155
bb	$1s_1$	9778	1.30	1.60
	$1p_1$	9944	0.90	0.59
	$2s_1$	10015	0.85	0.31
	$1d_{1,2,3}$	10123	0.49	0.59
	$2p_1$	10132	0.65	0.215

the vertex of the diquark-gluon interaction (6), which is calculated through the overlap integral of the diquark wave functions [12]. It is expressed by

$$F(r) = 1 - e^{-\xi r - \zeta r^2} \quad (10)$$

and takes the internal structure of a diquark into account [12] smearing the diquark-gluon interaction. In this table, we use the lowercase letters to denote diquark quantum numbers. This is done to distinguish them from the quark-diquark excitations for which we reserve the uppercase letters. Here, $n = n_r + 1$, where n_r is the radial quantum number (the number of nodes of the wave function); $l = s, p, d, \dots$ is the orbital momentum and j the total momentum of the diquark. The calculations show that the masses of the d states with $j = 1, 2, 3$ differ by less than 1 MeV; thus, we consider their masses to be equal and give only one value.

IV. TRIPLY HEAVY BARYONS

As the second step, we calculate the masses of the triply heavy baryons as the bound states of a heavy quark and doubly heavy diquark. Evaluating the baryon masses, we treat all relativistic contributions nonperturbatively. The quark-diquark quasipotential contains the relativistic contributions both to the spin-independent V_{SI} and spin-dependent V_{SD} parts

$$V(r) = V_{SI}(r) + V_{SD}(r). \quad (11)$$

The spin-independent part determines the position of centers of gravity of the baryon levels, while the spin-dependent part is responsible for their fine and hyperfine splittings. These parts are expressed through the static potential and its derivatives. The explicit expressions for these potentials can be found in Refs. [12,13]. It is important to point out that, as it was already noted in the previous section, the diquark form factor $F(r)$ smears the diquark-gluon interaction, thus accounting for its

internal structure. As a result, in the nonrelativistic limit, the one-gluon exchange part of the quark-diquark potential is modified and has the form of the smeared Coulomb-like potential

$$\hat{V}_{\text{Coul}}(r) = -\frac{4}{3}\alpha_s \frac{F(r)}{r}, \quad (12)$$

with $F(r)$ given by Eq. (10).

The spin-dependent part of the quasipotential contains the spin-orbit, tensor, and spin-spin interactions. It has the following form [13]:

$$V_{\text{SD}}(r) = a_1 \mathbf{L} \mathbf{S}_d + a_2 \mathbf{L} \mathbf{S}_Q + b \left[-\mathbf{S}_d \mathbf{S}_Q + \frac{3}{r^2} (\mathbf{S}_d \mathbf{r})(\mathbf{S}_Q \mathbf{r}) \right] + c \mathbf{S}_d \mathbf{S}_Q, \quad (13)$$

where $\mathbf{L}$ is the orbital angular momentum; $\mathbf{S}_d$ and $\mathbf{S}_Q$ are the diquark and quark spin operators, respectively. The coefficients a_1 , a_2 , b , and c are expressed through the corresponding derivatives of the smeared Coulomb and confining potentials [13]. The smearing of the one-gluon exchange potential (12) naturally softens singularities in the relativistic quasipotential in configuration space and allows us to solve numerically the quasipotential equation in its complete relativistic form. Note that both the one-gluon exchange and confining potentials contribute to the quark-diquark spin-orbit interaction. The presence of the spin-orbit $\mathbf{L} \mathbf{S}_Q$ and of tensor terms in the quark-diquark potential leads to a mixing of states with the same total angular momentum J and parity P but different values of the diquark angular momentum ($\mathbf{L} + \mathbf{S}_d$). We consider such mixing in the same way as in the case of doubly heavy baryons [14].

We calculate masses of all triply heavy baryons: Ω_{ccc} , Ω_{bbb} , Ω_{ccb} , and Ω_{cbb} . Their calculated spectra are given in Tables II–V. In the left hand half of these tables, we give

TABLE II. Masses of the Ω_{ccc} states.

