Magnetogenesis from baryon asymmetry during an early matter dominated era

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In this paper, we study the simultaneous evolution of baryon asymmetry and hypermagnetic field amplitude assuming an early matter domination. We contrast our results to the conventional case where radiation domination during early universe is assumed. We show that the baryon asymmetry and the hypermagnetic field amplitude can change by orders of magnitude if we assume a nonstandard history of cosmology. That is because the Hubble rate determines which processes are efficient. We find that a change in Hubble rate can have a significant impact on when the weak sphalerons become active. As a result of a change in the evolution of baryonic asymmetry, alters the evolution of hypermagnetic field amplitude. It is known that if the hypermagnetic field amplitude is large enough, it can save the baryon asymmetry from diminishing. We show that whether a small seed of hypermagnetic field amplitude can be amplified to a large enough value will strongly depend on the history of cosmology.

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I. INTRODUCTION

According to our knowledge of the big bang nucleosynthesis (BBN) [1] and the cosmic microwave background (CMB) [2], we expect the universe to be in the stage of radiation domination (RD) around the temperature of (10 MeV-eV). On the other hand, we know that the energy densities of radiation, matter, and dark energy evolve as a^{-4} , a^{-3} , and a^0 respectively, with a being the scale factor. Therefore, the most natural and simplistic assumption about the early universe is that it was in the stage of RD after reheating up until BBN and CMB. In this assumption, if we have any matter particle with a sizable mass, it decays instantly once the temperature falls below its mass. However, it is possible to have a matter particle Φ with a relatively long lifetime before it decays. In this case, the assumption of instantaneous decay of Φ is no longer valid and for some interval in temperature, the energy density of Φ dominates the energy density of the universe [3]. The most important consequence of early matter domination (EMD) is that the Hubble rate is faster:

$$H_M \equiv \sqrt{\frac{8\pi(\rho_R + \rho_\Phi)}{3M_{\text{Pl}}^2}}, \quad (1)$$

where ρ_R is the energy density of radiation, ρ_Φ is the energy density of Φ , and $M_{\text{Pl}} = 1.2 \times 10^{19}$ GeV is the Planck mass. If $\rho_\Phi \gg \rho_R$, we are in the era of matter domination. The phenomenological importance of early matter domination (EMD) scenario was first realized long ago in the context of supersymmetry [3,4]. However, it has gained a lot of attention recently, especially to revive some of the well-motivated dark sector proposals which had not survived the experimental scrutiny [5–11] (for more details see [12] and references therein). If one of these models is correct and we are indeed living in a universe with such cosmological history, it is important to analyze the effects of EMD on other phenomena. Specifically, since the Hubble rate determines which processes are efficient in early universe, a change in Hubble rate can have important phenomenological outcomes. One place that this feature can have significant effects is in the abundance of baryon asymmetry, which is the focus of this paper.

The value of matter-antimatter asymmetry has been measured independently by BBN [1] and the Planck [2] to

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq 8.5 \times 10^{-11}, \quad (2)$$

with $s = 2\pi^2 g_* T^3/45$ being the entropy density, and g_* is the relativistic degrees of freedom.

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A study that wants to explain baryogenesis must respect the famous Sakharov conditions: (1) Baryon number violation, (2) C and CP violation, and (3) out of thermal equilibrium process [13]. Even though in the Standard Model (SM) we are aware of a CP violation in weak interactions, the amount of the CP violation is not enough to explain the baryon asymmetry. Among the mentioned requirements, only the first one is satisfied in the SM. Baryon (and lepton) symmetries are two of the known anomalous accidental symmetries of the SM. The baryon number in the SM is violated through the interactions of the weak sphalerons, which are highly active before the electroweak phase transition (EWPT), but have a suppressed interaction rate after the EWPT. Among the SM fermions, right-handed electrons are special, because they have the smallest Yukawa couplings. Therefore, if right-handed electrons acquire an asymmetry, they will keep it relatively longer, until their Yukawa interaction enters thermal equilibrium ($\sim 10^4$ – 10^5 GeV) [14,15]. After which, the asymmetry from right-handed electrons will be transferred to left-handed electrons. Then, weak sphalerons will distribute the asymmetry between all left-handed particles and eventually wash out the asymmetry [16]. Studies have shown that in the standard cosmology, if we start even with a large asymmetry in the right-handed electron ($\mu/T \sim 1$, with μ being the chemical potential), then the sphalerons still have enough time to eat up the asymmetry to a much smaller value than observed [17]. Therefore, some extra handles are needed. One way is to employ nonstandard cosmology [18].

If we have matter domination in the early universe, the Hubble rate will increase. Therefore, the temperature at which the electron Yukawa interaction exceeds the Hubble rate is smaller than the standard cosmology case. That will give sphalerons less time to eat up the asymmetry. Notice that in this case, the matter domination should not be too long such that the electron Yukawa enters equilibrium after EWPT. In other words, we do need the sphalerons to distribute the asymmetry, but we need to give them *less* time to eat up the asymmetry.

Similar to weak sphalerons, the Abelian anomaly may also lead to fermion number violation before EWPT: $\partial_\mu j_F^\mu \sim \frac{g'^2}{4\pi^2} \vec{E}_Y \cdot \vec{B}_Y$, with g' being the hypercharge and $\vec{E}_Y$ ($\vec{B}_Y$) being the hyperelectric (hypermagnetic) field [19,20]. Before EWPT hyperelectric and hypermagnetic field couple to chiral fermions, whereas after EWPT, the ordinary electromagnetic field's coupling to fermions is vectorlike. Hence, this fermion number violating effect is important only before EWPT. The intimate relationship between hypermagnetic field and baryon asymmetry has been discussed in the literature [17,21–32]. Abelian anomaly can lead to fermion number violation only if we have a nonzero seed of hypermagnetic field in the early universe. Interestingly, the observation of magnetic field with similar amplitudes and wave length in causally disconnected patches of the universe may also indicate that there was

a primordial seed of hypermagnetic field in the early universe [21,22,27,28,33–49]. Even though there are some uncertainties in the evolution of the hypermagnetic field amplitude, a rough estimate suggests that if the amplitude of the hypermagnetic field is on the order of 10^{20} G by the time of EWPT, the amplitude of the observed magnetic fields can be justified [22,26–28,50–52]. In this paper, we will consider the effect of the hypermagnetic field on the evolution of baryon asymmetry in the cases of both standard cosmology and a universe with an EMD. The Boltzmann equations describing the evolution of baryon asymmetry and hypermagnetic field is coupled, and therefore, we do a numerical study of a few benchmarks. To gain some intuition about the solutions, we present (an approximate) analytical solutions to some of the evolution equations. We show that their behavior in standard cosmology (SC) and EMD can be vastly different and nonintuitive. We emphasize that we are oblivious to the origin of EMD. Since EMD arise naturally in many beyond the SM solutions, we merely focus on how the evolution of baryon asymmetry and hypermagnetic field amplitude will change in the era of early matter domination.

