Collinear factorization of diphoton photoproduction at next to leading order

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We calculate in the framework of collinear factorization the amplitude for the photoproduction of a near forward large mass diphoton at leading twist and next to leading order (NLO) in α_s . We demonstrate the validity of factorization at this order, which was never achieved for such a reaction where the coefficient function describes a $2 \rightarrow 3$ hard process. While the Born order amplitude was purely imaginary and only probed the $x = \pm\xi$ cross-over line of generalized parton distributions (GPD) domain, the NLO result contains both a real and an imaginary part and probes the whole domain of definition of quark GPDs. The phenomenology of our results for medium (JLab) and higher energy (EIC) experiments will be developed in a future study.

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I. INTRODUCTION

In our recent works [1,2] we studied at leading order (LO) in the QCD coupling constant α_s the photoproduction of a large mass diphoton on a nucleon target

$$\gamma(q, \epsilon) + N(p_1, s_1) \rightarrow \gamma(q_1, \epsilon_1) + \gamma(q_2, \epsilon_2) + N'(p_2, s_2), \quad (1)$$

in the kinematical domain suitable to a factorization of generalized parton distributions (GPDs) and a hard amplitude. This domain is defined by large diphoton invariant mass and center of mass energy and by a small momentum transfer squared $t = (p_2 - p_1)^2$ between the initial and final nucleons. This reaction, which superficially looks like timelike Compton scattering (TCS) [3–7] with the dilepton replaced by a diphoton, has a quite different structure since the hard amplitude describes a $2 \rightarrow 3$ process, contrarily to more familiar reactions—deeply virtual Compton scattering (DVCS), deeply virtual meson production (DVMP) or TCS—which involve a $2 \rightarrow 2$ process. While other $2 \rightarrow 3$ processes have already been studied at leading order [8–12], assuming QCD collinear factorization at leading twist, there is yet no proof of this very basic property, even at the next to leading order (NLO) in α_s .

It is thus deemed appropriate to calculate at NLO in α_s the amplitude of the simplest $2 \rightarrow 3$ process, namely the photoproduction of a diphoton (1). This is the aim of this paper

where we demonstrate the factorization at NLO of the amplitude, showing in particular that the divergent parts of the NLO amplitude can be absorbed in the GPDs. We use dimensional regularization as the main tool of our proof. The phenomenological consequences of our NLO description of process (1) will be analyzed in a forthcoming publication.

In Sec. II we present the kinematics of our process, then we briefly recall in Sec. III the structure of a factorization proof at NLO. Section IV discusses in some details the calculation of the vector GPD contribution for our process. In Section V we discuss the corresponding results for the axial amplitude. Section VI contains some remarks and conclusions. Some of the results of this paper have been already presented in [13,14].

II. KINEMATICS

We use the following light-cone four-vectors:

$$p^\mu = \frac{\sqrt{s}}{2}(1, 0, 0, 1), \quad n^\mu = \frac{\sqrt{s}}{2}(1, 0, 0, -1), \quad (2)$$

the parameter $s = 2p \cdot n$ will be described in more detail later. We parametrize the momenta in the Sudakov basis:

$$\begin{aligned} p_1^\mu &= (1 + \xi)p^\mu + \frac{M^2}{s(1 + \xi)}n^\mu, \\ p_2^\mu &= (1 - \xi)p^\mu + \frac{M^2 + \vec{\Delta}_\perp^2}{s(1 - \xi)}n^\mu + \Delta_\perp^\mu, \quad q^\mu = n^\mu, \\ q_1^\mu &= \alpha n^\mu + \frac{(\vec{p}_\perp - \frac{1}{2}\vec{\Delta}_\perp)^2}{\alpha s}p^\mu + p_\perp^\mu - \frac{1}{2}\Delta_\perp^\mu, \\ q_2^\mu &= \bar{\alpha} n^\mu + \frac{(\vec{p}_\perp + \frac{1}{2}\vec{\Delta}_\perp)^2}{\bar{\alpha} s}p^\mu - p_\perp^\mu - \frac{1}{2}\Delta_\perp^\mu. \end{aligned} \quad (3)$$

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Here, M is the mass of the nucleon, ξ is the skewness variable and $\vec{v} \cdot \vec{u}$ denotes the Euclidean product of transverse vectors. Polarization vectors of the photons are written in the $\epsilon \cdot p = 0$ gauge as:

$$\begin{aligned}\epsilon^\mu(q_1) &= \epsilon_\perp^\mu(q_1) - \frac{2\epsilon_\perp(p_\perp - \frac{1}{2}\Delta_\perp)}{\alpha s} p^\mu, \\ \epsilon^\mu(q_2) &= \epsilon_\perp^\mu(q_2) + \frac{2\epsilon_\perp(p_\perp + \frac{1}{2}\Delta_\perp)}{\bar{\alpha} s} p^\mu, \\ \epsilon^\mu(q) &= \epsilon_\perp^\mu(q).\end{aligned}\quad (4)$$

The process can be described in terms of the following 4 invariants:

$$\begin{aligned}S_{\gamma N} &= (p_1 + q)^2, & t &= (p_1 - p_2)^2, \\ M_{\gamma\gamma}^2 &= (q_1 + q_2)^2, & u' &= (q_2 - q)^2.\end{aligned}\quad (5)$$

For further convenience we relate them to other invariants:

$$\begin{aligned}S_{\gamma N} &= (1 + \xi)s + M^2 \rightarrow s = \frac{S_{\gamma N} - M^2}{1 + \xi}, \\ t' &= (q_1 - q)^2 = t - M_{\gamma\gamma}^2 - u'.\end{aligned}\quad (6)$$