J^P	$NLnl$	Mass (MeV)	J^P	$NLnl$	Mass (MeV)
$\frac{1}{2}^+$	$2S1s_-, 1P1p$	5230	$\frac{1}{2}^-$	$1S1p, 1P1s$	5010
	$1D1s$	5278		$1P2s_-, 2S1p_-$	5370
$\frac{3}{2}^+$	$1S1s$	4712		$2S1p_+$	5385
	$1S2s_+$	5137		$1D1p_+$	5520
	$1D1s_+$	5267	$\frac{3}{2}^-$	$1S1p, 1P1s$	5029
	$1D1s_-, 1P1p$	5277		$1P2s_-, 2S1p_-$	5379
	$2S2s$	5541		$2S1p_+$	5394
$\frac{5}{2}^+$	$1D1s_-, 1P1p$	5278		$1D1p_+$	5522
	$1D1s_+$	5290	$\frac{5}{2}^-$	$1F1s$	5519
$\frac{7}{2}^+$	$1D1s_+$	5291		$1P1d_+$	5523
			$\frac{7}{2}^-$	$1F1s$	5517
				$1P1d_+$	5526

TABLE III. Masses of the Ω_{bbb} states.

J^P	$NLnl$	Mass (MeV)	J^P	$NLnl$	Mass (MeV)
$\frac{1}{2}^+$	$2S1s_-, 1P1p$	14877	$\frac{1}{2}^-$	$1S1p, 1P1s$	14698
	$1D1s$	14912		$1P2s_-, 2S1p_-$	14991
$\frac{3}{2}^+$	$1S1s$	14468		$2S1p_+$	15042
	$1S2s_+$	14815		$1D1p_+$	15088
	$1D1s_-, 1P1p$	14893	$\frac{3}{2}^-$	$1S1p, 1P1s$	14702
	$1D1s_+$	14905		$1P2s_-, 2S1p_-$	14922
$\frac{5}{2}^+$	$2S2s$	15123		$2S1p_+$	15031
	$1D1s_-, 1P1p$	14895		$1D1p_+$	15089
$\frac{7}{2}^+$	$1D1s_+$	14907	$\frac{5}{2}^-$	$1F1s$	15081
		14909		$1P1d_+$	15086
			$\frac{7}{2}^-$	$1F1s$	15082
				$1P1d_+$	15089

TABLE IV. Masses of the Ω_{ccb} states.

J^P	$NLnl$	Mass (MeV)	J^P	$NLnl$	Mass (MeV)
$\frac{1}{2}^+$	$1S1s$	7984	$\frac{1}{2}^-$	$1P1s$	8250
	$1S2s$	8361		$1S1p$	8266
	$2S1s$	8405		$1P1s$	8268
	$1D1s$	8472		$1S2p$	8550
	$1P1p$	8505		$2P1s$	8583
	$1P1p$	8511		$2P1s$	8591
	$1S1d$	8531		$1P2s$	8592
$\frac{3}{2}^+$	$1S1s$	7999	$\frac{3}{2}^-$	$1P2s$	8595
	$1S2s$	8366		$1P1s$	8262
	$2S1s$	8412		$1P1s$	8268
	$1D1s$	8474		$1S1p$	8273
	$1D1s$	8476		$1S2p$	8554
	$1P1p$	8506		$2P1s$	8587
	$1P1p$	8510		$2P1s$	8591
$\frac{5}{2}^+$	$1S1d$	8534		$1P2s$	8591
	$1D1s$	8473	$\frac{5}{2}^-$	$1P2s$	8594
	$1D1s$	8476		$1P1s$	8267
	$1P1p$	8508		$2P1s$	8590
	$1S1d$	8536		$1P2s$	8592
$\frac{7}{2}^+$	$1D1s$	8473		$1F1s$	8647
	$1S1d$	8538			

states with the positive parity and in the right one the negative parity. We use the standard notations for the baryon states J^P , where J and P are the baryon total spin and parity, respectively. The composition of the baryon state is given by $NLnl_j$, where the capital letters denote quantum numbers of the quark-diquark system and the lowercase letters the diquark state. N or n is the radial quantum number (the number of nodes of the wave function) plus one, and L or l is the orbital quantum number.

