Remarks on the QCD-electroweak phase transition in a supercooled universe

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In extensions of the Standard Model with no dimensionful parameters the electroweak phase transition can be delayed to temperatures of order 100 MeV. Then the chiral phase transition of QCD can proceed with six massless quarks. The top-quark condensate destabilizes the Higgs potential through the Yukawa interaction, triggering the electroweak transition. Based on the symmetries of massless QCD, it has been argued that the chiral phase transition is first order. We point out that the top-Higgs Yukawa interaction is nonperturbatively large at the QCD scale, and that its effect on the chiral phase transition may not be negligible, violating some of the symmetries of massless QCD. The remaining symmetries indicate that top quarks condense in a second-order phase transition, but top condensation might also be triggered by a first-order symmetry-breaking transition in the light-quark sector.

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I. INTRODUCTION

First-order phase transitions are interesting for cosmology because they can leave observable relics such as the baryon asymmetry of the Universe [1,2], gravitational waves [3], or large-scale magnetic fields [4]. In the Standard Model there is neither an electroweak, nor a QCD, or quark-hadron phase transition: both the electroweak symmetry [5,6] and the approximate symmetries of QCD get broken in a smooth crossover [7,8].

However, in nearly scale-invariant extensions of the Standard Model there can be a first-order electroweak phase transition and, furthermore, it can have very peculiar features: The decay rate of the metastable high-temperature phase can be tiny down to very low temperatures, so that the Universe supercools well below the critical temperature [9–12]. Important examples are Coleman-Weinberg-type models [13], which contain no (negative) mass squared term for the Higgs field(s). The thermal mass squared for the Higgs field H is positive, so that the state with $H = 0$ remains metastable until $T = 0$. In some cases the temperature becomes as low as $O(100)$ MeV while the electroweak symmetry is still unbroken, and all six quark flavors are still massless. QCD with six massless quarks is expected to have a first-order chiral phase transitions [14], in which quark condensates $\langle \bar{q}q \rangle \neq 0$ form at a

critical temperature of about 85 MeV [15]. If this also happens in the strongly supercooled universe, the top condensate would give rise to a linear term in the effective Higgs potential through the top Yukawa interaction

$$\mathcal{L}_{t\text{-Yukawa}} = y_t \bar{Q}_3 \tilde{H} t_R + \text{H.c.}, \quad (1)$$

where $\bar{Q}_3 = (\bar{t}_L, \bar{b}_L)$ is the doublet of third-family left-handed quarks, t_R is the right-handed top quark, and $\tilde{H} = i\sigma^2 H^*$. The linear term destabilizes the minimum of the effective potential at $H = 0$ and triggers electroweak symmetry breaking [9]. After the top condensation the universe either evolves into a metastable state, followed by another phase transition, or it could directly roll to the minimum of the effective potential. Similarly, patterns arise in other scale invariant extension, such as Randall-Sundrum models [16–18].

At a QCD phase transition the QCD coupling is large, and perturbation theory cannot be applied. Nonperturbative lattice computations with light quarks are very difficult (see [19,20] for simulations with large number of flavors). Insights into the nature of a phase transition can be gained from symmetry considerations. The Lagrangian of massless QCD with six quarks is invariant under chiral $SU(6)_L \times SU(6)_R$ flavor transformations. A quark condensate breaks the axial subgroup, leaving a vector $SU(6)$ symmetry. If this happens in a second-order phase transition, then at the critical temperature some fields become effectively massless,¹ and the corresponding effective theory becomes scale invariant. Then the renormalization group for the couplings of this effective theory has an infrared-stable fixed point.

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¹Or, equivalently, some correlations lengths diverge.

The nonexistence of an infrared-stable fixed point, on the other hand, is an indication that the phase transition is first order. Near a second-order phase transition, the effective long-distance degrees of freedom are scalar fields describing the quark condensates. The effective Lagrangian, and thus the beta functions are determined by the symmetries. The beta functions can be computed in the so-called ϵ expansion: They are treated perturbatively near $d = 4$ dimensions by writing $d = 4 - \epsilon$, and expanding in powers of ϵ . The effective theory describing the long-distance degrees of freedom near a second order is three-dimensional, so the ϵ expansion is applied to $\epsilon = 1$. For massless QCD with $N_f \geq 3$ quark flavors one finds no infrared-stable fixed point, which hints at a first-order chiral phase transition [14]. At nonzero, but small quark masses, a first-order phase transition is expected, even though it has not been observed in lattice simulations so far [20–23].

II. SYMMETRIES WITH TOP-HIGGS YUKAWA INTERACTION

Previous discussions of an electroweak phase transition triggered by top-quark condensation [11] were based on the symmetries of QCD alone. However, neither Yukawa nor electroweak gauge interactions respect the $SU(6)_L \times SU(6)_R$ symmetry. One may assume that electroweak interactions can be neglected, but the top Yukawa coupling becomes nonperturbatively large near the QCD scale [24,25]. In Ref. [25] the running couplings from the three-loop renormalization group of the massless Standard Model are shown: the top Yukawa coupling grows as fast as the QCD coupling and blows up when the renormalization scale approaches the QCD scale parameter, Λ_{QCD} .² Therefore, it is likely that the top Yukawa interaction in Eq. (1) would affect the chiral phase transition. When the Higgs field and the top Yukawa interaction are included, the remaining symmetry is

$$G = U(1)_B \times U(4)_L \times U(5)_R \times G_t, \quad (2)$$

where $U(1)_B$ represents baryon number, $U(4)_L$ acts on $u_L, \dots, c_L$, $U(5)_R$ acts on $u_R, \dots, b_R$, and $G_t = SU(2) \times U(1)$ acts like the electroweak gauge group, but only on Q_3 , t_R , and H .

