Lepton-specific inert two-Higgs-doublet model confronted with the new results for muon and electron $g - 2$ anomalies and multilepton searches at the LHC

Xiao-Fang Han,¹ Tianjun Li,^{2,3} Hong-Xin Wang¹,^{*} Lei Wang¹,^{*} and Yang Zhang⁴

¹Department of Physics, Yantai University, Yantai 264005, People's Republic of China

²CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People's Republic of China

³School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China

⁴School of Physics and Microelectronics, Zhengzhou University, Zhengzhou 450001, People's Republic of China

(Received 2 August 2021; accepted 16 November 2021; published 1 December 2021)

Combining the multilepton searches at the LHC, we study the possibilities of accommodating the new data of muon and electron $g - 2$ anomalies in the lepton-specific inert two-Higgs-doublet model. We take the heavy CP -even Higgs as the 125 GeV Higgs, and find the muon and electron $g - 2$ anomalies can be explained simultaneously in the region of $5 \text{ GeV} < m_h < 60 \text{ GeV}$, $200 \text{ GeV} < m_A < 620 \text{ GeV}$, $190 \text{ GeV} < m_{H^\pm} < 620 \text{ GeV}$ for appropriate Yukawa couplings between leptons and inert Higgs. Meanwhile, the model can give a better fit to the data of lepton universality in τ decays than the SM. Further, the multilepton event searches at the LHC impose a stringent upper bound on m_h , $m_h < 35 \text{ GeV}$.

DOI: [10.1103/PhysRevD.104.115001](https://doi.org/10.1103/PhysRevD.104.115001)

I. INTRODUCTION

The Fermilab collaboration presented their new result for muon anomalous magnetic moment which now, combined with the data of the E821 [1,2], amounts to [3]

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}. \quad (1)$$

The experimental value has an approximate 4.2σ discrepancy from the SM prediction [4–7]. Besides, the measurement value of the fine-structure constant using ^{133}Cs atoms at Berkeley [8] makes the electron $g - 2$ to have 2.4σ deviation from the SM prediction [9,10],

$$\Delta a_e = a_e^{\text{exp}} - a_e^{\text{SM}} = (-87 \pm 36) \times 10^{-14}. \quad (2)$$

However, the newest measurement result of the fine-structure constant using ^{87}Rb atoms at Laboratoire Kastler Brossel gives a value of a_e which is well consistent with the SM prediction [11]. The discrepancy between these two

experimental results is not clear. If the Berkeley ^{133}Cs experiment result can be confirmed, it will be challenging to explain muon and electron $g - 2$ simultaneously since the two excesses have opposite sign. In this paper, we take the Berkeley ^{133}Cs results seriously and discuss the implication of muon and electron $g - 2$ on new physics model.

The lepton-specific two-Higgs-doublet model (2HDM) can explain muon $g - 2$ anomaly simply [12–27], but will raise the discrepancy in lepton flavor universality (LFU) in τ decay [22–24]. In addition, the muon and electron $g - 2$ anomalies cannot be explained simultaneously in lepton-specific 2HDM since there is an opposite sign between them. There have been some new physics models which attempt to explain the muon and electron $g - 2$ simultaneously [28–55]. In Ref. [32], we propose a lepton-specific inert 2HDM to explain the muon and electron $g - 2$ anomalies, and the key point is that these Yukawa couplings for μ and e/τ have opposite sign. In the model, the extra Higgses will decay into leptons mainly. Therefore, the multilepton event searches at the LHC can impose stringent constraints on the model. Motivated by the new results for muon $g - 2$ and multilepton event searches at the LHC, we revisit the possibilities of accommodating muon and electron $g - 2$ anomalies in the lepton-specific inert 2HDM. In this paper, we study a different scenario from that of Ref. [32], and take the inert CP -even Higgs h to be lighter than the SM-like Higgs H . In such scenario, a very light CP -even Higgs h plays important roles in explaining

^{*}Corresponding author.
leiwang@ytu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

the muon and electron $g-2$ anomalies, and is expected to more easily avoid the constraints from the multi-lepton searches at the LHC than the scenario of Ref. [32] in which h is heavier than the 125 GeV Higgs.

Our work is organized as follows. In Sec. II we will give a brief introduction on the model. In Sec. III and Sec. IV, we introduce the numerical calculations, and show the allowed and excluded parameter space. Finally, we give our conclusion in Sec. V.

