One-loop divergences in the six-dimensional $\mathcal{N} = (1,0)$ hypermultiplet self-coupling model

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We study the six-dimensional $\mathcal{N} = (1,0)$ supersymmetric model of the interacting gauge multiplet and hypermultiplet with arbitrary self-coupling. Using the background-field method in the harmonic superspace, we calculate the divergent part of the one-loop effective action and discuss the possible finite contribution to the low-energy effective action. We demonstrate that the one-loop divergences do not vanish even in the case of the on-shell background superfields. An application of the developed technique to the hypermultiplet model in four dimensions is briefly discussed.

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I. INTRODUCTION

The study of the scalar-field models in diverse dimensions has attracted much interest in the last few years. The $O(N)$ vector model of the scalar fields with the relevant interactions in six dimensions has been extensively studied in the context of a critical phenomenon and large N expansion [1,2] (for a review see [3–5]). Supersymmetry restricts possible interaction of the scalar fields, described in the general case by the supersymmetric nonlinear sigma models. These theories highlight the close connections between supersymmetry and complex geometry. For the simplest supersymmetric sigma model with four supercharges, the target space is known to be a Kähler manifold [6]. The rigid supersymmetric sigma model with eight supercharges in diverse dimensions possessed by hyper-Kähler manifolds as target space. Such a sigma model has $\mathcal{N} = 2$ supersymmetry in four dimensions and $\mathcal{N} = (1,0)$ supersymmetry in six dimensions [7,8]. In its turn, the target space of the locally supersymmetric sigma model with eight supercharges is a quaternionic Kähler manifold [9] (see also [10,11] and references therein).

The formulation of the supersymmetric sigma models in terms of unconstrained superfields allows one to deeply understand the relationship between geometry and supersymmetry. The superfield technique is a convenient tool to

study quantum properties of the model, which preserves explicit supersymmetry during quantization (see, e.g., [12]).¹ We study the six-dimensional $\mathcal{N} = (1,0)$ supersymmetric hypermultiplet model minimally interacting with an Abelian gauge multiplet. We consider the case of the arbitrary gauge-invariant potential function for the hypermultiplet [14]. For the vanishing gauge multiplet, this theory describes the supersymmetric sigma-model with a general multicenter hyper-Kähler metric [15] (for a review see [16]). We formulate the model in the six-dimensional $\mathcal{N} = (1,0)$ harmonic superspace [16–19] and develop the covariant technique to study the effective action. We construct the one-loop effective action in the theory under consideration using the proper-time method in the harmonic superspace [20–23].

The study of the ultraviolet behavior of supersymmetric field theories in six dimensions has received considerable attention in recent years. The analysis of the UV divergences is carried out using both symmetry considerations [24–28] and direct calculations of the effective action [20,21,29–33] (see also earlier analysis [34–37]) and scattering amplitudes [38–43]. As an application of the developed technique, we calculate the one-loop divergent contributions to the effective action in the model under consideration. The result depends on both the background gauge multiplet and hypermultiplet. We also discuss the possible leading finite contribution to the effective action and calculate it for the case of the vanishing background hypermultiplet.

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¹The one-loop effective action for a general four-dimensional $\mathcal{N} = 1$ supersymmetric sigma model was studied using the superfield approach in the work [13].

The work is organized as follows. In Sec. II we discuss the basic properties of six-dimensional $\mathcal{N} = (1, 0)$ harmonic superspace and formulate the models. Section III contains the quantization procedure in the $\mathcal{N} = (1, 0)$ harmonic superspace and the evaluation of the one-loop effective action. Analysis of divergent contributions to the one-loop effective action is collected in Sec. IV. Finally, in Sec. V, we summarize the results obtained and briefly discuss the application of the developed technique to the $\mathcal{N} = 2$ supersymmetric hypermultiplet model in four dimensions.

II. HYPERMULTIPLY THEORY IN 6D

Throughout the paper, we follow the notations and conventions from the work [25]. We denote the 6D Minkowski space coordinates by x^M and the Grassmann coordinates by θ_i^a , where $M = 0, \dots, 5$, $a = 1, \dots, 4$. The additional $SU(2)$ index $i = 1, 2$ corresponds to the R -symmetry group of the simplest $\mathcal{N} = (1, 0)$ supersymmetry in six dimensions.