For the baryons composed of identical quarks (Ω_{ccc} and Ω_{bbb}), there is an additional complication. It is necessary to

TABLE V. Masses of the Ω_{cbb} states.

J^P	$NLnl$	Mass (MeV)	J^P	$NLnl$	Mass (MeV)
$\frac{1}{2}^+$	1S1s	11198	$\frac{1}{2}^-$	1P1s	11414
	1S2s	11507		1S1p	11506
	1S1d	11622		1P1s	11540
	2S1s	11690		1S2p	11654
	1P1p	11692		1P2s	11778
	1P1p	11714		1P2s	11796
	1D1s	11796		2S1p	11893
$\frac{3}{2}^+$	1S1s	11217	$\frac{3}{2}^-$	1S1p	11424
	1S2s	11515		1P1s	11535
	1S1d	11629		1P1s	11541
	2S1s	11700		1S2p	11660
	1P1p	11707		1P2s	11788
	1P1p	11717		1P2s	11795
	1D1s	11797		2S1p	11897
	1D1s	11807	$\frac{5}{2}^-$	1P1s	11543
				1P2s	11795
$\frac{5}{2}^+$	1S1d	11632	$\frac{7}{2}^-$	1P1d	11903
	1P1p	11715			
	1D1s	11806			
	1D1s	11807			
$\frac{7}{2}^+$	1S1d	11635			

take into account the Pauli principle not only for the diquark but also for the entire baryon. It requires the total wave function to be antisymmetric. The color wave function of the baryon is antisymmetric. The flavor part is symmetric. This means that the spin-momentum part of the wave function must be fully symmetric. For the ground 1S1s state, this part

is symmetric in momentum, and thus, the spin wave functions must be also fully symmetric, which corresponds to the total baryon spin $3/2$. Therefore, only the $3/2^+$ ground state is possible. The lightest $1/2^+$ state should contain excitations and has a significantly larger mass. The excited states are combinations of orbital and/or radial excitations of the diquark and/or quark-diquark bound systems with the fully symmetric ($3/2$) or mixed symmetry ($1/2$) spin wave functions. Symmetric combinations, such as, e.g., $|2S1s\rangle_+ = (|2S1s\rangle + |1S2s\rangle)/\sqrt{2}$, $|1D1s\rangle_+ = (|1D1s\rangle + |1S1d\rangle)/\sqrt{2}$, $|2S2s\rangle$, are combined with a fully symmetric spin $3/2$ wave function. While antisymmetric combinations, such as, e.g., $|2S1s\rangle_- = (|2S1s\rangle - |1S2s\rangle)/\sqrt{2}$, $|1D1s\rangle_- = (|1D1s\rangle - |1S1d\rangle)/\sqrt{2}$, $|1P2s\rangle_- = (|1P2s\rangle - |1S2p\rangle)/\sqrt{2}$, are combined with the mixed symmetry spin $1/2$ wave functions. The details can be found in Ref. [17].

Masses of the ground states of Ω_{ccc} , Ω_{bbb} , Ω_{ccb} , and Ω_{cbb} baryons were calculated in many papers based on different approaches [17–46]. The predicted masses of the ground state $3/2^+$ Ω_{ccc} baryons range from 4670 to 4990 MeV, and masses of the $3/2^+$ Ω_{bbb} baryons range from 13280 to 14834 MeV. Our predictions for these masses, 4712 MeV and 14468 MeV, respectively, are well inside both ranges.

Excited states received significantly less attention. In Tables VI–IX, we compare our predictions with previous calculations [17–28] for the masses of Ω_{ccc} , Ω_{bbb} , Ω_{ccb} , and Ω_{cbb} baryons. Masses of the triply heavy baryons were calculated using lattice QCD with dynamical light quark

TABLE VI. Comparison with previous theoretical predictions for the masses of the Ω_{ccc} states.