II. LEPTON-SPECIFIC INERT 2HDM

We add an inert Higgs doublet field to the SM, and the scalar potential is written as,

$$\begin{aligned} V = & Y_1(\Phi_1^\dagger \Phi_1) + Y_2(\Phi_2^\dagger \Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right]. \end{aligned} \quad (3)$$

Here we impose a discrete Z_2 symmetry under which Φ_2 is odd and the SM fields are even. We study the CP -conserving case where all λ_i are real. The two complex scalar doublets with the hypercharge $Y = 1$ can be given as

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H + iG_0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h + iA) \end{pmatrix}. \quad (4)$$

The Φ_1 field has the vacuum expectation value (VEV) $v = 246$ GeV, and the VEV of Φ_2 field is zero. The Y_1 is determined by the scalar potential minimum condition,

$$Y_1 = -\frac{1}{2}\lambda_1 v^2. \quad (5)$$

The Nambu-Goldstone bosons G^0 and $G^\pm$ are eaten by the gauge bosons. The $H^\pm$ and A are the mass eigenstates of the charged Higgs boson and CP -odd Higgs boson. Their masses are given as

$$m_{H^\pm}^2 = Y_2 + \frac{\lambda_3}{2}v^2, \quad m_A^2 = m_{H^\pm}^2 + \frac{1}{2}(\lambda_4 - \lambda_5)v^2. \quad (6)$$

The H is the SM-like Higgs, and has no mixing with the inert CP -even Higgs h . Their masses are given as

$$m_H^2 = \lambda_1 v^2 \equiv (125 \text{ GeV})^2, \quad m_h^2 = m_A^2 + \lambda_5 v^2. \quad (7)$$

The fermions obtain the mass terms from the Yukawa interactions with Φ_1 ,

$$-\mathcal{L} = y_u \bar{Q}_L \tilde{\Phi}_1 u_R + y_d \bar{Q}_L \Phi_1 d_R + y_l \bar{L}_L \Phi_1 e_R + \text{H.c.}, \quad (8)$$

where $Q_L^T = (u_L, d_L)$, $L_L^T = (\nu_L, l_L)$, $\tilde{\Phi}_1 = i\tau_2 \Phi_1^*$, and y_u , y_d and y_e are 3×3 matrices in family space. In addition, in the lepton sector we introduce the Z_2 symmetry-breaking Yukawa interactions of Φ_2 ,

$$\begin{aligned} -\mathcal{L} = & \frac{\sqrt{2}m_e}{v} \kappa_e \bar{L}_{1L} \Phi_2 e_R + \frac{\sqrt{2}m_\mu}{v} \kappa_\mu \bar{L}_{2L} \Phi_2 \mu_R \\ & + \frac{\sqrt{2}m_\tau}{v} \kappa_\tau \bar{L}_{3L} \Phi_2 \tau_R + \text{H.c.} \end{aligned} \quad (9)$$

We can obtain the lepton Yukawa couplings of extra Higgses (h , A , and $H^\pm$) from Eq. (9). The neutral Higgses h and A have no couplings to ZZ , WW . At the tree-level, the SM-like Higgs H has the same couplings to fermions and gauge bosons as the SM.

III. NUMERICAL CALCULATIONS

In the lepton-specific 2HDM and aligned 2HDM, κ_τ equals to κ_μ . As a result, the decay $\tau \rightarrow \mu\nu\nu$ will obtain negative contribution from the diagram mediated by the charged Higgs. In the model we take κ_μ to be opposite in sign from κ_τ . Thus, the diagrams mediated by the charged Higgs produce positive contribution to the decay $\tau \rightarrow \mu\nu\nu$, and the model can give better fit to the data of the LUF in the τ decay. When κ_μ is opposite in sign from κ_τ , the contributions of the CP -even (CP -odd) Higgses to muon $g-2$ are positive (negative) at the two-loop level and positive (negative) at one-loop level. Therefore, we take the heavy CP -even Higgs as the 125 GeV Higgs, $m_H = 125$ GeV, and make the light CP -even Higgs h to be light enough to enhance muon $g-2$. Since electron $g-2$ is opposite in sign from muon $g-2$, we will take κ_e to have a opposite sign from κ_μ .

In our calculations, we take $\lambda_2, \lambda_3, m_H, m_h, m_A$ and $m_{H^\pm}$ as the input parameters. According to Eqs. (6), (7), the values of λ_1, λ_5 and λ_4 can be determined. λ_2 controls the quartic couplings of extra Higgses, and does not affect the observables considered in our paper. We take $\lambda_3 = \lambda_4 + \lambda_5$ to make the Hhh coupling to be absent.

We scan over several key parameters in the following ranges:

$$\begin{aligned} 5 \text{ GeV} < m_h < 60 \text{ GeV}, \quad 200 \text{ GeV} < m_A < 800 \text{ GeV}, \\ 90 \text{ GeV} < m_{H^\pm} < 800 \text{ GeV}, \\ 1 < \kappa_\tau < 140, \quad -200 < \kappa_\mu < -1, \quad 1 < \kappa_e < 500. \end{aligned} \quad (10)$$

In such ranges of κ_τ, κ_μ and κ_e , the corresponding Yukawa couplings do not become nonperturbative.

