***CP* violation in quasi-two-body $D \rightarrow VP$ decays and three-body D decays mediated by vector resonances**

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We reexamine direct *CP* violation in the quasi-two-body $D \rightarrow VP$ decays and study *CP* asymmetries in three-body D decays proceeding through intermediate vector resonances within the framework of the topological amplitude approach for tree amplitudes and the QCD factorization approach for penguin amplitudes. Using the same mechanism of incorporating long-distance penguin-exchange amplitude induced from final-state rescattering that nicely accounts for the *CP* asymmetry difference between $D^0 \rightarrow K^+K^-$ and $\pi^+\pi^-$ modes, we find that *CP* asymmetry can also occur at the per mille level in many of the $D \rightarrow VP$ channels or otherwise be negligibly small. We point out six golden modes which have sufficiently large branching fractions and direct *CP* violation of order 10^{-3} . In particular, the direct *CP* asymmetry difference Δa_{CP}^{VP} between $D^0 \rightarrow K^+K^{*-}$ and $\pi^+\rho^-$ is predicted to be $(-1.61 \pm 0.33) \times 10^{-3}$, very similar to the counterpart in the *PP* sector. Besides, we find a positive and large *CP* asymmetry of order 10^{-3} in $D^0 \rightarrow K_S K^{*0}$, an example of the asymmetry induced at tree level. We take into account the flavor-singlet QCD-penguin contributions to calculate their effects on the modes such as $D \rightarrow \pi\phi$ and $D \rightarrow \eta\phi$. We compare our results with the other approach in detail. For three-body D decays, we perform the Dalitz plot analysis of those decays mediated by several vector resonances with the same mass but different electric charges. Local *CP* asymmetry varies in magnitude and sign from region to region and can reach the percent level in certain local regions of phase space due to interference.

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I. INTRODUCTION

In 2019 the LHCb Collaboration has announced the measured difference between the time-integrated *CP* asymmetries of the decays $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ based on pion and muon tagged analyses [1]. Combining these with previous LHCb results in 2014 [2] and 2016 [3] leads to a nonzero value of ΔA_{CP} [1],

$$\begin{aligned} \Delta A_{CP} &\equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \\ &= (-1.54 \pm 0.29) \times 10^{-3}. \end{aligned} \quad (1.1)$$

The time-integrated asymmetry can be further decomposed into a direct *CP* asymmetry a_{CP}^{dir} and a mixing-induced indirect *CP* asymmetry characterized by the parameter A_Γ which measures the asymmetry between $D^0 \rightarrow f$ and $\bar{D}^0 \rightarrow f$ effective decay widths,

$$A_{CP}(f) \approx a_{CP}^{\text{dir}}(f) - \frac{\langle t \rangle}{\tau(D^0)} A_\Gamma(f), \quad (1.2)$$

where $\langle t \rangle$ is the average decay time in the sample. To a good approximation, A_Γ is independent of the decay mode, and hence,

$$\Delta A_{CP} = \Delta a_{CP}^{\text{dir}} - \frac{\Delta \langle t \rangle}{\tau(D^0)} A_\Gamma, \quad (1.3)$$

where $\Delta \langle t \rangle$ is the difference in the mean decay times of the two decays. Based on the LHCb average of A_Γ , it follows that the direct *CP* asymmetry difference is given by [1]

$$\Delta a_{CP}^{\text{dir}} = (-1.57 \pm 0.29) \times 10^{-3}. \quad (1.4)$$

This is the first observation of *CP* violation at 5.3σ in the charm sector.

The most important question to be addressed is whether or not the observed *CP* violation in the charm sector is consistent with the standard model. This is the main issue discussed in most of the recent papers appearing after the

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2019 LHCb measurement [4–17].¹ One may naively argue that it is difficult to accommodate the LHCb's measurement of CP asymmetry within the standard model. To see this, consider the tree T and penguin P contributions to $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$. Their interference gives rise to $\Delta a_{CP}^{\text{dir}}$. A simplified expression of the CP asymmetry difference between them is given by²

$$\Delta a_{CP}^{\text{dir}} \approx -1.3 \times 10^{-3} \left(\left| \frac{P}{T} \right|_{KK} \sin \theta_{KK} + \left| \frac{P}{T} \right|_{\pi\pi} \sin \theta_{\pi\pi} \right), \quad (1.5)$$

where the factor of 1.3×10^{-3} comes from the imaginary part of a certain combination of CKM matrix elements, $2\text{Im}(\lambda_d \lambda_s^*)/|\lambda_d|^2$, with $\lambda_p \equiv V_{cp}^* V_{up}$ for $p = d, s, b$, and θ_{KK} is the strong phase of $(P/T)_{KK}$ and likewise for $\theta_{\pi\pi}$. Since $|P/T|$ is naively expected to be of order $(\alpha_s(\mu_c)/\pi) \sim \mathcal{O}(0.1)$, it appears that $\Delta a_{CP}^{\text{dir}}$ is most likely of order 10^{-4} even if the strong phases are allowed to be close to 90° . Indeed, using the results of $|P/T|$ obtained from light-cone sum rules [20],

$$\left| \frac{P}{T} \right|_{\pi\pi} = 0.093 \pm 0.011, \quad \left| \frac{P}{T} \right|_{KK} = 0.075 \pm 0.015, \quad (1.6)$$

Khodjamirian and Petrov argued an upper bound in the standard model, namely, $|\Delta a_{CP}^{\text{SM}}| \leq (2.0 \pm 0.3) \times 10^{-4}$.

