Suppressions of dijet azimuthal correlations in the future EIC

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Quark-antiquark pair (or dijet) production at the Electron-Ion Collider has been argued to be one of the most important processes that allows access to the Weizsäcker-Williams (WW) gluon distributions in the small- x limit. Within the framework of color glass condensate effective field theory, we calculate the dijet cross sections and azimuthal correlations at the partonic level by including the Sudakov resummations. Our numerical results show that the back-to-back correlations are significantly suppressed when the Sudakov resummations are taken into account. In addition, by using the solutions of the running-coupling Balitsky-Kovchegov equation, the unpolarized and linearly polarized WW gluon distributions in both coordinate and momentum space are presented.

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I. INTRODUCTION

Exploring the multidimensional structure and detailed dynamics of a hadron has been one of the primary goals of hadronic physics. In recent years, the so-called transverse momentum distributions (TMDs) associated with their evolutions, which have revealed essential aspects of the partonic structures in momentum space (parton distributions are not only functions of the longitudinal momentum fraction x , but also involve the intrinsic transverse momentum $k_{\perp}$), have received a lot of theoretical and experimental attention. The planned deep inelastic scattering (DIS) facilities, including the Electron Ion Collider (EIC) [1–3], the Large Hadron Electron Collider [4,5], and the Electron Ion Collider in China [6], will provide the cleanest environment to determine these fundamental quantities over a broad range of kinematics at high accuracy.

Quark-antiquark pair (or dijet) production in DIS has been argued to be an important process that allows to access partonic structures in momentum space. In particular, when the produced jets are mostly back to back, the novel process can be used as a proxy to probe the Weizsäcker-Williams (WW) gluon distribution [7]. The WW gluon distribution, as one type of gluon TMD, features intrinsic gauge-link structures (either future-pointing or past-pointing gauge links in light-cone gauge) and can be interpreted as the

number density of gluons inside the scattered target [7–9]. As addressed in Ref. [10], a nontrivial feature of these gluons is that they can be linearly polarized even if the hadron is unpolarized; if sizable, this can have a large impact on physical observables, such as (pseudo)scalar particle production in hadronic collisions [11–13] and the azimuthal correlations of dijets in DIS [7,14–22]. In Refs. [23,24], for dijet production in DIS, it has been shown that the unpolarized WW gluon distribution associated with its partner (the linearly polarized one) enters the cross section through an azimuthal modulation. Recent developments have also shown that the two gluon distributions not only exist in DIS processes, but also appear in other processes, such as dijets or quark-antiquark pairs [25–27], photon pairs [28], muon pairs [16], and quarkonium production in hadronic collisions [29–32]. A particular feature is that the unpolarized WW gluon distributions are always accompanied by their linearly polarized parts through an azimuthal modulation, which suggests that these gluon distributions (especially for the linearly polarized parts) can be extracted by measuring the azimuthal observables. Another interesting feature is that the linearly polarized WW gluon distribution will tend to its saturated bound at large transverse momenta, while it is suppressed at small transverse momenta [23,24], which was further confirmed by numerical calculations in Refs. [21,22]; for a recent review, please see Ref. [33] and references therein. Up to now, much about the two gluon distributions (apart from the positive bound of the linearly polarized part) is still experimentally unknown. Therefore, it is necessary to precisely extract and quantitatively understand the behaviors of the two gluon distributions in the high-energy limit, which will allow us to gain insight into the transverse

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tomography of protons and nuclei as well as the underlying dynamics in high-energy scatterings.

In this paper, we continue to concentrate on quark-antiquark pair (or dijet) production in the special process where a virtual photon scatters off of a hadronic target; this process typically corresponds to quark-antiquark (or dijet) production in DIS. Due to the broad range of kinematics that can be accessed at the future EIC, this process allows to map out the evolutions of gluon TMDs. In addition, when the scattering energy is sufficiently large (or, equivalently, for small x) it is possible to probe the high-density regime in which gluon saturation effects become more prominent, and therefore it is suitable to investigate the behavior of gluon TMDs in the dense regime. In the correlation limit, prior works have shown that this process can be consistently described in the framework of color glass condensate (CGC) theory, and that the relevant two WW gluon distributions can be expressed in terms of Wilson line correlators, which allows them to be calculated using solutions of the Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner (JIMWLK) equation [21–24]. However, this limitation places a strong restriction on the dijets in that it requires the transverse momentum imbalance $q_\perp$ to be much smaller than the relative transverse momentum $P_\perp$; in this case, there is another class of effects—known as Sudakov resummations—that can give sizable contributions to the cross section, and this should be taken into account when exploring the behaviors of the two gluon distributions in the considered process. Other phenomenological studies have shown that it is essential to include such resummations when quantitatively describing experimental observables [34–46]. Therefore, in the present work we continue in this direction by including the Sudakov resummations, and show how the resummations affect the azimuthal correlations of the dijets. Furthermore, as shown in Refs. [21,22,27,47], thanks to the numerical implementation of the JIMWLK equation, the two gluon distributions that appear in the considered process can be obtained. However, numerical implementations can only be achieved by fixing the coupling constant. The running-coupling corrections, despite being an important part of QCD dynamics, have yet to be calculated. In this paper, we prefer to work within the Gaussian approximation of JIMWLK evolution and use the running-coupling Balitsky-Kovchegov (rcBK) equation to perform the relevant calculations, as we expect the running-coupling corrections here to

be much more important than that of the Gaussian approximation. In addition, we stress that the rcBK solutions are well constrained by DIS data [48] and have been successful in describing particle spectra in hadronic collisions [49,50]. Therefore, one can be firmly convinced that the two gluon TMDs can be well studied by using the solutions of the rcBK equation.

The paper is organized as follows. In the next section, we begin with a brief review of the CGC results for quark-antiquark pair production in DIS and show that the cross sections are sensitive probes of gluon TMDs (unpolarized and linearly polarized WW gluon distributions) inside the scattered target at small x . In Sec. III, we discuss the detailed behaviors of unpolarized and linearly polarized WW gluon distributions in the saturation model and then present the numerical results using the solutions of the rcBK equation. In Sec. IV, we give our numerical results for the dijet cross sections, and discuss how the effects of Sudakov resummations and gluon saturation affect their azimuthal correlations. Finally, a short summary is given in Sec. V.

II. DIS DIJET PRODUCTION IN COLOR GLASS CONDENSATE FORMALISM

In this section, we first give a brief review of the quark-antiquark pair (or dijet) differential cross section in the CGC formalism and show that, by applying the “back-to-back correlation limit,” the cross sections are expressed in terms of WW gluon distributions.

To begin with, we consider a high-energy virtual photon γ^* with four-momentum p scattering off of a hadronic target, after which a quark-antiquark pair (or dijet¹) is produced in DIS via the exchange of a virtual photon. For this process, the hadronic target can be regarded as a classical gluon field due to the dominance of gluon saturation effects [52–56]. Therefore, it is convenient to calculate the cross section in the CGC formalism: the standard way is to consider that the virtual photon γ^* fluctuates into a quark-antiquark pair $q\bar{q}$, and then traverses the background field in an eikonal way; simultaneously, the fluctuated quarks receive a contribution to the scattering amplitude with a Wilson line. After averaging over the photon’s polarization and summing over the quarks’ helicities and colors, the differential cross section for this process can be expressed as follows [7,23]:

$$\begin{aligned} \frac{d\sigma_{T,L}^{\gamma^*A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} &= N_c \alpha_{\text{em}} e_q^2 \delta(p^+ - k_1^+ - k_2^+) \int \frac{d^2x_1}{(2\pi)^2} \frac{d^2x'_1}{(2\pi)^2} \frac{d^2x_2}{(2\pi)^2} \frac{d^2x'_2}{(2\pi)^2} e^{-ik_{1\perp} \cdot (x_1 - x'_1)} e^{-ik_{2\perp} \cdot (x_2 - x'_2)} \\ &\quad \times \sum_{\lambda\alpha\beta} \psi_{\alpha\beta}^{T,L\lambda}(x_1 - x_2) \psi_{\alpha\beta}^{T,L\lambda*}(x'_1 - x'_2) [1 + S_{x_g}^{(4)}(x_1, x_2; x'_2, x'_1) - S_{x_g}^{(2)}(x_1, x_2) - S_{x_g}^{(2)}(x'_2, x'_1)], \end{aligned} \quad (1)$$

¹Strictly speaking, for dijet production, the channel $\gamma^*q \rightarrow \gamma^*qg$ should be included. However, since we are only interested in the small- x regime, in which the gluon density is very large compared to the quarks, in this sense the contributions of quarks can be neglected. Otherwise, the quark part enters and gives an additional contribution to the cross section [16,51].

where α_{em} is the electromagnetic structure constant, and N_c and e_q are the number of colors and the electrical charge of the produced quark. The two-dimensional variables $\mathbf{x}_1$ and $\mathbf{x}_2$ denote the transverse coordinates of the outgoing quark and antiquark in the amplitude, respectively. Clearly, the variables $\mathbf{u} = \mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{v} = z\mathbf{x}_1 + (1-z)\mathbf{x}_2$ are the transverse size of the produced quark-antiquark pair and the transverse position of the incoming virtual photon, where $z = k_1^+/p^+$ is the longitudinal momentum fraction

of the outgoing quark with respect to the incoming virtual photon. Similar notations hold for the complex-conjugate amplitude. The splitting function $\psi_{\alpha\beta}^{T,L\lambda}(p^+, z, \mathbf{u})$ denotes the fact that the virtual photon γ^* with longitudinal momentum p^+ and virtuality Q^2 fluctuates into a quark-antiquark pair with transverse size $\mathbf{u}$, and this function can be calculated in light-cone perturbation theory and be expressed in the following form [57]:

$$\psi_{\alpha\beta}^{T\lambda}(p^+, z, \mathbf{u}) = 2\pi\sqrt{\frac{2}{p^+}} \begin{cases} i\epsilon_f K_1(\epsilon_f|\mathbf{u}|) \frac{u \cdot \epsilon_+^{(1)}}{|\mathbf{u}|} [\delta_{\alpha+}\delta_{\beta+}(1-z) + \delta_{\alpha-}\delta_{\beta-}z] + \delta_{\alpha-}\delta_{\beta+}m_q K_0(\epsilon_f|\mathbf{u}|), & \lambda = 1, \\ i\epsilon_f K_1(\epsilon_f|\mathbf{u}|) \frac{u \cdot \epsilon_+^{(2)}}{|\mathbf{u}|} [\delta_{\alpha-}\delta_{\beta-}(1-z) + \delta_{\alpha+}\delta_{\beta+}z] + \delta_{\alpha+}\delta_{\beta-}m_q K_0(\epsilon_f|\mathbf{u}|), & \lambda = 2, \end{cases} \quad (2)$$

$$\psi_{\alpha\beta}^{L\lambda}(p^+, z, \mathbf{u}) = 2\pi\sqrt{\frac{4}{p^+}} z(1-z) Q K_0(\epsilon_f|\mathbf{u}|) \delta_{\alpha\beta}, \quad (3)$$

where α, β are the helicities of the produced quark and antiquark, λ is the photon's polarization, T, L denote the transverse and longitudinal components of the wave function, and $\epsilon_f^2 = z(1-z)Q^2 + m_q^2$, where m_q is the mass of the produced quark.