The model gives the new contributions to muon $g-2$ via the one-loop diagrams and the two-loop Barr-Zee diagrams involving extra Higgses. For the one-loop contributions [12] we have

$$\Delta a_\mu^{2\text{HDM}}(1 \text{ loop}) = \frac{m_\mu^2}{8\pi^2 v^2} \sum_i \kappa_\mu^2 r_\mu^i F_j(r_\mu^i), \quad (11)$$

where $i = h, A, H^\pm$, $r_\mu^i = m_\mu^2/M_j^2$. For $r_\mu^i \ll 1$ we have $F_h(r) \simeq -\ln r - 7/6$, $F_A(r) \simeq \ln r + 11/6$, $F_{H^\pm}(r) \simeq -1/6$. (12)

The contributions of the two-loop diagrams with a closed fermion loop are given by

$$\Delta a_\mu^{2\text{HDM}}(2 \text{ loop}) = \frac{m_\mu^2}{8\pi^2 v^2} \frac{\alpha_{\text{em}}}{\pi} \sum_{i,\ell} Q_\ell^2 \kappa_\mu \kappa_\ell r_\ell^i G_i(r_\ell^i), \quad (13)$$

where $i = h, A$, $\ell = \tau$, and m_ℓ and Q_ℓ are the mass and electric charge of the lepton ℓ in the loop. The functions $G_i(r)$ are [13,14]

$$G_h(r) = \int_0^1 dx \frac{2x(1-x)-1}{x(1-x)-r} \ln \frac{x(1-x)}{r}, \quad (14)$$

$$G_A(r) = \int_0^1 dx \frac{1}{x(1-x)-r} \ln \frac{x(1-x)}{r}. \quad (15)$$

In our calculation, we also include the contributions of the two-loop diagrams with a closed charged Higgs loop, and find that their contributions are much smaller than those of fermion loop. The calculation of Δa_e is similar to that of Δa_μ , and we include the contributions of the two-loop diagrams with closed μ loop and τ loop.

The HFAG collaboration reported three ratios from pure leptonic processes, and two ratios from semihadronic processes, $\tau \rightarrow \pi/K \nu$ and $\pi/K \rightarrow \mu \nu$ [56]:

$$\begin{aligned} \left(\frac{g_\tau}{g_\mu}\right) &= 1.0011 \pm 0.0015, & \left(\frac{g_\tau}{g_e}\right) &= 1.0029 \pm 0.0015, \\ \left(\frac{g_\mu}{g_e}\right) &= 1.0018 \pm 0.0014, & \left(\frac{g_\tau}{g_\mu}\right)_\pi &= 0.9963 \pm 0.0027, \\ \left(\frac{g_\tau}{g_\mu}\right)_K &= 0.9858 \pm 0.0071, \end{aligned} \quad (16)$$

with the following definitions

$$\begin{aligned} \left(\frac{g_\tau}{g_\mu}\right)^2 &\equiv \bar{\Gamma}(\tau \rightarrow e \nu \bar{\nu}) / \bar{\Gamma}(\mu \rightarrow e \nu \bar{\nu}), \\ \left(\frac{g_\tau}{g_e}\right)^2 &\equiv \bar{\Gamma}(\tau \rightarrow \mu \nu \bar{\nu}) / \bar{\Gamma}(\mu \rightarrow e \nu \bar{\nu}), \\ \left(\frac{g_\mu}{g_e}\right)^2 &\equiv \bar{\Gamma}(\tau \rightarrow \mu \nu \bar{\nu}) / \bar{\Gamma}(\tau \rightarrow e \nu \bar{\nu}). \end{aligned} \quad (17)$$

Where $\bar{\Gamma}$ denotes the partial width normalized to its SM value. The correlation matrix for the above five observables is

$$\begin{pmatrix} 1 & +0.53 & -0.49 & +0.24 & +0.12 \\ +0.53 & 1 & +0.48 & +0.26 & +0.10 \\ -0.49 & +0.48 & 1 & +0.02 & -0.02 \\ +0.24 & +0.26 & +0.02 & 1 & +0.05 \\ +0.12 & +0.10 & -0.02 & +0.05 & 1 \end{pmatrix}. \quad (18)$$

In the model, we have the ratios

$$\begin{aligned} \left(\frac{g_\tau}{g_\mu}\right)^2 &\approx \frac{1 + 2\delta_{\text{loop}}^\tau}{1 + 2\delta_{\text{loop}}^\mu}, \\ \left(\frac{g_\tau}{g_e}\right)^2 &\approx \frac{1 + 2\delta_{\text{tree}} + 2\delta_{\text{loop}}^\tau}{1 + 2\delta_{\text{loop}}^\mu}, \\ \left(\frac{g_\mu}{g_e}\right)^2 &\approx 1 + 2\delta_{\text{tree}}, \\ \left(\frac{g_\tau}{g_\mu}\right)_\pi^2 &= \left(\frac{g_\tau}{g_\mu}\right)_K^2 = \left(\frac{g_\tau}{g_\mu}\right)^2. \end{aligned} \quad (19)$$