The organization of the paper is as the following: In Sec. II, we explain the basics of early matter domination; Sec. III is dedicated to the evolution of baryon asymmetry. Specifically, subsection III A is the evolution of η_B assuming a primordial asymmetry in the right-handed electron. The subsection III B is with the same assumption plus a nonzero seed of hypermagnetic field amplitude. In subsection III C, we show the evolution of η_B and B_Y when the asymmetry in the right-handed electron is from a gradual decay of another particle. Finally, the concluding remarks are presented in Sec. IV.

II. EARLY MATTER DOMINATION

According to the SC, we expect to be in the era of radiation domination after reheating and until CMB. However, it is theoretically motivated to assume an early matter domination (EMD) that affects the Hubble rate for a limited range of temperature [3,5,6,12,53–64]. It is important to note that the dominating particle has to decay so that by the time of BBN we are in the radiation-dominated epoch again. There are several ways to get EMD. For simplicity and for the sake of working with an explicit model, let us introduce a weakly interacting scalar Φ with a decay width Γ_Φ . Because Φ is weakly interacting, then it goes through coherent oscillation and thus redshifts as a^{-3} . Hence, it is conceivable that at $T_\star > m_\Phi$, the energy density of Φ dominates over radiation.¹ In other words, $T_\star$ is defined when $\rho_\Phi = \rho_R$. The Boltzmann equations

¹That is assuming $v_\Phi \gg T_\star$ such that $v_\Phi^2 m_\Phi^2 \simeq \rho_R$ at $T_\star$. This assumption is reasonable in the context of many motivated extensions of the Standard Model (e.g., Ref. [18,65]). Furthermore, it is important that the production of Φ freezes out at $T > T_\star$, so that Φ stays in a *coherent* oscillation.

(BEs) describing the evolution of Φ and radiation are the following:

$$\begin{aligned}\dot{\rho}_\Phi + 3H_M \rho_\Phi &= -\Gamma_\Phi \rho_\Phi, \\ \dot{\rho}_R + 4H_M \rho_R &= \Gamma_\Phi \rho_\Phi,\end{aligned}\quad (3)$$

with $H_M = \sqrt{\frac{8\pi}{3M_{\text{Pl}}^2}(\rho_\Phi + \rho_R)}$, being the Hubble rate, and Γ_Φ is the total decay width of Φ . The decay of Φ is radiation, and therefore increases the energy density of radiation. Eventually, ρ_Φ becomes negligible compared with ρ_R and we return to radiation domination, as shown in Fig. 1. The approximate analytical solution to Eq. (3) in the regime $t_\star < t < \Gamma_\Phi^{-1}$ is the following [66]:

$$\begin{aligned}\rho_\Phi^a &\simeq \frac{M_{\text{Pl}}^2}{6\pi t^2} e^{-\Gamma_\Phi t} \\ \rho_R^a &\simeq \frac{M p l^2 t_\star^{2/3}}{6\pi t^{8/3}} + \frac{\Gamma_\Phi M_{\text{Pl}}^2}{10\pi t},\end{aligned}\quad (4)$$

where $t_\star$ is the time corresponding to $T_\star$.

This phenomenon leads to a dilution factor that has been discussed extensively in the literature. In this work, however, we want to highlight some of the other consequences that have been given less attention but lead to nontrivial effects on the abundance of baryonic matter.

III. BARYOGENESIS ABUNDANCE

Explaining the matter-antimatter asymmetry has been the focus of many studies (e.g., [56,67–75]). If the fermionic sector gains an asymmetry before the electroweak phase transition (EWPT), weak sphalerons become one of the key elements in the dynamics of the asymmetry. Weak sphalerons act on left-handed chiral fermions and rapidly eat the asymmetry preserving $B - L$ with an interaction per unit volume proportional to $\alpha_w^5 T^4$, where α_w is the coupling constant of the weak interaction. After EWPT, however, this rate is exponentially suppressed, and weak sphalerons become inactive [76,77].

Requiring weak sphalerons to preserve $B - L$ limits its domain of interaction. More specifically, conserving $B - L$ and requiring the plasma to be neutral,² leads to some equilibrium conditions among the chemical potential of fermions (see the Appendix A for more details). Therefore, the chemical potentials of SM fermions are related to each other. If we have an asymmetry in one of the fermions, we have an asymmetry in the rest of the fermions as well. In particular, we can express the chemical potential of baryons in terms of the chemical potential of the right-handed electron: $\mu_B = \frac{198}{481} \mu_{e_R}$ [18,78]. Therefore, if we only study

²The plasma needs to be neutral of $SU(3) \times SU(2) \times U(1)_Y$ charges.

the BE of the right-handed electron, we can find the asymmetry of the baryons.

Right-handed electron because of its limited interactions plays a crucial role in keeping the asymmetries. It only participates in hypermagnetic³ and Yukawa interaction. That means that the strong and weak sphalerons cannot act on right-handed electrons. Furthermore, the Yukawa coupling of the right-handed electron is the smallest⁴; thus, the chirality flip of the electrons freezes into equilibrium last. In standard cosmology, the temperature at which the rate of chirality flip of the electron (Γ_{LR}) matches the Hubble rate $\Gamma_{LR} = H_R$ is denoted by T_{cf} . The rate of the chirality flip of the electron has been estimated by Refs. [79–81]: $\Gamma_{LR} \simeq 10^{-2} y_e^2 T$. Hence, in the SC, $T_{\text{cf}} \simeq 10^4 - 10^5$ GeV [14,15]. The small Yukawa coupling of the electron tames the weak sphalerons to wash out the asymmetry until T_{cf} . However, it is shown that the difference between T_{cf} and T_{EW} , in SC, is long enough that the weak sphalerons can still wash out the asymmetry significantly [79,82,83]. In the following, we will discuss how changing the history of early universe cosmology can affect the phenomenology of baryon asymmetry.