We use the harmonic superspace formulation [16] for the theory. The six-dimensional $\mathcal{N} = (1, 0)$ harmonic superspace coordinates $(x^M, \theta^{ai}, u^{\pm i})$ include additional harmonic variables $u^{\pm i}$. They parametrize the coset $SU(2)/U(1)$ and obey the constraints $u^+ u^- = 1$, $u_i^- \equiv (u^+)^*$. The harmonic superspace naturally admits the analytic basis, which is parametrized by the coordinates $\zeta = (x_A^M, \theta^{\pm a})$

$$x_A^M = x^M + \frac{i}{2} \theta^{+a} \gamma_{ab}^M \theta^{-b}, \quad \theta^{\pm a} = u_k^{\pm} \theta^{ak}. \quad (1)$$

In the analytic basis the spinor and harmonic derivatives are defined as follows [25]:

$$D_a^+ = \partial_{-a}, \quad D_a^- = -\partial_{+a} - 2i\theta^{-b} \partial_{ab}, \quad (2)$$

$$D^0 = u^{+i} \partial_{-i} - u^{-i} \partial_{+i} + \theta^{+a} \partial_{+a} - \theta^{-a} \partial_{-a},$$

$$D^{\pm\pm} = \partial^{\pm\pm} + i\theta^{\pm a} \theta^{\pm b} \partial_{ab} + \theta^{\pm a} \partial_{\mp a}. \quad (3)$$

Here we denoted $\partial_{\pm a} \theta^{\mp b} = \delta_a^b$, $\partial^{\pm\pm} = u^{\pm i} \partial_{\pm i}$, $\partial_{\pm i} = \frac{\partial}{\partial u^{\mp i}}$ and $\partial_{ab} = \frac{1}{2} (\gamma^M)_{ab} \partial_M$. The six-dimensional Weyl matrices $(\gamma^M)_{ab}$ are chosen as

$$(\gamma^M)_{ab} = -(\gamma^M)_{ba} \tilde{\gamma}_M^{ab} = \frac{1}{2} \varepsilon^{abcd} (\gamma_M)_{cd}, \quad (\gamma_M)_{ac} (\tilde{\gamma}_N)^{cb} \\ + (\gamma_N)_{ac} (\tilde{\gamma}_M)^{cb} = -2\delta_a^b \eta_{MN}, \quad (4)$$

where ε^{abcd} is the totally antisymmetric symbol. The derivatives listed above satisfy the algebra

$$\{D_a^+, D_b^-\} = 2i\partial_{ab}, \quad [D^{++}, D^{--}] = D^0, \\ [D^{\pm\pm}, D_a^{\pm}] = 0, \quad [D^{\pm\pm}, D_a^{\mp}] = D_a^{\pm}. \quad (5)$$

We include the integration over harmonic variables $u^{\pm i}$ only into the analytic superspace integration measure,

$$d\zeta^{(-4)} = d^6 x_A du (D^-)^4, \quad (6)$$

where $(D^\pm)^4 = -\frac{1}{24} \varepsilon^{abcd} D_a^\pm D_b^\pm D_c^\pm D_d^\pm$. The integration measure over the full superspace includes $d^{14}z = d^6 x_A (D^+)^4 (D^-)^4$ and the integration over harmonics du .

We consider the $\mathcal{N} = (1, 0)$ supersymmetric model of interacting q -hypermultiplet and Abelian gauge multiplet V^{++} in the six-dimensional harmonic superspace. We assume a general gauge-invariant self-coupling potential $L^{(+4)}$ for the superfield q^+ [16]. The action for the model is

$$S_0[q^+, V^{++}] \\ = \frac{1}{4f^2} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} V^{++}(z, u_1) V^{++}(z, u_2) \\ - \int d\zeta^{(-4)} (\tilde{q}^+ D^{++} q^+ + i\tilde{q}^+ V^{++} q^+ + L^{(+4)}), \quad (7)$$

where the coupling constant f has the dimension of the inverse mass. The gauge transformations

$$\delta V^{++} = -D^{++}\lambda, \quad \delta q^+ = i\lambda q^+, \quad \delta \tilde{q}^+ = -i\lambda \tilde{q}^+, \quad (8)$$

with a real analytic gauge parameter $\lambda = \lambda(\zeta, u)$, hold the action (7) invariant when the potential function for a hypermultiplet depends on the invariant combination of superfields, $L^{(+4)} = L^{(+4)}(\tilde{q}^+ q^+)$. The potential function $L^{(+4)}$ is a real one under the Grassmann conjugation [16], $\tilde{L}^{(+4)} = L^{(+4)}$.

The action (7) describes the minimal interaction of the dynamical Abelian gauge multiplet and hypermultiplet with an arbitrary potential function. The aim of the paper is to study the one-loop divergent contributions to the effective action for the model (7). In the case of the vanishing self-coupling potential, $L^{(+4)} = 0$, the model (7) was studied in [20].

III. ONE-LOOP EFFECTIVE ACTION

The effective action for the model (7) can be constructed using the background-field method in harmonic superspace [20–23]. We take the superfields V^{++}, q^+ as a sum of the “background” V^{++}, Q^+ and the “quantum” superfields v^{++}, q^+ ,

$$V^{++} \rightarrow V^{++} + f v^{++}, \quad q^+ \rightarrow Q^+ + q^+, \quad (9)$$

and then expand the action in a power series in quantum fields. To study the one-loop effective action in the

model (7) we will use the Faddeev-Popov procedure and the loop expansion at leading order. Hence, to quantize the theory with the classical action (7) we have to introduce the gauge-fixing action for the quantum field v^{++}

$$S_{\text{gf}} = -\frac{1}{4} \int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} v^{++}(z, u_1) v^{++}(z, u_2) + \frac{1}{8} \int d^{14}z du v^{++} (D^{--})^2 v^{++}. \quad (10)$$

The ghost superfields also contribute to effective action. However, in the Abelian case, such terms do not depend on the background superfields and can be omitted (see, e.g., [20] for details).