It should be noted that the interactions between the produced quark-antiquark pair and the scattered target are fully characterized by Wilson line correlators, as shown in the last line of Eq. (1). Specifically, the term equal to 1 means that the virtual photon freely passes through the target, and the quark-antiquark pair without scattering in the amplitude and complex-conjugate amplitude. The interference terms $S_{x_g}^{(2)}(\mathbf{x}_1, \mathbf{x}_2)$ and $S_{x_g}^{(2)}(\mathbf{x}'_1, \mathbf{x}'_2)$, known as dipoles, denote the fact that the quark-antiquark pair scatters off of the target either in the amplitude or the complex-conjugate amplitude. The four-point function $S_{x_g}^{(4)}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}'_1, \mathbf{x}'_2)$, called the quadrupole, describes the fact that the quark-antiquark pair undergoes multiscattering with the target in both the amplitude and complex-conjugate amplitude. The dipole and quadrupole are defined as

$$S_{x_g}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{N_c} \langle \text{Tr} U(\mathbf{x}_1) U^\dagger(\mathbf{x}_2) \rangle_{x_g}, \quad (4)$$

$$S_{x_g}^{(4)}(\mathbf{x}_1, \mathbf{x}_2; \mathbf{x}'_1, \mathbf{x}'_2) = \frac{1}{N_c} \langle \text{Tr} U(\mathbf{x}_1) U^\dagger(\mathbf{x}'_1) U(\mathbf{x}'_2) U^\dagger(\mathbf{x}_2) \rangle_{x_g}, \quad (5)$$

$$\text{with } U(\mathbf{x}) = \mathcal{P} \exp \left[ig_s \int dx^+ T^c A_c^-(x^+, \mathbf{x}) \right]. \quad (6)$$

The notation $\langle \cdots \rangle_{x_g}$ used here denotes the averaging over all possible distributions of color sources, and x_g is the fraction of the longitudinal momentum in the scattered target.

As shown above, quark-antiquark pair (or dijet) production in the DIS process can be quite well described in the CGC formalism, and the cross section directly probes the internal dynamics of the scattered target. It should be noted that the CGC approach does not depend on momentum scales and, in principle, can be applied in any scattering process when a high-density or “saturation” regime is accessed. This treatment is different from the TMD factorization approach, since the latter is strongly dependent on momentum scales. In order to recover the TMD results, it is useful to change variables as follows:

$$\mathbf{x}_1 = \mathbf{v} + (1-z)\mathbf{u} \quad \text{and} \quad \mathbf{x}_2 = \mathbf{v} - z\mathbf{u}. \quad (7)$$

Then, Eq. (1) becomes

$$\begin{aligned} \frac{d\sigma_{T,L}^{A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} &= N_c \alpha_{\text{em}} e_q^2 \delta(q^+ - k_1^+ - k_2^+) \int \frac{d^2\mathbf{u}}{(2\pi)^2} \frac{d^2\mathbf{u}'}{(2\pi)^2} \frac{d^2\mathbf{v}}{(2\pi)^2} \frac{d^2\mathbf{v}'}{(2\pi)^2} e^{-iq_\perp \cdot (\mathbf{v}-\mathbf{v}')} e^{-iP_\perp \cdot (\mathbf{u}-\mathbf{u}')} \sum_{\lambda\alpha\beta} \psi_{\alpha\beta}^{L,T\lambda}(\mathbf{u}) \psi_{\alpha\beta}^{L,T\lambda*}(\mathbf{u}') \\ &\times \left[1 + \frac{1}{N_c} \langle \text{Tr} U(\mathbf{v} + (1-z)\mathbf{u}) U^\dagger(\mathbf{v}' + (1-z)\mathbf{u}') U(\mathbf{v}' - z\mathbf{u}') U^\dagger(\mathbf{v} - z\mathbf{u}) \rangle_{x_g} \right. \\ &\left. - \frac{1}{N_c} \langle \text{Tr} U(\mathbf{v} + (1-z)\mathbf{u}) U^\dagger(\mathbf{v} - z\mathbf{u}) \rangle_{x_g} - \frac{1}{N_c} \langle \text{Tr} U(\mathbf{v}' - z\mathbf{u}') U^\dagger(\mathbf{v}' + (1-z)\mathbf{u}') \rangle_{x_g} \right], \quad (8) \end{aligned}$$

where $q_\perp = k_{1\perp} + k_{2\perp}$ and $P_\perp = (1-z)k_{1\perp} - zk_{2\perp}$ are the transverse momentum imbalance and relative momentum of the quark pair. Obviously, the two variables are the conjugate momenta with respect to $\mathbf{v}$ and $\mathbf{u}$. In what follows, we are only interested in a special kinematic limit,² also known as the “back-to-back correlation limit” [7], in which the transverse momentum imbalance is much

smaller than the relative momentum, i.e., $|q_\perp| \ll |P_\perp|$. With this limitation, the corresponding variables $\mathbf{u}$ and $\mathbf{u}'$ are much smaller than $\mathbf{v}$ and $\mathbf{v}'$; in this sense, terms involving multipoint Wilson line correlators can be expanded in a Taylor series in coordinate space. To leading order in $\mathbf{u}$ and $\mathbf{u}'$,³ Eq. (8) can be rewritten as

$$\begin{aligned} \frac{d\sigma_{T,L}^{*A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} &= N_c \alpha_{\text{em}} e_q^2 \delta(q^+ - k_1^+ - k_2^+) \int \frac{d^2\mathbf{u}}{(2\pi)^2} \frac{d^2\mathbf{u}'}{(2\pi)^2} \frac{d^2\mathbf{v}}{(2\pi)^2} \frac{d^2\mathbf{v}'}{(2\pi)^2} \\ &\times e^{-iq_\perp \cdot (\mathbf{v}-\mathbf{v}')} e^{-iP_\perp \cdot (\mathbf{u}-\mathbf{u}')} \sum_{\lambda\alpha\beta} \psi_{\alpha\beta}^{L,T\lambda}(\mathbf{u}) \psi_{\alpha\beta}^{L,T\lambda*}(\mathbf{u}') \\ &\times \left[-\mathbf{u}_i \mathbf{u}'_j \frac{1}{N_c} \langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g} \right]. \end{aligned} \quad (9)$$

From the above expression, one can see that the differential cross section can be factorized into two parts: one involves hard scale $P_\perp$ (which is referred to as the hard part), and the other is related to $q_\perp$, and it is this quantity that directly probes the transverse momentum of the incoming gluon from the hadronic target. Together with the saturation scale Q_{sg} , there are three scales in this scattering process. Notice that, although $|q_\perp| \ll |P_\perp|$, the value of $q_\perp$ can be of the order of (or softer than) the saturation scale Q_{sg} ; in this sense, gluon saturation effects still remain important and this process can indeed provide information on the internal dynamics in the dense regime.

Within the back-to-back correlation limit, due to the two separated scales, i.e., $|P_\perp| \gg |q_\perp|$, there is another class of effects with a double logarithmic form $\ln \frac{P_\perp^2}{q_\perp^2}$ that can give a large contribution to the cross section, and this should be included. This factor, also known as Sudakov resummations, comes from the resummation of soft collinear gluon radiation from the produced quark pair. In the considered process, on the cross section level, which can be obtained by calculation one-loop correction of the quark pair [43,44]. After including this contribution, Eq. (9) becomes

$$\begin{aligned} \frac{d\sigma_{T,L}^{*A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} &= N_c \alpha_{\text{em}} e_q^2 \delta(q^+ - k_1^+ - k_2^+) \int \frac{d^2\mathbf{u}}{(2\pi)^2} \frac{d^2\mathbf{u}'}{(2\pi)^2} \frac{d^2\mathbf{v}}{(2\pi)^2} \frac{d^2\mathbf{v}'}{(2\pi)^2} \\ &\times e^{-iq_\perp \cdot (\mathbf{v}-\mathbf{v}')} e^{-iP_\perp \cdot (\mathbf{u}-\mathbf{u}')} \sum_{\lambda\alpha\beta} \psi_{\alpha\beta}^{L,T\lambda}(\mathbf{u}) \psi_{\alpha\beta}^{L,T\lambda*}(\mathbf{u}') e^{-S_{\text{sud}}(P_\perp, \mathbf{v}-\mathbf{v}')} \\ &\times \left[-\mathbf{u}_i \mathbf{u}'_j \frac{1}{N_c} \langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g} \right]. \end{aligned} \quad (10)$$

To leading order, the Sudakov factor can be expressed as [44]

$$S_{\text{sud}}(P_\perp, \mathbf{v}-\mathbf{v}') = \int_{c_0^2/(\mathbf{v}-\mathbf{v}')^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2} \frac{N_c}{2} \frac{\alpha_s}{\pi} \ln \frac{P_\perp^2}{\mu^2}, \quad (11)$$

where $c_0 = 2e^{-\gamma_E}$, with γ_E being Euler's constant. In the following, the running coupling factor α_s is computed at the scale $\mu^2 = c_0^2/(\mathbf{v}-\mathbf{v}')^2$.

It should be noted that the Sudakov factor only resums the soft radiation from the produced quark pair, and this contribution is independent of the variables $\mathbf{u}$ and $\mathbf{u}'$. Therefore, one can straightforwardly integrate over $\mathbf{u}$ and $\mathbf{u}'$ by collecting Eqs. (2) and (3); then, Eq. (10) becomes

²Beyond the correlation limit, this process has been studied in Ref. [58], which gives insight into the dynamics of multipoint Wilson line correlators in the small- x limit.