A. Primordial asymmetry

In this subsection, we assume that the right handed electron has a large primordial asymmetry and we study the evolution of its asymmetry in SC and EMD. More specifically, we assume $\mu_{e_R}^0 = T_\star/2$, where the superscript 0 indicates the initial value.⁵ The conversion between μ_{e_R} and n_{e_R} is simply done using $n_{e_R} = \mu_{e_R} T^2/6$. The BE governing the asymmetry of the right-handed electron is

$$\dot{n}_{e_R} + 3H n_{e_R} = -\Gamma_{LR} \left(n_{e_R} - n_{e_L} + \frac{n_\phi}{2} \right), \quad (5)$$

where n_i with $i = \{e_R, e_L, \phi, \dots\}$ is the difference between the number densities of a particle and its antiparticle,⁶ and ϕ represents the Higgs. Here, we use H to represent a general Hubble rate (RD or EMD). From the conservation of $B - L$ and neutrality of the plasma, we can find n_{e_L} and n_ϕ in terms of n_{e_R} :

$$n_{e_L} = -\frac{415}{962} n_{e_R} \quad n_\phi = \frac{45}{481} n_{e_R}. \quad (6)$$

³The hypermagnetic interaction will also play an important role in the dynamics of the right-handed asymmetry, which we will discuss later.

⁴That is if we ignore the neutrinos for now.

⁵The initial value of μ_{e_R} is chosen such that we can still use perturbation theory. As we will discuss shortly, the large value of μ_{e_R} is needed to amplify a small seed of hypermagnetic field amplitude.

⁶The ellipses mean that this definition applies to any particle.

FIG. 1. The Hubble rates H_M (shown in green) and in the standard cosmology H_R (red) are shown. As the plot demonstrates, the Hubble rate is increased until Φ decays into radiation, after which the two Hubble rates match ($\rho_\Phi \ll \rho_R$). This plot is with the assumption that $T_\star = 10^7$ GeV and $\Gamma_\Phi = 5 \times 10^{-15}$ GeV.

To solve Eq. (5) in the case of SC, it is more convenient⁷ to do a change of variables:

$$\eta_{e_R} = \eta_{e_R}^0 \text{Exp} \left[-\frac{2133 M_{\text{Pl}} y_e^2 (\frac{1}{T} - \frac{1}{T_\star})}{19240 \sqrt{5} g_\star \pi^{3/2}} \right], \quad (7)$$

where $\eta_{e_R} = n_{e_R}/s$, with s being the entropy. Since we are only considering the temperatures greater than T_{EW} , $g_\star$ is a fixed number and it is equal to 106.75. As Eq. (7) shows, the comoving number density of the asymmetry is exponentially sensitive to the Yukawa coupling, and thus with a larger Yukawa coupling (e.g., muons), it would decrease much faster. Solving Eq. (5) in the case of EMD is less trivial. Partially because of a more complicated Hubble rate, but also because the relation between temperature and time is a more complex relation:

$$T(t) \equiv \left(\frac{\rho_R(t)}{30 g_\star} \right)^{1/4}. \quad (8)$$

However, one can find an approximate solution to Eq. (5) in the case of EMD, by using the analytical equations given in Eq. (4) and assuming $\rho_\Phi \gg \rho_R$, when evaluating the Hubble rate (See Appendix B for details). For the sake of accuracy, in the following, we work with the numerical solution of $n_{e_R}^M$.

⁷One can easily solve Eq. (5) with respect to time:

$$n_{e_R}^R(t) = n_{e_R}^0 \left(\frac{t_\star}{t} \right)^{3/2} \text{Exp} \left[-\frac{711 y_e^2 \sqrt{3} M_{\text{Pl}} (\sqrt{t} - \sqrt{t_\star})}{9620 (5\pi)^{3/4} g_\star^{1/4}} \right].$$

FIG. 2. In the upper panel, the Hubble rates H_M (green) and H_R (red) as well as Γ_{LR} (blue) are shown. In the lower panel, the evolution of η_B according to Eq. (5) is shown, where the green line corresponds to $H = H_M$ and the red line is when $H = H_R$. The initial asymmetry in the right-handed electron is assumed to be $\mu_{e_R}^0/T_\star = 1/2$ to respect perturbation. The horizontal dashed black line is the approximate observed value of baryonic asymmetry, η_B^{obs} . The dashed gray line indicates T_{EW} , and the sphalerons become inactive for $T < T_{\text{EW}}$. Therefore, the value of η_B freezes at T_{EW} . As the plots show, in SC, the chirality flip of the electron comes into equilibrium earlier, and therefore the resulting asymmetry is much smaller than the observed value, $\eta_B^R(T = T_{\text{EW}}) \ll \eta_B^{\text{obs}}$. In the case of EMD, on the other hand, the asymmetry is kept for longer and $\eta_B^M(T = T_{\text{EW}})$ matches the observed value.

The evolution of η_B is shown⁸ in the lower panel of Fig. 2. As long as the Hubble rate is greater than the term involving Γ_{LR} , the comoving number density of asymmetry η_B stays constant. However, once the rate of the chirality flip of the electron exceeds the Hubble rate, the asymmetry starts to decrease. As the upper panel of Fig. 2 shows, $T_{\text{cf}}^M < T_{\text{cf}}^R$ meaning the asymmetry is preserved for longer. Another consequence of EMD is a dilution coming from the decay of Φ into the thermal bath. Therefore, we expect η_B^M to decrease as a result of this dilution. This effect is shown by the rate at which η_B decreases for $T < T_{\text{cf}}$. In the

⁸Even though Eq. (5) describes η_{e_R} , we can find η_B from the equilibrium conditions: $\eta_B = \frac{198}{481} \eta_{e_R}$.

case of EMD, we see that the rate of the drop in η_B is much greater than the SC case.⁹

B. Primordial asymmetry + hypermagnetic field

Here, we still assume that the fermions have a primordial asymmetry, but we include the effects of the hypermagnetic field as well. The baryon number violation in the SM is proportional to $\vec{E}_Y \cdot \vec{B}_Y$, where $\vec{E}_Y$ and $\vec{B}_Y$ are the hyper-electric and hypermagnetic field, respectively. Therefore, it is crucial to study the effect of the hypermagnetic field on the evolution of the baryon asymmetry as well. [17,21–32].