The one-loop effective action for the theory (7) is defined by the quadratic with respect to quantum fields part of a total action

$$S_{2,\text{total}} = S_2[q^+] + \frac{1}{4} \int d\zeta^{(-4)} v^{++} \partial^2 v^{++} - i f \int d\zeta^{(-4)} \{ \tilde{Q}^+ v^{++} q^+ + \tilde{q}^+ v^{++} Q^+ \}, \quad (11)$$

where $\partial^2 = \partial^M \partial_M$. The action $S_2[q^+]$ introduced in (11) includes only the quantum superfield q^+ and do not involve the quantum gauge multiplet v^{++} ,

$$S_2[q^+] = - \int d\zeta^{(-4)} \{ \tilde{q}^+ \mathcal{D}^{++} q^+ + \frac{1}{2} \Psi ((\tilde{Q}^+ q^+)^2 + 2 \tilde{Q}^+ Q^+ \tilde{q}^+ q^+ + (\tilde{q}^+ Q^+)^2) \}. \quad (12)$$

Here we introduce the covariant harmonic derivative

$$\mathcal{D}^{++} = D^{++} + iV^{++} + i\Psi^{++}, \quad (13)$$

and use the notations

$$\Psi^{++} = -i \frac{\partial L^{(+4)}(\tilde{Q}^+ Q^+)}{\partial(\tilde{Q}^+ Q^+)}, \quad \Psi = \frac{\partial^2 L^{(+4)}(\tilde{Q}^+ Q^+)}{\partial(\tilde{Q}^+ Q^+)^2}. \quad (14)$$

We rewrite the action (12) into the matrix form

$$S_2[Q^+; V^{++}] = -\frac{1}{2} \int d\zeta^{(-4)} (q^+ \widetilde{\tilde{q}^+}) (\mathcal{D}^{++} \cdot \mathbf{1} + Q^{++}) \times \begin{pmatrix} q^+ \\ \tilde{q}^+ \end{pmatrix}, \quad (15)$$

where $\mathbf{1}$ is the second-order identity matrix and Q^{++} is defined as follows:

$$Q^{++} = \Psi \begin{pmatrix} \tilde{Q}^+ Q^+ & (Q^+)^2 \\ -(\tilde{Q}^+)^2 & -\tilde{Q}^+ Q^+ \end{pmatrix} \quad (16)$$

The model (7) contains the cubic interaction vertex, which leads to the mixed term for the quantum gauge multiplet and hypermultiplet in (11). For further consideration, it is convenient to separate the contributions from the quantum gauge multiplet and the hypermultiplet. We remove such terms by the change of the quantum hypermultiplet variables in the path integral (see, e.g., [20])

$$\begin{pmatrix} q^+ \\ \tilde{q}^+ \end{pmatrix} \rightarrow \begin{pmatrix} q^+ \\ \tilde{q}^+ \end{pmatrix} - f \int d\zeta_2^{(-4)} \mathbf{G}^{(1,1)}(1|2) \begin{pmatrix} iQ^+ \\ -i\tilde{Q}^+ \end{pmatrix}_2 v_2^{++}. \quad (17)$$

The Green's function $\mathbf{G}(1|2)$ satisfies the equation

$$(\mathcal{D}^{++} \cdot \mathbf{1} + Q^{++}) \mathbf{G}^{(1,1)}(1|2) = \delta^{(3,1)}(1|2) \cdot \mathbf{1}. \quad (18)$$

A formal solution of the equation (18) has the form

$$\mathbf{G}^{(1,1)} = \mathcal{G}^{(1,1)} \cdot \mathbf{1} - \mathcal{G}^{(1,1)} Q^{++} \mathcal{G}^{(1,1)} + \dots, \quad (19)$$

where the Green's function $\mathcal{G}^{(1,1)}$ satisfies the equation

$$\mathcal{D}^{++} \mathcal{G}^{(1,1)}(1|2) = \delta^{(3,1)}(1|2). \quad (20)$$

Applying the change of variables (17), we obtain

$$S_{2,\text{total}} = S_2[q^+] + \frac{1}{4} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} v_1^{++} \{ \partial^2 \delta_A^{(2,2)}(1|2) - 4f^2 \tilde{Q}_1^+ \mathcal{G}^{(1,1)}(1|2) Q_2^+ + \dots \} v_2^{++}, \quad (21)$$

where ellipsis means the contributions with Q^{++} and high powers of Green's function $\mathcal{G}^{(1,1)}$ coming from the decomposition (19). The action (21) produces the following one-loop quantum correction to the classical action (7)

$$\Gamma^{(1)} = \frac{i}{2} \text{Tr}_{(3,1)} \ln(\mathcal{D}^{++} \cdot \mathbf{1} + Q^{++}) + \frac{i}{2} \text{Tr}_{(2,2)} \ln(\partial^2 - 4f^2 \tilde{Q}^+ \mathcal{G}^{(1,1)} Q^+ + \dots). \quad (22)$$