Before the LHCb observation of Δa_{CP} in 2019, there existed two independent studies in 2012 in which direct CP violation in charmed meson decays was explored based on the topological diagram approach for tree amplitudes and the QCD-inspired approach for penguin amplitudes [21–23]. Interestingly, both works predicted a Δa_{CP} of order 0.1% with a negative sign seven years before the LHCb observation of CP asymmetry in the charm sector. It followed that

$$\left(\frac{P}{T} \right)_{\pi\pi} \approx 0.23 e^{-i150^\circ}, \quad \left(\frac{P}{T} \right)_{KK} \approx 0.22 e^{-i150^\circ} \quad (1.7)$$

were found in the QCD factorization (QCDF) calculation [9,21,22], while

$$\left(\frac{P}{T} \right)_{\pi\pi} \approx 0.30 e^{i110^\circ}, \quad \left(\frac{P}{T} \right)_{KK} \approx 0.24 e^{i110^\circ} \quad (1.8)$$

¹Various models including many new physics ones (see, for example, Refs. [2–23] listed in [9]) had been proposed in the past to explain the large $\Delta a_{CP} = -(0.82 \pm 0.21 \pm 0.11)\%$ reported by the LHCb in 2012 [18]. However, this large Δa_{CP} was not seen anymore in subsequent LHCb analyses [2,3,19].

²A complete expression of $\Delta a_{CP}^{\text{dir}}$ will be given in Eq. (1.9) below.

were obtained in the so-called factorization-assisted topological amplitude (FAT) approach [23]. Contrary to the sum-rule calculation, the magnitude of P/T turns out to be of order $(0.22 \sim 0.30)$ in the QCD-inspired approaches. However, it does not suffice to explain the measured value of $\Delta a_{CP}^{\text{dir}}$.

The complete expression of $\Delta a_{CP}^{\text{dir}}$ is given by [22]

$$\Delta a_{CP}^{\text{dir}} = -1.30 \times 10^{-3} \left(\left| \frac{P^d + PE^d + PA^d}{T + E^s - \Delta P} \right|_{KK} \sin \delta_{KK} + \left| \frac{P^s + PE^s + PA^s}{T + E^d + \Delta P} \right|_{\pi\pi} \sin \delta_{\pi\pi} \right), \quad (1.9)$$

where E is the W -exchange amplitude, PE is the QCD-penguin exchange amplitude, PA is the QCD-penguin annihilation amplitude, the superscript d or s refers to the quark involved in the associated penguin loop, and δ_{KK} is the strong phase of $(P^d + PE^d + PA^d)_{KK}$ relative to $(T + E^s - \Delta P)_{KK}$ and likewise for the phase $\delta_{\pi\pi}$. The parameter ΔP is defined by $\Delta P \equiv (P^d + PE^d + PA^d) - (P^s + PE^s + PA^s)$. The observation of the decay $D^0 \rightarrow K^0 \bar{K}^0$ indicates that SU(3) symmetry must be broken in the topological amplitude E . A fit to the data yields two possible solutions [9]:

$$\begin{aligned} \text{I: } E^d &= 1.10 e^{i15.1^\circ} E, & E^s &= 0.62 e^{-i19.7^\circ} E; \\ \text{II: } E^d &= 1.10 e^{i15.1^\circ} E, & E^s &= 1.42 e^{-i13.5^\circ} E, \end{aligned} \quad (1.10)$$

where E^q refers to the W -exchange amplitude associated with $c\bar{u} \rightarrow q\bar{q}$ ($q = d, s$).

It follows that

$$\begin{aligned} \left(\frac{P^s + PE^s + PA^s}{T + E^d + \Delta P} \right)_{\pi\pi} &= 0.40 e^{i176^\circ}, \\ \left(\frac{P^d + PE^d + PA^d}{T + E^s - \Delta P} \right)_{KK} &= \begin{cases} 0.29 e^{-i164^\circ} \\ 0.29 e^{i178^\circ} \end{cases} \end{aligned} \quad (1.11)$$

in QCDF for solutions I and II of W -exchange amplitudes E^d and E^s [9], and

$$\begin{aligned} \left(\frac{P^s + PE^s + PA^s}{T + E^d + \Delta P} \right)_{\pi\pi} &= 0.66 e^{i134^\circ}, \\ \left(\frac{P^d + PE^d + PA^d}{T + E^s - \Delta P} \right)_{KK} &= 0.45 e^{i131^\circ} \end{aligned} \quad (1.12)$$

in FAT [23]. Now we see a crucial difference between the two calculations. The calculated phases $\delta_{\pi\pi}$ and δ_{KK} are close to 180° in QCDF, while they deviate from 90° by only around 40° in FAT. As a consequence, Eq. (1.12) leads to $\Delta a_{CP}^{\text{dir}} \approx -1.0 \times 10^{-3}$, but not so for QCDF, where $\Delta a_{CP}^{\text{dir}} \approx 6.8 \times 10^{-5}$ and -4.9×10^{-5} for solutions I and II of W -exchange amplitudes, respectively.