Interestingly, the observed large-scale magnetic field in the void between the galaxies is yet another puzzle in astrophysics and cosmology [33,48,49]. The observation of magnetic field with similar magnitudes in a causally disconnect patches of universe, might be a hint of the existence of a primordial magnetic field [21,22,27,28,34–47]. If we insist that the existence of these magnetic fields roots in cosmology, a rough conservative estimate suggest that we need to have a hypermagnetic field by the time of EWPT with an amplitude of 10^{19} G to explain the current magnetic field in the inter Galactic medium [22,26–28,50–52].

The details of the interdependencies of the evolution of the hypermagnetic field amplitude (HMFA) and baryon asymmetry has been discussed in Refs. [17,28,31,84–88]. The Lagrangian describing $U(1)_Y$ in the Minkowski space is the following [28,31,84,86].

$$\mathcal{L} = -\frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} - J_Y^\mu Y_\mu - c'_E \frac{\alpha'}{8\pi}(2Y \cdot B_Y), \quad (9)$$

where $Y_{\mu\nu}$ is the field tensor of the hypercharge, and $\alpha' = 0.01$ is the structure constant of the hypercharge interaction. The first term in the Lagrangian (9) is the kinetic term, J_Y refers to the Ohmic current, and the term involving c'_E is the Chern-Simons term. The value of c'_E can be written in term of the chemical potential of the chiral fermions [31,84]:

$$c'_E = \sum_{i=1}^{n_G} \left[-2\mu_{R_i} + \mu_{L_i} - \frac{2}{3}\mu_{d_{R_i}} - \frac{8}{3}\mu_{u_{R_i}} + \frac{1}{3}\mu_{Q_i} \right] \\ = -99/37\mu_{e_R}, \quad (10)$$

where the last equality comes from applying the equilibrium conditions. To study the evolution of the HMFA in the early universe, we must consider the expansion factor as well (the details are given in Refs. [87,88]). The doing so, we can obtain the hyperelectric field:

⁹There is a difference between T_{cf} in the upper panel of Fig. 2 and the temperature at which η_B starts to decrease in the lower panel of Fig. 2 coming from the different coefficients of H and Γ_{LR} in Eq. (5).

$$\vec{E}_Y = \frac{1}{a\sigma} \vec{\nabla} \times \vec{B}_Y + \frac{\alpha'}{2\pi\sigma} c'_E \vec{B}_Y - \vec{v} \times \vec{B}_Y, \quad (11)$$

where a is the scale factor, $\sigma \simeq 100T$ is the electrical hyperconductivity of the plasma, and v is the velocity of the plasma. Let us ignore the term containing the velocity of the plasma for now. This is justified, because the length scale of the change in the bulk velocity is much smaller than the correlation length of the hypermagnetic field. In other words, the infrared modes of the hypercharge are unaffected by the velocity of the plasma [25]. With this assumption, we can find the evolution of the hypermagnetic field as following:

$$\partial_t \vec{B}_Y + 2H\vec{B}_Y = \frac{1}{a^2\sigma} \nabla^2 \vec{B}_Y - \frac{\alpha'}{2\pi a\sigma} c'_E \vec{\nabla} \times \vec{B}_Y. \quad (12)$$

One of the Maxwell equation of $U(1)_Y$ is that $\vec{\nabla} \cdot \vec{B}_Y = 0$. Therefore, we can write $\vec{B}_Y = (1/a)\nabla \times \vec{A}_Y$, with $\vec{A}_Y$ being the vector potential. One particular nontrivial configuration of the vector potential that can result in fully helical hypermagnetic field¹⁰ is [17,22,26,27,30,31,90–92]:

$$\vec{A}_Y = \gamma(t)(\sin kz, \cos kz, 0), \quad (13)$$

where $\gamma(t)$ is the time-dependent amplitude of $\vec{A}_Y$, and k is the comoving wave number. This configuration has been extensively studied in the literature [14,17,22,22,30,31,87,90–93]. It appears as if this configuration violates the assumption of homogeneity and isotropy condition. However, it can be shown that as long as the amplitude of the hypermagnetic field is smaller than 10^{23} G, the magnetic pressure ($B_Y^2/8\pi$) is orders of magnitude smaller than the fluid pressure. Thus, this violation of homogeneity and isotropy is negligible [88,94,95].

With the configuration given in Eq. (13), we can solve for hypermagnetic field. Notice that since the time dependent part of $\vec{A}_Y$ has been factored out of the vector part, the time dependent part of $\vec{B}_Y$ can similarly be separated, $B_Y(t) = k/a\gamma(t)$ (The details are given in Ref. [88]). Therefore, we can find the BE of the amplitude of the hypermagnetic field

$$\partial_t B_Y + 2HB_Y = -\frac{k'}{\sigma} B_Y \left(k' + \frac{\alpha'}{2\pi} c'_E \right), \quad (14)$$

where $k' = k/a \simeq kT$. To continue the discussion, it is useful to note that $-\frac{k'^2}{\sigma} B_Y$ leads to a decrease in B_Y , where as $-\frac{\alpha'k'}{2\pi\sigma} c'_E$ leads to an increase in B_Y . As Eq. (14) shows, if $\mu_{e_R}/T \simeq 1$, then B_Y gets amplified. On the other hand, if

¹⁰The magnetic field in the inter Galactic medium has been inferred to be helical, and therefore, a fully helical configuration is desirable [89].