At leading twist, the calculation of the hard part uses the approximation $\Delta_\perp = 0$, justified by the fact that in the considered kinematics one has $|p_\perp^2| \gg |\Delta_\perp^2|$. The simplified kinematical relations then read:

$$\begin{aligned}\tau &= \frac{M_{\gamma\gamma}^2 - t}{S_{\gamma N} - M^2}, & \xi &= \frac{\tau}{2 - \tau}, \\ -t' &= \frac{\vec{p}_\perp^2}{\alpha}, & -u' &= \frac{\vec{p}_\perp^2}{\bar{\alpha}}, & \alpha + \bar{\alpha} &= 1.\end{aligned}\quad (7)$$

To reduce the number of considered diagrams, we will consider a process with three incoming photons with momenta (k_1, k_2, k_3) and polarization vectors $(\epsilon_1, \epsilon_2, \epsilon_3)$, and then sum the amplitudes over $\{(k_1, \epsilon_1), (k_2, \epsilon_2), (k_3, \epsilon_3)\}$ corresponding to all permutations of $\{(-q_1, \epsilon^*(q_1)), (-q_2, \epsilon^*(q_2)), (q, \epsilon(q))\}$. We introduce the parameters β_i, κ_i defined as:

$$\begin{aligned}2pk_i &:= \beta_i s, & 2k_1 k_2 &:= \kappa_3 s, \\ 2k_2 k_3 &:= \kappa_1 s, & 2k_1 k_3 &:= \kappa_2 s.\end{aligned}\quad (8)$$

It is easy to check that $\kappa_i = 2\xi\beta_i$. Scalar products of corresponding momenta are

$$-2pq_1 = -\alpha s, \quad -2pq_2 = -\bar{\alpha} s, \quad 2pq = s. \quad (9)$$

Adding the different diagrams amounts to let the β_i and κ_i parameters be subject to the following permutations:

$$\begin{aligned}\{\beta_i\} &= \text{perm of } \{-\alpha, -\bar{\alpha}, 1\}, \\ \{\kappa_i\} &= \text{perm of } \left\{\frac{u'}{s}, \frac{t'}{s}, \frac{M_{\gamma\gamma}^2}{s}\right\}.\end{aligned}\quad (10)$$

In the calculation of the hard part it is assumed that $p_1 - p_2 \approx 2\xi p$, so that $2\xi p = q_1 + q_2 - q$. Hence, for each polarization vector ϵ_i :

$$\epsilon_i(k_1 + k_2 + k_3) = 0. \quad (11)$$

Moreover, $k_1 \epsilon_1 = k_2 \epsilon_2 = k_3 \epsilon_3 = 0$. In the considered gauge, only the transverse parts of polarizations vectors contribute to their products with momenta. We have:

$$\begin{aligned}q_1 \epsilon(q) &= -q_2 \epsilon(q) = -\vec{p}_t \vec{\epsilon}_t(q), \\ q \epsilon(q_1) &= q_2 \epsilon(q_1) = \frac{1}{\alpha} \vec{p}_t \vec{\epsilon}_t(q_1), \\ q \epsilon(q_2) &= q_1 \epsilon(q_2) = -\frac{1}{\bar{\alpha}} \vec{p}_t \vec{\epsilon}_t(q_2).\end{aligned}\quad (12)$$

III. FACTORIZATION

In the collinear factorization regime, the scattering amplitude of our process (1) reads:

$$\mathcal{A} = \sum_q \int_{-1}^1 dx [T_B^q(x) F_B^q(x) + \tilde{T}_B^q(x) \tilde{F}_B^q(x)], \quad (13)$$

where F_B^q and $\tilde{F}_B^q$ are bare vector and axial quark GPDs; T_B^q and $\tilde{T}_B^q$ are bare vector and axial coefficient functions containing infrared divergences at $O(\alpha_s)$, and are written as:

$$T_B^q = \text{Tr} \left[\mathcal{M} \frac{\not{p}}{4} \right] = C_0^q + \left(\frac{M_{\gamma\gamma}^2 e^\gamma}{4\pi\mu_R^2} \right)^{-\epsilon/2} \left(-\frac{2}{\epsilon} C_{\text{coll}}^q + C_1^q \right), \quad (14)$$

$$\tilde{T}_B^q = \text{Tr} \left[\mathcal{M} \gamma^5 \frac{\not{p}}{4} \right] = \tilde{C}_0^q + \left(\frac{M_{\gamma\gamma}^2 e^\gamma}{4\pi\mu_R^2} \right)^{-\epsilon/2} \left(-\frac{2}{\epsilon} \tilde{C}_{\text{coll}}^q + \tilde{C}_1^q \right), \quad (15)$$

where $\mathcal{M}$ is the $q\gamma \rightarrow q\gamma\gamma$ hard scattering amplitude with the quark spinors removed.

Only nonsinglet quark GPDs contribute to our process and the $O(\alpha_s)$ relation between the bare and renormalized GPDs in the $\overline{\text{MS}}$ scheme reads:

$$F_B^q(x) = F^q(x) + \left(\frac{2}{\epsilon} + \ln \frac{e^\gamma \mu_F^2}{4\pi\mu_R^2} \right) K_{NS}^{qq}(x, x') \otimes F^q(x'), \quad (16)$$

where μ_F and μ_R are respectively the factorization and renormalization scales, γ is the Euler constant and K_{NS}^{qq} is

the nonsinglet evolution kernel (see, e.g., Eq. (101) of [15]). The cancellation of divergences—a necessary requirement for a consistent factorization framework—occurs, if:

$$\begin{aligned} C_{\text{coll}}^q(x') &= C_0^q(x) \otimes K_{NS}^{qq}(x, x'), \\ \tilde{C}_{\text{coll}}^q(x') &= \tilde{C}_0^q(x) \otimes K_{NS}^{qq}(x, x'), \end{aligned} \quad (17)$$

which leads to:

$$\mathcal{A} = \sum_q \int_{-1}^1 dx [T^q(x) F^q(x) + \tilde{T}^q(x) \tilde{F}^q(x)], \quad (18)$$

where:

$$T^q = C_0^q + C_1^q + \log\left(\frac{M_{\gamma\gamma}^2}{\mu_F^2}\right) C_{\text{coll}}^q, \quad (19)$$

and the corresponding equation for the axial amplitude.