The first term in the effective action (22) corresponds to the contribution from quantum hypermultiplet. The last term in (22) conditioned by the quantum vector multiplet contribution. The functional trace in (22) includes the matrix trace and integration over harmonic superspace

$$\text{Tr}_{(q,4-q)} \mathcal{O} = \text{tr} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)} \delta_A^{(4-q,q)}(1|2) \mathcal{O}^{(q,4-q)}(1|2),$$

where $\delta_A^{(q,4-q)}(1|2)$ is an analytic delta function [16] and $\mathcal{O}^{(q,4-q)}$ is the kernel of the operator $\mathcal{O}$ acting in the space

of analytic superfields with the harmonic U(1) charge q [16].

Let us consider the first contribution in the effective action (22) in detail,

$$\begin{aligned} & \frac{i}{2} \text{Tr}_{(3,1)} \ln(\mathcal{D}^{++} \cdot \mathbf{1} + \mathcal{Q}^{++}) \\ &= i \text{Tr}_{(3,1)} \ln \mathcal{D}^{++} + \frac{i}{2} \text{Tr}_{(3,1)} \ln(\mathbf{1} + \mathcal{G}^{(1,1)} \mathcal{Q}^{++}). \end{aligned} \quad (23)$$

The second term in (23) is defined as the series

$$\text{Tr}_{(3,1)} \ln(\mathbf{1} + \mathcal{G}^{(1,1)} \mathcal{Q}^{++}) = \text{Tr}_{(3,1)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} (\mathcal{G}^{(1,1)} \mathcal{Q}^{++})^n. \quad (24)$$

The matrix power in the last expression also includes the integration over analytic subspace. Every term in the series (24) including $(\mathcal{Q}^{++})^n = \mathcal{Q}^{++}_1 \mathcal{Q}^{++}_2 \dots \mathcal{Q}^{++}_{n-1} \mathcal{Q}^{++}_n$, $n > 1$, contains the second power of the matrix $\mathcal{Q}^{++}$ depending on the same argument due to the trace Tr . However, the matrix $\mathcal{Q}^{++}$ is traceless and nilpotent, $(\mathcal{Q}^{++})^2 = 0$. Hence the contribution (24) equals zero.

Thus the one-loop effective action (22) is reduced to

$$\begin{aligned} \Gamma^{(1)} &= i \text{Tr}_{(3,1)} \ln \mathcal{D}^{++} \\ &+ \frac{i}{2} \text{Tr}_{(2,2)} \ln(\partial^2 - 4f^2 \tilde{Q}^+ \mathcal{G}^{(1,1)} Q^+ + \dots). \end{aligned} \quad (25)$$

We have to note that the effective action (25) depends on the two superfields, namely the analytic gauge superfield V^{++} and the background hypermultiplet Q^+ . We do not assume any restriction on these fields. Our aim is to study one-loop divergencies of the effective action (25) but first of all, we consider the properties of the covariant harmonic derivative $\mathcal{D}^{++}$.

The operator $\mathcal{D}^{++}$ includes, beside the analytic gauge connection V^{++} , as well as the superfield Ψ^{++} (14). The analytic superfield Ψ^{++} by construction is an ordinary real function of the hypermultiplet. One can consider the superfield Ψ^{++} as an additional analytic gauge connection. Hence, we have the harmonic covariant derivative depending on the two Abelian-gauge connections.

Next, we have to derive the algebra of covariant derivatives, including both gauge-superfield connections. For convenience, we introduce a notation

$$\mathcal{V}^{++} = V^{++} + \Psi^{++}. \quad (26)$$

Then following standard procedure [16], we consider the nonanalytic gauge connection $\mathcal{V}^{--}$ by the rule

$$D^{--} \mathcal{V}^{++} = D^{++} \mathcal{V}^{--}, \quad (27)$$

which can be solved exactly in the Abelian case

$$\mathcal{V}^{--} = \int du_1 \frac{\mathcal{V}^{++}(u_1)}{(u^+ u_1^+)^2}. \quad (28)$$