$\mu_{e_R} = 0$, it is *not* generated by the hypermagnetic field. Similarly, the large HMFA has an effect on the evolution of baryon asymmetry as discussed in [30,31,88]:

$$\dot{n}_{e_R} + 3Hn_{e_R} = -\Gamma_{LR} \left(n_{e_R} - n_{e_L} + \frac{n_\phi}{2} \right) + \frac{\alpha'}{\pi} \vec{E}_Y \cdot \vec{B}_Y, \quad (15)$$

where

$$\vec{E}_Y \cdot \vec{B}_Y = \frac{B_Y^2}{\sigma} \left(k' + \frac{\alpha'}{2\pi} c'_E \right). \quad (16)$$

Notice that if $B_Y = 0$, the asymmetry in the right-handed electron cannot generate a nonzero B_Y . However, as long as both μ_{e_R} and B_Y are nonzero, they have a nontrivial influence on each other. The intricate connection between HMFA and baryon asymmetry has been discussed in Refs. [30,31]. As Eq. (16) shows, the existence of a nonzero B_Y has two contributions on the evolution of n_{e_R} : one that is proportional to $k'B_Y^2$ which leads to an increase in n_{e_R} , and another one proportional to $c'_E B_Y^2$ that leads to a decrease in the asymmetry. Therefore, a combination of large B_Y and small n_{e_R} is necessary for B_Y to save n_{e_R} from decreasing exponentially.

Figure 3 shows the temperature evolution of $\eta_B(T)$ and $B_Y(T)$ assuming

$$\begin{aligned} \Gamma &= 2 \times 10^{-15} \text{ GeV}, & T_\star &= 10^7 \text{ GeV}, & \mu_{e_R}^0 &= \frac{T_\star}{2}, \\ B_Y^0 &= 10^4 \text{ G}, & c_{k_0} &\equiv \frac{k}{10^{-7}} = 1.73, \end{aligned} \quad (17)$$

in the two cases of SC and EMD. Initially, B_Y starts to decrease, because the Hubble rate is higher than the term with c'_E . As soon as $-\frac{\alpha'k'}{2\pi\sigma}c'_E > 2H$, then $B_Y(T)$ starts to increase. Since the Hubble rate in EMD is greater than the Hubble rate in SC, the decrease in B_Y goes on for a longer time. The increase in B_Y continues until $k'^2/\sigma > |\frac{\alpha'k'}{2\pi\sigma}c'_E|$. As Fig. 3 shows, if B_Y increases to a large enough value, it can prevent η_B from decreasing exponentially. The higher Hubble rate in EMD does not allow B_Y to increase high enough, and thus η_B in EMD continues to decrease exponentially.¹¹ As far as we are aware, this effect was not discussed in the literature so far. It is worth mentioning that any results we discuss here are highly sensitive to the initial conditions. As was illustrated in Appendix B, η_B and B_Y are exponentially sensitive to some of the free parameters. Any small change in the initial conditions may change the final results drastically.

¹¹Furthermore, since the higher Hubble rate keeps Γ_{LR} out of equilibrium for a longer time, μ_{e_R} in EMD is larger as B_Y is increasing. This is another reason that B_Y cannot help n_{e_R} from decreasing exponentially.

FIG. 3. In the upper panel, the evolution of η_B and in the lower panel the evolution of B_Y as a function of temperature for the benchmark indicated is presented. The large asymmetry in the right-handed electron causes B_Y to increase. If $\frac{\alpha' B_Y^2 k'}{\pi \sigma}$ gets higher than $\Gamma_{LR} n_{e_R}$, the asymmetry is prevented from diminishing. In EMD, due to the higher Hubble rate, B_Y starts to increase at lower temperatures, and therefore it never gets large enough to compete with $\Gamma_{LR} n_{e_R}$.

C. Gradual increase of η_{e_R}

In the following, we want to repeat the same analysis, but assume the injection of asymmetry into e_R is gradual: from the decay of another particle, which we call S for now. To keep our discussion general, we are going to be oblivious about the quantum numbers of S . The BE of n_{e_R} due to the decay of $S \rightarrow e_R \dots$ is

$$\begin{aligned} \dot{n}_{e_R} + 3Hn_{e_R} &= -\Gamma_{LR} \left(n_{e_R} - n_{e_L} + \frac{n_\phi}{2} \right) \\ &\quad + \frac{\alpha'}{\pi} \vec{E}_Y \cdot \vec{B}_Y + \Gamma_e n_S, \end{aligned} \quad (18)$$

where Γ_e is the decay width of S into e_R and n_S is the asymmetric number density of S . It is more convenient to express in terms of ρ_S : $n_S \equiv \eta_S \rho_S / m_S$, where η_S is a dimensionless parameter quantifying the amount of asymmetry in S . Therefore, the new parameters compared to the previous section are

FIG. 4. The evolution of η_B as a function of temperature for the two cases of SC (red) and EMD (green). In this plot, the asymmetry is due to the gradual decay of a particle S into a right-handed electron for the benchmark shown. In SC, once the sphalerons become active, the rate of injection of asymmetry matches the wash-out rate, and thus η_B becomes constant. In EMD, sphalerons become active much later, and thus η_B increases for a long period. The decrease in η_B is due to the dilution of the universe from the decay of S . Eventually, η_B in the two cases become comparable.

$$\Gamma_e, \quad \eta_S, \quad m_S, \quad \rho_S.$$

Even though ρ_S is a free parameter, we will restrict it to (1) $\rho_S = \rho_R$, (2) $\rho_S = \rho_\Phi$, (3) $\rho_S = \rho_R e^{-\frac{T-m_S}{T/2}}$. As mentioned earlier, we are not going to think about the explicit models where each of these densities are realized, though the model building for the second and third scenario are easily feasible (e.g., [18]).

The BEs of our interest [Eqs. (3), (18), and (14)] are highly coupled and it is more convenient to show the effect of MD era through a numerical study of a few benchmarks. Let us start our analysis with a simple case where all of the parameters are the same in SC and EMD, except for the Hubble rate. Specifically, the first benchmark we consider is the following,

$$\begin{aligned} \rho_S &= \rho_R, & \Gamma_e &= 5 \times 10^{-15} \text{ GeV}, \\ \eta_S &= 1, & m_S &= 10^4 \text{ GeV}. \end{aligned} \quad (19)$$

In the case of EMD, we will assume $\Gamma_S = 5 \times 10^{-15} \text{ GeV}$ as well. Figure 4 shows the evolution of η_B as a function of temperature for the specified benchmark. In the case of SC, η_B becomes constant after sphalerons become active. That is because the rate of injection of asymmetry becomes comparable with the rate of wash-out of the asymmetry by the sphaleron. The larger Hubble rate delays the effects of sphalerons. Hence, η_B increases for a longer period. Once S decays and injects entropy to the universe, η_B starts decreasing. Once sphalerons become active, η_B decreases further until ρ_S becomes negligible and it merges the SC

FIG. 5. The evolution of η_B (upper panel) and B_Y (lower panel) as a function of temperature for the two cases of SC (red) and EMD (green). In this plot, the asymmetry is due to the gradual decay of a particle S into a right-handed electron for the benchmark shown. Here, we are highlighting the effect of B_Y and its effect on the evolution of η_B . Since η_B reaches a large value in EMD at higher temperatures, it leads to an earlier amplification of B_Y , such that by T_{EW} , we have reached the desired value of B_Y in EMD, but B_Y is very small in SC.

case. This plot is assuming $B_Y^0 = 0$ and therefore the hypermagnetic field does not affect η_B .