J^P	Our (MeV)	[18] (MeV)	[19] (MeV)	[17] (MeV)	[20] (MeV)	[21] (MeV)	[22] (MeV)	[23] (MeV)
$\frac{1}{2}^+$	5230	5397(13)	5376	5358	5473	5325	5352	
	5278	5403(14)				5332	5373	
$\frac{3}{2}^+$	4712	4761(6)	4798	4797	4806	4965	4828	4760
	5137	5315(31)	5286	5309	5300	5313	5285	5150
	5267	5428(13)	5376	5358	5448		5368	
	5277	5463(13)					5412	
$\frac{5}{2}^+$	5278	5404(15)	5376	5358	5416	5329	5392	
	5290	5462(15)				5343	5433	
$\frac{7}{2}^+$	5291	5395(49)	5376	5358	5375	5331	5418	
$\frac{1}{2}^-$	5010	5118(9)	5129	5103	5012	5155	5142	
	5370	5610(31)	5525		5607			
	5385	5629(43)						
$\frac{3}{2}^-$	5029	5122(13)	5129	5103	4991	5160	5162	5027
	5379	5660(31)	5525		5584			
	5394	5722(44)						
$\frac{5}{2}^-$	5519	5514(64)	5558		4965			
	5523	5707(25)			5584			
$\frac{7}{2}^-$	5517	5679(28)			5829			

TABLE VII. Comparison with previous theoretical predictions for the masses of the Ω_{bbb} states.

J^P	Our (MeV)	[24] (MeV)	[19] (MeV)	[17] (MeV)	[20] (MeV)	[21] (MeV)	[22] (MeV)	[23] (MeV)
$\frac{1}{2}^+$	14877	14938(18)	14894	14896	15306	15097	14971	
	14912	14953(17)				15102	14959	
$\frac{3}{2}^+$	14468	14371(12)	14396	14347	14496	14834	14432	14370
	14815	14840(14)	14805	14832	15154	15089	14848	14980
	14893	14958(18)	14894	14896	15300		14975	
	14905	15005(20)					15016	
$\frac{5}{2}^+$	14895	14964(18)	14894	14896	15293	15101	14981	
	14907	15007(20)				15109	15022	
$\frac{7}{2}^+$	14909	14969(17)	14894	14896	15286	15101	14988	
$\frac{1}{2}^-$	14698	14706(9)	14688	14645	14944	14975	14773	
	14991		15016					
$\frac{3}{2}^-$	14702	14714(9)	14688	14645	14937	14976	14779	14771
	14922		15016					
$\frac{5}{2}^-$	15081		15038		14931			
$\frac{7}{2}^-$	15082				15641			

TABLE VIII. Comparison with previous theoretical predictions for the masses of the Ω_{ccb} states.

J^P	Our (MeV)	[25] (MeV)	[26] (MeV)	[19] (MeV)	[17] (MeV)	[27] (MeV)	[21] (MeV)	[23] (MeV)
$\frac{1}{2}^+$	7984	8005(17)	8007(29)	8004	8301	8005	8245	7867
	8361			8455	8647	8621	8537	8337
	8405			8536		8848		
$\frac{3}{2}^+$	7999	8026(18)	8037(29)	8023	8301	8049	8265	7963
	8366			8468	8600	8637	8553	8427
	8412			8536	8647	8831		
$\frac{5}{2}^+$	8473			8536	8647	8808	8568	
	8476						8571	
$\frac{7}{2}^+$	8473			8538	8647	8780	8568	
	8538						8653	
$\frac{1}{2}^-$	8250			8306	8491	8400	8418	8164
	8266						8422	
$\frac{3}{2}^-$	8262			8306	8491	8383	8420	8275
	8268						8422	
$\frac{5}{2}^-$	8267			8311	8491	8365	8432	

fields in Refs. [18,24–26]. Our predictions for the masses of the ground and excited states of the Ω_{ccc} baryon are lower than lattice [18] results by about 50–150 MeV; however, the structure of our excited spectrum is close to the lattice one. For the Ω_{bbb} baryon, the agreement of our predictions with lattice results [24] is even better. Masses of only ground states $1/2^+$ and $3/2^+$ of Ω_{ccb} and Ω_{cbb} baryons were calculated on the lattice [25,26]. They agree well with our results. The constituent quark model, which employs the Gaussian expansion method and the variational principle to solve the nonrelativistic three-body problem, was used in Ref. [19] to compute the mass spectra of triply heavy

baryons. The renormalization group procedure for effective particles was applied for studying baryons with heavy quarks in Ref. [17]. The hypercentral constituent quark model was employed in Refs. [20,27,28]. For the calculation of the triply heavy baryon masses, Refs. [21,22] used the nonrelativistic quark model with the harmonic oscillator wave functions and perturbative account of the relativistic corrections. The relativistic Faddeev equation with the rainbow-ladder truncated kernel was employed in Ref. [23].

