Magnetic dipole moments of the T_{cc}^+ and Z_V^{++} tetraquark states

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(Received 6 September 2021; accepted 22 November 2021; published 3 December 2021)

Inspired by the observation of the doubly charmed state T_{cc} and following theoretical studies on its spectroscopic parameters, we investigate its magnetic dipole moment, assigning it the quantum numbers $J^P = 1^+$, and both the compact diquark-antidiquark and molecular structures in the framework of the light cone QCD. We also calculate the magnetic dipole moment of the theoretically predicted singly charmed state, Z_V^{++} , with two units of electric charge and the quantum numbers $J^P = 1^-$ in diquark-antidiquark picture. The numerical results are obtained as $\mu_{T_{cc}^+-Di} = 0.66_{-0.23}^{+0.34} \mu_N$, $\mu_{T_{cc}^+-Mol} = 0.43_{-0.22}^{+0.23} \mu_N$ and $\mu_{Z_V^{++}} = 3.35_{-0.73}^{+0.89} \mu_N$. These results may be checked via other phenomenological approaches. The obtained results may be useful in exact determinations of the natures of these states.

DOI: [10.1103/PhysRevD.104.114002](https://doi.org/10.1103/PhysRevD.104.114002)

I. INTRODUCTION

Besides the standard mesons and baryons/antibaryons, formed of $q\bar{q}$ and $qqq/\bar{q}\bar{q}\bar{q}$, there exist hadrons made of more than three quarks/antiquarks or valence gluons. These categories of hadrons contain exotic states including glueballs, hybrids, tetraquarks, pentaquarks, hexaquarks, etc. In last two decades, a large number of these exotic states have been observed in the particle factories. Although some of them are now well established, there are some doubts on the existence of some other members. Exact determinations of the nature, structure, and quantum numbers of these states need more experimental efforts. Following the experiments, and sometimes before the experimental results, these states are investigated by many theoretical and phenomenological models and approaches. There are many exotic states, proposed in theory, waiting to be confirmed by the experiments. For the recent experimental and theoretical progresses on the exotic states see, for instance, Refs. [1–13]. Let us make a note here that most of heavy multiquark particles discovered in the experiment, to date, have hidden-charm or hidden-bottom quark structure; i.e., they contain $c\bar{c}$ or $b\bar{b}$ in their inner structures.

Very recently, the LHCb collaboration reported observation of a first doubly charmed tetraquark state T_{cc}^+ in the $D^0 D^0 \pi^+$ mass spectrum with over 10σ significance [14,15]. The simplest assumption on its valence quark component is $cc\bar{u}\bar{d}$ and its spin parity was suggested by the experiment as $J^P = 1^+$. Its mass, with respect to the $D^0 D^{*+}$ threshold, and width have been measured to be

$$\begin{aligned} \delta m &= m_{T_{cc}^+} - (m_{D^0} + m_{D^{*+}}), \\ &= -273 \pm 61 \pm 5_{-14}^{+11} \text{ KeV}, \end{aligned} \quad (1)$$

$$\Gamma = 410 \pm 165 \pm 43_{-38}^{+18} \text{ KeV}. \quad (2)$$

As we see, its decay width is very small and its decay to $D^0 D^0 \pi^+$ occur very slowly and with delay. Because of that this particle is the longest living exotic state discovered till now. The discovery of the first doubly charmed tetraquark state will, undoubtedly, usher in a new era in the study of hadron spectroscopy and improve our understanding of the nonperturbative nature of the strong interaction. Before and after the experimental discovery, some spectroscopic properties and decay channels of the T_{cc}^+ state have been investigated within different theoretical models [16–27]. Note that the spectroscopic parameters and different decay modes of the scalar and pseudoscalar T_{cc} states as well as the T_{bc} and T_{bb} states of different quantum numbers were already investigated in Refs. [28–34], which may be in the agenda of future experiments. Another important subclass of doubly charmed tetraquarks includes particles that bear two units of electric charge. Spectroscopic parameters and

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decay widths of such exotic states with $cc\bar{d}\bar{s}$ and $cc\bar{s}\bar{s}$ quark contents were investigated in Ref. [35]. These particles have not been observed experimentally yet, but their existence is important for understanding the nature of exotic states. We calculate the magnetic dipole moment (MDM) of the T_{cc}^+ state newly detected by LHCb collaboration [14,15] in order to shed light on its nature and physical properties.