³Following this procedure, higher orders can be obtained, and the next-to-leading power order can be found in Ref. [59].

$$\frac{d\sigma_{\gamma^*A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} = \alpha_{\text{em}} e_q^2 \delta(x_{\gamma^*} - 1) z(1-z) \left\{ 2[z^2 + (1-z)^2] \left[\frac{\delta^{ij}}{(P_\perp^2 + \epsilon_f^2)^2} - \frac{4\epsilon_f^2 P_\perp^i P_\perp^j}{(P_\perp^2 + \epsilon_f^2)^4} \right] + \frac{8m_q^2 P_\perp^i P_\perp^j}{(P_\perp^2 + \epsilon_f^2)^4} \right\} \\ \times \int \frac{d^2\mathbf{v}}{(2\pi)^2} \frac{d^2\mathbf{v}'}{(2\pi)^2} e^{-S_{\text{sud}}(P_\perp, \mathbf{v}-\mathbf{v}')} e^{-iq_\perp \cdot (\mathbf{v}-\mathbf{v}')} [-\langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g}], \quad (12)$$

$$\frac{d\sigma_{\gamma_L^*A \rightarrow q\bar{q}X}}{d^3k_1 d^3k_2} = \alpha_{\text{em}} e_q^2 \delta(x_{\gamma^*} - 1) z(1-z) \left[4z^2(1-z)^2 Q^2 \frac{4P_\perp^i P_\perp^j}{(P_\perp^2 + \epsilon_f^2)^4} \right] \\ \times \int \frac{d^2\mathbf{v}}{(2\pi)^2} \frac{d^2\mathbf{v}'}{(2\pi)^2} e^{-S_{\text{sud}}(P_\perp, \mathbf{v}-\mathbf{v}')} e^{-iq_\perp \cdot (\mathbf{v}-\mathbf{v}')} [-\langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g}]. \quad (13)$$

Due to the fact that $P_\perp$ is the conjugate momentum with respect to $\mathbf{u}$, the orientation of $\mathbf{u}_i$ holds in the above expressions, and is represented by $P_\perp^i$ after taking the Fourier transformation. Another piece is the double derivation of the quadrupole, which is related to the operator definition of the WW gluon distribution. When averaging over the impact parameter $B_\perp = \frac{1}{2}(\mathbf{v} + \mathbf{v}')$, it can be expressed as

$$xW^{ij}(x_g, \mathbf{v} - \mathbf{v}') = -\frac{2S_\perp}{(2\pi)^2 \alpha_s} \langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g}, \quad (14)$$

in which $S_\perp = \int d^2B_\perp$ is the transverse area of the scattered target. Generally speaking, xW^{ij} is a 2×2 matrix that can be decomposed into various tensor structures. In what follows, it is convenient to separate it into trace and traceless parts. For positive definite, the corresponding tensor structure in coordinate space takes the form

$$xW^{ij}(x_g, r_\perp) = \frac{1}{2} \delta^{ij} xG(x_g, r_\perp) + \frac{1}{2} \left(\delta^{ij} - 2 \frac{r_\perp^i r_\perp^j}{r_\perp^2} \right) xH(x_g, r_\perp), \quad (15)$$

where $xG(x_g, r_\perp)$ and $xH(x_g, r_\perp)$ are the unpolarized and linearly polarized WW gluon distributions in coordinate space. Combining Eqs. (14) and (15), the cross section becomes

$$\frac{d\sigma_{\gamma^*A \rightarrow q\bar{q}X}}{d\mathcal{P}.S.} = \alpha_{\text{em}} \alpha_s e_q^2 \delta(x_{\gamma^*} - 1) z(1-z) \frac{1}{(P_\perp^2 + \epsilon_f^2)^4} \{ [(z^2 + (1-z)^2)(P_\perp^4 + \epsilon_f^4) + 2m_q^2 P_\perp^2] \widetilde{xG}(x_g, P_\perp, q_\perp) \\ - [2(z^2 + (1-z)^2) P_\perp^2 \epsilon_f^2 - 2m_q^2 P_\perp^2] \cos(2\Delta\phi) \widetilde{xH}(x_g, P_\perp, q_\perp) \}, \quad (16)$$

$$\frac{d\sigma_{\gamma_L^*A \rightarrow q\bar{q}X}}{d\mathcal{P}.S.} = 8\alpha_{\text{em}} \alpha_s e_q^2 Q^2 \delta(x_{\gamma^*} - 1) z^3(1-z)^3 \frac{P_\perp^2}{(P_\perp^2 + \epsilon_f^2)^4} (\widetilde{xG}(x_g, P_\perp, q_\perp) + \cos(2\Delta\phi) \widetilde{xH}(x_g, P_\perp, q_\perp)), \quad (17)$$

where $\Delta\phi$ is the angle between $P_\perp$ and $q_\perp$, $d\mathcal{P}.S. = d^3k_1 d^3k_2$, and

$$\widetilde{xG}(x_g, P_\perp, q_\perp) = \int \frac{d^2r_\perp}{(2\pi)^2} e^{-S_{\text{sud}}(P_\perp, r_\perp)} e^{-iq_\perp \cdot r_\perp} xG(x_g, r_\perp), \quad (18)$$

$$\widetilde{xH}(x_g, P_\perp, q_\perp) = \int \frac{d^2r_\perp}{(2\pi)^2} e^{-S_{\text{sud}}(P_\perp, r_\perp)} e^{-iq_\perp \cdot r_\perp} xH(x_g, r_\perp). \quad (19)$$

$\widetilde{xG}(x_g, P_\perp, q_\perp)$ and $\widetilde{xH}(x_g, P_\perp, q_\perp)$ are the resummed WW gluon distributions in momentum space. From the above expressions, one can see that the two distributions have

resummed small- x logarithms and Sudakov double logarithms simultaneously. We would like to point out that the resummations performed here are similar to those in Refs. [43,44]; alternatively, a more general description of TMD evolutions was given in Ref. [60] in which the two resummations can be performed without choosing a specific scattering process in the CGC formalism. Notice that, for the linearly polarized WW gluon distribution, $\widetilde{xH}(x_g, P_\perp, q_\perp)$, one cannot perform this transformation directly; this is mainly due to the fact that the tensor structure $\frac{1}{2}(\delta^{ij} - 2 \frac{r_\perp^i r_\perp^j}{r_\perp^2})$ is hidden in $xH(x_g, r_\perp)$. In practice, when performing this transformation, a factor of

$-\cos(2\theta)$ will be generated, where θ is the angle between $q_\perp$ and $r_\perp$.

From Eqs. (16) and (17), one can see that the differential cross section can be formulated in terms of $\widetilde{xG}$ and $\widetilde{xH}$. In particular, the coefficients of $\cos(2\Delta\phi)$ terms in the above expressions can provide us with information on linearly polarized WW gluon distributions, $\widetilde{xH}$. If one integrates out ϕ , the coefficients will disappear and the cross section only depends on the unpolarized ones, $\widetilde{xG}$. This indicates that the two gluon distributions can be extracted by measuring the total cross section and the coefficient of $\cos(2\Delta\phi)$, respectively. As argued in Refs. [23,24], since we are working in coordinate space ($\Delta\phi = \phi_u - \phi_v$, with ϕ_u and ϕ_v being the azimuthal angles of $\mathbf{u}$ and $\mathbf{v}$, while $P_\perp$ and $q_\perp$ are the conjugate momenta of $\mathbf{u}$ and $\mathbf{v}$), after the Fourier transformation it is possible to explore the properties of $\widetilde{xH}$ by measuring the asymmetry of $P_\perp$ and $q_\perp$. Furthermore, we note that for transversely and longitudinally polarized photons there exists a sign change for the coefficients of $\cos(2\Delta\phi)$, as shown in Eqs. (16) and (17). In particular, if $Q \approx 0$ (for quasiphoton scattering processes), this process corresponds to dijet production in an ultra-peripheral hadronic collision, and the coefficient in Eq. (16) remains positive; this argument is consistent with a recent measurement by the CMS Collaboration [61].

III. GENERAL PROPERTIES OF WW GLUON DISTRIBUTIONS IN THE MCLERRAN- VENUGOPALAN MODEL

In the last section, we discussed the quark-antiquark pair (or dijet) cross section that can be formulated in terms of $\widetilde{xG}$ and $\widetilde{xH}$, and pointed out that the $\cos(2\Delta\phi)$ coefficients directly relate to the linearly polarized ones. In order to give numerical results of the cross sections, the explicit expressions of $\widetilde{xG}$ and $\widetilde{xH}$ should be specified. In what follows, we do not include the Sudakov factor and apply the original McLerran-Venugopalan (MV) model [54–56] to explore the behaviors of the two gluon distributions. In this model, the color charge density $\rho_a(x^+, \mathbf{x})$ conforms to a Gaussian distribution and satisfies the following relation:

$$\langle \rho_a(x^+, \mathbf{x}) \rho_b(y^+, \mathbf{y}) \rangle_{x_g} = \delta_{ab} \delta^{(2)}(\mathbf{x} - \mathbf{y}) \mu_{x_g}^2(x^+), \quad (20)$$

where $\mu_{x_g}^2$ is the squared density of color charges per area. The above relation only holds when quantum evolution effects are not large, but x_g is sufficiently small such that the high-density regime can be accessed. In this sense, the color charge density $\rho_a(x^+, \mathbf{x})$ is regarded as a static source, while for small- x gluons, due to the fact that the occupation numbers are very large, these gluons are treated as a classical gauge field A_a^- and can be expressed in terms of $g_s \rho$ via the Yang-Mills equation,

$$A_a^-(x^+, \mathbf{x}) = g_s \int d^2z G_0(\mathbf{x} - \mathbf{z}) \rho_a(x^+, \mathbf{z}),$$

$$G_0(\mathbf{x}) = \int \frac{d^2k_\perp}{(2\pi)^2} \frac{e^{ik_\perp \cdot \mathbf{x}}}{k_\perp^2}, \quad (21)$$

where $G_0(\mathbf{x})$ is the gluon propagator. Using Eq. (20), one can obtain the two-point function of the gluon field A_a^- ,

$$\langle A_a^-(x^+, \mathbf{x}) A_b^-(y^+, \mathbf{y}) \rangle_{x_g} = \frac{1}{g_s^2} \delta_{ab} \delta(x^+ - y^+) \mu_{x_g}^2 L_{xy}, \quad (22)$$

with

$$L_{xy} = g_s^4 \int d^2z G_0(\mathbf{x} - \mathbf{z}) G_0(\mathbf{y} - \mathbf{z}) = g_s^4 \int \frac{d^2l_\perp}{(2\pi)^2} \frac{1}{l_\perp^4} e^{il_\perp \cdot (\mathbf{x} - \mathbf{y})}. \quad (23)$$

By introducing the dimensional function

$$\Gamma(\mathbf{x} - \mathbf{y}) \equiv \mu_{x_g}^2 (L_{xx} + L_{yy} - 2L_{xy}), \quad (24)$$

it is straightforward to show that the quark dipole reads

$$S_{q\bar{q}}^{(2)}(\mathbf{x}, \mathbf{y}) \equiv \frac{1}{N_c} \langle \text{Tr} U(\mathbf{x}) U^\dagger(\mathbf{y}) \rangle_{x_g} = e^{-\frac{C_F}{2} \Gamma(\mathbf{x} - \mathbf{y})} \quad (25)$$

and the gluon dipole is

$$S_{gg}^{(2)}(\mathbf{x}, \mathbf{y}) \equiv \frac{1}{N_c^2 - 1} \langle \text{Tr} W(\mathbf{x}) W^\dagger(\mathbf{y}) \rangle_{x_g} = e^{-\frac{N_c}{2} \Gamma(\mathbf{x} - \mathbf{y})}, \quad (26)$$

with

$$\begin{aligned} \Gamma(\mathbf{x} - \mathbf{y}) &= 2g_s^4 \mu_{x_g}^2 \int \frac{d^2l_\perp}{(2\pi)^2} \frac{1}{l_\perp^4} (1 - e^{il_\perp \cdot (\mathbf{x} - \mathbf{y})}) \\ &\approx \alpha_s (g_s \mu_{x_g})^2 (\mathbf{x} - \mathbf{y})^2 \ln \frac{1}{|\mathbf{x} - \mathbf{y}| \Lambda_{\text{QCD}}} \\ &= \frac{1}{2C_F} (\mathbf{x} - \mathbf{y})^2 Q_s^2 \ln \frac{1}{|\mathbf{x} - \mathbf{y}| \Lambda_{\text{QCD}}} \\ &= \frac{1}{2N_c} (\mathbf{x} - \mathbf{y})^2 Q_{sg}^2 \ln \frac{1}{|\mathbf{x} - \mathbf{y}| \Lambda_{\text{QCD}}}, \end{aligned} \quad (27)$$

where $Q_s^2 = 2C_F \alpha_s g_s^2 \mu_{x_g}^2$ and $Q_{sg}^2 = 2N_c \alpha_s g_s^2 \mu_{x_g}^2$ are the quark and gluon saturation scales, respectively. Λ_{QCD} introduced here is the QCD infrared scale that regulates the nonperturbative Coulomb tail at large distances. It should be noted that the logarithmic term in the above expressions is very important and is the crucial difference between the Golec-Biernat–Wüsthoff [62] and MV models. In particular, the logarithmic terms in the MV model restore the correct high- $k_\perp$ perturbative behaviors for the WW gluon distributions, $\sim Q_s^2/k_\perp^2$ (see, for example, Refs. [63,64]). For the remainder of this work, we will use the functional form

$$\Gamma(\mathbf{x} - \mathbf{y}) = \frac{1}{2C_F} (\mathbf{x} - \mathbf{y})^2 Q_s^2 \ln \left(\frac{1}{|\mathbf{x} - \mathbf{y}| \Lambda_{\text{QCD}}} + e \right), \quad (28)$$

where the factor e used here regulates the divergence of the logarithmic term for large distances, $|\mathbf{x} - \mathbf{y}| \geq 1/\Lambda_{\text{QCD}}$.