The δ_{tree} and $\delta_{\text{loop}}^{\tau,\mu}$ are respectively corrections from the tree-level diagrams and the one-loop diagrams mediated by the charged Higgs. They are given as [22,24,25]

$$\delta_{\text{tree}} = \frac{m_\tau^2 m_\mu^2}{8m_{H^\pm}^4} \kappa_\tau^2 \kappa_\mu^2 - \frac{m_\mu^2}{m_{H^\pm}^2} \kappa_\tau \kappa_\mu \frac{g(m_\mu^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}, \quad (20)$$

$$\delta_{\text{loop}}^{\tau,\mu} = \frac{1}{16\pi^2} \frac{m_{\tau,\mu}^2}{v^2} \kappa_{\tau,\mu}^2 \left[1 + \frac{1}{4} (H(x_A) + H(x_h)) \right], \quad (21)$$

where $f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$, $g(x) \equiv 1 + 9x - 9x^2 - x^3 + 6x(1+x) \ln(x)$ and $H(x_\phi) \equiv \ln(x_\phi)(1+x_\phi)/(1-x_\phi)$ with $x_\phi = m_\phi^2/m_{H^\pm}^2$.

We perform χ_τ^2 calculation for the five observables. The covariance matrix constructed from the data of Eq. (16) and Eq. (18) has a vanishing eigenvalue, and the corresponding degree is removed in our calculation. In our discussions we require the value of χ_τ^2 to be smaller than the SM value, $\chi_\tau^2(\text{SM}) = 12.3$.

The measured values of the ratios of the leptonic Z decay branching fractions are given as [57]:

$$\begin{aligned} \frac{\Gamma_{Z \rightarrow \mu^+ \mu^-}}{\Gamma_{Z \rightarrow e^+ e^-}} &= 1.0009 \pm 0.0028, \\ \frac{\Gamma_{Z \rightarrow \tau^+ \tau^-}}{\Gamma_{Z \rightarrow e^+ e^-}} &= 1.0019 \pm 0.0032, \end{aligned} \quad (22)$$

with a correlation of +0.63. In the model, the width of $Z \rightarrow \tau^+ \tau^-$ can have sizable deviation from the SM value due to the loop contributions of the extra Higgs bosons,

because they strongly interact with charged leptons. The quantities of Eq. (22) are calculated in the model are similar to Refs. [22,24,25].

$$\begin{aligned}\frac{\Gamma_{Z \rightarrow \mu^+ \mu^-}}{\Gamma_{Z \rightarrow e^+ e^-}} &\approx 1.0 + \frac{2g_L^e \text{Re}(\delta g_L^{2\text{HDM}}) + 2g_R^e \text{Re}(\delta g_R^{2\text{HDM}})}{g_L^e{}^2 + g_R^e{}^2} \frac{m_\mu^2 \kappa_\mu^2}{m_\tau^2 \kappa_\tau^2}, \\ \frac{\Gamma_{Z \rightarrow \tau^+ \tau^-}}{\Gamma_{Z \rightarrow e^+ e^-}} &\approx 1.0 + \frac{2g_L^e \text{Re}(\delta g_L^{2\text{HDM}}) + 2g_R^e \text{Re}(\delta g_R^{2\text{HDM}})}{g_L^e{}^2 + g_R^e{}^2}.\end{aligned}\quad (23)$$

where the SM value $g_L^e = -0.27$ and $g_R^e = 0.23$. $\delta g_L^{2\text{HDM}}$ and $\delta g_R^{2\text{HDM}}$ are given as

$$\begin{aligned}\delta g_L^{2\text{HDM}} &= \frac{1}{16\pi^2} \frac{m_\tau^2}{v^2} \kappa_\tau^2 \left\{ -\frac{1}{2} B_Z(r_A) - \frac{1}{2} B_Z(r_h) - 2C_Z(r_A, r_h) \right. \\ &\quad \left. + s_W^2 [B_Z(r_A) + B_Z(r_h) + \tilde{C}_Z(r_A) + \tilde{C}_Z(r_h)] \right\}, \\ \delta g_R^{2\text{HDM}} &= \frac{1}{16\pi^2} \frac{m_\tau^2}{v^2} \kappa_\tau^2 \left\{ 2C_Z(r_A, r_h) - 2C_Z(r_{H^\pm}, r_{H^\pm}) \right. \\ &\quad \left. + \tilde{C}_Z(r_{H^\pm}) - \frac{1}{2} \tilde{C}_Z(r_A) - \frac{1}{2} \tilde{C}_Z(r_h) \right. \\ &\quad \left. + s_W^2 [B_Z(r_A) + B_Z(r_h) + 2B_Z(r_{H^\pm}) \right. \\ &\quad \left. + \tilde{C}_Z(r_A) + \tilde{C}_Z(r_h) + 4C_Z(r_{H^\pm}, r_{H^\pm})] \right\},\end{aligned}\quad (24)$$

where $r_\phi = m_\phi^2/m_Z^2$ with $\phi = A, h, H^\pm$, and

$$B_Z(r) = -\frac{\Delta_\epsilon}{2} - \frac{1}{4} + \frac{1}{2} \log(r), \quad (25)$$