IV. THE VECTOR AMPLITUDE

Let us first study the vector GPD contribution to the scattering amplitude.

A. Lowest order analysis

At leading (i.e., zeroth) order in α_s , the contribution to the vector amplitude coming from the diagram shown in Fig. 1, with a given permutation of photons, reads [1]:

$$\text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] = -\frac{ie_q^3}{s^2 \beta_1 \beta_2 \beta_3} \frac{1}{x + \xi + i0_1} \frac{1}{x - \xi - i0_3} 4\beta_2 \mathcal{A}_{1,2,3}, \quad (20)$$

where we denote $i0_i = i0 \cdot \text{sgn}(\beta_i)$ and define the trace structure:

$$4\mathcal{A}_{1,2,3} := \text{Tr}(\not{\epsilon}_3(-\not{k}_3)\not{\epsilon}_2\not{k}_1\not{\epsilon}_1\not{p}), \quad (21)$$

FIG. 1. The general form of the LO diagram. As described in the text, to simplify computations we first consider a diagram with 3 incoming photons, and then we use the appropriate substitutions.

since it turns out that it appears in all diagrams. The full amplitude of the hard subprocess is obtained by summing over all permutations of photons k_i :

$$\text{Tr}[i\mathcal{M}^0 \not{p}] = \sum_{\text{perm}} \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}]. \quad (22)$$

The trace structure has the important property:

$$\sum_{\text{perm}} \beta_2 \mathcal{A}_{1,2,3} = 0, \quad (23)$$

which results in the vanishing of the real part of the amplitudes. Let us observe that the interchange of indices $1 \leftrightarrow 3$ in Eq. (20) does not change it if $\text{sgn}(\beta_1) = \text{sgn}(\beta_3)$, and results in complex conjugation of the term $\frac{1}{x+\xi+i0_1} \frac{1}{x-\xi-i0_3}$ if $\text{sgn}(\beta_1) = -\text{sgn}(\beta_3)$. The only terms proportional to the imaginary part of propagators which survive, are those from diagrams in which the incoming photon is the middle one (so that $k_2 \rightarrow q$). Finally:

$$C_0^q(x, \xi) = \frac{ie_q^3 \mathcal{A}}{M_{\gamma\gamma}^2 \alpha \bar{\alpha}} \text{Im}\left(\frac{1}{x + \xi - i0}\right) + (x \rightarrow -x), \quad (24)$$

where:

$$\begin{aligned} \mathcal{A} &= [(\alpha - \bar{\alpha})(\vec{\epsilon}_t^*(\mathbf{q}_1)\vec{\epsilon}_t^*(\mathbf{q}_2))(\vec{p}_t\vec{e}_t(\mathbf{q})) \\ &\quad - (\vec{p}_t\vec{\epsilon}_t^*(\mathbf{q}_1))(\vec{e}_t(\mathbf{q})\vec{\epsilon}_t^*(\mathbf{q}_2)) \\ &\quad + (\vec{p}_t\vec{\epsilon}_t^*(\mathbf{q}_2))(\vec{e}_t(\mathbf{q})\vec{\epsilon}_t^*(\mathbf{q}_1))]. \end{aligned} \quad (25)$$

This reproduces the result of [1].

B. Self-energy insertions at $\mathcal{O}(\alpha_s)$

The contributions of self-energy diagrams shown in Fig. 2 read:

$$\begin{aligned} \text{Tr}[i\mathcal{M}_{1,2,3}^{2,L} \not{p}] &= \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \cdot \frac{\alpha_s C_F}{4\pi} \left(\frac{M_{\gamma\gamma}^2 e^\gamma}{4\pi\mu^2}\right)^{-\frac{\epsilon}{2}} \\ &\quad \times \left(-\frac{2}{\epsilon} + \log\left(-\frac{x+\xi}{2\xi}\beta_1 - i0\right) - 1\right), \end{aligned} \quad (26)$$

for the left panel diagram, and

$$\begin{aligned} \text{Tr}[i\mathcal{M}_{1,2,3}^{2,R} \not{p}] &= \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \cdot \frac{\alpha_s C_F}{4\pi} \left(\frac{M_{\gamma\gamma}^2 e^\gamma}{4\pi\mu^2}\right)^{-\frac{\epsilon}{2}} \\ &\quad \times \left(-\frac{2}{\epsilon} + \log\left(\frac{x-\xi}{2\xi}\beta_3 - i0\right) - 1\right), \end{aligned} \quad (27)$$

for the right panel diagram.

FIG. 2. Self-energy insertion contributions to the NLO amplitude: diagrams 2.L and 2.R.

FIG. 3. Vertex insertion contributions to the NLO amplitude: diagrams 3.L and 3.R.