Recently, the LHCb collaboration announced the observation of two new structures $X_0(2900)$ and $X_1(2900)$ in the process $B^+ \rightarrow D^+ D^- K^+$ [36,37]. These resonances appear as the intermediate states of the decay chain $B^+ \rightarrow D^+ X \rightarrow D^+ D^- K^+$, and they are neutral four-quark states. These decays take place due to color-favored and color-suppressed modes of the B^+ meson, as also stated in Ref. [38]. But weak decays of B^+ may provoke also the decays $B^+ \rightarrow D^- Z^{++} \rightarrow D^- D^+ K^+$, where Z^{++} is a doubly charged exotic state with inner structure of $cu\bar{s}\bar{d}$ (for more information see [39]). It is expected that the LHCb collaboration observe the doubly charged tetraquarks $Z^{++} = [cu][\bar{s}\bar{d}]$ with different spin-parities in the above processes. Thus, the possible scalar and vector four-quark resonances Z_S^{++} and Z_V^{++} may be found as resonances in the $D^+ K^+$ invariant mass. Motivated by this, such states are proposed in Ref. [39] and the spectroscopic properties and possible decay channels of Z_V^{++} state are investigated using two-point QCD sum rules. Because of the importance of these states, in the second part of this study, we investigate the electromagnetic properties of Z_V^{++} state. Because of two units of electric charge, the MDM of this state is expected to be large and easily accessible in the future experiments. Note that the MDM of the charmed-strange $Z_{cs}(3985)$ tetraquark was calculated in Ref. [40] using the light cone QCD sum rule formalism.

The electromagnetic multipole moments of hadrons, besides their spectroscopic parameters, can help us determine their exact nature, substructure, and quantum numbers. These parameters give useful information about the charge and magnetization distributions inside the hadrons that help us gain information on their geometric shapes. Recall that the magnetic and higher order moments of particles that contain information about the spatial distributions of charge and magnetization within the particles are directly related to the spatial distributions of quarks and gluons in hadrons.

In the present study, we calculate the MDMs of T_{cc}^+ and Z_V^{++} states in the compact diquark-antidiquark picture using the light cone QCD sum rule formalism. For the case of T_{cc}^+ , we also extract its MDM considering it as the $D^* D + DD^*$ molecular state. We employ the distribution amplitudes (DAs) of the on shell real photon state at light cone, which are available in terms of different twists.

The paper is organized in the following way: in Sec. II, we calculate the MDM of the T_{cc}^+ and Z_V^{++} states. In Sec. III, we numerically analyze the MDM of the T_{cc}^+ and

Z_V^{++} states. The last section is devoted to the summary of the results and conclusions.

II. FORMALISM

In the light cone QCD sum rule method, we calculate a correlation function as a building block of the method, once in terms of hadronic parameters such as electromagnetic form factors and, second, in terms the QCD parameters and DAs of the on shell photon. By matching the coefficients of suitable Lorentz structures from both sides and using the assumption of quark-hadron duality, we will be able to evaluate the related hadronic observables in terms of QCD degrees of freedom as well as auxiliary parameters entering the calculations at different steps.

A. MDM of the T_{cc}^+ state

To evaluate the MDM of the T_{cc}^+ state within the light cone sum rules, we start with the correlation function

$$\Pi_{\mu\nu}^{T_{cc}^+}(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_{\mu}^{T_{cc}^+}(x) J_{\nu}^{T_{cc}^+ \dagger}(0) \} | 0 \rangle_{\gamma}, \quad (3)$$

where the subindex γ represents the background electromagnetic field and $J_{\mu}(x)$ is the interpolating current of the T_{cc}^+ state with the quantum numbers $J^P = 1^+$ and the quark content $cc\bar{u}\bar{d}$. The T_{cc}^+ state can be interpolated by either compact axial vector diquark-light scalar antidiquark or molecular $D^* D + DD^*$ structures as