III. PHASE TRANSITIONS

In a chiral phase transition G is broken by expectation values of

$$\Phi_{ij} \sim \overline{q_{Li}} q_{Rj}. \quad (3)$$

If such a phase transition was second order, some components of Φ would become effectively massless at the critical temperature. Unlike in the $SU(6)_L \times SU(6)_R$ symmetric case [14], not all Φ_{ij} become massless at the same

temperature, but only those related by the symmetry G . Thus the breaking of G might proceed in a sequence of phase transitions.

When the top condensate forms, Φ_{i6} with $i = 5, 6$ (corresponding to b and t) gets a nonzero expectation value. It transforms as a doublet under $SU(2)$.³ The long-distance effective theory is equivalent to an $O(N)$ model with $N = 4$, which is known to have a second-order phase transition (see e.g., [26]). Thus, one would conclude that the breaking of G_t through top-quark condensation happens in a second-order phase transition.

However, this is not the only possibility. Coming from high temperatures, there could first be one or several of the following phase transitions involving the light-quark sector, which break different symmetries: (i) when Φ_{ij} with $i = 1, \dots, 4$, $j = 1, \dots, 5$ gets an expectation value, or (ii) Φ_{ij} with $i = 5, 6$, $j = 1, \dots, 5$, or (iii) Φ_{i6} with $i = 1, \dots, 4$. Under the symmetry-group G in Eq. (2), these fields transform like $\Phi \rightarrow U^\dagger \Phi V$ with $U \in U(4)_L$, $V \in U(5)_R$ in case (i), with $U \in SU(2) \subset G_t$, $V \in U(5)_R$ in case (ii). In case (iii) the field transforms like $\Phi \rightarrow U^\dagger \Phi$ with $U \in U(4)_L$. Then the long-distance behavior is again described by an $O(N)$ model, this time with $N = 8$, which has a second-order phase transition [26]. Results applicable to cases (i) and (ii) were obtained in Ref. [27], where the fixed points for complex scalar fields $\Phi = (\Phi_{ij})$, $i = 1, \dots, M$, $j = 1, \dots, N$, with the $U(M) \times U(N)$ -symmetric Lagrangian

$$\mathcal{L} = \text{Tr}(\partial_\mu \Phi^\dagger)(\partial^\mu \Phi) + r \text{Tr} \Phi^\dagger \Phi + \frac{u}{4} (\text{Tr} \Phi^\dagger \Phi)^2 + \frac{v}{4} \text{Tr}(\Phi^\dagger \Phi)^2 \quad (4)$$

were studied at leading order in the ϵ expansion. There it was found that there is an infrared-stable fixed point for $MN < 2$, and that there are additional fixed points when $(N + M)^2 \geq 12(MN - 2)$, i.e., for $N > N_+$ or $N < N_-$ with $N_\pm = 5M \pm [24(M^2 - 1)]^{1/2}$. This indicates that there are no infrared-stable fixed points for $(M, N) = (4, 5)$ and $(M, N) = (2, 5)$, corresponding to case (i) and (ii), respectively. One would therefore conclude that in these two cases the phase transition is first order. The analysis of [27] was extended to five [28] and recently to six loops [29]. The ϵ expansion turns out to be not particularly well behaved for $\epsilon = 1$. The result of a resummation performed in Ref. [28] indicate that N_+ could be slightly less than five when $M = 2$, so there might be an infrared-stable fixed point for $(M, N) = (2, 5)$. This was confirmed in [29]. On the other hand, Refs. [27–29] all find that $N_+ > 5$ for $M = 4$, indicating that there is a first-order phase transition for $(M, N) = (4, 5)$.

If there is a first-order phase transition as described in the previous paragraph, there are two possibilities. The first is that in this phase transition a top condensate emerges as well, not for symmetry reasons, but because there is a discontinuous change of the properties of the system. That would mean

²The Higgs self-coupling blows up as well, but it does not affect the symmetries discussed here.

³It can mix with $\tilde{H}$, which has the same quantum numbers.

that the top condensate forms in a first-order phase transition. Another possibility is that the top condensate forms later in a second-order transition as described above. Which of these two possibilities is realized can probably not be inferred from symmetry considerations, but only from lattice simulations of the QCD + Higgs system.

If electroweak interactions cannot be neglected, the Φ_{ij} are not gauge invariant, and cannot serve as an order parameter indicating the breaking of a global symmetry. In particular, there is no *global* symmetry which gets broken when $\bar{t}t$ gets an expectation value from which one could infer information about the existence or the order of a phase transition.⁴ The baryon number is violated due to

⁴This situation is similar to the breaking of the electroweak gauge symmetry through the Higgs expectation value, and for the measured value of the Higgs boson there is no phase transition in the Standard Model.

electroweak sphalerons. A remaining global symmetry is $SU(2)_Q \times SU(2)_u \times SU(3)_d$ acting on the left-handed quark doublets and the right-handed up-type quarks of the first two families, and on all right-handed down-type quarks. This symmetry can be broken by expectation values of the gauge invariant operators $\bar{Q}_\alpha \tilde{H} u_j$ ($\alpha = 1, 2; j = 1, 2$), or $\bar{Q}_\alpha H d_j$ ($\alpha = 1, 2; j = 1, 2, 3$). In a phase transition in which these operators acquire an expectation value the Higgs field would also have to become nonzero. Then it would still have to overcome the barrier of the effective potential, without assistance from the top quark.

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