C. Vertex corrections at $O(\alpha_s)$

The left and right vertex corrections, shown in Fig. 3 denoted respectively as $\mathcal{M}^{3,L}$ and $\mathcal{M}^{3,R}$, read:

$$\text{Tr}[i\mathcal{M}_{1,2,3}^{3,L}\not{p}] = \text{Tr}[i\mathcal{M}_{1,2,3}^0\not{p}] \cdot \frac{\alpha_s C_F}{4\pi} \left(\frac{M_{\gamma\gamma}^2 e^\gamma}{4\pi\mu^2} \right)^{-\frac{\epsilon}{2}} \times \left(-\frac{2}{\epsilon} + \log\left(-\frac{x+\xi}{2\xi} \beta_1 - i0 \right) - 4 \right), \quad (28)$$

$$\text{Tr}[i\mathcal{M}_{1,2,3}^{3,R}\not{p}] = \text{Tr}[i\mathcal{M}_{1,2,3}^0\not{p}] \cdot \frac{\alpha_s C_F}{4\pi} \left(\frac{M_{\gamma\gamma}^2 e^\gamma}{4\pi\mu^2} \right)^{-\frac{\epsilon}{2}} \times \left(-\frac{2}{\epsilon} + \log\left(\frac{x-\xi}{2\xi} \beta_3 - i0 \right) - 4 \right), \quad (29)$$

The middle vertex correction, shown in Fig. 4, has a more complicated structure. Introducing $D_1 = (x+\xi)\beta_1$, $D_3 = -(x-\xi)\beta_3$, we can write it as:

$$\begin{aligned} \text{Tr}[i\mathcal{M}_{1,2,3}^{3,M}\not{p}] = & \text{Tr}[i\mathcal{M}_{1,2,3}^0\not{p}] \cdot \frac{\alpha_s C_F}{4\pi} \left\{ \left(\frac{M_{\gamma\gamma}^2 e^\gamma}{4\pi\mu^2} \right)^{-\frac{\epsilon}{2}} \left(+\frac{2}{\epsilon} + 1 - \frac{D_1 \log\left(\frac{-D_1-i0}{2\xi} \right) - D_3 \log\left(\frac{-D_3-i0}{2\xi} \right)}{D_1 - D_3} \right) \right. \\ & + \left(\frac{\mathcal{B}_{1,2,3}}{\mathcal{A}_{1,2,3}} \right) \frac{D_1}{D_1 - D_3} \left(1 + \frac{-2D_1 + D_3}{D_1 - D_3} \log \frac{D_1 + i0}{D_3 + i0} \right) \\ & \left. + \left(\frac{\mathcal{C}_{1,2,3}}{\mathcal{A}_{1,2,3}} \right) \frac{D_3}{D_1 - D_3} \left(1 + \frac{D_1 - 2D_3}{D_1 - D_3} \log \frac{D_1 + i0}{D_3 + i0} \right) \right\}, \quad (30) \end{aligned}$$

where:

$$\mathcal{B}_{1,2,3} = \text{Tr}\{\not{\epsilon}_3 \not{k}_3 \not{k}_1 \not{\epsilon}_2 \not{p} \not{\epsilon}_1\}, \quad \mathcal{C}_{1,2,3} = \text{Tr}\{\not{\epsilon}_3 \not{p} \not{\epsilon}_2 \not{k}_3 \not{\epsilon}_1\}.$$

D. The box contributions

There are two box contributions shown on Fig. 5. The divergent part of the amplitude 4.L reads:

$$\begin{aligned} \text{Tr}[i\mathcal{M}_{1,2,3}^{4,L}\not{p}]_{\text{div}} = & -\frac{4}{\epsilon} \frac{\alpha_s C_F}{4\pi} \text{Tr}[i\mathcal{M}_{1,2,3}^0\not{p}] \log\left(-\frac{x-\xi}{2\xi} + i0_3 \right), \quad (31) \end{aligned}$$

FIG. 4. Vertex insertion contributions to the NLO amplitude: diagram 3.M.

FIG. 5. Box contributions to the NLO amplitude: diagrams 4.L and 4.R.

while the divergent part of the second box diagram reads:

$$\text{Tr}[i\mathcal{M}_{1,2,3}^{4,R}\not{p}]_{\text{div}} = -\frac{4\alpha_S C_F}{\varepsilon 4\pi} \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \log\left(\frac{x+\xi}{2\xi} + i0_1\right). \quad (32)$$

The finite contribution resulting from these box diagrams is lengthy and will be included in formulas presented in Sec. IV G.

E. The pentagonal contribution

The pentagonal contribution to the amplitude, resulting from the diagram presented on Fig. 6, reads:

$$\text{Tr}[i\mathcal{M}_{1,2,3}^5 \not{p}] = C_F e_q^3 g^2 \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{\text{Tr}\{\gamma_\mu \not{p} \gamma^\mu (\not{p} + \not{k}) \phi_3(\not{p} + \not{k}_1 + \not{k}_2 + \not{k}) \phi_2(\not{p} + \not{k}_1 + \not{k}) \phi_1(\not{p} + \not{k})\}}{(k^2 + i0)(D_+ + i0)(D_- + i0)(D_1 + i0)(D_{12} + i0)}, \quad (33)$$

where $\bar{p} = (x + \xi)p$, $\underline{p} = (x - \xi)p$ and:

$$D_\pm = ((x \pm \xi)p + k)^2, \quad D_1 = ((x + \xi)p + k_1 + k)^2, \quad D_{12} = ((x + \xi)p + k_1 + k_2 + k)^2.$$

Observing that

$$2(pk) = \frac{1}{2\xi}(D_+ - D_-), \quad k^2 = \frac{1}{2\xi}((x + \xi)D_- - (x - \xi)D_+),$$

and using the following gamma-identities inside the trace: $\gamma^\mu \not{p} \gamma_\mu = (-2 + \varepsilon)\not{p}$, $\not{k} \not{p} \not{k} = 2(pk)\not{k} - k^2 \not{p}$, allows us to obtain the following identity:

$$2(pk)\not{k} - k^2 \not{p} = \frac{D_+}{2\xi}(\not{k} + \underline{p}) - \frac{D_-}{2\xi}(\not{k} + \bar{p}), \quad (34)$$

FIG. 6. The 5-point diagram contributing to the NLO amplitude.