$$\begin{aligned}C_Z(r_1, r_2) &= \frac{\Delta_\epsilon}{4} - \frac{1}{2} \int_0^1 dx \int_0^x dy \log[r_2(1-x) \\ &\quad + (r_1 - 1)y + xy],\end{aligned}\quad (26)$$

$$\begin{aligned}\tilde{C}_Z(r) &= \frac{\Delta_\epsilon}{2} + \frac{1}{2} - r[1 + \log(r)] + r^2[\log(r) \log(1 + r^{-1}) \\ &\quad - \text{Li}_2(-r^{-1})] - \frac{i\pi}{2} [1 - 2r + 2r^2 \log(1 + r^{-1})].\end{aligned}\quad (27)$$

The 125 GeV Higgs (H) has the same tree-level couplings to the fermions and gauge bosons as the SM, and the $H \rightarrow hh$ decay is absent for $\lambda_3 = \lambda_4 + \lambda_5$. Since the extra Higgses have no coupling to quarks, we can safely neglect the constraints from the meson observable. We employ the 2HDMC [58] to implement theoretical constraints from the vacuum stability, unitarity and coupling-constant perturbativity, as well as the constraints from the oblique parameters (S, T, U). Adopting the recent fit results in Ref. [59], we use the following values of S, T, U ,

$$S = 0.02 \pm 0.10, \quad T = 0.07 \pm 0.12, \quad U = 0.00 \pm 0.09. \quad (28)$$

The correlation coefficients are given by

$$\rho_{ST} = 0.92, \quad \rho_{SU} = -0.66, \quad \rho_{TU} = -0.86. \quad (29)$$

HiggsBounds [60] is employed to implement the exclusion constraints from the searches for the neutral and charged Higgs at the LEP at 95% confidence level.

The extra Higgs bosons are dominantly produced at the LHC via the following electroweak processes:

$$pp \rightarrow Z^* \rightarrow hA, \quad (30)$$

$$pp \rightarrow W^{\pm*} \rightarrow H^\pm h, \quad (31)$$

$$pp \rightarrow W^{\pm*} \rightarrow H^\pm A, \quad (32)$$

$$pp \rightarrow Z^*/\gamma^* \rightarrow H^+ H^-, \quad (33)$$

$$pp \rightarrow Z \rightarrow \tau^+ \tau^- h. \quad (34)$$

In our scenario, the main decay modes of the Higgs bosons are

$$h \rightarrow \tau^+ \tau^-, \quad \mu^+ \mu^-, \dots, \quad (35)$$

$$A \rightarrow \tau^+ \tau^-, \quad Zh, \mu^+ \mu^-, \dots, \quad (36)$$

$$H^\pm \rightarrow \tau^\pm \nu, \quad W^\pm h, \quad \mu^\pm \nu, \dots. \quad (37)$$

We perform simulations for the processes using MG5_aMC-2.4.3 [61] with PYTHIA6 [62] and DELPHES-3.2.0 [63], and impose the constraints from all the analysis at the 13 TeV LHC in the latest CheckMATE2.0.28 [64], as well as analysis we implemented in our previous works [32,65]. Besides, we also impose the recently published analyses of searching for events with three or more leptons, with up to two hadronical τ leptons, using 13 TeV LHC 137 fb $^{-1}$ data [66]. It improves the limits on chargino mass in a simplified SUSY model that wino-like chargino/neutralino decaying to τ s. The signal regions of 4lI, 4lJ, and 4lK give strongest constraints on our samples, which require 4 leptons with one or two hadronical τ leptons in the final states, because of the dominated multilepton final states in our model.

IV. RESULTS AND DISCUSSIONS

After imposing the constraints of “premuon $g-2$ ” (denoting the theory, the oblique parameters, the exclusion limits from the searches for Higgs at LEP), in Fig. 1 we show the surviving samples which are consistent with Δa_μ and Δa_e at the 2σ level. Both one-loop and two-loop

FIG. 1. The samples within 2σ ranges of Δa_μ (left panel), Δa_e (middle panel), and both Δa_μ and Δa_e (right panel). All the samples satisfy the constraints of “pre-muon $g-2$ ”.

diagrams give positive contributions to Δa_μ . For Δa_e , the contributions of one-loop are positive and those of two-loop are negative. Only the contributions of two-loop can make Δa_e to be within 2σ range for large enough $\kappa_\tau \kappa_e$. Δa_μ and Δa_e respectively favor $-\kappa_\mu$ and κ_e to increase with m_h . For $\kappa_\tau = 140$ and $m_h = 60$ GeV, $-\kappa_\mu$ and κ_e are respectively required to be larger than 40 and 250. Due to the constraints from the searches for $ee \rightarrow \tau\tau(h) \rightarrow \tau\tau\tau\tau$ at LEP [67], κ_τ is required to be smaller than 90 for $m_h < 27$ GeV. As a result, the relative large $-\kappa_\mu$ and κ_e are respectively required to explain Δa_μ and Δa_e for $m_h < 27$ GeV. The right panel shows that the upper limits of $-\kappa_\mu$ within the 2σ ranges of both Δa_μ and Δa_e are much smaller than those within the 2σ ranges of Δa_μ .