One of the most salient features of the topological approach for charm decays is that all the topological amplitudes except the tree amplitude T are dominated by nonfactorizable long-distance effects. That is, final-state interactions play an essential role in describing the hadronic decays of charmed mesons. In [21] we have pointed out the importance of a resonantlike final-state rescattering which has the same topology as the QCD-penguin exchange topological graph PE . Hence, penguin annihilation receives sizable long-distance contributions through final-state interactions. Making the ansatz that $(PE)^{\text{LD}}$ is of the same order of magnitude as E and flavor independent, we obtain

$$\begin{aligned} \left(\frac{P^s + PE^s + PA^s + PE^{\text{LD}}}{T + E^d + \Delta P} \right)_{\pi\pi} &= 0.81e^{i119^\circ}, \\ \left(\frac{P^d + PE^d + PA^d + PE^{\text{LD}}}{T + E^s - \Delta P} \right)_{KK} &= \begin{cases} 0.48e^{i143^\circ} \\ 0.48e^{i126^\circ} \end{cases} \end{aligned} \quad (1.13)$$

Comparing this with Eq. (1.11), we see that, in the presence of the long-distance penguin exchange, not only are the magnitudes of the ratios enhanced but the strong phases are also closer to 90° rather than to 180° . We have predicted that $\Delta a_{CP}^{\text{dir}}$ is about $(-0.139 \pm 0.004)\%$ and $(-0.151 \pm 0.004)\%$ for the two solutions of W -exchange amplitudes [22], which were the main predictions, among others, made in 2012. Nowadays, we know that the LHCb's new measurement (1.1) almost coincides with our second solution. This implies that one does not need new physics at all to understand the first observation of $\Delta a_{CP}^{\text{dir}}$ by the LHCb.

Although both QCDF and FAT approaches lead to $\Delta a_{CP}^{\text{dir}}$ at the per mille level [21–23], they can be discriminated in the $D \rightarrow VP$ sector. While our approach based on QCDF predicts CP asymmetries of $D^0 \rightarrow \pi^+\rho^-$ and $D^0 \rightarrow K^+K^{*-}$ similar to that of the corresponding $D \rightarrow PP$ decays [9], the FAT approach leads to very tiny values of CP violation for $D^0 \rightarrow \pi^\pm\rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ [24]. In this work, we examine the underlying reason for the discrepancy between the two approaches for $D \rightarrow VP$ decays and present a more detailed analysis. We also study the implications for CP asymmetry distributions in three-body D decays. For example, the experimental Dalitz plot analysis of $D^0 \rightarrow \pi^+\pi^-\pi^0$ indicates that this channel is predominated by the $\rho^\pm$ and ρ^0 resonances (see Sec. III B below). The interference between these resonances will yield rich information on local CP asymmetries.

The present work is organized as follows. In view of updates in several branching fractions, we perform a refit to the Cabibbo-favored modes to extract various topological tree amplitudes in Sec. II. Then, we elaborate on the calculation of direct CP violation in $D \rightarrow VP$ decays in detail and compare our results with those of FAT. In Sec. III we discuss CP asymmetry distributions in the following three-body D decays: $D^0 \rightarrow K^+K^-\pi^0$, $D^0 \rightarrow \pi^+\pi^-\pi^0$, $D^+ \rightarrow K^+K_S\pi^0$, and $D_s^+ \rightarrow K^0\pi^+\pi^0$. We focus on the

phase-space region where these three-body decays are governed by the quasi-two-body $D \rightarrow VP$ ones. Our conclusions are presented in Sec. IV.

II. DIRECT CP VIOLATION IN $D \rightarrow VP$ DECAYS

A. Topological tree amplitudes

For $D \rightarrow VP$ decays, there exist two different types of topological diagrams since the spectator quark of the charmed meson may end up in the pseudoscalar or vector meson. For the topological color-allowed tree amplitude T and color-suppressed amplitude C in $D \rightarrow VP$ decays, the subscript P (V) implies that the pseudoscalar (vector) meson contains the spectator quark of the charmed meson. For the W -exchange amplitude E and W -annihilation A with the final state $q_1\bar{q}_2$, the subscript P (V) denotes that the pseudoscalar (vector) meson contains the antiquark $\bar{q}_2$.