As mentioned above, although the Gaussian approximation in the MV model [or Eq. (20)] is only valid when the quantum evolution is not too large, the MV model can still be regarded as a good initial condition for quantum (or small- x) evolution. In the dipole picture, it is known that the evolution of the quark dipole $S_{q\bar{q}}^{(2)}$ toward small x is captured by the BK equation; by using this model, the needed initial condition at $x_0 = 0.01$ can be conventionally written as

$$S_{q\bar{q}}^{(2)}(\mathbf{x}, \mathbf{y})|_{x_0=0.01} = \exp \left[-\frac{1}{4} (\mathbf{x} - \mathbf{y})^2 Q_{s0}^2 \ln \left(\frac{1}{|\mathbf{x} - \mathbf{y}| \Lambda_{\text{QCD}}} + e \right) \right]. \quad (29)$$

For the initial condition, there are two free parameters in Eq. (29): one is the infrared scale Λ_{QCD} (which is often chosen to be $\Lambda_{\text{QCD}} \sim 1/R_0$, where R_0 is the radius of a nucleon), and the other is the initial saturation scale Q_{s0}^2 ; in practice, this value can be determined by fitting the structure function F_2 of the proton from HERA data. In the present work, we choose $\Lambda_{\text{QCD}} = 0.241$ GeV and $Q_{s0}^2 = 0.2$ GeV² associated with Eq. (29) as the inputs for the rcBK equation, and employ the same procedure performed in Ref. [48], including the running-coupling kernel (Balitsky's prescription) to obtain the numerical solutions. After this initialization, for small- x gluons ($x \leq x_0$), the dynamics can be obtained by using the numerical solutions. We stress that Q_{s0}^2 is the initial saturation scale of the proton at $x_0 = 0.01$ for a heavy nucleus target, $Q_{s0A}^2 = A^{1/3} Q_{s0}^2$.

In the CGC approach, it is well known that the saturation scale will increase as x_g decreases; in what follows, we use a model-independent definition via the relation [65]

$$S_{q\bar{q}}^{(2)}(x_g, |\mathbf{x} - \mathbf{y}| = \sqrt{2}/Q_s) = e^{-1/2}. \quad (30)$$

In this sense, Q_s^2 can be obtained by using the solutions of the rcBK equation, while, it is only the saturation scale in the fundamental representation, the gluon saturation scale can be obtained by using $Q_{sg}^2 = N_c/C_F Q_s^2$.

Another important ingredient is the double derivative of the quadrupole correlator, which is directly related to the definition of the WW gluon distribution [Eq. (14)] and can be expressed as [7,27]

$$\begin{aligned} & \frac{1}{N_c} \langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g} \\ &= \frac{C_F}{N_c} \frac{1 - e^{-\frac{N_c}{2} \Gamma(\mathbf{v}-\mathbf{v}')}}{\Gamma(\mathbf{v}-\mathbf{v}')} \frac{\partial}{\partial \mathbf{v}^i} \frac{\partial}{\partial \mathbf{v}'^j} \Gamma(\mathbf{v}-\mathbf{v}'), \end{aligned} \quad (31)$$

with

$$\frac{\partial}{\partial \mathbf{v}^i} \frac{\partial}{\partial \mathbf{v}'^j} \Gamma(\mathbf{v}-\mathbf{v}') = -2g_s^4 \mu_{x_g}^2 \int \frac{d^2 l_\perp}{(2\pi)^2} \frac{l_\perp^i l_\perp^j}{l_\perp^4} e^{il_\perp \cdot (\mathbf{v}-\mathbf{v}')}. \quad (32)$$

After that, it is now possible to take the following calculations by using the solutions of rcBK equation, as we show below.

A. Unpolarized and linearly polarized WW gluon distributions in coordinate space

In order to perform a systematic study, we first express the two gluon distributions in coordinate space. By using Eq. (14) and performing the projection onto $\frac{1}{2} \delta^{ij}$ and $\frac{1}{2} (\delta^{ij} - 2 \frac{r_\perp^i r_\perp^j}{r_\perp^2})$, one can directly obtain

$$\begin{aligned} xG(x_g, r_\perp = \mathbf{v} - \mathbf{v}') &= -\frac{2S_\perp}{\alpha_s (2\pi)^2} \delta^{ij} \langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g} \\ &= \frac{2C_F S_\perp}{\alpha_s \pi^2} \frac{1}{(\mathbf{v} - \mathbf{v}')^2} \left[1 - \exp \left(-\frac{1}{4} (\mathbf{v} - \mathbf{v}')^2 Q_{sg}^2 \ln \frac{1}{|\mathbf{v} - \mathbf{v}'| \Lambda_{\text{QCD}}} \right) \right] \end{aligned} \quad (33)$$

and

$$\begin{aligned} xH(x_g, r_\perp = \mathbf{v} - \mathbf{v}') &= -\frac{2S_\perp}{\alpha_s (2\pi)^2} \left[\delta^{ij} - 2 \frac{(\mathbf{v} - \mathbf{v}')^i (\mathbf{v} - \mathbf{v}')^j}{(\mathbf{v} - \mathbf{v}')^2} \right] \langle \text{Tr}[\partial^i U(\mathbf{v})] U^\dagger(\mathbf{v}') [\partial^j U(\mathbf{v}')] U^\dagger(\mathbf{v}) \rangle_{x_g} \\ &= \frac{C_F S_\perp}{\alpha_s \pi^2} \frac{1}{(\mathbf{v} - \mathbf{v}')^2 \ln \frac{1}{|\mathbf{v} - \mathbf{v}'| \Lambda_{\text{QCD}}}} \left[1 - \exp \left(-\frac{1}{4} (\mathbf{v} - \mathbf{v}')^2 Q_{sg}^2 \ln \frac{1}{|\mathbf{v} - \mathbf{v}'| \Lambda_{\text{QCD}}} \right) \right]. \end{aligned} \quad (34)$$

FIG. 1. Unpolarized and linearly polarized WW gluon distributions in coordinate space. Black and red curves denote the unpolarized and linearly polarized WW gluon distributions, while the solid, dashed, and dot-dashed lines represent the results at $x_g = 10^{-2}$, 10^{-3} , and 10^{-4} , respectively. The blue vertical line denotes the position $1/\Lambda_{\text{QCD}}$ and the purple one represents the typical position where the gluons are saturated at $x_g = 10^{-3}$, $r_s = 1/Q_{sg}$. The right panel shows the ratio $xH(x_g, r_\perp)/xG(x_g, r_\perp)$ as a function of $|r_\perp|$ for the three different values of x_g .

In the derivations of these two expressions, we have assumed that the logarithmic terms remain the same as in the MV model. With this assumption, the two expressions have similar functional forms as Eq. (29), and their evolutions toward small x are fully captured by the gluon saturation scale. Alternatively, following the method introduced in Refs. [66,67], one is allowed to express the two gluon distributions in terms of the full dipole amplitude.

To investigate the general properties of unpolarized and linearly polarized WW gluon distributions in coordinate space, we typically distinguish two regions. For the small dipole size limit, $|r_\perp| \ll 1/Q_{sg}$, one can easily check that $xG(x_g, r_\perp) \simeq \frac{C_F S_\perp Q_{sg}^2}{2\alpha_s \pi^2} \ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}$ and $xH(x_g, r_\perp) \simeq \frac{C_F S_\perp Q_{sg}^2}{4\alpha_s \pi^2}$. In the small- $r_\perp$ limit, $xG(x_g, r_\perp)$ behaves as $\ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}$ and $xH(x_g, r_\perp)$ to be a constant at given x_g , especially, in the ultraviolet limit ($|r_\perp| \rightarrow 0$), the unpolarized WW gluon distribution diverges logarithmically, while the linearly polarized one remains finite, the perturbative behaviors are actually in agreement with the properties at large transverse momentum [11]. Moreover, it is interesting to note that the two gluon distributions have a strong dependence on A since $Q_{sg}^2 \sim A^{1/3}$ and $S_\perp \sim R_A^2 \sim A^{2/3}$, which indicates that, in the small- $r_\perp$ limit, the two gluon distributions have no nuclear shadowing effect, and this feature also holds for large transverse momentum. Furthermore, it should be noted that $\frac{xH(x_g, r_\perp)}{xG(x_g, r_\perp)} \simeq \frac{1}{2} \ln^{-1} \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}$, the ratio seems to be zero when $r_\perp$ is small enough even though xH still remains nonzero value. Last but not least, the evolutions of the two gluon

distributions toward small x are completely captured by Q_{sg}^2 . However, for $|r_\perp| \gg 1/Q_{sg}$, but $|r_\perp| \leq 1/\Lambda_{\text{QCD}}$, one can check that $xG(x_g, r_\perp) \simeq \frac{2C_F S_\perp}{\alpha_s \pi^2} \frac{1}{r_\perp^2}$ and $xH(x_g, r_\perp) \simeq \frac{C_F S_\perp}{\alpha_s \pi^2} \frac{1}{r_\perp^2 \ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}}$, the two gluon distributions are independent of x_g and fall off as $r_\perp^2$ with increasing $|r_\perp|$, these behaviors are completely different from that in the small- $r_\perp$ limit. Also, the fact that the ratio $\frac{xH(x_g, r_\perp)}{xG(x_g, r_\perp)} \approx 1/2$ indicates that the linearly polarized WW gluon distribution will tend to its saturated boundary⁴ when $r_\perp$ is large enough.

In order to give numerical results for the unpolarized and linearly polarized WW gluon distributions in coordinate space, according to Eqs. (33) and (34) the gluon saturation scale Q_{sg}^2 should be known, and its value can be obtained by solving the rcBK equation. To avoid repetition, details can be found in Refs. [48,50]. We stress that the key input of the rcBK equation is the initial saturation scale Q_{s0}^2 ; in what follows, we choose $Q_{s0}^2 = 0.2 \text{ GeV}^2$ and present the results for the proton.