and to write the amplitude as the sum of two 4-point integrals:

$$\text{Tr}[i\mathcal{M}_{1,2,3}^5 \not{p}] = \text{Tr}[i\mathcal{M}_{1,2,3}^{5,L} \not{p}] + \text{Tr}[i\mathcal{M}_{1,2,3}^{5,R} \not{p}], \quad (35)$$

where

$$\text{Tr}[i\mathcal{M}_{1,2,3}^{5,L} \not{p}] := -\frac{-2+\varepsilon}{2\xi} C_F e_q^3 g^2 \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{\text{Tr}\{(\not{p} + \not{k}) \not{\epsilon}_3 (\not{p} + \not{k}_1 + \not{k}_2 + \not{k}) \not{\epsilon}_2 (\not{p} + \not{k}_1 + \not{k}) \not{\epsilon}_1\}}{(k^2 + i0)(D_+ + i0)(D_1 + i0)(D_{12} + i0)}, \quad (36)$$

$$\text{Tr}[i\mathcal{M}_{1,2,3}^{5,R} \not{p}] := -\frac{-2+\varepsilon}{2\xi} C_F e_q^3 g^2 \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{\text{Tr}\{(\not{p} + \not{k}) \not{\epsilon}_3 (\not{p} + \not{k}_1 + \not{k}_2 + \not{k}) \not{\epsilon}_2 (\not{p} + \not{k}_1 + \not{k}) \not{\epsilon}_1\}}{(k^2 + i0)(D_- + i0)(D_1 + i0)(D_{12} + i0)}. \quad (37)$$

The names *5.L* and *5.R* come from the structure of denominators in integrands, which are the same as in amplitudes *4.L* and *4.R*. The divergent part of the amplitude reads:

$$\text{Tr}[i\mathcal{M}_{1,2,3}^5 \not{p}]_{\text{div}} = -\frac{\alpha_s C_F}{4\pi} \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \frac{2}{\varepsilon} \left(\frac{x-\xi}{\xi} \log\left(-\frac{x-\xi}{2\xi} + i0_3\right) - \frac{x+\xi}{\xi} \log\left(\frac{x+\xi}{2\xi} + i0_1\right) \right). \quad (38)$$

The finite contribution resulting from this pentagonal diagram will be included in formulas presented in Sec. IV G.

F. Total divergent part

Summing the various contributions written above in Eqs. (26), (27), (28), (29), (30), (31), (32) and (38) we can write the divergent part of the amplitude corresponding to a given permutation of photons entering the hard part as:

$$\text{Tr}[i\mathcal{M}_{1,2,3} \not{p}]_{\text{div}} = -\frac{2}{\varepsilon} \cdot \frac{\alpha_s C_F}{4\pi} \cdot \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \left(3 + \frac{x+\xi}{\xi} \log\left(-\frac{x-\xi}{2\xi} + i0_3\right) - \frac{x-\xi}{\xi} \log\left(\frac{x+\xi}{2\xi} + i0_1\right) \right), \quad (39)$$

which after the summation over permutations gives [see the definition (14)]:

$$C_{\text{coll}}^q = \frac{ie_q^3 \mathcal{A}}{M_{\gamma\gamma}^2 \alpha \bar{\alpha}} \frac{\alpha_s C_F}{4\pi} \text{Im} \left[\frac{1}{x+\xi-i0} \left(-3 - 2 \log\left(\frac{x+\xi}{2\xi} - i0\right) \right) \right] + (x \rightarrow -x). \quad (40)$$

The resulting expressions for C_0^q and C_{coll}^q are, up to a common constant factor, equal to the imaginary part of the corresponding expressions in DVCS (see Eq. (13) in [6]). The nonsinglet and singlet kernels K_{qq} are at the α_s order equal to each other [15] and real, hence the condition for the cancellation of divergences expressed by Eq. (17) in the case of the considered process is just the imaginary part of the corresponding expression for DVCS.

G. The finite part of C_1

The finite parts of the contributions of self energy and vertex diagrams to C_1 are explicitly written in Eqs. (26)–(30). The finite parts emerging from the box and pentagon diagrams are more cumbersome and it is convenient to write them in terms of functions defined by the following equations:

$$\begin{aligned} (-1)^n s^{-n} \mathcal{F}_{\text{box}}(x, \xi, \{\beta_i\}) &:= \int_0^1 dr dz r^a z^b (-2(z\bar{p}k_1 + zr\bar{p}k_2 + rk_1 k_2) - i0)^{-n}, \\ &= (-1)^n s^{-n} \int_0^1 dr dz r^a z^b ((z(x+\xi)\beta_1 + zr(x+\xi)\beta_2 + r\kappa_3) + i0)^{-n}, \end{aligned} \quad (41)$$

$$\mathcal{G}(x, \xi, \{\beta_i\}) := \int_0^1 dr dz z ((z(x+\xi)\beta_1 + zr(x+\xi)\beta_2 + r\kappa_3) + i0)^{-2} \ln(-z(x+\xi)\beta_1 - zr(x+\xi)\beta_2 - r\kappa_3 - i0). \quad (42)$$