The partial decay width of the D meson into a vector and a pseudoscalar mesons is given by

$$\Gamma(D \rightarrow VP) = \frac{p_c}{8\pi m_D^2} \sum_{\text{pol}} |\mathcal{M}|^2, \quad (2.1)$$

with p_c being the c.m. momentum of either final-state particle. This expression can be simplified by the replacement $\epsilon \cdot p_D \rightarrow p_c(m_D/m_V)$ after the polarization states of the vector meson are summed over. Hence,

$$\Gamma(D \rightarrow VP) = \frac{p_c^3}{8\pi m_V^2} |\tilde{\mathcal{M}}|^2, \quad (2.2)$$

with $\mathcal{M} = \tilde{\mathcal{M}}(\epsilon \cdot p_D)$. By performing a χ^2 fit to the Cabibbo-favored (CF) $D \rightarrow VP$ decays, we have extracted the magnitudes and strong phases of the topological amplitudes T_V , C_V , E_V , A_V and T_P , C_P , E_P , A_P from the measured partial widths through Eq. (2.2) and found many possible solutions with local χ^2 minima [9,25]. Previously, we found six best χ^2 -fit solutions (S1)–(S6) as listed in Table IV of [9], when we restricted ourselves to $\chi_{\text{min}}^2 < 10$. Although solutions (S1)–(S6) generally fit the CF modes well, they led to very different predictions for some of the singly Cabibbo-suppressed (SCS) decays. In particular, the $D^0 \rightarrow \pi^0\omega$, $D^+ \rightarrow \pi^+\rho^0$ and $D^+ \rightarrow \pi^+\omega$ decays were very useful in discriminating among different solutions. For example, solutions (S1), (S2), (S4), and (S5) implied that the branching fraction of $D^+ \rightarrow \pi^+\rho^0$ should be smaller than that of $D^+ \rightarrow \pi^+\omega$, in contradiction with the experimental data. As shown in [9], solution (S6) gave a better description of the SCS modes.

In light of the recent new measurements of $D^0 \rightarrow \bar{K}^{*0}\eta$ by Belle [26], $D^0 \rightarrow \bar{K}^{*0}\phi$, $D_s^+ \rightarrow \bar{K}^{*0}K^+$, $\pi^+\phi$ by BESIII [27,28], we have performed a new fit to the data given in Table I and now find five solutions, (S1')–(S5'), listed in Table II. It is noted that there is a sign ambiguity if all the strong phases change their sign simultaneously. The

TABLE I. Flavor amplitude decompositions, experimental branching fractions, and predicted branching fractions for the Cabibbo-favored $D \rightarrow VP$ decays. Data are taken from the Particle Data Group [30], Belle [26], and BESIII [27,28]. Here $\lambda_{sd} \equiv V_{cs}^* V_{ud}$. The column of $\mathcal{B}_{\text{theory}}(S3')$ shows predictions based on solutions (S3') presented in Table II. All branching fractions are quoted in units of %.

Meson	Mode	Amplitude decomposition	$\mathcal{B}_{\text{exp}}$	$\mathcal{B}_{\text{theory}}(S3')$
D^0	$K^{*-}\pi^+$	$\lambda_{sd}(T_V + E_P)$	5.34 ± 0.41	5.41 ± 0.33
	$K^-\rho^+$	$\lambda_{sd}(T_P + E_V)$	11.3 ± 0.7	11.4 ± 0.6
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}\lambda_{sd}(C_P - E_P)$	3.74 ± 0.27	3.64 ± 0.17
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}\lambda_{sd}(C_V - E_V)$	$1.26^{+0.12}_{-0.16}$	1.30 ± 0.12
	$\bar{K}^{*0}\eta$	$\lambda_{sd}[\frac{1}{\sqrt{2}}(C_P + E_P)\cos\phi - E_V\sin\phi]$	1.352 ± 0.115^a	1.33 ± 0.08
	$\bar{K}^{*0}\eta'$	$-\lambda_{sd}[\frac{1}{\sqrt{2}}(C_P + E_P)\sin\phi + E_V\cos\phi]$	< 0.10	0.0034 ± 0.0003
	$\bar{K}^0\omega$	$-\frac{1}{\sqrt{2}}\lambda_{sd}(C_V + E_V)$	2.22 ± 0.12	2.24 ± 0.16
	$\bar{K}^0\phi$	$-\lambda_{sd}E_P$	0.866 ± 0.029^b	0.868 ± 0.028
D^+	$\bar{K}^{*0}\pi^+$	$\lambda_{sd}(T_V + C_P)$	1.57 ± 0.13	1.58 ± 0.13
	$\bar{K}^0\rho^+$	$\lambda_{sd}(T_P + C_V)$	$12.3^{+1.2}_{-0.7}$	12.5 ± 0.4
D_s^+	$\bar{K}^{*0}K^+$	$\lambda_{sd}(C_P + A_V)$	3.91 ± 0.09^c	3.92 ± 0.13
	$\bar{K}^0K^{*+}$	$\lambda_{sd}(C_V + A_P)$	5.4 ± 1.2	2.95 ± 0.20
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}\lambda_{sd}(A_P - A_V)$	$\dots$	0.023 ± 0.014
	$\rho^+\eta$	$\lambda_{sd}[\frac{1}{\sqrt{2}}(A_P + A_V)\cos\phi - T_P\sin\phi]$	8.9 ± 0.8	8.95 ± 0.36
	$\rho^+\eta'$	$\lambda_{sd}[\frac{1}{\sqrt{2}}(A_P + A_V)\sin\phi + T_P\cos\phi]$	5.8 ± 1.5	3.43 ± 0.12
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}\lambda_{sd}(A_V - A_P)$	0.019 ± 0.012	0.023 ± 0.014
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}\lambda_{sd}(A_V + A_P)$	0.192 ± 0.030	0.19 ± 0.04
	$\pi^+\phi$	$\lambda_{sd}T_V$	4.53 ± 0.12^c	4.52 ± 0.11