We first show the small- x evolutions of the two gluon distributions in coordinate space. In the left panel of Fig. 1, we present the numerical results of the unpolarized and linearly polarized WW gluon distributions at different values of x_g . Black curves denote the unpolarized WW

⁴In practice, the saturated boundary is sensitive to the treatment of the logarithmic term $\ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}$. If one absorbs $\frac{1}{2}$ into the logarithmic term, the saturated boundary tends to 1.

gluon distributions and the red ones denote the linearly polarized distributions. Solid, dashed, and dot-dashed lines correspond to $x_g = 10^{-2}$, 10^{-3} , and 10^{-4} , respectively. From this figure, one can clearly see that in the small- $r_\perp$ region the unpolarized WW gluon distributions increase mildly with decreasing $r_\perp$, precisely behaves as $\ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}$. While for the linearly polarized ones, the results tend to be flat and remain nonzero values over a broad small- $r_\perp$ region. Besides that, one can observe that the two gluon distributions increase with decreasing x_g , this trend is expected since $xG, xH \sim Q_{sg}^2$ and the gluon saturation scale Q_{sg}^2 goes large when x_g becomes small. As in the large- $r_\perp$ region, e.g., $|r_\perp| \geq 1 \text{ GeV}^{-1}$, one can observe that the unpolarized (linearly polarized) WW gluon distributions for different values of x_g merge into a single curve; this is mainly due to the fact that the two gluon distributions are independent of x_g when $r_\perp$ is large enough. In particular, in the large- $r_\perp$ region, one can see that both two gluon distributions have the same behavior, precisely equal to $r_\perp^{-2}$.

In the right panel of Fig. 1, we present the ratio $xH(x_g, r_\perp)/xG(x_g, r_\perp)$ as a function of $r_\perp$. From this figure, one can observe that the numerical results for the three different values of x_g merge into a single curve; this is mainly due to the fact that the ratio becomes a function of $r_\perp$ by comparing Eqs. (33) and (34). Another feature is that the ratio increases for increasing $r_\perp$ and tends to the saturated boundary $1/2$ when $r_\perp$ is large enough, which indicates that $xH(x_g, r_\perp) \leq \frac{1}{2}xG(x_g, r_\perp)$. However, for physical aspect, $xH(x_g, r_\perp) \leq xG(x_g, r_\perp)$, this discrepancy mainly comes from the treatment of the logarithmic term in Eq. (34), $\ln^{-1} \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}$. In our calculations, we have $\ln^{-1} \frac{1}{|r_\perp| \Lambda_{\text{QCD}}} \rightarrow \ln^{-1} (\frac{1}{|r_\perp| \Lambda_{\text{QCD}}} + e)$, if one uses $\ln^{-1} \frac{1}{|r_\perp|^2 \Lambda_{\text{QCD}}^2} \rightarrow \ln^{-1} (\frac{1}{|r_\perp|^2 \Lambda_{\text{QCD}}^2} + e)$, which will give the saturated boundary to be 1. We stress that the different treatments do not alter the following analysis.

To better understand the attractive behaviors of the two gluon distributions in the different $r_\perp$ regions, we take the case of $x_g = 10^{-3}$ as an example, as shown by the dashed curves in the left panel of Fig. 1, for the given x_g , gluon saturation scale Q_{sg}^2 can be obtained by using the condition of Eq. (30), accordingly, the typical saturation length is $r_s = 1/Q_{sg}$, as depicted by the purple vertical line in this figure. These results presented here only denote the probability of the gluons being measured in coordinate space, or in other words, the two gluon distributions can be understood as the using of an external probe with transverse size $r_\perp$ to measure these gluons. Recalling that only those gluons with a transverse size larger than the transverse resolution size $r_\perp$ of the probe can be measured, gluons with a smaller transverse size ($< |r_\perp|$) cannot be seen by the probe and hence do not give contributions to the distributions. Following this manner, (i): in small $|r_\perp|$

region, i.e., $|r_\perp| \leq r_s$, when $|r_\perp|$ goes small, this equivalents to increasing the transverse resolution of the probe, $Q^2 \sim 1/r_\perp^2$, the high resolution probe can, indeed, measure much more small size gluons that leading the distribution goes large toward to small $r_\perp$. On the other hand, since the transverse sizes of measured gluons are smaller than the typical saturation length r_s , therefore, the measured gluons are locally distributed in the saturated regime and are strongly affected by gluon rescattering, in this sense, such the effects make the unpolarized WW gluon distribution xG increases logarithmically as $r_\perp$ decreases (or increasing the transverse resolution Q^2), as shown by the black dashed line in the left panel of Fig. 1. This argument is somewhat consistent with the description of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equations [68–70]. In the saturated regime, it is useful to recall that the measured gluons are in an equilibrium state of gluon radiation and recombination. These gluons are initially radiated from color sources and have polarizations, while the effects of recombination will constrain them to be polarized; as a consequence, their polarizations are balanced by the two effects. Therefore, in the small- $r_\perp$ region, the linearly polarized WW gluon distribution xH remains finite and constant, as shown by the red dashed line in the left panel of Fig. 1. In addition, although the linearly polarized WW gluon distribution xH remains constant, the unpolarized one xG increases logarithmically toward small $r_\perp$; in this sense, the ratio of xH/xG tends to be smaller, as clearly seen in the right panel of Fig. 1. (ii): When $r_s \leq |r_\perp| \leq 1/\Lambda_{\text{QCD}}$, since the transverse resolution size $|r_\perp|$ of the probe is larger than the saturation length r_s , although the density is very large, only few gluons out of the saturated domain can be measured, as shown by the dashed lines in the left panel of Fig. 1, both of unpolarized xG and linearly polarized WW gluon distributions are falling off rapidly toward large $r_\perp$. In this regime, due to the effect of gluon recombinations becomes weak, therefore, the ratio of xH/xG goes large as $r_\perp$ becomes large. In particular, one can expect that the linearly polarized part will tend to its saturated boundary if no gluon recombination effect. (iii): When $|r_\perp| \geq 1/\Lambda_{\text{QCD}}$, i.e., the transverse resolution size is larger than $1/\Lambda_{\text{QCD}}$. In the MV model, the color sources must vanish when the measured size is larger than the size of the nucleon $R_0 \sim 1/\Lambda_{\text{QCD}}$, which is also the requirement of the local Gaussian assumption in the MV model. Consequently, as shown by the dashed lines in the left panel of Fig. 1, unpolarized and linearly polarized WW gluon distributions drop quickly as $r_\perp$ increases. In addition, as argued in previous, although the density is very low, the polarizations are still held and eventually lead the ratio of xH/xG to be maximum, as shown in right panel of Fig. 1.

B. Unpolarized and linearly polarized WW gluon distributions in momentum space

Correspondingly, the unpolarized and linearly polarized WW gluon distributions can be obtained by performing a Fourier transformation, and they are expressed as follows:

$$\begin{aligned}
xG(x_g, k_\perp) &= \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} xG(x_g, r_\perp) \\
&= \frac{2C_F S_\perp}{\alpha_s \pi^2} \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} \frac{1}{r_\perp^2} \left[1 - \exp \left(-\frac{1}{4} r_\perp^2 Q_{sg}^2 \ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}} \right) \right]
\end{aligned} \tag{35}$$

and

$$\begin{aligned}
xH(x_g, k_\perp) &= \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} xH(x_g, r_\perp) \\
&= \frac{C_F S_\perp}{\alpha_s \pi^2} \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} \frac{-\cos(2\theta)}{r_\perp^2 \ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}} \left[1 - \exp \left(-\frac{1}{4} r_\perp^2 Q_{sg}^2 \ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}} \right) \right] \\
&= \frac{C_F S_\perp}{2\alpha_s \pi^3} \int d|r_\perp| \frac{J_2(|k_\perp| |r_\perp|)}{|r_\perp| \ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}}} \left[1 - \exp \left(-\frac{1}{4} r_\perp^2 Q_{sg}^2 \ln \frac{1}{|r_\perp| \Lambda_{\text{QCD}}} \right) \right].
\end{aligned} \tag{36}$$

It is worth noting that the linearly polarized WW gluon distribution cannot be transformed directly from coordinate space to momentum space, as the tensor structure $\frac{1}{2}(\delta^{ij} - 2\frac{r_\perp^i r_\perp^j}{r_\perp^2})$ is different from δ^{ij} . After performing this projection, the tensor structure will naturally generate a $-\cos(2\theta)$ factor, where θ is the relative angle between $r_\perp$ and $k_\perp$. Alternatively, $xG(x_g, k_\perp)$ and $xH(x_g, k_\perp)$ can be obtained by projecting xW^{ij} onto $\frac{1}{2}\delta^{ij}$ and $\frac{1}{2}(2\frac{k_\perp^i k_\perp^j}{k_\perp^2} - \delta^{ij})$, respectively, which was done in Refs. [23,24].

In the above expression, $J_2(|k_\perp| |r_\perp|)$ is the Bessel function of the first kind. The following analysis is similar to that for gluon distributions in coordinate space. For $k_\perp^2 \gg Q_{sg}^2$, the dominated contributions comes from the small dipole size $|r_\perp| \ll 1/Q_{sg}$, and can be evaluated by expanding the exponential term, then $xG(x_g, k_\perp) \simeq \frac{C_F S_\perp}{4\alpha_s \pi^3} \frac{Q_{sg}^2}{k_\perp^2}$ and $xH(x_g, k_\perp) \simeq \frac{C_F S_\perp}{4\alpha_s \pi^3} \frac{Q_{sg}^2}{k_\perp^2}$, in this limit, the two gluon distributions are completely same and behave as $\frac{Q_{sg}^2}{k_\perp^2}$. The perturbative property is agreement with other QCD results [63,64,71,72]. Moreover, due to $xG(x_g, k_\perp) \simeq xH(x_g, k_\perp) \simeq \frac{C_F N_c A}{2\pi^3 k_\perp^2}$, the two gluon distributions scale like A and have no nuclear shadowing effects; therefore, each nucleon can additively give a contribution to the distributions in the dilute region. This conclusion is also in agreement with that in coordinate space in the small- $r_\perp$ limit. In addition, $\frac{xH(x_g, k_\perp)}{xG(x_g, k_\perp)} \approx 1$ when $k_\perp$ is large enough, which gives a saturated boundary for the linearly polarized WW gluon distribution in the dilute regime [23]. For $\Lambda_{\text{QCD}}^2 \leq k_\perp^2 \ll Q_{sg}^2$, the dominant contribution comes from large distances $r_\perp \gg 1/Q_{sg}$, one can neglect the exponential and the logarithmic $r_\perp$ terms, and arrive at $xG(x_g, k_\perp) \simeq \frac{C_F S_\perp}{2\pi^3 \alpha_s} \ln \frac{Q_{sg}^2}{k_\perp^2}$ and $xH(x_g, k_\perp) \simeq \frac{N_c C_F A}{2\pi^3} \frac{1}{Q_{sg}^2} \ln \frac{Q_{sg}^2}{k_\perp^2}$,

one can check that the two gluon distributions scale like $A^{2/3}$, which means the two gluon distributions have strong nuclear shadowing effects in the dense regime. Another interesting feature is that the linearly polarized WW gluon distribution is weakly dependent on $k_\perp$ and approaches a constant when $k_\perp$ is of the order of the saturation scale Q_{sg} . Furthermore, one can find that $\frac{xH(x_g, k_\perp)}{xG(x_g, k_\perp)}$ is suppressed in the small- $k_\perp$ limit, in which the multigluon scattering effects play a role that constrains the gluons to be polarized.