It is worth stressing that the integrals in the above equations are finite. Using above definitions, and omitting for brevity the argument of functions $\mathcal{F}_{\text{box}}$ and $\mathcal{G}$, we can write the finite part of the box diagram to which this permutation contributes as:

$$\begin{aligned}
\text{Tr}[i\mathcal{M}_{1,2,3}^{4,L}\not{p}]_{\text{fin}} = & -i\frac{\alpha_S C_F}{4\pi} e_q^3 \frac{1}{2pk_3 - i0} s^{-2} \left\{ \mathcal{F}_{210}(\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 k_2 \not{\epsilon}_1 k_1) - \text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 k_1 \not{\epsilon}_1 k_2) - 2\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1 k_2)) \right. \\
& + \mathcal{F}_{201}(-\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1 k_1) - 2\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1 k_1) - 4\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1 \bar{p}) - 5\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 k_1 \not{\epsilon}_1 \bar{p})) \\
& + \mathcal{F}_{211}(-\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1 k_2) - \text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 k_2 \not{\epsilon}_1 \bar{p}) + \text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1 k_1) \\
& - 2\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1 k_2) - \text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1 \bar{p})) \\
& + \mathcal{F}_{220}(\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 k_2 \not{\epsilon}_1 k_2) + \text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1 k_2)) + \mathcal{F}_{221}\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1 k_2) \\
& + s\mathcal{F}_{100}(\text{Tr}(k_3 \not{\epsilon}_3 \not{p} \not{\epsilon}_2 \not{\epsilon}_1 k_1) - \text{Tr}(\not{p} \not{\epsilon}_3 k_3 \not{\epsilon}_2 k_1 \not{\epsilon}_1) - \text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_1 \not{\epsilon}_2 \not{\epsilon}_1) - 2\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 \not{\epsilon}_1)) \\
& + s\mathcal{F}_{110}(\text{Tr}(k_3 \not{\epsilon}_3 \not{p} \not{\epsilon}_2 \not{\epsilon}_1 k_2) + \text{Tr}(\not{p} \not{\epsilon}_3 k_3 \not{\epsilon}_2 k_2 \not{\epsilon}_1) + \text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_2 \not{\epsilon}_2 \not{\epsilon}_1)) + 2\mathcal{G}\text{Tr}(k_3 \not{\epsilon}_3 \not{p} k_3 \not{\epsilon}_2 k_1 \not{\epsilon}_1 \bar{p}) \Big\} \\
& - \frac{\alpha_S C_F}{2\pi} \cdot \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \log(2\xi) \log\left(\frac{1}{2\xi}(\xi - x + i0 \cdot \text{sgn}(\beta_3))\right) \quad (43)
\end{aligned}$$

The finite part of the pentagonal diagram reads:

$$\begin{aligned}
\text{Tr}[i\mathcal{M}_{1,2,3}^{5,L}\not{p}]_{\text{fin}} = & -i\frac{\alpha_S C_F}{4\pi} e_q^3 \frac{1}{\xi} s^{-2} \frac{1}{2} \left\{ \mathcal{F}_{201}(\text{Tr}(k_1 \not{\epsilon}_3 \bar{p} \not{\epsilon}_2 k_1 \not{\epsilon}_1) + \text{Tr}(k_1 \not{\epsilon}_3 k_1 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1) + 2\text{Tr}(k_1 \not{\epsilon}_3 k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1) \right. \\
& + 5\text{Tr}(\bar{p} \not{\epsilon}_3 k_1 \not{\epsilon}_2 k_1 \not{\epsilon}_1) + 4\text{Tr}(\bar{p} \not{\epsilon}_3 k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1)) \\
& + \mathcal{F}_{210}(-\text{Tr}(k_1 \not{\epsilon}_3 k_1 \not{\epsilon}_2 k_2 \not{\epsilon}_1) + \text{Tr}(k_2 \not{\epsilon}_3 k_1 \not{\epsilon}_2 k_1 \not{\epsilon}_1) + 2\text{Tr}(k_2 \not{\epsilon}_3 k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1)) \\
& + \mathcal{F}_{211}(\text{Tr}(\bar{p} \not{\epsilon}_3 k_1 \not{\epsilon}_2 k_2 \not{\epsilon}_1) + \text{Tr}(\bar{p} \not{\epsilon}_3 k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1) - \text{Tr}(k_1 \not{\epsilon}_3 \bar{p} \not{\epsilon}_2 k_2 \not{\epsilon}_1) \\
& - \text{Tr}(k_1 \not{\epsilon}_3 k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1) + \text{Tr}(k_2 \not{\epsilon}_3 \bar{p} \not{\epsilon}_2 k_1 \not{\epsilon}_1) + \text{Tr}(k_2 \not{\epsilon}_3 k_1 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1) + 2\text{Tr}(k_2 \not{\epsilon}_3 k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1)) \\
& + \mathcal{F}_{220}(-\text{Tr}(k_2 \not{\epsilon}_3 k_1 \not{\epsilon}_2 k_2 \not{\epsilon}_1) - \text{Tr}(k_2 \not{\epsilon}_3 k_2 \not{\epsilon}_2 k_1 \not{\epsilon}_1)) - \mathcal{F}_{221}(\text{Tr}(k_2 \not{\epsilon}_3 \bar{p} \not{\epsilon}_2 k_2 \not{\epsilon}_1) + \text{Tr}(k_2 \not{\epsilon}_3 k_2 \not{\epsilon}_2 \bar{p} \not{\epsilon}_1)) \\
& + \mathcal{F}_{100}(\text{Tr}(\not{\epsilon}_3 \not{\epsilon}_2 k_1 \not{\epsilon}_1) + \text{Tr}(\not{\epsilon}_3 k_1 \not{\epsilon}_2 \not{\epsilon}_1) - \text{Tr}(\not{\epsilon}_3 \not{\epsilon}_2 \not{\epsilon}_1 k_1) + 2\text{Tr}(\not{\epsilon}_3 k_2 \not{\epsilon}_2 \not{\epsilon}_1)) \\
& - \mathcal{F}_{110}(\text{Tr}(\not{\epsilon}_3 \not{\epsilon}_2 k_2 \not{\epsilon}_1) + \text{Tr}(\not{\epsilon}_3 k_2 \not{\epsilon}_2 \not{\epsilon}_1) + \text{Tr}(\not{\epsilon}_3 \not{\epsilon}_2 \not{\epsilon}_1 k_2)) + 2\mathcal{G}\text{Tr}(\bar{p} \not{\epsilon}_3 k_3 \not{\epsilon}_2 k_1 \not{\epsilon}_1) \\
& + 2\mathcal{A}_{1,2,3} \frac{s^2(x - \xi)}{(\underline{p}k_3 - i0)(\bar{p}k_1 + i0)} \log\left(\frac{1}{2\xi}(\xi - x + i0 \cdot \text{sgn}(pk_3))\right) \Big\} \\
& - \frac{\alpha_S}{4\pi} C_F \cdot \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \log(2\xi) \frac{x - \xi}{\xi} \log\left(\frac{1}{2\xi}(\xi - x + i0 \cdot \text{sgn}(\beta_3))\right). \quad (44)
\end{aligned}$$