^aThe new measurement from Belle $\mathcal{B}(D^0 \rightarrow \bar{K}^{*0}\eta) = (1.41^{+0.13}_{-0.12})\%$ [26] is taken into account in the world average.

^bThe new measurement from BESIII [27] is taken into account in the world average.

^cThe new measurement from BESIII [28] is taken into account in the world average.

topological amplitudes of all these solutions respect the hierarchy pattern:

$$|T_P| > |T_V| \gtrsim |C_P| > |C_V| \gtrsim |E_P| > |E_V| > |A_{P,V}|. \quad (2.3)$$

Among them, solution (S3') is very similar to solution (S6) and provides the best description of the SCS modes. The topological amplitude sizes and the strong phases for this solution are presented in Table II, where we have chosen the $\eta - \eta'$ mixing angle $\phi = 43.5^\circ$ [29], which is defined in the flavor basis

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (2.4)$$

with $\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $\eta_s = s\bar{s}$. Note that (S3') in Table II here and (S6) in Table IV of [9] are quite similar except for E_V and the phase of T_P . It is not surprising that the magnitude of $E_V(S3') = 0.58 \pm 0.06$ is about twice as large as that of $E_V(S6) = 0.26 \pm 0.04$ [9] because the new average of $\mathcal{B}(D^0 \rightarrow \bar{K}^{*0}\eta) = (1.352 \pm 0.115)\%$ is increased by 35% from the old one $(1.02 \pm 0.30)\%$ [30], recalling that its amplitude is proportional to $[\frac{1}{\sqrt{2}}(C_P + E_P)\cos\phi - E_V\sin\phi]$. As a consequence, the phase of E_V also changes significantly from $(224^{+22}_{-40})^\circ$

[9] to $(283 \pm 5)^\circ$ with a much improved accuracy. Since $A(D^0 \rightarrow K^-\rho^+) \propto (T_P + E_V)$, T_P is affected by a change of E_V accordingly.

In the PP sector, we need SU(3) breaking in the W -exchange diagrams [see Eq. (1.10)] in order to induce the observed $D^0 \rightarrow K_S K_S$ decay and to explain the large rate difference between $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ decays. Likewise, we also need SU(3) breaking in the W -exchange amplitudes in the VP sector because (a) the ratios, $\Gamma(D^0 \rightarrow K^+ K^{*-})/\Gamma(D^0 \rightarrow \pi^+ \rho^-) = 0.32 \pm 0.03$ and $\Gamma(D^0 \rightarrow K^- K^{*+})/\Gamma(D^0 \rightarrow \pi^- \rho^+) = 0.45 \pm 0.03$ [30], deviate substantially from the unity expected in the SU(3) limit and (b) the predicted rates of $D^0 \rightarrow K^0 \bar{K}^{*0}$ and $D^0 \rightarrow \bar{K}^0 K^{*0}$ modes are too large [9]. Writing

$$\begin{aligned} E_V^d &= e_V^d e^{i\delta_V^d} E_V, & E_V^s &= e_V^s e^{i\delta_V^s} E_V, \\ E_P^d &= e_P^d e^{i\delta_P^d} E_P, & E_P^s &= e_P^s e^{i\delta_P^s} E_P, \end{aligned} \quad (2.5)$$

we are able to determine the eight unknown parameters $e_V^d, e_P^d, e_V^s, e_P^s$ and $\delta e_V^d, \delta e_P^d, \delta e_V^s, \delta e_P^s$ from the branching fractions of the following eight modes: $D^0 \rightarrow \pi^+ \rho^-, \pi^- \rho^+, \pi^0 \rho^0, \pi^0 \omega$ and $D^0 \rightarrow K^+ K^{*-}, K^- K^{*+}, K^0 \bar{K}^{*0}, \bar{K}^0 K^{*0}$. Table III shows the four solutions of SU(3)-breaking effects in the W -exchange amplitudes for solution (S3'). Recall

TABLE II. Solutions with $\chi^2_{\min} \leq 10$ obtained using Eq. (2.2) and $\phi = 43.5^\circ$. We will take solution (S3') in subsequent analyses. The amplitude sizes are quoted in units of $10^{-6}(\epsilon \cdot p_D)$ and the strong phases in units of degrees.