In Fig. 2 we present the numerical results of the two gluon distributions in momentum space. In the left panel of Fig. 2, the black and red curves denote the unpolarized and linearly polarized WW gluon distributions, while the solid, dashed, and dot-dashed lines correspond to the results at $x_g = 10^{-2}, 10^{-3}$, and 10^{-4} , respectively. As shown in the left panel of Fig. 2, for each value of x_g , one can clearly see that the results for unpolarized and linearly polarized WW gluon distributions approach a single line when $k_\perp$ is very large, which implies that the linearly polarized WW gluon distribution tends to its saturated boundary, $xH(x_g, k_\perp) \approx xG(x_g, k_\perp)$. Another important feature is that both distributions have the same asymptotic behavior, i.e., an inverse power law, precisely equal to $k_\perp^{-2}$ when $k_\perp$ lies in the large transverse momentum region. Furthermore, with decreasing x_g , one can observe that the curves are shifted to large $k_\perp$; this is because, in the large- $k_\perp$ region, the evolutions of the two gluon distributions toward small x are fully characterized by the gluon saturation scale, Q_{sg}^2 , and which is the value becomes large when x_g decreases. As $k_\perp$ decreases, the two gluon distributions start to separate. Specifically, the unpolarized gluon distribution will increase wildly with decreasing $k_\perp$, while the linearly polarized ones tend to flat over a broad range of $k_\perp$ and are almost independent of x_g , this feature depicted here again confirms what we discussed in the above.

FIG. 2. Unpolarized and linearly polarized WW gluon distributions in momentum space. Black and red curves denote the unpolarized and linearly polarized WW gluon distributions, while the solid, dashed, and dot-dashed lines correspond to the results at $x_g = 10^{-2}$, 10^{-3} , and 10^{-4} , respectively. The right panel shows the ratio $xH(x_g, k_\perp)/xG(x_g, k_\perp)$ as a function of $|k_\perp|/Q_{sg}$ for the three different values of x_g .

In the right panel of Fig. 2, we present the ratio $xH(x_g, k_\perp)/xG(x_g, k_\perp)$ as a function of $|k_\perp|/Q_{sg}$ for three different values of x_g . The solid, dashed, and dot-dashed lines correspond to the results at $x_g = 10^{-2}$, 10^{-3} , and 10^{-4} , respectively. From this figure, one can see that these quantities are weakly dependent on x_g and approach 1 when $|k_\perp|/Q_{sg} > 4 \div 5$, this strongly indicates the geometric scaling behavior [73,74] is held for the two gluon

distributions and the saturated boundary of linearly polarized WW gluon distribution is reached at large $k_\perp$ region. When $k_\perp$ goes small, such as of order of Q_{sg} , in which gluon multiple rescattering (saturation) effects to be manifest and play a role that suppressing the gluons to be polarized. Therefore, as shown in this figure, the effects will naturally cause the ratio to be smaller. We note that, although the ratio goes small when decreasing $k_\perp$, it is still sizable even for

FIG. 3. Small- x evolutions for unpolarized and linearly polarized WW gluon distributions. Black and red curves denote the unpolarized and linearly polarized ones, the left and right panels correspond to the results for $|k_\perp| = 1, 2$ GeV, and the vertical dot-dashed lines represent the values of x_g with the condition that $Q_{sg} = |k_\perp|$.

small $k_\perp$ value, $xH(x_g, k_\perp)/xG(x_g, k_\perp) \geq 10\%$. In this sense, the dijet $\cos(2\Delta\phi)$ structure⁵ in the considered process is expected to be visible even in the small transverse momentum region.

For completeness, in Fig. 3 we also present small- x evolutions for the two gluon distributions. In the calculations, we have chosen two values of $k_\perp$: results presented in the left panel of Fig. 3 correspond to the case with $|k_\perp| = 1$ GeV, and the right panel is for $|k_\perp| = 2$ GeV. The solid (black) and dashed (red) curves denote unpolarized and linearly polarized WW gluon distributions, and the dot-dashed (blue) vertical lines represent the values of x_g by setting $Q_{sg} = |k_\perp|$. In the left panel of Fig. 3, one can see that the vertical line separates the domain into two regions, namely, the dilute regime in which $x_g \geq 0.003$ with $|k_\perp| \geq Q_{sg}$, and the dense (saturated) regime in which $x_g \leq 0.003$ with $|k_\perp| \leq Q_{sg}$. As discussed in the above, we recall that the gluon saturation effects will be very significant in the dense regime ($x_g \leq 0.003$), where gluon rescattering effects will be very important and constrain those gluons to be polarized, therefore, the linearly polarized part becomes flat as shown in the figure. While, in the dilute regime ($x_g \geq 0.003$), the effect is less manifest and makes the linearly polarized part close to the unpolarized one. This feature is more visible for the case with $|k_\perp| = 2$ GeV (see the right panel of Fig. 3).

IV. NUMERICAL RESULTS AND DISCUSSIONS

According to Eqs. (16) and (17), the quark-antiquark pair (or dijet) cross section is directly related to unpolarized and linearly polarized WW gluon distributions, as mentioned in previous, the two gluon distributions associated with their evolutions can be obtained by using the solutions of the rcBK equation. In this section, we mainly focus on quark-antiquark (or dijet) production in the $\gamma^*p(A)$ scattering processes, and show that the cross sections and their azimuthal correlations are strongly affected by parton shower and gluon saturation effects. In order to give the azimuthal correlations, we define the azimuthal correlation function in the following form:

$$C(\Delta\phi) = \frac{d\sigma^{\gamma^*p \rightarrow q\bar{q}X}}{d\mathcal{P.S.}} / \int d\Delta\phi \frac{d\sigma^{\gamma^*p \rightarrow q\bar{q}X}}{d\mathcal{P.S.}}, \quad (37)$$

where $\Delta\phi$ is the difference in azimuthal angle between $P_\perp$ and $q_\perp$. This definition has a clear physical interpretation that quantifying the probability distribution of the dijet versus the relative angle $\Delta\phi$.

To perform numerical calculations, we focus on typical EIC kinematics and take the center-of-mass energy of the scattering system to be $W = 140$ GeV. When the

kinematics are specified, x_g enters Eqs. (16) and (17) and can be determined via

$$x_g = \frac{1}{W^2 + Q^2 - M_p^2} \left(Q^2 + q_\perp^2 + \frac{P_\perp^2}{z(1-z)} \right), \quad (38)$$

where M_p is the mass of the scattering proton. When performing calculations, we also restrict $|P_\perp| \geq 2|q_\perp|$ in order to meet the back-to-back correlation limit. We stress that the chosen kinematics in the following is also relevant for probing gluon saturation effects of the scattered target, as we will demonstrate below.

In addition, according to Eq. (38), x_g increases with increasing $P_\perp$, while it has a weak dependence on $q_\perp$, which indicates that $d\sigma/d|P_\perp|$ can be used as a proxy to measure the evolutions of $xG(x_g, q_\perp)$ and $xH(x_g, q_\perp)$ toward x_g , while $d\sigma/d|q_\perp|$ can be used as a direct probe to measure the transverse momentum of gluons inside the scattered target.

A. Azimuthal correlations of dijets

In Fig. 4 we show the differential cross sections for longitudinally and transversely polarized photons as functions of transverse momentum $|P_\perp|$, in which we have set the transverse momentum imbalance $|q_\perp| = 0.5$ GeV and restrict the dijet to be purely back to back, $\Delta\phi = \pi$. For the phenomenological study, we also vary photon's virtualities Q to be the order of transverse momentum $|P_\perp|$, specifically, the left panel in Fig. 4 shows the results for the case of $Q = 2|P_\perp|$ and the right panel corresponds to the case of $Q = |P_\perp|$. It should also be noted that the presented results are for light flavor quark jets with quark mass $m_q = 0.01$ GeV and symmetric longitudinal jet $z = 1/2$. As can be seen from this figure, the cross sections rapidly decrease with increasing dijet transverse momentum $|P_\perp|$. For different virtualities Q , the contributions of longitudinally polarized photons and transversely polarized photons to the total cross sections are different. When $Q = 2|P_\perp|$ (see left panel of Fig. 4), one can observe that the longitudinal part is larger than the transverse part; in this case, the total cross section is mainly driven by the longitudinal part. On the other hand, when $Q = |P_\perp|$ (see right panel of Fig. 4), although the longitudinal part is still larger than the transverse part, the two parts seem to be comparable in the total cross section. By comparing the two figures, one can conclude that the contribution of the transverse part to the total cross section will increase with decreasing the photon's virtualities, Q^2 .

Similar to Fig. 4, in Fig. 5 we show the differential cross sections as functions of transverse momentum imbalance $|q_\perp|$, in which we have set the transverse momentum $|P_\perp| = 4$ GeV and restrict the dijet to be purely back to back, $\Delta\phi = \pi$, while the other variables are the same as in Fig. 4. From this figure, one can observe that the

⁵From Eqs. (16) and (17), one can check that the $\cos(2\Delta\phi)$ structure is sensitive to the ratio $xH(x_g, k_\perp)/xG(x_g, k_\perp)$.

FIG. 4. Total (black solid), longitudinal (red dashed), and transverse (blue dot-dashed) cross sections for light flavored quark dijets as a function of $|P_\perp|$, where $|q_\perp| = 0.5$ GeV and $z = 1/2$. The left panel corresponds to the case with $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$.

contributions of transversely and longitudinally polarized photons to the total cross sections are quite different compared to those in Fig. 4. Specifically, for the case of $Q = 2|P_\perp|$ (left panel of Fig. 5), one can observe that the distribution of the transverse part decreases much more sharply than the longitudinal part as $q_\perp$ increases; therefore, the total cross section is almost driven by the longitudinally polarized photon when $q_\perp$ is large enough. Actually, the peculiar behaviors exhibited by transversely and longitudinally photons in this figure are not surprising and can be understood as follows. Before to clarify that, it is

useful to remind that the transverse momentum imbalance $q_\perp$ is the only variable in the calculations, and as the quantity that directly probes the transverse momentum of incoming gluons from the scattered target. According to Eqs. (16) and (17), if only the unpolarized WW gluon distribution $xG(x_g, q_\perp)$ is considered or integrating over $\Delta\phi$, one can expect that the two cross sections are parallelly falling with increasing $q_\perp$. Moreover, it should be noted that the dijets are restricted to be purely back to back ($\Delta\phi = \pi$); in this sense, the $\cos(2\Delta\phi)$ terms will give a negative (positive) contribution for transversely [Eq. (16)]

FIG. 5. Total (black solid), longitudinal (red dashed), and transverse (blue dot-dashed) cross sections for light flavor quark jets as a function of $|q_\perp|$, where $|P_\perp| = 4$ GeV and $z = 1/2$. The left panel corresponds to the case with $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$. The vertical lines denote the values of gluon saturation scales at the specific kinematics.