Terms in the last lines of Eqs. (43) and (44) are not present in the work [14]—they result from using different definitions of the hard scale ($M_{\gamma\gamma}^2$ here versus s in [14]). The finite parts of diagrams 4.*R* and 5.*R* are obtained from (43) and (44) respectively by substituting $k_1 \rightarrow -k_3$, $k_2 \rightarrow -k_2$, $k_3 \rightarrow -k_1$, $\xi \rightarrow -\xi$, and $\epsilon_1 \leftrightarrow \epsilon_3$.

At NLO, the summation over photons' permutations does not yield as many simplifications as it did at the leading order, so that the calculation of the finite part will be performed numerically with the help of the PARTONS framework [16]. In Appendix A and B we rewrite Eqs. (43) and (44) with all traces computed, apart from $\text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}]$ which is given in Formula (20).

V. THE AXIAL AMPLITUDE

We shall not detail the corresponding analysis for the axial amplitude, which follows the same lines as the vector

part, but only quote the main results. The axial LO amplitude corresponding to a given permutation of photons reads [1]:

$$\begin{aligned}
\text{Tr}[i\mathcal{M}_{1,2,3}^0 \gamma^5 \not{p}] \\
= \frac{ie_q^3}{s^2 \beta_1 \beta_3} \frac{1}{(x + \xi) + i0_1} \frac{1}{(x - \xi) - i0_3} \text{Tr}(\not{\epsilon}_3 k_3 \not{\epsilon}_2 k_1 \not{\epsilon}_1 \gamma^5 \not{p}). \quad (45)
\end{aligned}$$

After summation over all permutations of photons one gets:

$$\tilde{C}_0 = -\frac{ie_q^3 \tilde{\mathcal{A}}}{M_{\gamma\gamma}^2 \alpha \tilde{\alpha}} \text{Im}\left(\frac{1}{x + \xi - i0}\right) - (x \rightarrow -x) \quad (46)$$

where:

$$\begin{aligned}\tilde{A} = & \frac{1}{2s} (\bar{\alpha} \text{Tr}(\not{\epsilon}(q) \not{q} \not{\epsilon}^*(q_2) \not{q}_1 \not{\epsilon}^*(q_1) \gamma^5 \not{p}) \\ & + \alpha \text{Tr}(\not{\epsilon}(q) \not{q} \not{\epsilon}^*(q_1) \not{q}_2 \not{\epsilon}^*(q_2) \gamma^5 \not{p})).\end{aligned}\quad (47)$$

At the first order in α_s , an analysis very similar to the one discussed above for the vector part, yields a divergent part equal to:

$$\begin{aligned}\tilde{C}_{\text{coll}} = & \frac{ie_q^3 \tilde{A}}{M_{\gamma\gamma}^2 \alpha \bar{\alpha}} \frac{\alpha_s C_F}{4\pi} \text{Im} \left[\frac{1}{x + \xi - i0} \right. \\ & \left. \times \left(-3 - 2 \log \left(\frac{x + \xi}{2\xi} - i0 \right) \right) \right] - (x \rightarrow -x)\end{aligned}\quad (48)$$

which allows to check factorization by explicit computation, the coefficient function divergences being exactly cancelled by those related to the evolution equations of the axial GPDs.

Since the smallness of the axial contribution has been demonstrated in [1] for the LO computation, we do not write down here the lengthy expressions for the finite part of the NLO axial amplitude. They have an analogous structure as those in the vector part, including the presence of the same functions $\mathcal{F}_{nab}$ and $\mathcal{G}$, the only difference being the traces.

VI. CONCLUSION

Our results on the NLO analysis of the scattering amplitude of process (1) is a first step toward a complete proof of QCD collinear factorization for the amplitude of such processes where the coefficient function describes a $2 \rightarrow 3$ hard scattering. We shall soon continue this study by examining the phenomenological consequences of our NLO results, both for reactions at electron machines at medium (JLab) or high energy [17,18] and in ultraperipheral reactions at LHC [19].

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APPENDIX A: BOX CONTRIBUTION WITH COMPUTED TRACES

To make the formulas more readable and easier to implement in numerical computations, we will denote products of polarization vectors and considered momenta as:

$$\epsilon_1 k_2 = -\epsilon_1 k_3 = ek_1, \quad \epsilon_2 k_1 = -\epsilon_2 k_3 = ek_2, \quad \epsilon_3 k_1 = -\epsilon_3 k_2 = ek_3. \quad (A1)$$

The product of two polarization vectors i and j will be denoted as $ei j = \epsilon_i \cdot \epsilon_j$.