The commutation relations of the algebra (5) should be rewritten and up to the definitions coincide with the algebra of covariant derivatives of six-dimensional $\mathcal{N} = (1, 0)$ supersymmetric gauge theory [25, 28]

$$\begin{aligned} [\nabla^{--}, D_a^+] &= \nabla_a^-, & [\nabla^{++}, \nabla_a^-] &= D_a^+, \\ [\nabla^{++}, D_a^+] &= [\nabla^{--}, \nabla_a^-] = 0, \\ [D_a^+, \nabla_b^-] &= 2i \nabla_{ab}, & [D_a^+, \nabla_{bc}] &= \frac{i}{2} \varepsilon_{abcd} \mathcal{W}^{+d}, \\ [\nabla_a^-, \nabla_{bc}] &= \frac{i}{2} \varepsilon_{abcd} \mathcal{W}^{-d}. \end{aligned} \quad (29)$$

Here we have introduced the gauge field strength $\mathcal{W}^{+a}$

$$\mathcal{W}^{+a} = -\frac{i}{6} \varepsilon^{abcd} D_b^+ D_c^+ D_d^+ \mathcal{V}^{--}, \quad \mathcal{W}^{-a} := \nabla^{--} \mathcal{W}^{+a}. \quad (30)$$

Let us define the analytic superfield $\mathcal{F}^{++} = D_a^+ \mathcal{W}^{+a}$. The superfields $\mathcal{F}^{++}$ and $\mathcal{W}^{+a}$ possess some useful properties [28]

$$D^{++} \mathcal{F}^{++} = 0, \quad D^{++} \mathcal{W}^{+a} = 0, \quad D^{++} \mathcal{W}^{-a} = 0, \quad (31)$$

where $\mathcal{W}^{-a} = D^{--} \mathcal{W}^{+a}$. We have to note that superfields $\mathcal{F}^{++}$ and $\mathcal{W}^{+a}$ depend on the superfields V^{++} and Q^+ . One can separate both $\mathcal{F}^{++}$ and $\mathcal{W}^{+a}$ in two independent parts

$$\mathcal{W}^{+a} = \mathcal{W}_V^{+a} + \mathcal{W}_Q^{+a}, \quad \mathcal{F}^{++} = \mathcal{F}_V^{++} + \mathcal{F}_Q^{++}. \quad (32)$$

Each of these superfields corresponds to the connection V^{++} and Ψ^{++} respectively.

The covariant analytic d'Alembertian $\widehat{\square}$ [28] in six dimensions reads

$$\widehat{\square} = \frac{1}{2} (D^+)^4 (D^{--})^2. \quad (33)$$

Acting in the space of analytic superfields, the operator (33) transforms to [25]

$$\widehat{\square} = \eta^{MN} \mathcal{D}_M \mathcal{D}_N + \mathcal{W}^{+a} \mathcal{D}_a^- + \mathcal{F}^{++} D^{--} - \frac{1}{2} (D^{--} \mathcal{F}^{++}), \quad (34)$$

where η_{MN} is 6D Minkowski metric and $\mathcal{D}_M = \partial_M - i \mathcal{A}_M$ is a space-time covariant derivative, $\mathcal{A}_M = \frac{i}{2} (\tilde{\gamma}_M)^{ab} D_a^+ D_b^+ \mathcal{V}^{--}$.

Also, we have introduced the covariant harmonic derivative $\mathcal{D}^{--} = D^{--} + i\mathcal{V}^{--}$, which is constructed using the non-analytic connection $\mathcal{V}^{--}$.

IV. ONE-LOOP DIVERGENCIES

The one-loop contribution to the effective action (25) contains the trace of the logarithm of the first-order differential operator, $\Gamma_q = i\text{Tr} \ln \mathcal{D}^{++}$. To calculate the divergent contributions of such the term, we follow the work [23] and vary it with respect to the superfield $\mathcal{V}^{++}$

$$\delta\Gamma_q = -\text{Tr}_{(3,1)} \delta\mathcal{V}^{++} \mathcal{G}^{(1,1)} = - \int d\zeta_1^{(-4)} \delta\mathcal{V}_1^{++} \mathcal{G}^{(1,1)}(1|2)|_{2 \rightarrow 1}. \quad (35)$$

The Green's function $\mathcal{G}^{(1,1)}(1|2)$ solves Eq. [4] and has the following formal expression (see, e.g., [20,30])

$$\mathcal{G}^{(1,1)}(1|2) = \frac{(D_1^+)^4 (D_2^+)^4 \delta^{14}(z_1 - z_2)}{\widehat{\square}_1 (u_1^+ u_2^+)^3}, \quad (36)$$

where $\delta^{14}(z_1 - z_2) = \delta^6(x_1 - x_2) \delta^8(\theta_1 - \theta_2)$ is a full $\mathcal{N} = (1, 0)$ superspace delta function.