If we include the effect of $B_Y(T)$, the evolution of η_B changes significantly. Recall that large η_B leads¹² to amplification in B_Y . A combination of large B_Y and η_B , on the other hand, leads to a drop in η_B [see Eq. (16)]. The decrease in η_B continues until either the Hubble rate dominates the process or $k' \simeq \frac{a'}{2\pi} c'_E$. As we can see from Fig. 5, since η_B in EMD is larger than the SC case, the amplification of $B_Y(T)$ occurs faster, as opposed to Section III B. The increase in B_Y leads to a drop in η_B . For the case of EMD, η_B decreases further due to the dilution of the universe. Interestingly, we see that when the asymmetry is injected gradually, the two cases of SC and EMD merge. That is if we were oblivious to EWPT. However, since after the EW phase transition, sphalerons become inactive, we are only sensitive to η_B at T_{EW} .

¹²Large refers to larger than the Hubble rate.

FIG. 6. The evolution of η_B as a function of temperature for the specified benchmark is presented. The green line is assuming $\rho_S = \rho_\Phi$, and $\Gamma_e = \Gamma$. The solid red line corresponds to $\rho_S = \rho_R e^{\frac{T-m_\Phi}{T/2}}$ and $\Gamma_e = \Gamma$. The decay width of S in the case of SC is not limited from above. Therefore, the case where $\rho_S = \rho_R e^{\frac{T-m_\Phi}{T/2}}$ and $\Gamma_e = 10^3 \times \Gamma$ is also shown (dashed red).

So far, we have only included the effect of the change in the Hubble rate. However, it is possible that the injection of asymmetry into e_R is from the decay of Φ . In other words, $\rho_S = \rho_\Phi$. This case has been discussed in the context of a specific model in Ref. [18] with $B_Y^0 = 0$ and [88], where $B_Y^0 = 10^{-2}$ G. It is, however, difficult to compare such scenario with a similar scenario in SC. Maybe the closest reasonable assumption is to assume $\rho_S = \rho_R e^{\frac{T-m_\Phi}{T/2}}$ in SC and $\rho_S = \rho_\Phi$ in EMD. Thus, Γ_e in EMD must be smaller than Γ ; but there is no such requirement for the SC case and we can have $\Gamma_{e_{SC}} \gg \Gamma$. To illustrate the effect of early MD in this case, let us consider the following benchmarks:

$$\begin{aligned} \eta_S &= 1, & m_S &= 10^4 \text{ GeV}, \\ \text{EMD: } \rho_S &= \rho_\Phi, & \Gamma_e &= 5 \times 10^{-15} \text{ GeV}, \\ \text{SC: } \rho_S &= \rho_R e^{\frac{T-m_\Phi}{T/2}}, & \Gamma_e &= \begin{cases} 5 \times 10^{-15} \text{ GeV} \\ 5 \times 10^{-12} \text{ GeV} \end{cases} \end{aligned} \quad (20)$$

The evolution of η_B as a function of temperature assuming $B_Y^0 = 0$, is shown in Fig. 6. As the figure shows, η_B increases due to the injection of asymmetry from S , and then it decreases because of the work of the sphaleron (and the dilution of the universe in the case of EMD).

A more interesting case is when we include the effect of B_Y in the evolution of η_B as well. As discussed earlier, large¹³ η_B can lead to an amplification of B_Y and in this process η_B decreases to a smaller value. The amplified B_Y on the other hand prevents η_B from diminishing to zero. This behavior is shown in Fig. 7. The case where η_B does

FIG. 7. The evolution of η_B (upper panel) and B_Y (lower panel) as a function of temperature for the specified benchmark is presented. The green line assumes $\rho_S = \rho_\Phi$ and the red lines correspond to $\rho_S = \rho_R e^{\frac{T-m_\Phi}{T/2}}$. The Γ_e in the solid lines is Γ and the dashed line has $\Gamma_e = 10^3 \Gamma$. As the plots show, if η_B in the early universe hits a large value, it can cause an amplification in B_Y which in return can save η_B from diminishing.

not hit a large value (solid red), end up having negligible η_B and B_Y . In the other cases, the value of η_B is such that it can lead to the amplification of B_Y . The values of η_B and B_Y at T_{EW} in these cases are significant.

IV. CONCLUSION

The history of the early universe can have significant phenomenological impacts on the abundance of matter. In this paper, we studied the effect of early matter domination on the abundance of baryonic asymmetry (η_B). Since the evolution of η_B is intimately connected with the amplitude of the hypermagnetic field, we studied the effect of the hypermagnetic field as well.

Naively, one expects that the baryon asymmetry to be diluted in the case of early matter domination. Through considering a few benchmarks, we show that the effect of early matter domination can be highly nontrivial. In the case of early matter domination, η_B can be orders of magnitudes greater, smaller, or even equal to η_B in the standard cosmology.

¹³By large, we mean $\eta_B \gtrsim 10^{-4}$.