This is because κ_τ is required to be large enough to explain Δa_e , and for such large κ_τ , Δa_μ favors a relative small κ_μ .

After imposing the constraints of pre-muon $g-2$, we show the surviving samples with $\chi_\tau^2 < 12.3$ in Fig. 2. Because κ_μ is opposite in sign from κ_τ , the second term of δ_{tree} in Eq. (20) is positive, which gives a well fit to g_τ/g_e . Such case is not realized in the lepton-specific 2HDM and aligned 2HDM. From Fig. 2, we find that χ_τ^2 increases with $m_{H^\pm}$, and obtains a relative small value for the moderate $-\kappa_\mu \kappa_\tau$. The χ_τ^2 can be as low as 7.4, which is much smaller than the SM value.

In Fig. 3 we show the surviving samples after imposing the constraints of pre-muon $g-2$, Δa_μ , Δa_e , $\chi_\tau^2 < 12.3$,

FIG. 2. The surviving samples fit the data of LFU in τ decay with $\chi_\tau^2 < 12.3$. All the samples satisfy the constraints of “pre-muon $g-2$ ”.

FIG. 3. The allowed samples (squares) and excluded samples (bullets) by the direct search limits from the LHC at 95% confidence level. All the samples satisfy the constraints of pre-muon $g-2$, Δa_μ , Δa_e , $\chi_\tau^2 < 12.3$, and Z decays.

Z decays, and the direct searches at LHC. The Fig. 3 shows that the points with relatively larger $m_A/m_{H^\pm}$ or lower m_h can escape the direct searches. The production cross sections at the LHC decrease with heavier $A/H^\pm$, and this region can be further detected with higher luminosity and collision energy. For the light h , the τ s from h in decays become too soft to be distinguished at detector, while the τ s from h in $A/H^\pm$ decays are collinear because of the large mass splitting between h and $A/H^\pm$. Meanwhile, the $A/H^\pm \rightarrow hZ/W^\pm$ decay modes dominate the $A/H^\pm$ decays in the low m_h region. Thus, in the region of $m_h < 20$ GeV, the acceptance of above signal region for final state of two collinear $\tau + Z/W$ boson quickly decreases.

The upper-right panel of Fig. 3 shows that the χ_τ^2 is required to be closed to the SM value since the multilepton event searches at the LHC favor large $m_{H^\pm}$ or small m_h . For small m_h , the muon and electron anomalies favor small κ_τ and $-\kappa_\mu$. As a result, the new contribution to χ_τ^2 are sizably reduced. The χ_τ^2 is allowed to be as low as 11.5 for $m_{H^\pm}$ around 200 GeV, which is visibly smaller than

the SM value, 12.3. The Fig. 3 shows that the parameters of the Yukawa couplings are favored in the region of $15 < -\kappa_\mu < 90$, $5 < \kappa_\tau < 50$, and $160 < \kappa_e < 500$.

We introduce explicit Z_2 breaking terms in the Eq. (9). For a natural theory model, such breaking terms should be tiny if the theory respects the original symmetry. Although the κ_e , κ_μ , and κ_τ are substantial, the breaking terms of Eq. (9) still can be significantly suppressed by $\frac{m_e}{v}$, $\frac{m_\mu}{v}$, and $\frac{m_\tau}{v}$. In addition, Fig. 3 shows that κ_e , κ_μ , and κ_τ are allowed to have very small values for a very light h . Especially for that κ_τ is allowed to be smaller than 10. Therefore, for a light h , we may introduce the tiny breaking terms to accommodate the LFU in τ decay, the muon and electron $g-2$ anomalies.

In the Eq. (9), we make the lepton-flavor-conserving (violation) terms to break (respect) the Z_2 symmetry. Such case may be naturally achieved by introducing an additional Z'_4 symmetry. Under Z'_4 symmetry, the three generation lepton fields respectively have charge $i, -i, -1$, and other fields including Φ_2 have charge 1. Thus, in the model with explicit Z_2 breaking, the lepton-flavor-conserving

(violation) terms are still allowed (forbidden) by the additional Z'_4 symmetry.

In order to avoid the constraint from the $H \rightarrow hh$ decay, we simply take $\lambda_3 = \lambda_4 + \lambda_5$ to make the tree-level Hhh coupling to be absent. The Hhh coupling can be corrected at one-loop by the exchange of SM leptons in the loop, and the correction is proportional to $\frac{1}{16\pi^2}(\frac{m_\ell}{v})^3\kappa_\ell^2$. When κ_ℓ is $\mathcal{O}(10^2)$, the correction at the one-loop is still small. In addition, we can tune the parameter λ_3 so that the contributions of the tree-level and one-loop to the Hhh coupling are canceled to some extent. Meanwhile, the value of λ_3 does not affect the results of the LFU in τ decay, the muon and electron $g-2$ anomalies.