FIG. 6. Normalized azimuthal correlations $C(\Delta\phi)$ as functions of $\Delta\phi$, in which $W_{\gamma^*p} = 140$ GeV, $|P_\perp| = 2$ GeV, and $z = 1/2$. Results in the left panel correspond to the case of different photon virtualities Q and the right panel is for different $q_\perp$.

or longitudinally [Eq. (17)] polarized photons, and therefore the two cross sections will naturally be separated and the separation between the two is directly related to the quantity $xH(x_g, q_\perp)$ or the ratio $xH(x_g, q_\perp)/xG(x_g, q_\perp)$. Furthermore, in the specific kinematics, since x_g is weakly dependent on $q_\perp$ and $x_g \approx 0.0065$, correspondingly, the gluon saturation scale $Q_{sg} \approx 0.88$ GeV as shown by the vertical line in the left panel of Fig. 5. We recall that the ratio $xH(x_g, q_\perp)/xG(x_g, q_\perp)$ increases when $q_\perp$ increases (see also the right panel of Fig. 2). As a consequence, the separation between the two cross sections will become more significant as $|q_\perp| \geq Q_{sg}$, which is shown in the left panel of Fig. 5. Similar trends hold for the case with $Q = |P_\perp|$ (right panel of Fig. 5) as well.

As mentioned previously, the longitudinally polarized photons [Eq. (17)] will give a $\cos(2\Delta\phi)$ structure in azimuthal correlations, in contrast, the transversely polarized photons [Eq. (16)] gives a $-\cos(2\Delta\phi)$ structure,⁶ and this part plays a role that partially washing out the $\cos(2\Delta\phi)$ structure in azimuthal correlations. Therefore, one can expect that the $\cos(2\Delta\phi)$ structure will become more visible if the total cross section is dominated by longitudinally polarized photons; otherwise, if the total cross section is mainly driven by the transverse part, it is a challenge to observe the $\cos(2\Delta\phi)$ structure.

In Fig. 6 we plot the normalized azimuthal correlations $C(\Delta\phi)$ defined in Eq. (37) as functions of the relative angle $\Delta\phi$. In the left panel of Fig. 6, the represented results are for different photon virtualities Q when setting $|P_\perp| = 2$ GeV and $|q_\perp| = 0.5$ GeV, while the other kinematics are same as in Fig. 4. The solid (black), dashed (red), and dot-dashed

(blue) lines correspond to the cases of $Q = 2|P_\perp|$, $|P_\perp|$, and 1 GeV, respectively. As can be seen from this figure, the $\cos(2\Delta\phi)$ structure in azimuthal correlations will be suppressed and broadens with decreasing virtualities Q ; this is mainly due to the fact that the contribution of longitudinally polarized photons to the total cross section will decrease as Q decreases. In the right panel of Fig. 6, we show the azimuthal correlations for different $q_\perp$, where in the calculations we set $Q = 2|P_\perp|$ with $|P_\perp| = 2$ GeV, while the other kinematics are same as before. The solid (black) and dashed (red) lines correspond to $|q_\perp| = 0.5$ and 1 GeV, respectively. As discussed previously, the $\cos(2\Delta\phi)$ structure in azimuthal correlation is sensitive to the ratio $xH(x_g, q_\perp)/xG(x_g, q_\perp)$, and the ratio will increase as $q_\perp$ increases; therefore, one can observe that the $\cos(2\Delta\phi)$ structure will become more visible for $|q_\perp| = 1$ GeV compared to that of $|q_\perp| = 0.5$ GeV.

Furthermore, we have also checked that for the heavy flavor jets, where the heavy quark mass $m_q = 1.4$ GeV, the results are very similar to those for the light quark jets. For this reason, we do not include separate plots.

B. Sudakov factor suppresses the azimuthal correlations

In order to further investigate how the Sudakov resummations impact the cross sections, we perform calculations using the same kinematics as in Fig. 4, and the results are shown in Fig. 7. Similarly, the left and right panels in Fig. 7 correspond to the cases of $Q = 2|P_\perp|$ and $Q = |P_\perp|$, and the black and red curves denote without and with Sudakov resummations, respectively. By comparing the black and red curves depicted in Fig. 7, one can observe that the behavior of the cross sections is not changed after including the Sudakov resummations, and the cross sections are

⁶If the quark mass is considered, only $Q^2 \geq \frac{2m_q^2}{z^2+(1-z)^2}$ will give a negative contribution.

FIG. 7. Total (solid), longitudinal (dashed), and transverse (dot-dashed) cross sections for dijets as functions of $|P_\perp|$, where $|q_\perp| = 0.5$ GeV and $z = 1/2$. The black and red curves denote without and with Sudakov resummations, left panel corresponds to the case of $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$.

tamely falling with increasing $|P_\perp|$. However, a difference between the two (black and red curves) is that the Sudakov resummations will suppress the cross sections, and the suppression tends to be significant when $|P_\perp|$ increases, as revealed by the magnitude in the figure. This naturally reflects the fact that the parton shower effect is relatively weaker in the low- $|P_\perp|$ region, while the effect becomes stronger in the large- $|P_\perp|$ region. This argument is consistent with other perturbative calculations; see, e.g., Ref. [41]. Another interesting feature seen by comparing the black and red curves, such as for the case with $Q = 2|P_\perp|$ (left panel of Fig. 7), is that the relative difference between transverse and longitudinal parts decreases when the Sudakov resummations are taken into account. More precisely, for the specific kinematics, the ratio of the transverse component in the total cross section $\sigma_T/\sigma_{\text{Total}} \approx 0.32$ when including the Sudakov resummations, while $\sigma_T/\sigma_{\text{Total}} \approx 0.25$ if the Sudakov resummations are not included. As discussed previously, transversely polarized photons give a contribution that partially washing out the $\cos(2\Delta\phi)$ structure in azimuthal correlations, therefore, one can straightforwardly expect that the $\cos(2\Delta\phi)$ structure will be suppressed and broaden if the Sudakov resummations are taken into account. We further notice that, after including Sudakov resummations, the transverse part seems to be parallelly falling with increasing $P_\perp$ respect to the longitudinal part, in this sense, the $\cos(2\Delta\phi)$ structure in azimuthal correlations will not be changed for different values of $P_\perp$. Similar features hold for the case with $Q = |P_\perp|$ (see right panel of Fig. 7).

In Fig. 8 we show the cross sections as functions of the transverse momentum imbalance $|q_\perp|$, while the other kinematics are the same as in Fig. 5. Similarly, the black

and red curves denote without and with Sudakov resummations, the left and right panels correspond to the case of $Q = 2|P_\perp|$ and $Q = |P_\perp|$, respectively. As shown in this figure, the red curves are significantly suppressed compared to the black one in the small- $q_\perp$ region, while with increasing $q_\perp$ one can observe that this suppression decreases.⁷ This actually reflects the fact that Sudakov effects will reduce the distribution in the small- $q_\perp$ region and make the distributions to be flattened over a broad $q_\perp$ region. We notice that the distributions presented here are somewhat consistent with the recent study in Ref. [75] in which the authors pointed out the Sudakov resummations are very important in describing the dijet transverse momentum imbalance distributions at E791 [76,77], and shown only the Sudakov effects is considered that can make the theoretical results meet to the experimental data. Another important feature seen in the figure is that the difference between the transverse and longitudinal cross sections will become smaller if the Sudakov resummations are considered; therefore, the $\cos(2\Delta\phi)$ structure in azimuthal correlations is expected to be suppressed and broaden.

In Fig. 9 we show the normalized azimuthal correlations $C(\Delta\phi)$ as functions of $\Delta\phi$. In the left panel of Fig. 9, we present the results for different $P_\perp$, in which we have set $Q = 2|P_\perp|$, $|q_\perp| = 0.5$ GeV, and $z = 1/2$. The solid,

⁷For the transverse part, although the distribution is suppressed for small $q_\perp$, one can observe that the distribution with Sudakov resummations is still larger than that without Sudakov resummations. One should not be confused, as this is because we have set the dijet to be purely back to back in the calculations, and the result for the transverse part only quantifies the difference between xG and xH [see Eq. (16)].

FIG. 8. Total (solid), longitudinal (dashed), and transverse (dot-dashed) cross sections for the dijet as functions of $|q_\perp|$, where $|P_\perp| = 4$ GeV and $z = 1/2$. The black and red curves denote without and with Sudakov resummations, left panel corresponds to the case of $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$.

dashed, and dot-dashed lines correspond to $|P_\perp| = 4, 2$, and 1 GeV, and the red and black curves denote with and without Sudakov resummations, respectively. From this figure, one can observe that the back-to-back ($\Delta\phi = \pi$) correlations are indeed suppressed when we include Sudakov resummations; in particular, this suppression becomes stronger when $P_\perp$ increases. Moreover, in the specific kinematics, one can observe that the red curves completely overlap, which indicates that the $\cos(2\Delta\phi)$ structure in azimuthal correlations will be unchanged after including Sudakov resummations. This feature can be

traced back to the fact that the transverse cross sections will be parallelly falling with respect to the longitudinal ones as $P_\perp$ increases (see, for example, Fig. 7).

As mentioned previously, the effect of Sudakov resummations is strongly sensitive to the difference between $|P_\perp|$ and $|q_\perp|$; when Q and $|P_\perp|$ are fixed, one may expect that the effect will decrease with increasing $|q_\perp|$. In order to verify this, in the right panel of Fig. 9 we present results for $Q = 4$ GeV and $|P_\perp| = 2$ GeV, where the solid and dashed curves correspond to $|q_\perp| = 0.5$ and 1 GeV, and the black and red curves denote without and with Sudakov

FIG. 9. Normalized azimuthal correlations $C(\Delta\phi)$ as functions of $\Delta\phi$. Left panel is for difference values of $P_\perp$, and the right panel is for different $q_\perp$.

resummations, respectively. From this figure, besides the fact that the Sudakov resummations suppress the back-to-back correlations, one can observe that the suppression for $|q_\perp| = 0.5$ GeV is stronger than that of $|q_\perp| = 1$ GeV (by comparing the differences between the away-side peaks for the two cases.). Furthermore, it is worth noting that, after we include Sudakov resummations, the away-side ($\Delta\phi = \pi$) peak for $|q_\perp| = 1$ GeV is still larger than that of $|q_\perp| = 0.5$ GeV; the main reason for this comes from the fact that the $\cos(2\Delta\phi)$ structure is sensitive to $xH(x_g, q_\perp)/xG(x_g, q_\perp)$ and the ratio becomes large as $q_\perp$ increases.

C. Gluon saturation further suppresses the azimuthal correlations of the dijets

As mentioned previously, the $\cos(2\Delta\phi)$ terms in the cross sections directly quantify the values of linearly polarized WW gluon distributions [Eqs. (16) and (17)], and this distribution is strongly suppressed when the gluon saturation dominates.