$$\begin{aligned} J_{\mu}^{T_{cc}^+ - Di}(x) &= c_a^T(x) \gamma_{\mu} C c_b(x) \bar{u}_a(x) C \gamma_5 \bar{d}_b^T(x), \\ J_{\mu}^{T_{cc}^+ - Mol}(x) &= \frac{1}{\sqrt{2}} \{ [\bar{u}_a(x) i \gamma_5 c_a(x)] [\bar{d}_b(x) \gamma_{\mu} c_b(x)] \\ &\quad + [\bar{u}_a(x) \gamma_{\mu} c_a(x)] [\bar{d}_b(x) i \gamma_5 c_b(x)] \}. \end{aligned} \quad (4)$$

In Eq. (4), $u(x)$, $c(x)$, and $d(x)$ are the quark fields, and C stands for the charge-conjugation operator. In this equation the currents $J_{\mu}^{T_{cc}^+ - Di}(x)$ and $J_{\mu}^{T_{cc}^+ - Mol}(x)$ stand for compact diquark-antidiquark and molecular structures, respectively. In the following, we will explicitly present the calculations only for $J_{\mu}^{T_{cc}^+ - Di}(x)$ case and for simplicity we will omit the label Di from the current. However, we do all the calculations in this subsection in both pictures and will extract the numerical values of MDM of T_{cc}^+ in both pictures.

In hadronic side, the correlation function is evaluated by its saturation with the intermediate hadronic states. By performing the four integral over x we get

$$\Pi_{\mu\nu}^{\text{Had}-T_{cc}^+}(p, q) = \frac{\langle 0 | J_{\mu}^{T_{cc}^+} | T_{cc}^+(p) \rangle \langle T_{cc}^+(p) | T_{cc}^+(p+q) \rangle_{\gamma}}{p^2 - m_{T_{cc}^+}^2} \times \frac{\langle T_{cc}^+(p+q) | J_{\nu}^{T_{cc}^+} | 0 \rangle}{(p+q)^2 - m_{T_{cc}^+}^2} + \dots, \quad (5)$$

where q is the photon's momentum and dots stand for the contributions coming from the higher states and continuum. The matrix element $\langle 0 | J_{\mu}^{T_{cc}^+} | T_{cc}^+ \rangle$ is parametrized as

$$\langle 0 | J_{\mu}^{T_{cc}^+} | T_{cc}^+ \rangle = \lambda_{T_{cc}^+} \varepsilon_{\mu}^{\theta}, \quad (6)$$

with $\lambda_{T_{cc}^+}$ being the residue of the T_{cc}^+ state and $\varepsilon_{\mu}^{\theta}$ is its polarization vector.

In the existence of the external electromagnetic background field, the matrix element $\langle T_{cc}^+(p) | T_{cc}^+(p+q) \rangle_{\gamma}$ can be expressed in terms of the Lorentz invariant form factors as follows [41]:

$$\langle T_{cc}^+(p, \varepsilon^{\theta}) | T_{cc}^+(p+q, \varepsilon^{\delta}) \rangle_{\gamma} = -\varepsilon^{\tau} (\varepsilon^{\theta})^{\alpha} (\varepsilon^{\delta})^{\beta} \left[G_1(Q^2) (2p+q)_{\tau} g_{\alpha\beta} + G_2(Q^2) (g_{\tau\beta} q_{\alpha} - g_{\tau\alpha} q_{\beta}) - \frac{1}{2m_{T_{cc}^+}^2} G_3(Q^2) (2p+q)_{\tau} q_{\alpha} q_{\beta} \right], \quad (7)$$

where ε^{δ} and ε^{θ} are the polarization vectors of the initial and final T_{cc}^+ states and ε^{τ} is the polarization vector of the photon. Here, $G_1(Q^2)$, $G_2(Q^2)$, and $G_3(Q^2)$ are invariant form factors, with $Q^2 = -q^2$.