V. CONCLUSION

In the lepton-specific inert two-Higgs-doublet model, we consider relevant theoretical and experimental constraints,

especially for the multilepton event searches at the LHC, and discuss the possibilities of explaining the new muon $g-2$ anomaly reported by the Fermilab and electron $g-2$ anomaly. We find that the muon and electron $g-2$ anomalies can be explained simultaneously in the region of $5 \text{ GeV} < m_h < 35 \text{ GeV}$, $200 \text{ GeV} < m_A < 620 \text{ GeV}$, $190 \text{ GeV} < m_{H^\pm} < 620 \text{ GeV}$, $15 < -\kappa_\mu < 90$, $5 < \kappa_\tau < 50$, and $160 < \kappa_e < 500$.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grants No. 11975013, No. 11875062, No. 12105248, and by the Project of Shandong Province Higher Educational Science and Technology Program under Grants No. 2019KJJ007.

-
- [1] G. W. Bennett *et al.* (Muon $g-2$ Collaboration), *Phys. Rev. Lett.* **86**, 2227 (2001).
 - [2] G. W. Bennett *et al.* (Muon $g-2$ Collaboration), *Phys. Rev. D* **73**, 072003 (2006).
 - [3] B. Abi *et al.* (Fermilab Collaboration), *Phys. Rev. Lett.* **126**, 141801 (2021).
 - [4] T. Aoyama, M. Hayakawa, T. Kinoshita, and M. Nio, *Phys. Rev. Lett.* **109**, 111808 (2012).
 - [5] A. Czarnecki, W. J. Marciano, and A. Vainshtein, *Phys. Rev. D* **67**, 073006 (2003).
 - [6] G. Eichmann, C. S. Fischer, and R. Williams, *Phys. Rev. D* **101**, 054015 (2020).
 - [7] M. Davier, A. Hoecker, B. Malaescu, and Z. Zhang, *Eur. Phys. J. C* **80**, 241 (2020).
 - [8] R. H. Parker, C. Yu, W. Zhong, B. Estey, and H. Mueller, *Science* **360**, 191 (2018).
 - [9] D. Hanneke, S. Fogwell, and G. Gabrielse, *Phys. Rev. Lett.* **100**, 120801 (2008).
 - [10] D. Hanneke, S. F. Hoogerheide, and G. Gabrielse, *Phys. Rev. A* **83**, 052122 (2011).
 - [11] L. Morel, Z. Yao, P. Clad, and S. Guellati-Khlifa, *Nature (London)* **588**, 61 (2020).
 - [12] A. Dedes and H. E. Haber, *J. High Energy Phys.* **05** (2001) 006.
 - [13] D. Chang, W.-F. Chang, C.-H. Chou, and W.-Y. Keung, *Phys. Rev. D* **63**, 091301 (2001).
 - [14] K. M. Cheung, C. H. Chou, and O. C. W. Kong, *Phys. Rev. D* **64**, 111301 (2001).
 - [15] J. Cao, P. Wan, L. Wu, and J. M. Yang, *Phys. Rev. D* **80**, 071701 (2009).
 - [16] L. Wang and X. F. Han, *J. High Energy Phys.* **05** (2015) 039.
 - [17] E. J. Chun, Z. Kang, M. Takeuchi, and Y.-L. Tsai, *J. High Energy Phys.* **11** (2015) 099.
 - [18] T. Han, S. K. Kang, and J. Sayre, *J. High Energy Phys.* **02** (2016) 097.
 - [19] V. Ilisie, *J. High Energy Phys.* **04** (2015) 077.
 - [20] A. Cherchiglia, P. Kneschke, D. Stockinger, and H. Stockinger-Kim, *J. High Energy Phys.* **01** (2017) 007.
 - [21] X. Liu, L. Bian, X.-Q. Li, and J. Shu, *Nucl. Phys.* **B909**, 507 (2016).
 - [22] T. Abe, R. Sato, and K. Yagyu, *J. High Energy Phys.* **07** (2015) 064.
 - [23] A. Crivellin, J. Heeck, and P. Stoffer, *Phys. Rev. Lett.* **116**, 081801 (2016).
 - [24] E. J. Chun and J. Kim, *J. High Energy Phys.* **07** (2016) 110.
 - [25] L. Wang, J. M. Yang, M. Zhang, and Y. Zhang, *Phys. Lett. B* **788**, 519 (2019).
 - [26] D. Sabatta, A. S. Cornell, A. Goyal, M. Kumar, and B. Mellado, *Chin. Phys. C* **44**, 063103 (2020).
 - [27] E. J. Chun and T. Mondal, *Phys. Lett. B* **802**, 135190 (2020).
 - [28] H. Davoudiasl and W. J. Marciano, *Phys. Rev. D* **98**, 075011 (2018).
 - [29] J.-J. Zhang, M. He, X.-G. He, and G. Li, *Nucl. Phys.* **B953**, 114968 (2020).
 - [30] A. Crivellin, M. Hoferichter, and P. Schmidt-Wellenburg, *Phys. Rev. D* **98**, 113002 (2018).
 - [31] J. Liu, C. E. M. Wagner, and X.-P. Wang, *J. High Energy Phys.* **03** (2019) 008.
 - [32] X.-F. Han, T. Li, L. Wang, and Y. Zhang, *Phys. Rev. D* **99**, 095034 (2019).
 - [33] M. Endo and W. Yin, *J. High Energy Phys.* **08** (2019) 122.
 - [34] H. Davoudiasl and W. J. Marciano, *Phys. Rev. D* **98**, 075011 (2018).
 - [35] M. Abdullah, B. Dutta, S. Ghosh, and T. Li, *Phys. Rev. D* **100**, 115006 (2019).
 - [36] S. Gardner and X. Yan, *Phys. Rev. D* **102**, 075016 (2020).