One key player in the evolution of η_B is the weak sphaleron that leads to the washout of the asymmetry. Weak Sphalerons only act on left-handed fermions. An asymmetry in right-handed particles can be transferred to left-handed particles through Yukawa interaction. Hence, the right-handed electron because of its small Yukawa coupling has a special role in saving the asymmetry. If the right-handed electron gains an asymmetry, it will keep it for a relatively long time until its Yukawa interaction enters equilibrium. In standard cosmology, the Yukawa interaction of the right-handed electron is not slow enough to keep the asymmetry in the right-handed electron for a sufficiently long time. Studies have shown that assuming standard cosmology the sphalerons have enough time to wash out the asymmetry. In a universe with early matter domination, on the other hand, due to the change in Hubble rate, the sphalerons remain inactive for a longer period. Therefore, η_B in a universe with a modified history can be many orders of magnitude greater than η_B in standard cosmology.

An asymmetry in right-handed electrons also affects the evolution of the hypermagnetic field. That is true only if the amplitude of the hypermagnetic field has a nonzero initial value. An asymmetry with values $\eta_B \gtrsim 10^{-4}$ may amplify a seed of hypermagnetic field amplitude. The grown amplitude will also feed to the asymmetry and save the asymmetry from diminishing. In a universe with an enhanced Hubble rate, the amplitude of the hypermagnetic field gets diluted. Whether the diluted magnetic field can get amplified or not is strongly dependent on the benchmark of the interest. Our numerical results show that in the scenario where the asymmetry in the right-handed electron is from a slow decay, the hypermagnetic field amplitude has a high chance of getting amplified. Hence, the evolution of both baryon asymmetry and the amplitude of hypermagnetic field are highly nontrivial and strongly depend on the benchmark of the interest.

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APPENDIX A: EQUILIBRIUM CONDITIONS

The equilibrium values are derived from the following constraints (See [96] and Sec. 2 of [83] for further details):

- (i) Due to flavor mixing the quark sector, all of the quarks regardless of their handedness have the same chemical potential

$$\mu_Q \equiv \mu_{u_{R_i}} = \mu_{d_{R_i}} = \mu_{Q_i}, \quad (\text{A1})$$

where $i = 1, 2, 3$ indicates the generation of the quark. It is easy to see that

$$\mu_B = \sum_i 2\mu_{Q_i} + \mu_{u_{R_i}} + \mu_{d_{R_i}} = 12\mu_Q. \quad (\text{A2})$$

It is worth mentioning that we are working in the limit of massless neutrinos, where there is no flavor mixing in the lepton sector. A more careful analysis where the lepton mixing is considered is beyond the scope of this paper.

- (ii) The work of weak sphalerons is so fast, that we can approximate it as being instantaneous:

$$9\mu_Q + \mu_{L_1} + \mu_{L_2} + \mu_{L_3} = 0. \quad (\text{A3})$$

- (iii) The Yukawa interaction of all particles, except for e_R , are in thermal equilibrium:

$$\mu_{u_{R_i}} - \mu_{Q_i} = \mu_\phi, \quad i = 1, 2, 3$$

$$\mu_{d_{R_i}} - \mu_{Q_i} = -\mu_\phi, \quad i = 1, 2, 3$$

$$\mu_{e_{R_i}} - \mu_{L_i} = -\mu_\phi, \quad i = 2, 3$$

- (iv) The plasma does not have any net hypermagnetic charge.

$$Q = 6\mu_Q - \mu_{R_1} - \mu_{L_1} - 2\mu_{L_2} - 2\mu_{L_3} + 13\mu_0 = 0 \quad (\text{A4})$$

- (v) $B - L$ is respected in each generation:

$$\frac{1}{3}\mu_B - L_i = C_i, \quad (\text{A5})$$

where i indicates the generation again. Here, C_i s are a constant and indicate the primordial values of $B/3 - L$ in each generation.

These equilibrium conditions are such that we only have one unknown which is μ_{e_R} . If we want to relax some of these conditions and end up with two unknowns, it is best to relax the condition where the Yukawa interaction of muon is in equilibrium. Similar to right-handed electron, right-handed muon is not involved in the work of strong and weak sphalerons, and it has a small Yukawa coupling. In the following, we have relaxed this condition, and have looked at the evolution equation of the right-handed muon. We compare the evolution with the equilibrium value of the right handed muon in Fig. 8. Notice that the right-handed muon quickly merges with its equilibrium value. In Fig. 8, we had chosen the initial value of $\mu_{\mu_R}^0 = T_*/2$. For this figure, we have assumed EMD, with $\Gamma_\Phi = 5 \times 10^{-15}$ GeV.

FIG. 8. The evolution of μ_R as a function of temperature. The initial value of $\mu_{\mu_R}^0$ is equal to $T_*/2$, and $\Gamma_\Phi = 5 \times 10^{-15}$ GeV. Notice that the right-handed muon approaches its equilibrium value before very rapidly.

Notice that the right-handed muon quickly merges with its equilibrium value.

APPENDIX B: ANALYTIC VS NUMERIC

In this section, we compare the analytical solution to the numerical ones. Figure 9 shows $n_{e_R}^M$ for the case where

FIG. 9. The evolution of n_{e_R} as a function of temperature, where the green line is the evolution obtained using numerical approach, and the blue dashed line is the analytical approach. Notice that the analytical approach is good estimate up to the time when the decay from Φ produces enough radiation that $\rho_\Phi \simeq \rho_R$, and thus violating our assumptions of the analytical approach. For this plot, we have assumed $\Gamma_\Phi = 5 \times 10^{-15}$ GeV.

$\Gamma_\Phi = 5 \times 10^{-15}$ GeV, and the hypermagnetic field amplitude is turned off. The analytical solution [Eq. (B1)] fits the numerical one up to the time when the decay of Φ becomes efficient.

$$n_{e_R}^M(t) \simeq n_0 \text{Exp} \left[-2 \text{ExpIntEi} \left(-\frac{t\Gamma_\Phi}{2} \right) + 2 \text{ExpIntEi} \left(-\frac{t_*\Gamma_\Phi}{2} \right) + \frac{79y_e^2 3^{1/4}}{48100(10\pi)^{3/4}} \left[-8 \times 5^{3/4} \pi (tT(t) - t_*T_*) \right. \right. \\ \left. \left. - \frac{25M_{\text{Pl}}^2 t_*^{2/3} (5 + \frac{3t_*^{5/3}\Gamma_\Phi}{t_*^2})^{3/4} \text{HG1F2}[\frac{1}{5}, \frac{3}{4}, \frac{6}{5}, -\frac{3t_*^{5/3}\Gamma_\Phi}{5t_*^{2/3}}]}{2\pi t^{5/3} T(t)} - \frac{25(t_*M_{\text{Pl}})^{1/2} (5 + 3t_*\Gamma_\Phi)^{1/4} \text{HG1F2}[\frac{1}{5}, \frac{3}{4}, \frac{6}{5}, -\frac{3t_*\Gamma_\Phi}{5}]}{(5 + 3t_*\Gamma)^{1/4}} \right] \right], \quad (\text{B1})$$

where “ExpIntEi(z)” are the exponential Integral function $Ei(z) = -\int_{-z}^{\infty} \frac{e^t}{t} dt$, and “HG1F2” are the hypergeometric function ${}_2F_1$. As Eq. (B1) illustrates, the baryon asymmetry is extremely sensitive to Γ_Φ . Furthermore, since the baryon asymmetry is exponentially dependent on temperature, once our assumptions breakdown, the analytical approximation may differ significantly from the actual n_{e_R} .