Set	$ T_V $ $ E_P $	$ T_P $ δ_{E_P}	δ_{T_P} $ A_V $	$ C_V $ δ_{A_V}	δ_{C_V} $ A_P $	$ C_P $ δ_{A_P}	δ_{C_P} $\chi^2_{\min}$	$ E_V $ Fit quality	δ_{E_V}
(S1')	2.19 ± 0.03 1.69 ± 0.03	3.32 ± 0.06 109 ± 2	100 ± 3 0.20 ± 0.02	1.75 ± 0.04 32^{+8}_{-11}	312 ± 3 0.21 ± 0.03	2.10 ± 0.02 357^{+12}_{-9}	201 ± 1 5.88	0.29 ± 0.04 11.74%	334^{+14}_{-22}
(S2')	2.19 ± 0.03 1.69 ± 0.03	3.23 ± 0.06 109 ± 2	264 ± 3 0.20 ± 0.02	1.74 ± 0.04 349^{+9}_{-7}	50 ± 3 0.22 ± 0.03	2.07 ± 0.02 23^{+9}_{-12}	201 ± 1 6.39	0.36 ± 0.05 9.41%	8^{+9}_{-8}
(S3')	2.19 ± 0.03 1.69 ± 0.03	3.56 ± 0.06 108 ± 3	61 ± 5 0.23 ± 0.02	1.69 ± 0.04 77 ± 5	220 ± 3 0.18 ± 0.03	2.02 ± 0.02 111^{+13}_{-10}	201 ± 1 7.06	0.58 ± 0.06 6.99%	283 ± 5
(S4')	2.19 ± 0.03 1.69 ± 0.03	3.50 ± 0.06 109 ± 2	106^{+3}_{-4} 0.25 ± 0.02	1.74 ± 0.04 62^{+6}_{-7}	262^{+4}_{-3} 0.14 ± 0.03	2.09 ± 0.02 44^{+50}_{-16}	201 ± 1 7.34	0.38 ± 0.05 6.19%	308^{+8}_{-10}
(S5')	2.19 ± 0.03 1.69 ± 0.03	3.54 ± 0.06 108 ± 2	268^{+5}_{-4} 0.26 ± 0.02	1.67 ± 0.04 324 ± 5	107^{+3}_{-4} 0.13 ± 0.02	2.04 ± 0.02 329^{+24}_{-32}	201 ± 1 8.57	$0.62^{+0.06}_{-0.07}$ 3.56%	43 ± 4

TABLE III. Solutions of the parameters $e_{V,P}^{d,s}$ and the phases $\delta e_{V,P}^{d,s}$ (in units of degrees) describing SU(3)-breaking effects in the W -exchange amplitudes.

	e_V^d	δe_V^d	e_P^d	δe_P^d	e_V^s	δe_V^s	e_P^s	δe_P^s
(i)	0.34	347	0.62	343	0.24	101	0.62	149
(ii)	0.34	347	0.62	343	0.24	101	0.40	349
(iii)	0.34	347	0.62	343	1.63	229	0.62	149
(iv)	0.34	347	0.62	343	1.63	229	0.40	349

that in our previous work, there are six solutions for the parameters $e_{V,P}^{d,s}$ and the associated phases (see Table VII of [9]). As we shall show in Sec. II C below, the four different solutions of SU(3) breaking in W -exchange amplitudes can be discriminated using the SCS mode $D^0 \rightarrow \eta\phi$. It turns out that solution (iv) is preferred.

B. Penguin amplitudes in QCD factorization

Although the topological tree amplitudes T , C , E , and A for two-body hadronic D decays can be extracted from the data, information on penguin amplitudes (QCD penguin, penguin annihilation, etc.) needed for inducing CP violation in the SCS decays is still absent. To consider the penguin contributions, we take the QCDF approach [31,32] to evaluate the hadronic matrix elements. Although the QCD-inspired approaches such as QCDF and pQCD [33] have been applied rather successfully to describe weak hadronic decays of B mesons, it should be kept in mind that these approaches provide only a crude estimate of the penguin amplitudes in charm decays because the charm quark mass is not heavy enough and $1/m_c$ power corrections are so large that a sensible heavy quark expansion is not allowed.

For penguin effects in $D \rightarrow M_1 M_2$ decays, we consider the QCD-penguin P , flavor-singlet QCD-penguin S , QCD-penguin exchange PE , and QCD-penguin annihilation PA

as depicted in Fig. 1 (see [34] for details).³ In QCDF, the amplitudes of these QCD-penguin diagrams read [32]

$$\begin{aligned}
 S_{M_1}^P &= \frac{G_F}{\sqrt{2}} [a_3^p(M_1 M_2) + \eta a_5^p(M_1 M_2)] X^{(DM_1, M_2)}, \\
 P_{M_1}^P &= \frac{G_F}{\sqrt{2}} [a_4^p(M_1 M_2) + \eta r_\chi^{M_2} a_6^p(M_1 M_2)] X^{(DM_1, M_2)}, \\
 PE_{M_1}^P &= \frac{G_F}{\sqrt{2}} (f_D f_{M_1} f_{M_2}) \frac{m_V}{m_D p_c} [b_3^p]_{M_1 M_2} (\epsilon \cdot p_D), \\
 PA_{M_1}^P &= \frac{G_F}{\sqrt{2}} (f_D f_{M_1} f_{M_2}) \frac{m_V}{m_D p_c} [b_4^p]_{M_1 M_2} (\epsilon \cdot p_D), \quad (2.6)
 \end{aligned}$$