To further demonstrate that the gluon saturation effects will actually suppress the back-to-back correlations, we perform a calculation to compare the dijet azimuthal correlations in γ^*p and γ^*A scattering systems with the same kinematics, in which the only difference is the initial saturation scale, for γ^*p scattering process, the initial saturation scale for proton $Q_{s0p}^2 = 0.2$ GeV², for γ^*A , we choose $Q_{s0A}^2 = 1.2$ GeV² due to $Q_{sA}^2 \sim A^{1/3} Q_{sp}^2$ with $A \approx 200$. Under this assumption, one can expect that the back-to-back correlation in γ^*A scattering process will be suppressed as compared to that in γ^*p if gluon saturation effects is the actual reason that smearing the correlations.

In Fig. 10 we present the normalized azimuthal correlations for different scattering systems. The black and red curves denote the γ^*p and γ^*A scattering processes, and the solid and dashed lines correspond to without and with Sudakov resummations. The left panel is for $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$, in which we have set $|P_\perp| = 2$ GeV, $|q_\perp| = 0.5$, and $z = 1/2$. From this figure, by comparing the solid lines with the dashed lines, one observes that the back-to-back ($\Delta\phi = \pi$) correlations are suppressed for each colliding system, as discussed in previous, this suppression mainly comes from Sudakov resummations. It should be noted that the chosen kinematics for γ^*p and γ^*A systems are the same in the calculations, and therefore the effects of Sudakov resummations should be the same in the two scattering systems. Furthermore, by comparing the red and black curves, one can see that the dijets back-to-back ($\Delta\phi = \pi$) correlations in γ^*A are significantly suppressed compared to those in γ^*p . This suppression is mainly due to the gluon saturation effects for heavy ion A is more predominant than the proton p . To demonstrate this, it is useful to recall that the enhancement of back-to-back correlations is proportional to the ratio $|q_\perp|/Q_{sg}$, and with the fact of gluon saturation scale of A is larger than that of proton ($Q_{sA}^2 = A^{1/3} Q_{sp}^2$), therefore, when the kinematics are fixed, the values of $|q_\perp|/Q_{sg}$ in γ^*A scattering system is smaller than that in γ^*p , as a consequence, the back-to-back correlations in γ^*A is naturally suppressed as compared to that in γ^*p scattering process.

In addition, we also investigate the $\cos(2\Delta\phi)$ anisotropy in the two scattering systems, which is the quantity that can be directly measured in experiments and is defined as follows:

FIG. 10. Normalized azimuthal correlation function $C(\Delta\phi)$ for different scattering systems, in which $|P_\perp| = 2$ GeV, $|q_\perp| = 0.5$ GeV, and $z = 1/2$. The black and red curves correspond to γ^*p and γ^*A , and the solid and dashed curves denote without and with Sudakov resummations, respectively. The left panel corresponds to the case with $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$.

FIG. 11. Elliptic anisotropy v_2 as a function of $q_\perp$ for different scattering systems, in which $|P_\perp| = 4$ GeV and $z = 1/2$. The black and red curves correspond to γ^*p and γ^*A , and the solid and dashed curves denote without and with Sudakov resummations, respectively. The left panel corresponds to the case with $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$.

$$v_2 \equiv \frac{\int d\Delta\phi \cos(2\Delta\phi) d\sigma}{\int d\Delta\phi d\sigma}. \quad (39)$$

Clearly, the numerator $\int d\Delta\phi \cos(2\Delta\phi) d\sigma \sim xH(x_g, q_\perp)$, while the denominator $\int d\Delta\phi d\sigma \sim xG(x_g, q_\perp)$; hence, $v_2 \sim xH(x_g, q_\perp)/xG(x_g, q_\perp)$. Therefore, by measuring the elliptic anisotropy v_2 of the dijets, we can get information about xH/xG in the scattered target.

In Fig. 11 we present results for the elliptic anisotropy v_2 as a function of the transverse momentum imbalance $q_\perp$ for different scattering systems, in which we have set $|P_\perp| = 4$ GeV and $z = 1/2$. The black and red curves correspond to γ^*p and γ^*A processes, and the solid and dashed lines denote without and with Sudakov resummations, respectively. The left panel corresponds to the case with $Q = 2|P_\perp|$ and the right panel is for $Q = |P_\perp|$. From this figure (for example, in the left panel of Fig. 11), one can observe that the elliptic anisotropy v_2 for γ^*A is significantly lower than that for γ^*p , the main reason comes from the fact that the gluon saturation effects for heavy ion A is stronger than that of proton p . By comparing the solid and dashed lines, one can see that the soft gluon resummations significantly reduce the elliptic anisotropy v_2 as well. In particular, in the present kinematic regime, we stress that, for γ^*A , the rather strong suppression of v_2 caused by the effects of gluon saturation and soft gluon resummations will make it very difficult to measure in experiments. On the other hand, the strong suppression for γ^*A as compared to that of γ^*p also as a signature of gluon saturation for heavy nucleus ion. One possible way to give a sizable elliptic anisotropy v_2 that can be measured in experiments is to enlarge $q_\perp$ or increase x_g that making $|q_\perp|/Q_{sg}$ not even small.

Although the elliptic anisotropy v_2 can be used to probe xH/xG of the scattered target, it involves the other hard factors, such as $Q, P_\perp$, etc. In the same way, the polarized anisotropy can be expressed as [21,22]

$$v_2^L \equiv \frac{\int d\Delta\phi \cos(2\Delta\phi) d\sigma^L}{\int d\Delta\phi d\sigma^L} = \frac{1}{2} \frac{xH}{xG} \quad (40)$$

and

$$v_2^T \equiv \frac{\int d\Delta\phi \cos(2\Delta\phi) d\sigma^T}{\int d\Delta\phi d\sigma^T} = -\frac{1}{2} R' \frac{xH}{xG}, \quad (41)$$

where R' is determined by the kinematics in Eq. (16),

$$R' = \frac{2(z^2 + (1-z)^2)P_\perp^2 \epsilon_f^2 - 2m_q^2 P_\perp^2}{(z^2 + (1-z)^2)(P_\perp^4 + \epsilon_f^4) + 2m_q^2 P_\perp^2}. \quad (42)$$

From Eqs. (40) and (41), one can observe that the longitudinally polarized elliptic anisotropy v_2^L directly probes the ratio xH/xG and has no dependence on other additional variables, while for the transversely polarized anisotropy v_2^T , which involves a factor of R' and has a sign change as compared to v_2^L . One should note that these polarized anisotropies cannot be directly measured in experiments, one possible way to extract the polarized anisotropies following the relation

$$v_2 = \frac{Rv_2^L + v_2^T}{1 + R} \quad (43)$$

where R is determined by comparing Eqs. (16) and (17)

$$R = \frac{8Q^2 z^3 (1-z)^3 P_\perp^2}{(z^2 + (1-z)^2)(P_\perp^4 + \epsilon_f^4) + 2m_q^2 P_\perp^2}. \quad (44)$$

In addition, we have also computed the longitudinally polarized anisotropy v_2^L in γ^*p and γ^*A scattering processes, the results are very similar to that of v_2 , therefore, we do not include it as a separated plot.

V. SUMMARY

In this paper we have investigated the inclusive dijet cross sections and their azimuthal correlations in ep and eA collisions within the framework of CGC EFT. By applying the so-called “back-to-back correlation limit,” the cross section can be expressed in terms of unpolarized and linearly polarized WW gluon distributions. Although the unpolarized and linearly polarized WW gluon distributions have little known in experiments, the novel scattering processes provide the cleanest environment to probe the two gluon TMDs, as well as allow one to quantitatively probe the gluon saturation effect inside the scattered target.

As compared to previous related studies [21,22,27,47], in this work we employed the nonlinear Gaussian approximation, using the solutions of the rcBK equation to give the numerical results of unpolarized and linearly polarized WW gluon distributions in both coordinate and momentum space. Our results show that the two gluon distributions have strong geometrical scaling behaviors over a broad range of transverse momentum. In particular, the linearly polarized WW gluon distribution tends to be saturated and has the same asymptotic behavior as the unpolarized one in the large transverse momentum region, while in the small transverse momentum region, in which gluon saturation effects become manifest, this distribution remains nonzero even at small transverse momentum and seems to be constant when the transverse momentum is of the order of the gluon saturation scale (see, for example, Figs. 2 and 3). We stress that, although only the two gluon TMDs were presented in the work, the other gluon TMDs (such as those that enter the cross sections in pp and pA collisions [27,47,78]) can also be obtained following this procedure, and we hope that the features depicted in the present work can be further validated by experiments in the future.

Moreover, we calculated the quark-antiquark pair (or dijet) cross sections and their azimuthal correlations by including the parton shower effects (or Sudakov resummations) in the scattering processes. In the calculations, we restricted the transverse momentum $P_\perp$ and the transverse momentum imbalance $q_\perp$ of the dijet with the condition that $|P_\perp| \geq 2|q_\perp|$ to meet the “back-to-back correlation limit.” For the cross sections, although the dijets are restricted to be purely back to back ($\Delta\phi = \pi$), we found that the Sudakov resummations can strongly affect the transverse momentum distributions. Specifically, for $d\sigma/d|P_\perp|$, numerical results show that the Sudakov

resummations suppress the cross sections and the suppression becomes stronger with increasing $P_\perp$ (shown in Fig. 7), while for $d\sigma/d|q_\perp|$ the Sudakov effects will significantly reduce the distributions as $q_\perp$ decreases, and seems to make the distributions to be flattened over a broad small $q_\perp$ region (Fig. 8). As for the dijet azimuthal correlations, numerical results have shown that the back-to-back correlations are significantly suppressed when the Sudakov effects are taken into account (Fig. 9).

Furthermore, we also presented the results for dijet azimuthal correlations and the elliptic anisotropy v_2 in the γ^*p and γ^*A scattering processes. By comparing the back-to-back correlations in the two scattering processes, we found that a significant suppression caused by gluon saturation effects is visible, especially when including the Sudakov effects, which further suppresses the back-to-back correlations. We also found that a similar suppression can be observed in the elliptic anisotropy v_2 as well. In particular, in the studied kinematic regime, we stress that, for γ^*A , the rather strong suppression of v_2 caused by the effects of gluon saturation and soft gluon resummations will make it very difficult to measure for experiments as compared to that of γ^*p (see Fig. 11). On the other hand, as compared to γ^*p , the rather strong suppressions of back-to-back correlations and the elliptic anisotropy in γ^*A is a solid signature of gluon saturation for heavy nuclei, which can be further validated or tested at the future EIC.

It should be noted that, in the present work, our numerical results were only calculated at the parton level, and any other final-state effects were not taken into account. To be able to compare these results with future EIC data and other studies, there are still some issues that should be resolved. For final-state dijet production, the jet cone size dependence should be taken into account. In that case, as introduced in Ref. [45], more refined versions of Sudakov factors (including the jet cone size dependence as well as the next-to-leading logarithmic corrections) should be employed. Also, as pointed out in Refs. [79,80], final-state soft gluon radiation can have a large impact on the dijet anisotropies at the EIC; therefore, it would be very interesting to incorporate these effects into the CGC approach, which can be used as a cross check for the perturbative calculations. Another issue is for dihadron productions, this can be done by convoluting the dijet cross section with fragmentation functions, similar results as shown in Figs. 10 and 11 are expected to hold for dihadron systems. We leave this issue for future work.

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