To demonstrate the necessity of the relativistic treatment of the triply heavy baryons, we calculated the values of

TABLE IX. Comparison with previous theoretical predictions for the masses of the Ω_{ccb} states.

J^P	Our (MeV)	[25] (MeV)	[26] (MeV)	[19] (MeV)	[17] (MeV)	[28] (MeV)	[21] (MeV)	[23] (MeV)
$\frac{1}{2}^+$	11198	11194(17)	11195(28)	11200	11218	11231	11535	11077
	11507			11607	11585	11757	11787	11603
	11622			11677	11626	11934		
$\frac{3}{2}^+$	11217	11211(18)	11229(28)	11221	11218	11296	11554	11167
	11515			11622	11585	11779	11798	11703
	11629			11677	11626	11928		
$\frac{5}{2}^+$	11632			11677	11626	11919	11823	
	11715						11831	
$\frac{7}{2}^+$	11635			11688	11626	11909	11810	
$\frac{1}{2}^-$	11414			11482	11438	11573	11710	11413
	11506						11757	
$\frac{3}{2}^-$	11424			11482	11438	11566	11711	11523
	11535						11759	
$\frac{5}{2}^-$	11543			11569	11601	11558	11762	

$\sqrt{\langle v^2/c^2 \rangle}$ using obtained triply heavy baryon wave functions. For the ground state of the Ω_{ccc} baryon, we found the above values of 0.51 and 0.28 for the charm quark and doubly charmed diquark, respectively. For the Ω_{bbb} baryon, the corresponding values are 0.29 and 0.14, for the Ω_{ccb} are 0.20 and 0.30, and for the Ω_{cbb} 0.58 and 0.10. For the excited states, these values are even larger. As was expected, the velocities of quarks in these baryons are close to velocities of heavy quarks in quarkonia. We see that even for the heaviest triply bottom baryons relativistic effects are important. They produce additional shifts and splittings of the levels. This calculation demonstrates the need for a relativistic treatment of triply heavy baryons.

V. CONCLUSIONS

In this paper, we applied the relativistic quark model based on the quasipotential approach in QCD for the calculation of the mass spectra of triply heavy baryons. The relativistic quark-diquark approximation was used to reduce a very complicated relativistic three-body problem for the subsequent solution of two more simple two-body problems: first calculation of the diquark properties and then considering baryons as a quark-diquark bound system. Such an approach was previously successfully applied for the calculation of the masses of the ground and excited states of doubly heavy [14], heavy [13], and strange

baryons [15]. It is important to emphasize that all parameters of the model were kept fixed from the previous calculations, and no new parameters were introduced. We assumed that two identical heavy quarks form a doubly heavy diquark. Masses and wave functions of the ground and excited states of such diquarks were calculated. The internal structure of the diquark was taken into account by the form factor of the diquark-gluon interaction which was calculated as the overlap integral of the diquark wave functions. The internal structure of the diquark was found to be very important for obtaining the correct prediction for the mass of the doubly heavy baryon Ξ_{cc} [14]. It also allows us to obtain local completely relativistic quark-diquark quasipotential without fictitious singularities.

We solved numerically the corresponding quasipotential equation and obtained the masses of the ground and excited states of Ω_{ccc} , Ω_{bbb} , Ω_{ccb} , and Ω_{cbb} baryons. Excited states with total spin up to $J = 7/2$ both with positive and negative parities were considered. The calculated masses were compared with previous lattice QCD [18,24–26] and quark model calculations. Reasonable agreement with lattice results was found.

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