where $p = d, s, p_c$ is the c.m. momentum, and $\eta = 1$ (-1) for $M_1 M_2 = PV$ (VP). We have inserted an additional factor of $m_V/(m_D p_c)$ in the PE and PA amplitudes so that they are expressed in units of $\epsilon \cdot p_D$. In our convention, M_2 is an emitted meson and M_1 shares the same spectator quark with the charmed meson. For the PA diagram with $q\bar{q}$ produced from the exchanged gluon, M_1 refers to the meson containing the antiquark $\bar{q}$. The factorizable matrix element X reads

$$\begin{aligned}
 X^{(DP,V)} &\equiv \langle V | J^\mu | 0 \rangle \langle P | J'_\mu | D \rangle = 2f_V m_V F_1^{DP}(m_V^2) (\epsilon \cdot p_D), \\
 X^{(DV,P)} &\equiv \langle P | J^\mu | 0 \rangle \langle V | J'_\mu | D \rangle = 2f_P m_V A_0^{DV}(m_P^2) (\epsilon \cdot p_D), \quad (2.7)
 \end{aligned}$$

where we have followed the conventional Bauer-Stech-Wirbel definition for form factors [35]. The chiral factors $r_\chi^{M_2}$ in Eq. (2.6) are given by

³Notice the notation difference (especially PE and PA) in [23] and [24]. The notation of P_T , P_C , PE , and PA in [23] was changed to PT , PC , PA , and PE , respectively, in [24]. In terms of the latter notation, PT , PC , PA , and PE correspond to our S , P , PE , and PA , respectively.

FIG. 1. Penguin-type topological diagrams: (a) QCD-penguin P , (b) flavor-singlet QCD-penguin S diagrams with 2 (3) gluon lines for M_2 being a pseudoscalar meson P (vector meson V), (c) QCD-penguin exchange PE , and (d) QCD-penguin annihilation PA diagram. In our convention, M_1 shares the same spectator quark with the charmed meson, while M_2 is an emitted meson. For the PA diagram with $q\bar{q}$ produced from the exchanged gluon, M_1 is the meson containing the antiquark $\bar{q}$.

$$r_{\chi}^P(\mu) = \frac{2m_P^2}{m_c(\mu)(m_2 + m_1)(\mu)}, \quad r_{\chi}^V(\mu) = \frac{2m_V}{m_c(\mu)} \frac{f_V^\perp(\mu)}{f_V}, \quad (2.8)$$

with $f_V^\perp(\mu)$ being the scale-dependent transverse decay constant of the vector meson V . The flavor operators a_i^p have the expressions [32]

$$a_i^p(M_1 M_2) = \left(c_i + \frac{c_{i\pm 1}}{N_c} \right) N_i(M_2) + \frac{c_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \left[V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2) \right] + \mathcal{P}_i^p(M_2), \quad (2.9)$$

where $C_F = (N_c^2 - 1)/(2N_c)$ with N_c denoting the number of colors, and contributions from vertex corrections V_i , hard spectator interactions H_i , and penguin contractions $\mathcal{P}_i$ have been taken into account [32]. Note that $a_i(M_2 M_1) \neq a_i(M_1 M_2)$, in general. The annihilation operators $b_{3,4}^p$ are given by

$$b_3^p = \frac{C_F}{N_c^2} [c_3 A_1^i + c_5 (A_3^i + A_3^f) + N_c c_6 A_3^f], \quad b_4^p = \frac{C_F}{N_c^2} [c_4 A_1^i + c_6 A_2^i], \quad (2.10)$$

where the annihilation amplitudes $A_{1,2,3}^{i,f}$ are defined in [32].

Flavor amplitude decompositions of the SCS $D \rightarrow VP$ decays are updated in Table IV. Comparing with Table VI in [9], we have made the following improvements: (i) The notation of PE_P and PE_V in our previous publications [9,21] should be interchanged, i.e., $PE_P \leftrightarrow PE_V$.⁴

⁴We see from Fig. 1 that PE_P (PE_V) is always accompanied by P_P (P_V). In our previous publications, we had, for example, $A(D^+ \rightarrow K^+ \bar{K}^{*0}) = \lambda_d A_V + \lambda_s T_V + \lambda_p (P_V^p + PE_P)$ and $A(D^+ \rightarrow \bar{K}^0 K^{*+}) = \lambda_d A_P + \lambda_s T_P + \lambda_p (P_P^p + PE_V)$ (see Table II of [21]). Hence, in order to be consistent with the notation in Fig. 1, we need to interchange PE_P and PE_V . This is done in Table IV.