The analytical solution to n_{e_R} and B_Y is more complicated once we assume a nonzero hypermagnetic field

amplitude. Solving such highly coupled differential equations, even in the case of SC is nontrivial. We first change the differential to temperature, using¹⁴:

$$\frac{d}{dt} = -HT \frac{d}{dT}.$$

Then, we can get the following approximate solutions:

$$n_{e_R}^R = \frac{\mathbf{n}_{e_R}^0 T^3}{T_*^3} \text{Exp} \left[-\frac{21.33\sqrt{5}y_e^2 M_{\text{Pl}}}{962\pi^{3/2} g_*^{1/2} (T - T_*)} \right] + f(T) - f(T_*) \\ f(T) = \int_T^{T_*} dT \frac{3\mathbf{c}_{\mathbf{k}_0} M_{\text{Pl}} \mathbf{B}_i^2 T^2}{5^7 T_*^4} \text{Exp} \left[\frac{-3M_{\text{Pl}} \mathbf{c}_{\mathbf{k}_0}^2}{40\sqrt{5}\pi^{3/2} (g_*)^{1/2} T} \right] \text{Exp} \left[\frac{3\sqrt{5}y_e^2 A M_{\text{Pl}}}{2\pi^{3/2} (g_*)^{1/2} T} \right] \quad (\text{B2})$$

where $A = 7.11/481$.

¹⁴We emphasize that this relation is true for radiation domination, but it is only an approximation for EMD era.

FIG. 10. The evolutions of n_{e_R} and B_Y as a function of temperature, where the red lines represent the numerical approach, and the blue dashed lines are the analytical solutions.

Similarly, we can find the expression for B_Y in SC:

$$\begin{aligned}
 B_Y^R = B_Y^0 \text{Exp} & \left[\frac{M_{\text{Pl}}^3}{555000 \sqrt{5} A^6 M_{\text{Pl}}^8 \pi^{3/2} (g_*)^{3/2} T_*^5 T_*^7 y_e^{12}} (-\mathbf{c}_{\mathbf{k}_0} \alpha' (-41625 A^6 \mathbf{c}_{\mathbf{k}_0} M_{\text{Pl}}^6 T_*^4 (-g_* T T_*^3 + g_* T^2 T_*^2) y_e^{12})) \right. \\
 & + 22 \sqrt{g_*} \sqrt{\pi} \left(-\text{Exp} \left[\frac{3 \sqrt{5} A M_{\text{Pl}} (T - T_*) y_e^2}{2 \sqrt{g_*} \pi^{3/2} T T_*} \right] + g_* T^5 T_*^3 (256 g_*^{5/2} \pi^{15/2} T_*^4 - 384 \sqrt{5} A g_*^2 M_{\text{Pl}} \pi^6 T_*^3 y_e^2 \right. \\
 & + 1440 A^2 g_*^{3/2} M_{\text{Pl}}^2 \pi^{9/2} T_*^2 y_e^4 - 720 \sqrt{5} A^3 g_* M_{\text{Pl}}^3 \pi^3 T_* y_e^6 + 1350 A^4 \sqrt{g_*} M_{\text{Pl}}^4 \pi^{3/2} y_e^8 - 2025 \sqrt{5} A^5 M_{\text{Pl}}^5 \pi^3 T_* T_* y_e^{10}) \\
 & + M_{\text{Pl}} \left(-384 \sqrt{5} A g_*^3 \pi^6 T_*^4 T_*^7 y_e^2 - 720 \sqrt{5} A^3 g_*^2 M_{\text{Pl}}^2 \pi^3 T_*^2 T_*^7 y_e^6 + 1350 A^4 g_*^{3/2} M_{\text{Pl}}^3 \pi^{3/2} T T_*^7 y_e^8 + 405 \sqrt{5} A^5 g_* M_{\text{Pl}}^4 T_*^7 y_e^{10} \right. \\
 & + 405 \sqrt{5} A^5 g_* M_{\text{Pl}}^4 T^5 T_*^2 (1 - 5 \mathbf{n}_{\mathbf{e}_R}^0 T_*^2) y_e^{10} + 32 g_*^{5/2} M_{\text{Pl}} \pi^{9/2} T^3 T_*^5 \left(8 \pi^3 \frac{g_* T^2 T_*^2}{M_{\text{Pl}}^2} + 45 A^2 T_*^2 y_e^4 \right) \left. \left. \right) \right) \\
 & + 555000 \sqrt{5} A^6 g_*^{3/2} M_{\text{Pl}}^5 \pi^{3/2} T^5 T_*^7 y_e^{12} \text{Log} \left[\frac{T^2}{T_*^2} \right] \left. \right]. \tag{B3}
 \end{aligned}$$

As can be seen, the expressions get extremely complicated once we include the hypermagnetic field amplitude. The comparison between analytical solutions and numerical solution is shown in Fig. 10. The analytical solution in EMD is much more complicated and could not be obtained. Again, that is mainly because the relationship between temperature and time is more complicated. Nonetheless, it

is easy to see that because of the larger Hubble rate during EMD era, the amplification of hypermagnetic field amplitude (specifically satisfying the condition $-\frac{\alpha' K'}{2\pi\sigma} c_E' > 2H$) occurs at a later time. Therefore, it is more difficult to amplify the hypermagnetic field amplitude to a desired value in the case of EMD.

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