$$\begin{aligned}\text{Tr}[i\mathcal{M}_{1,2,3}^{4L} \not{p}]_{\text{fin}} = & -i \frac{\alpha_s C_F}{4\pi} e_q^3 \frac{1}{2pk_3 - i0} 2 \{ -\mathcal{F}_{210} s^{-1} [\beta_3 (\kappa_1 s (e13 \cdot ek_2 - 3e12 \cdot ek_3) + 4ek_1 \cdot ek_2 \cdot ek_3) \\ & - s(e23 \cdot ek_1 - e13 \cdot ek_2) (\beta_1 (\kappa_1 + 2\kappa_2) - 2\beta_2 (\kappa_1 + \kappa_3))] \\ & + \mathcal{F}_{201} (x + \xi) [\beta_1^2 (e13 \cdot ek_2 - e23 \cdot ek_1) + \beta_1 (2\beta_2 e13 \cdot ek_2 - 2\beta_2 e23 \cdot ek_1 \\ & + 3\beta_3 e12 \cdot ek_3 - 9\beta_3 e13 \cdot ek_2 + 4\beta_3 e23 \cdot ek_1) + 2\beta_2 \beta_3 (e12 \cdot ek_3 - e13 \cdot ek_2)] \\ & - \mathcal{F}_{211} (x + \xi) [\beta_1 \beta_3 (e23 \cdot ek_1 - e12 \cdot ek_3) + 2\beta_2^2 (e23 \cdot ek_1 - e13 \cdot ek_2) \\ & + \beta_2 \beta_3 (-3e12 \cdot ek_3 + 3e13 \cdot ek_2 + 2e23 \cdot ek_1)] + \mathcal{F}_{220} s^{-1} [\beta_2 \kappa_1 s (e23 \cdot ek_1 - e13 \cdot ek_2) \\ & + \beta_3 (-e12 \cdot ek_3 \cdot \kappa_1 s + e23 \cdot ek_1 \cdot \kappa_1 s + 4ek_1 \cdot ek_2 \cdot ek_3)] \\ & + \mathcal{F}_{221} (x + \xi) \beta_2 [-\beta_2 e13 \cdot ek_2 + \beta_2 e23 \cdot ek_1 - \beta_3 e12 \cdot ek_3 + \beta_3 e23 \cdot ek_1] \\ & + \mathcal{F}_{100} [-\beta_1 e13 \cdot ek_2 + \beta_1 e23 \cdot ek_1 - 2\beta_2 e13 \cdot ek_2 \\ & + 2\beta_2 e23 \cdot ek_1 - 3\beta_3 e12 \cdot ek_3 + 3\beta_3 e13 \cdot ek_2 - 2\beta_3 e23 \cdot ek_1] \\ & + \mathcal{F}_{110} [3\beta_2 e13 \cdot ek_2 - 3\beta_2 e23 \cdot ek_1 + \beta_3 e12 \cdot ek_3 - \beta_3 e23 \cdot ek_1] \\ & + 2\mathcal{G}(x + \xi) \beta_3 [-\beta_1 e13 \cdot ek_2 + \beta_1 e23 \cdot ek_1 - \beta_3 e12 \cdot ek_3 + \beta_3 e13 \cdot ek_2] \} \\ & - \frac{\alpha_s C_F}{2\pi} \cdot \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \log(2\xi) \log \left(\frac{1}{2\xi} (\xi - x + i0 \cdot \text{sgn}(\beta_3)) \right).\end{aligned}\quad (A2)$$

APPENDIX B: PENTAGONAL CONTRIBUTION WITH COMPUTED TRACES

$$\begin{aligned}
\text{Tr}[i\mathcal{M}_{1,2,3}^{5,L}\not{p}]_{\text{fin}} = & -\frac{\alpha_S C_F}{4\pi} e_q^3 \frac{1}{\xi} s^{-2} 2 \left\{ +\mathcal{F}_{210}[ek2 \cdot (2ek1 \cdot ek3 - e13 \cdot \kappa_1 s) - 3ek3 \cdot (2ek1 \cdot ek2 - e12 \cdot \kappa_1 s)] \right. \\
& + s\mathcal{F}_{201}(x + \xi)[2\beta_1 e12 \cdot ek3 - 5\beta_1 e13 \cdot ek2 + 3\beta_1 e23 \cdot ek1 + \beta_2 e12 \cdot ek3 - \beta_2 e13 \cdot ek2] \\
& + \mathcal{F}_{211}(x + \xi)[\beta_1 e12 \cdot ek3 - \beta_1 e23 \cdot ek1 + 3\beta_2 e12 \cdot ek3 - 3\beta_2 e13 \cdot ek2] \\
& + \mathcal{F}_{220}[ek3 \cdot (2ek1 \cdot ek2 - e12 \cdot \kappa_1 s) + ek1 \cdot (e23 \cdot \kappa_1 s + 2ek2 \cdot ek3)] \\
& + s\mathcal{F}_{221}(x + \xi)[\beta_2 e23 \cdot ek1 - \beta_2 e12 \cdot ek3] \\
& - s\mathcal{F}_{100}[3e12 \cdot ek3 - 3e13 \cdot ek2 + 2e23 \cdot ek1] \\
& + s\mathcal{F}_{110}[e12ek3 - e23ek1] \\
& - s\mathcal{G}(x + \xi)[\beta_1 e13 \cdot ek2 - \beta_1 e23 \cdot ek1 + \beta_3 e12 \cdot ek3 - \beta_3 e13 \cdot ek2] \\
& + \frac{1}{2} \mathcal{A}_{1,2,3} \frac{s^2(x - \xi)}{(\underline{pk}_3 - i0)(\bar{p}k_1 + i0)} \log\left(\frac{1}{2\xi}(\xi - x + i0 \cdot \text{sgn}(pk_3))\right) \Big\} \\
& - \frac{\alpha_S}{4\pi} C_F \cdot \text{Tr}[i\mathcal{M}_{1,2,3}^0 \not{p}] \log(2\xi) \frac{x - \xi}{\xi} \log\left(\frac{1}{2\xi}(\xi - x + i0 \cdot \text{sgn}(\beta_3))\right). \tag{B1}
\end{aligned}$$

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