- [37] M. Badziak and K. Sakurai, *J. High Energy Phys.* **10** (2019) 024.
- [38] A. Hernandez, S. F. King, H. Lee, and S. J. Rowley, *Phys. Rev. D* **101**, 115016 (2020).
- [39] A. Hernandez, Y. Velasquez, S. Kovalenko, H. N. Long, N. Perez-Julve, and V. V. Vien, *Eur. Phys. J. C* **81**, 191 (2021).
- [40] S. Jana and S. Saad, *Phys. Rev. D* **101**, 115037 (2020).
- [41] L. Calibbi, M. Ibanez, A. Melis, and O. Vives, *J. High Energy Phys.* **06** (2020) 087.
- [42] C. Chen and T. Nomura, *Nucl. Phys.* **B964**, 115314 (2021).
- [43] C. Chua, *Phys. Rev. D* **102**, 055022 (2020).
- [44] C. Hati, J. Kriewald, J. Orloff, and A. M. Teixeira, *J. High Energy Phys.* **07** (2020) 235.
- [45] B. Dutta, S. Ghosh, and T. Li, *Phys. Rev. D* **102**, 055017 (2020).
- [46] F. J. Botella, F. Cornet-Gomez, and M. Nebot, *Phys. Rev. D* **102**, 035023 (2020).
- [47] I. Dorsner, S. Fajfer, and S. Saad, *Phys. Rev. D* **102**, 075007 (2020).
- [48] S. Jana, W. Rodejohann, and S. Saad, *Phys. Rev. D* **102**, 075003 (2020).
- [49] E. J. Chun and T. Mondal, *J. High Energy Phys.* **11** (2020) 077.
- [50] S. Li, X.-Q. Li, Y. Li, Y.-D. Yang, and X. Zhang, *J. High Energy Phys.* **01** (2021) 034.
- [51] L. D. Rose, S. Khalil, and S. Moretti, *Phys. Lett. B* **816**, 136216 (2021).
- [52] A. Hernandez, S. F. King, and H. Lee, *Phys. Rev. D* **103**, 115024 (2021).
- [53] N. Chen, B. Wang, and C. Yao, [arXiv:2102.05619](#).
- [54] A. Bodas, R. Coy, and S. King, [arXiv:2102.07781](#).
- [55] J. Cao, Y. He, J. Lian, D. Zhang, and P. Zhu, *Phys. Rev. D* **104**, 055009 (2021).
- [56] Y. Amhis *et al.* (Heavy Flavor Averaging Group (HFAG) Collaboration), [arXiv:1412.7515](#).
- [57] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak Working Group, SLD Electroweak Group, and SLD Heavy Flavour Group Collaborations), *Phys. Rep.* **427**, 257 (2006).
- [58] D. Eriksson, J. Rathsmann, and O. Stål, *Comput. Phys. Commun.* **181**, 189 (2010).
- [59] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018).
- [60] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein, and K. E. Williams, *Comput. Phys. Commun.* **181**, 138 (2010).
- [61] J. Alwall *et al.*, *J. High Energy Phys.* **07** (2014) 079.
- [62] P. Torrielli and S. Frixione, *J. High Energy Phys.* **04** (2010) 110.
- [63] J. de Favereau *et al.* (DELPHES 3 Collaboration), *J. High Energy Phys.* **02** (2014) 057.
- [64] D. Dercks, N. Desai, J. S. Kim, K. Rolbiecki, J. Tattersall, and T. Weber, *Comput. Phys. Commun.* **221**, 383 (2017).
- [65] G. Pozzo and Y. Zhang, *Phys. Lett. B* **789**, 582 (2019).
- [66] A. M. Sirunyan *et al.* (CMS Collaboration), Report No. CMS-PAS-SUS-19-012.
- [67] J. Abdallah *et al.* (DELPHI Collaboration), *Eur. Phys. J. C* **38**, 1 (2004).