Using Eqs. (5)–(7), the correlation function takes the form

$$\Pi_{\mu\nu}^{\text{Had}-T_{cc}^+}(p, q) = \frac{\varepsilon_{\rho} \lambda_{T_{cc}^+}^2}{[m_{T_{cc}^+}^2 - (p+q)^2][m_{T_{cc}^+}^2 - p^2]} \left[G_2(Q^2) \left(q_{\mu} g_{\rho\nu} - q_{\nu} g_{\rho\mu} - \frac{p_{\nu}}{m_{T_{cc}^+}^2} \left(q_{\mu} p_{\rho} - \frac{1}{2} Q^2 g_{\mu\rho} \right) + \frac{(p+q)_{\mu}}{m_{T_{cc}^+}^2} \left(q_{\nu} (p+q)_{\rho} + \frac{1}{2} Q^2 g_{\nu\rho} \right) - \frac{(p+q)_{\mu} p_{\nu} p_{\rho}}{m_{T_{cc}^+}^4} Q^2 \right) + \text{other independent structures} \right] + \dots \quad (8)$$

To determine the MDM, the value of the form factor $G_2(Q^2)$ is needed only at $Q^2 = 0$. The magnetic form factor $F_M(Q^2)$ is written as

$$F_M(Q^2) = G_2(Q^2), \quad (9)$$

the value of which at $Q^2 = 0$, i.e., $F_M(0)$, is proportional to the MDM, $\mu_{T_{cc}^+}$:

$$\mu_{T_{cc}^+} = \frac{e}{2m_{T_{cc}^+}} F_M(0). \quad (10)$$

In QCD side, the correlation function in Eq. (3), is calculated in deep Euclidean region in terms of QCD degrees of freedom as well as the DAs of the photon. To this end, we substitute the explicit forms of the interpolating currents in the correlation function and contract the corresponding quark fields with the help of the Wick's theorem. As a result, we get

$$\Pi_{\mu\nu}^{\text{QCD}-T_{cc}^+}(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | \left\{ \text{Tr}[\gamma_5 \tilde{S}_d^{b'b}(-x) \gamma_5 S_u^{a'a}(-x)] \text{Tr}[\gamma_{\nu} \tilde{S}_c^{aa'}(x) \gamma_{\mu} S_c^{bb'}(x)] - \text{Tr}[\gamma_5 \tilde{S}_d^{b'b}(-x) \gamma_5 S_u^{a'a}(-x)] \text{Tr}[\gamma_{\nu} \tilde{S}_c^{ba'}(x) \gamma_{\mu} S_c^{ab'}(x)] \right\} | 0 \rangle_{\gamma}, \quad (11)$$

where

$$\tilde{S}_{c(q)}^{ij}(x) = C S_{c(q)}^{ijT}(x) C,$$

with $S_{q(c)}(x)$ being the full light (charm) quark propagator. They are given as

$$S_q(x) = i \frac{\not{x}}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} \left(1 - i \frac{m_q \not{x}}{4} \right) - \frac{\langle \bar{q}q \rangle}{192} m_0^2 x^2 \left(1 - i \frac{m_q \not{x}}{6} \right) - \frac{ig_s}{32\pi^2 x^2} G^{\mu\nu}(x) [\not{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\not{x}], \quad (12)$$

$$S_c(x) = \frac{m_c^2}{4\pi^2} \left[\frac{K_1(m_c \sqrt{-x^2})}{\sqrt{-x^2}} + i \frac{\not{x} K_2(m_c \sqrt{-x^2})}{(\sqrt{-x^2})^2} \right] - \frac{g_s m_c}{16\pi^2} \int_0^1 dv G^{\mu\nu}(vx) \left[(\sigma_{\mu\nu} \not{x} + \not{x} \sigma_{\mu\nu}) \right. \\ \left. \times \frac{K_1(m_c \sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma_{\mu\nu} K_0(m_c \sqrt{-x^2}) \right], \quad (13)$$

where $\langle \bar{q}q \rangle$ is quark condensate, m_0 is defined through the quark-gluon mixed condensate $\langle 0 | \bar{q} g_s \sigma_{\alpha\beta} G^{\alpha\beta} q | 0 \rangle = m_0^2 \langle \bar{q}q \rangle$, $G^{\mu\nu}$ is the gluon field strength tensor, $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$, and K_i s are modified Bessel functions of the second kind.