The analysis of divergent contributions of the expression (35) is similar to what was carried out for six-dimensional $\mathcal{N} = (1, 0)$ supersymmetric Abelian gauge theory in [20]. First of all, we use the proper-time technique for the inverse operator $\widehat{\square}^{-1}$ and rewrite the expression (35) as follows:

$$\delta\Gamma_q = - \int d\zeta_1^{(-4)} \delta\mathcal{V}^{++} \int_0^\infty d(is) (is\mu^2)^{\frac{\epsilon-2}{2}} e^{is\widehat{\square}_1} (D_1^+)^4 (D_2^+)^4 \times \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3} \Big|_{2=1}. \quad (37)$$

In the last expression, we have introduced the proper-time parameter s and the arbitrary mass dimension parameter μ . We use a dimensional regularization scheme, where divergencies appear as a pole $\frac{1}{\epsilon}$ under the condition $\epsilon \rightarrow 0$.

The integrand in the expression (37) contains eight D -factors acting on the Grassmann delta function $\delta^8(\theta_1 - \theta_2)$ in the coinciding-points limit. To calculate this limit, one can use the identity

$$(D_1^+)^4 (D_2^+)^4 \delta^8(\theta_1 - \theta_2)|_{2 \rightarrow 1} = (u_1^+ u_2^+)^4. \quad (38)$$

After that, it transforms to

$$\delta\Gamma_{q,\text{div}} = - \int d\zeta_1^{(-4)} \delta\mathcal{V}^{++} \times \int_0^\infty d(is) (is\mu^2)^{\frac{\epsilon-2}{2}} e^{is\widehat{\square}_1} (u_1^+ u_2^+) \delta^6(x_1 - x_2)|_{2=1}. \quad (39)$$

The operator $\widehat{\square}$ contains the harmonic derivative D^{--} , which acting on the factor $(u_1^+ u_2^+)$ produces a nonzero contribution due to the identity $D_1^{--}(u_1^+ u_2^+)|_{2 \rightarrow 1} = (u_1^- u_2^+)|_{2 \rightarrow 1} = -1$. Commuting the operator $e^{is\widehat{\square}}$ with the factor $(u_1^+ u_2^+)$

$$e^{is\widehat{\square}}(u_1^+ u_2^+) e^{-is\widehat{\square}}|_{2=1,\text{div}} = -i \frac{(is)^2}{2} (\partial^2 F^{++}) - i \frac{(is)^3}{6} \{4(\partial^M \partial^N F^{++}) \partial_M \partial_N\}, \quad (40)$$

where we act on the space-time delta function in the momentum representation and take the coincident points limit

$$e^{is\partial^2} \delta^6(x_1 - x_2)|_{2=1} = -\frac{i}{(4\pi is)^3}, \quad (41)$$

$$\partial_M \partial_N e^{is\partial^2} \delta^6(x_1 - x_2)|_{2=1} = \frac{i\eta_{MN}}{(4\pi)^3 (is)^4}. \quad (42)$$

Then we count integral over proper time (see the details in [20]) in accordance with

$$i \int_0^\infty ds (is)^{\frac{\epsilon-2}{2}} e^{-ism^2} = \Gamma\left[\frac{\epsilon}{2}\right] \approx \frac{2}{\epsilon}, \quad \epsilon \rightarrow 0, \quad (43)$$

and finally, we have

$$\delta\Gamma_{q,\text{div}} = \frac{1}{3(4\pi)^3 \epsilon} \int d\zeta^{(-4)} \delta\mathcal{V}^{++} \partial^2 \mathcal{F}^{++}. \quad (44)$$

One can show that the last expression can be evaluated from the functional

$$\Gamma_{q,\text{div}} = \frac{1}{6(4\pi)^3 \epsilon} \int d^4 z du \mathcal{V}^{--} \partial^2 \mathcal{V}^{++} = \frac{1}{6(4\pi)^3 \epsilon} \int d\zeta^{(-4)} (\mathcal{F}^{++})^2, \quad (45)$$

taking into account the connection of the variation $\delta\mathcal{V}^{++}$ with $\delta\mathcal{V}^{--}$ [25]

$$\delta\mathcal{V}^{--} = \frac{1}{2} (D^{--})^2 \delta\mathcal{V}^{++} - \frac{1}{2} D^{++} (D^{--} \delta\mathcal{V}^{--}), \quad (46)$$

and the properties (31) for the superfield $\mathcal{F}^{++}$.

The one-loop divergent contribution (45) to the effective action of the theory under consideration (7) depends on the external gauge multiplet V^{++} and background hypermultiplet Q^+ . Using the definitions, (32) we can rewrite (45) in the form

$$\Gamma_{q,\text{div}} = \frac{1}{6(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} (\mathcal{F}_V^{++} + \mathcal{F}_Q^{++})^2. \quad (47)$$

Now, we return to the second term in the effective action (25) and consider the divergent contributions to the one-loop effective action coming from the quantum gauge multiplet,

$$\Gamma_v = \frac{i}{2} \text{Tr}_{(2,2)} \ln \{ \partial^2 - 4f^2 \tilde{Q}^+ \mathcal{G}^{(1,1)} Q^+ + \dots \}. \quad (48)$$