Finally, we choose the Lorentz invariant structure $q_\mu \epsilon_\nu$ from the both sides and match its coefficients from both the hadronic and QCD sides. To eliminate the contributions of the higher states and continuum, we perform the Borel transformation and continuum subtraction. The procedures are lengthy but standard, we do not present the steps here and refer the reader for instance to Ref. [42]. As a results, we obtain the light cone QCD sum rule for the MDM as

$$\mu_{T_{cc}^+} \lambda_{T_{cc}^+}^2 = e^{\frac{m_c^2}{M^2}} \Pi^{\text{QCD}-T_{cc}^+}(M^2, s_0), \quad (14)$$

where M^2 and s_0 are auxiliary parameters stemming from the applications of the Borel transformation and continuum procedures. The $\Pi^{\text{QCD}-T_{cc}^+}(M^2, s_0)$ function is quite lengthy, so the explicit expression of it is not presented here.

B. MDM of the Z_V^{++} state

As we mentioned in the previous subsection, calculations are started by writing the appropriate correlation function,

$$\Pi_{\mu\nu}^{Z_V^{++}}(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | \mathcal{T} \{ J_\mu^{Z_V^{++}}(x) J_\nu^{Z_V^{++}\dagger}(0) \} | 0 \rangle_\gamma, \quad (15)$$

$$\Pi_{\mu\nu}^{\text{Had}-Z_V^{++}}(p, q) = \frac{\epsilon_\rho \lambda_{Z_V^{++}}^2}{[m_{Z_V^{++}}^2 - (p+q)^2][m_{Z_V^{++}}^2 - p^2]} \left[G_2(Q^2) \left(q_\mu g_{\rho\nu} - q_\nu g_{\rho\mu} - \frac{p_\nu}{m_{Z_V^{++}}^2} \left(q_\mu p_\rho - \frac{1}{2} Q^2 g_{\mu\rho} \right) \right. \right. \\ \left. \left. + \frac{(p+q)_\mu}{m_{Z_V^{++}}^2} \left(q_\nu(p+q)_\rho + \frac{1}{2} Q^2 g_{\nu\rho} \right) - \frac{(p+q)_\mu p_\nu p_\rho}{m_{Z_V^{++}}^4} Q^2 \right) + \text{other independent structures} \right] + \dots. \quad (18)$$

The QCD side of the sum rules, $\Pi_{\mu\nu}^{\text{QCD}-Z_V^{++}}(p, q)$, is obtained by inserting the interpolating current $J_\mu(x)$ into Eq. (15), and contracting proper quark fields via Wick's theorem. After these steps, we obtain

where the $J_\mu^{Z_V^{++}}(x)$ is interpolating current of the Z_V^{++} state and it is given in the diquark-antidiquark picture with the quantum number $J^P = 1^-$ as

$$J_\mu^{Z_V}(x) = \epsilon \tilde{\epsilon} [u_b^T(x) C \gamma_5 c_c(x)] [\bar{s}_d(x) \gamma_\mu \gamma_5 \bar{d}_e^T(x)], \quad (16)$$

where $\epsilon \tilde{\epsilon} = \epsilon^{abc} \epsilon^{dec}$, and a, b, c, d , and e denote quark colors. In Eq. (16), $u(x)$, $c(x)$, $s(x)$, and $d(x)$ are the quark fields, and C stands for the charge-conjugation operator.

To obtain required sum rules for parameters of Z_V^{++} state, we have to represent the correlation function $\Pi_{\mu\nu}^{Z_V^{++}}(p, q)$ with respect to these parameters, and get the hadronic side of the sum rules $\Pi_{\mu\nu}^{\text{Had}-Z_V^{++}}(p, q)$. In connection with the hadronic parameters the correlation function has the subsequent form

$$\Pi_{\mu\nu}^{\text{Had}-Z_V^{++}}(p, q) = \frac{\langle 0 | J_\mu^{Z_V^{++}} | Z_V^{++}(p) \rangle \langle Z_V^{++}(p) | Z_V^{++}(p+q) \rangle_\gamma}{p^2 - m_{Z_V^{++}}^2} \\ \times \frac{\langle Z_V^{++}(p+q) | J_\nu^{Z_V^{++}} | 0 \rangle}{(p+q)^2 - m_{Z_V^{++}}^2} + \dots, \quad (17)$$

where the dots stand for contributions of higher resonances and continuum states. The matrix element of the electromagnetic current between the Z_V^{++} states is described by three form factors and is presented in Eq. (7). Using the corresponding equations, the result for the hadronic side is obtained as follows:

$$\Pi_{\mu\nu}^{\text{QCD}-Z_V^{++}}(p, q) = i \int d^4x e^{ipx} \varepsilon \tilde{e} e' \tilde{e}' \langle 0 | \text{Tr}[\gamma_5 \tilde{S}_u^{bb'}(x) \gamma_5 S_c^{cc'}(x)] \text{Tr}[\gamma_\mu \gamma_5 \tilde{S}_d^{e'e}(-x) \gamma_5 \gamma_\nu S_s^{d'd}(-x)] | 0 \rangle_\gamma. \quad (19)$$

The sum rules for the MDM of the Z_V^{++} state can be obtained by matching the hadronic and QCD sides of the corresponding correlation function and performing usual operations required in QCD sum rule calculations. As a result of these processes, we get

$$\mu_{Z_V^{++}} \lambda_{Z_V^{++}}^2 = e \frac{m_{Z_V^{++}}^2}{M^2} \Pi^{\text{QCD}-Z_V^{++}}(M^2, s_0). \quad (20)$$

The analytical calculations are ended here. Now, we are ready to move on the numerical analysis part.

III. NUMERICAL ANALYSIS

In this section, we perform the numerical analysis of the sum rules for the MDMs of the T_{cc}^+ and Z_V^{++} states obtained in the previous section. We use $m_u = m_d = 0$, $m_s = 96_{-4}^{+8}$ MeV, $m_c = (1.275 \pm 0.025)$ GeV, $m_{T_{cc}^+} = 3868 \pm 124$ MeV [20], $m_{Z_V^{++}} = 3515 \pm 125$ MeV [39], $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$ with $\langle \bar{u}u \rangle = (-0.24 \pm 0.01)^3$ GeV³ [43], $m_0^2 = 0.8 \pm 0.1$ GeV² [43], $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.012 \pm 0.004)$ GeV⁴ [44], $\lambda_{T_{cc}^+-Di} = (1.96 \pm 0.44) \times 10^{-2}$ GeV⁵ [20], $\lambda_{T_{cc}^+-Mol} = (1.56 \pm 0.22) \times 10^{-2}$ GeV⁵ [27], and $\lambda_{Z_V^{++}} = (1.85 \pm 0.45) \times 10^{-2}$ GeV⁵ [39]. The wave functions inside the DAs of the photon and all the related parameters are borrowed from Ref. [45].

The next step in the numerical analysis is to find the working regions of the auxiliary parameters known as the continuum threshold s_0 and the Borel parameter M^2 . To this end, we enforce the following conditions: convergence of operator product expansion, pole dominance, as well as minimal sensitivity of the resulting observables to auxiliary parameters. As a result of these constraints, the following working regions are obtained for these parameters:

$$\begin{aligned} 4.0 \text{ GeV}^2 &\leq M^2 \leq 6.0 \text{ GeV}^2 && \text{for } T_{cc}^+ \text{ state,} \\ 4.0 \text{ GeV}^2 &\leq M^2 \leq 6.0 \text{ GeV}^2 && \text{for } Z_V^{++} \text{ state,} \\ 19.5 \text{ GeV}^2 &\leq s_0 \leq 21.5 \text{ GeV}^2 && \text{for } T_{cc}^+ \text{ state,} \\ 15.0 \text{ GeV}^2 &\leq s_0 \leq 17.0 \text{ GeV}^2 && \text{for } Z_V^{++} \text{ state.} \end{aligned}$$

Having determined the working regions of M^2 and s_0 , we now study the dependence of MDM of the T_{cc}^+ and Z_V^{++} states on M^2 , at several fixed values of s_0 in Fig. 1. We notice that MDM of the T_{cc}^+ and Z_V^{++} states show good stability with respect to the variation in M^2 in its working region, especially at lower values of s_0 . We also find that these MDMs exhibit residual dependence on the variation

of s_0 , which appear as the main sources of uncertainties in the results.