To extract the divergent contribution from the expression above, we expand the logarithm and collect all terms with the third power of the inverse ∂^2 operator acting on the delta function in the coincident-points limit. Indeed, in the frame of dimension regularization scheme, we have

$$\frac{1}{(\partial^2)^3} \delta^6(x_1 - x_2)|_{2=1} = \frac{i}{(4\pi)^3 \varepsilon}, \quad \varepsilon \rightarrow 0. \quad (49)$$

However, the Green's function $\mathcal{G}^{(1,1)}$ contains the inverse power of the covariant d'Alembertian operator (34), and it is enough to expand the logarithm in (48) up to the first order over $\mathcal{G}^{(1,1)}$

$$\Gamma_{v,\text{div}} = -2if^2 \int d\zeta^{(-4)} \tilde{Q}^+ Q^+ \frac{1}{\partial^2} \mathcal{G}^{(1,1)}(1|2)|_{2=1}. \quad (50)$$

Let us consider in detail the Green's function $\mathcal{G}^{(1,1)}$ under the Grassmann variables limit $\theta_2 \rightarrow \theta_1$. We use the explicit form of the Green's function (36) and the identity (38),

$$\mathcal{G}^{(1,1)}(1|2)|_{\text{div}, \theta_2 \rightarrow \theta_1} = \frac{1}{\square} (u_1^+ u_2^+) \delta^6(x_1 - x_2). \quad (51)$$

We substitute the last expression into (50)

$$\Gamma_{v,\text{div}} = -2if^2 \int d\zeta^{(-4)} \tilde{Q}^+ Q^+ \frac{1}{\partial^2 \square} (u_1^+ u_2^+) \delta^6(x_1 - x_2)|_{2=1}, \quad (52)$$

and as the next step we calculate the coincident harmonic-points limit, $u_2^+ \rightarrow u_1^+$, taking into account that $(u_1^+ u_2^+)|_{2 \rightarrow 1} = 0$ and $(u_1^- u_2^+)|_{2 \rightarrow 1} = -1$. We decompose the inverse operator $\square$ (34) up to the first order over the harmonic derivative $\mathcal{D}^{--}$ and act on the factor $(u_1^+ u_2^+)$ in (52)

$$\begin{aligned} & \frac{1}{\partial^2(\partial^2 + i\mathcal{F}^{++}\mathcal{D}^{--} + \dots)} (u_1^+ u_2^+) \delta^6(x_1 - x_2)|_{2=1} \\ &= -i\mathcal{F}^{++} \frac{(u_1^- u_2^+)}{(\partial^2)^3} \delta^6(x_1 - x_2)|_{2=1}. \end{aligned} \quad (53)$$

As a result, we get the following expression for the divergent contribution

$$\Gamma_{v,\text{div}} = \frac{2if^2}{(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} \tilde{Q}^+ \mathcal{F}^{++} Q^+. \quad (54)$$

Finally, we combine the two contributions (45) and (54)

$$\begin{aligned} \Gamma_{\text{div}}^{(1)} &= \frac{1}{6(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} (\mathcal{F}_V^{++} + \mathcal{F}_Q^{++}) \\ &\quad \times (\mathcal{F}_V^{++} + \mathcal{F}_Q^{++} + 12if^2 \tilde{Q}^+ Q^+), \end{aligned} \quad (55)$$

where we substitute $\mathcal{F}^{++}$ introduced in (32) as a sum of the gauge multiplet-dependent superfield $\mathcal{F}_V^{++}$ and the hypermultiplet one $\mathcal{F}_Q^{++}$. The presence of the contributions with $\mathcal{F}_Q^{++}$ is caused by self-interacting term for the hypermultiplet in the classical action (7). If we take $L^{(+4)} = 0$ in the action (7), the superfield $\mathcal{F}_Q^{++}$ vanishes, which is in agreement with [20].

Let us discuss the final result (55) and consider possible constraints on the background superfields. The classical equations of motion for the background superfields have the form

$$\mathcal{F}_V^{++} - 2if^2 \tilde{Q}^+ Q^+ = 0, \quad \mathcal{D}^{++} Q^+ = 0, \quad (56)$$

where the covariant harmonic derivative $\mathcal{D}$ was introduced in (13). One can exclude the superfield $\mathcal{F}_V^{++}$ from (55), assuming (56). Nevertheless, the one-loop divergencies (55) in the theory under consideration (7) do not vanish in case of the general self-interaction $L^{(+4)}$.