The final results obtained from the analyses for the MDMs, in both the compact tetraquark or diquark-anti-diquark and molecular pictures for T_{cc}^+ state and the

FIG. 1. The dependence of MDMs of the T_{cc}^+ and Z_V^{++} states on M^2 at three fixed values of s_0 .

compact tetraquark of diquark-antidiquark picture for Z_V^{++} state are given as

$$\begin{aligned}\mu_{T_{cc}^+-Di} &= 0.66_{-0.23}^{+0.34} \mu_N, \\ \mu_{T_{cc}^+-Mol} &= 0.43_{-0.22}^{+0.23} \mu_N, \\ \mu_{Z_V^{++}} &= 3.35_{-0.73}^{+0.89} \mu_N.\end{aligned}\quad (21)$$

Errors in the results are due to the uncertainties related to the calculations of the working regions for s_0 and M^2 as well as those of other input parameters. It should be noted again that the main source of errors in the results is the variations of the results with respect to s_0 . From Eq. (21), we see that although the central values of the MDM obtained for the T_{cc}^+ state in two diquark-antidiquark and molecular pictures differ from each other considerably; however, they are consistent with each other within the presented uncertainties. This is usual situation in the case of tetraquarks against the standard baryons of three quarks: the Σ and Λ baryons contain the same quark content and they are members of the same octet, but they have very different MDM. This means that the MDM is not able to distinguish between the two pictures in T_{cc}^+ channel from theoretical point of view. Future possible experimental or lattice results on the MDM of this doubly charmed state and their comparison with the results obtained in the present study as well as comparison of the existing and future experimental results on the mass, width, and other decay parameters, related to the various interactions of this state with other particles, with the theoretical predictions of different phenomenological models will shed light on the inner structure and nature of this state and help us fix its quantum numbers and quark-gluon organization. The result for the MDM of Z_V^{++} state, due to its double charge, is considerably large compared to other tetraquarks. Any future measurements of its electromagnetic properties and comparison of the results with theoretical predictions will be very useful in our understanding of tetraquarks and as a result the QCD as theory of strong interaction.

IV. DISCUSSION AND CONCLUDING REMARKS

The MDM, together with other electromagnetic observables, of the composite particles encode important information on their nature, substructure, and geometric shape. They contain information on the spatial distributions of charge and magnetization inside the particles that directly give information on the spatial distributions of quarks and gluons inside hadrons.

In this study, the MDMs of the recently observed doubly charmed exotic state T_{cc}^+ as well as the recently proposed doubly charged four-quark candidate Z_V^{++} were investigated within the framework of light cone QCD. We considered both the compact diquark-antidiquark and molecular pictures for T_{cc}^+ but only the diquark-antidiquark organization for the Z_V^{++} state. They were treated as states with $J^P = 1^+$ and $J^P = 1^-$ quantum numbers, respectively.

We extracted the values of the MDMs as $\mu_{T_{cc}^+-Di} = 0.66_{-0.23}^{+0.34} \mu_N$, $\mu_{T_{cc}^+-Mol} = 0.43_{-0.22}^{+0.23} \mu_N$, and $\mu_{Z_V^{++}} = 3.35_{-0.73}^{+0.89} \mu_N$. The obtained results for the $\mu_{T_{cc}^+}$ in two pictures are consistent within the presented uncertainties although their central values differ from each other considerably. This indicates that the MDM is not able to distinguish between the two pictures. The obtained results in this study for the MDM of this state together with the results of other theoretical models on the mass, width, and other decay properties of this state and comparison of the obtained results with the existing and future experimental data will shed light on the nature of the newly seen T_{cc}^+ tetraquark. The result for the MDM of Z_V^{++} state, due to its double charge, is considerably large. The orders of MDMs, especially for $\mu_{Z_V^{++}}$, indicate that they are accessible in the experiment. Our result on MDM of Z_V^{++} together with its spectroscopic parameters calculated in Ref. [39], may help experimental groups in the search for this state and its properties in the experiment. Our results on MDMs may be checked via alternative phenomenological approaches.

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