As an example, we consider the case when the potential function $L^{(+4)}$ in the model (7) has a simple form $L^{(+4)} = \frac{1}{2}(\tilde{Q}^+ Q^+)^2$. The analytic Ψ^{++} and nonanalytic Ψ^{--} connections for this potential are

$$\Psi^{++} = -i\tilde{Q}^+ Q^+, \quad \Psi^{--} = \int du_2 \frac{\Psi_2^{++}}{(u_1^+ u_2^+)^2}. \quad (57)$$

We also assume that the classical equations of motion (56) are satisfied. Hence, nonanalytic superfield Ψ^{--} in (57) can be exactly evaluated, $\Psi^{--} = -i\tilde{Q}^- Q^-$, where $Q^- = \mathcal{D}^{--} Q^+$. The divergent contribution (55) in the theory under consideration with such self-interaction takes the form

$$\begin{aligned} \Gamma_{\text{div}}^{(1)}[Q^+] &= \frac{1}{6(4\pi)^3 \varepsilon} \int d^{14}z du (\tilde{Q}^- Q^- \partial^2 (\tilde{Q}^+ Q^+) \\ &\quad + 16f^2 \tilde{Q}^- Q^- \tilde{Q}^+ Q^+) \\ &\quad - \frac{14f^4}{3(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} (\tilde{Q}^+ Q^+)^2, \end{aligned} \quad (58)$$

and, as one can see, it does not vanish even if we suppose the slowly varying background hypermultiplet, $\partial Q^+ \approx 0$.

The leading low-energy finite contribution to effective action (25) we obtain assuming the vanish background hypermultiplet, $Q^+ = 0$. In this case the classical equation of motion for the background gauge multiplet (56) is reduced to

$$\mathcal{F}_V^{++} = 0. \quad (59)$$

We use the method developed in [44] to evaluate the leading finite contribution to the effective action (25). The result is

$$\Gamma_{\text{lead}}^{(1)}[Q^+] = \frac{1}{32\pi^3 m^2} \int d\zeta^{(-4)} (\mathcal{W}_V^+)^4. \quad (60)$$

We have to note that in general the superfield $\mathcal{W}_V^{+a}$ is not analytic by construction (30). However, due to the on-shell condition (59), it becomes analytic superfield $D_a^+ \mathcal{W}_V^{+a} = \mathcal{F}_V^{++} = 0$. The contribution (60) includes the forth power of the Abelian gauge field strength, $F_{MN} = \partial_M A_N - \partial_N A_M$, and describe the effective action for the background gauge multiplet with the constant gauge field strength.

The parameter m^2 appears in (60) after integration over proper time and should be passed to zero, $m^2 \rightarrow 0$. The calculation of the leading low-energy contribution to the effective action in the case of $Q^+ \neq 0$ is a nontrivial question, which is beyond the scope of the present paper. However, we can mention that due to analyticity and dimensional analysis, the structure of the leading finite contribution in the $Q^+ \neq 0$ case will be the same (60). Moreover, the superfield $\mathcal{W}_Q^{+a}$ introduced in (30) do not appear in the low-energy effective action. Indeed, the superfield $\mathcal{W}_Q^{+a}$ includes the term $D_a^+ D_b^+ Q^-$, which after some algebra transforms to $2i\partial_{ab} Q^+$ and vanishes for the constant background. In this case, as a infrared regulator it is natural to choose the gauge-invariant uncharged combination of the constant background hypermultiplets, e.g., $m^2 = \tilde{Q}^- Q^+ - Q^- \tilde{Q}^+$, which is real under Grassmann conjugation.

V. CONCLUDING REMARKS

In the present paper, we studied the divergent contributions to the one-loop effective action in the model of interacting gauge multiplet and hypermultiplet with arbitrary self-interaction potential function in six

dimensions (7). Using the background-field method and proper-time technique in the six-dimensional $\mathcal{N} = (1, 0)$ harmonic superspace, we derived the one-loop contribution to the effective action in the theory under consideration and calculate the divergent contributions to the effective action (55). Also, we briefly discussed the possible leading finite contributions to low-energy effective action in the model.

The developed above technique is universal and can be applied to hypermultiplet models with rigid supersymmetry in lower dimensions. Technically, the main distinction in these cases will be in the calculating of the momentum and proper-time integrals in expression (39), because the term (50) does not contribute to the divergent part of the effective action in lower dimensions. The five-dimensional hypermultiplet theory does not have one-loop logarithmic divergencies. Hence, it is interesting to study the finite contribution to the effective action. However, in four dimensions² the one-loop effective action of the hypermultiplet with gauge-invariant self-interacting term contains the divergent contribution of the following form

$$\Gamma_{4\text{D},\text{div}}^{(1)} = \frac{1}{32\pi^2 \epsilon} \int d^{12}z du \mathcal{V}^{--} \mathcal{V}^{++}. \quad (61)$$

In the present paper, we considered the interaction of the hypermultiplet with the Abelian gauge multiplet in six dimensions. It is natural to generalize the above consideration to the arbitrary non-Abelian case and study the divergent contribution to the effective action of such a model. It would be interesting to calculate the finite contributions to the effective action of the non-Abelian hypermultiplet theory in six dimensions and consider the dimensional reduction of such contributions to dimensions five and four. We are going to examine these problems in upcoming works.

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²Many aspects of the $\mathcal{N} = 2$ hypermultiplet theory in four dimensions were considered in the works [45–50].

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