- (ii) Both QCD-penguin annihilation diagrams PA_P and PA_V contribute to $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ decays.
- (iii) Singlet QCD-penguin contributions denoted by S_V and S_P for SU(3)-flavor singlets η, η', ω , and ϕ are included.
- (iv) SU(3) breaking in W -exchange amplitudes is taken into account through Eq. (2.5). Values of SU(3)-breaking effects are explicitly shown in the following modes: $D^0 \rightarrow \pi^0 \phi$, $D^+ \rightarrow \pi^+ \phi$, $K^+ \bar{K}^{*0}$, $\bar{K}^0 K^{*+}$ and $D_s^+ \rightarrow \pi^+ K^{*0}$, $\pi^0 K^{*+}$, $K^+ \rho^0$, $K^0 \rho^+$, $K^+ \omega$.

Calculations based on QCDF give the hierarchical relation for QCD-penguin amplitudes (see, e.g., Table VII below)

$$|P_P| \sim |P_V| > |S_V| \sim |PE_V| \gtrsim |S_P| \sim |PE_P| > |PA_V| \sim |PA_P|. \quad (2.11)$$

Roughly speaking, $|PE/P| \sim \mathcal{O}(10^{-1})$ and $|PA/P| \sim \mathcal{O}(10^{-2})$.

C. Branching fractions and SU(3) breaking

In the topological-diagram approach, we apply the topological amplitudes extracted from the CF modes to the SCS decays to predict their rates. It is well known that there exists significant symmetry-breaking effects in some of the SCS modes from the flavor SU(3) symmetry limit. In principle, SU(3)-breaking effects in the color-allowed T and color-suppressed C tree amplitudes can be estimated provided they are factorizable (in units of $\epsilon \cdot p_D$),

$$\begin{aligned} T_V &= \frac{G_F}{\sqrt{2}} a_1(\bar{K}^* \pi) 2f_\pi m_{K^*} A_0^{DK^*}(m_\pi^2), \\ C_P &= \frac{G_F}{\sqrt{2}} a_2(\bar{K}^* \pi) 2f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2), \\ T_P &= \frac{G_F}{\sqrt{2}} a_1(\bar{K} \rho) 2f_\rho m_\rho F_1^{DK}(m_\rho^2), \\ C_V &= \frac{G_F}{\sqrt{2}} a_2(\bar{K} \rho) 2f_K m_\rho A_0^{D\rho}(m_K^2). \end{aligned} \quad (2.12)$$

The topological amplitude sizes of T_V , C_P , T_P , and C_V given in Table II are extracted from a fit to the data of Cabibbo-allowed $D \rightarrow VP$ decays. They are dominated by

TABLE IV. Flavor amplitude decompositions for singly Cabibbo-suppressed $D \rightarrow VP$ decays. Summation over $p = d, s$ is understood. The symbol $S_V^{\eta_q(\eta_s)}$ implies a singlet amplitude with $M_1 = V$ and $M_2 = \eta_q(\eta_s)$. Values of SU(3)-breaking effects in $D^0 \rightarrow \pi^0 \phi$ and some of the D^+ and D_s^+ decays are shown.

Mode	Representation
D^0	
$\pi^+ \rho^-$	$\lambda_d(T_V + E_P^d) + \lambda_p(P_V^p + PE_V + PA_V + PA_P)$
$\pi^- \rho^+$	$\lambda_d(T_P + E_V^d) + \lambda_p(P_P^p + PE_P + PA_V + PA_P)$
$\pi^0 \rho^0$	$\frac{1}{2} \lambda_d(-C_P - C_V + E_P^d + E_V^d) + \frac{1}{2} \lambda_p(P_P^p + P_V^p + PE_P + PE_V + PA_P + PA_V)$
$K^+ K^{*-}$	$\lambda_s(T_V + E_P^s) + \lambda_p(P_V^p + PE_V + PA_P + PA_V)$
$K^- K^{*+}$	$\lambda_s(T_P + E_V^s) + \lambda_p(P_P^p + PE_P + PA_P + PA_V)$
$K^0 \bar{K}^{*0}$	$\lambda_d E_V^d + \lambda_s E_P^s + \lambda_p(PA_P + PA_V)$
$\bar{K}^0 K^{*0}$	$\lambda_d E_P^d + \lambda_s E_V^s + \lambda_p(PA_P + PA_V)$
$\pi^0 \omega$	$\frac{1}{2} \lambda_d(-C_V + C_P - E_P^d - E_V^d) + \frac{1}{2} \lambda_p(P_P^p + P_V^p + PE_P + PE_V + 2S_P)$
$\pi^0 \phi$	$\frac{1}{\sqrt{2}} \lambda_s(1.23C_P) + \frac{1}{\sqrt{2}} \lambda_p S_P$
$\eta \omega$	$\frac{1}{2} [\lambda_d(C_V + C_P + E_V^d + E_P^d) \cos \phi - \sqrt{2} \lambda_s C_V \sin \phi$ $+ \lambda_p(P_P^p + P_V^p + PE_P + PE_V + PA_P + PA_V + (2S_{\eta_q} + 2S_V^{\eta_q}) \cos \phi - \sqrt{2} S_V^{\eta_s} \sin \phi)]$
$\eta' \omega$	$\frac{1}{2} [\lambda_d(C_V + C_P + E_V^d + E_P^d) \sin \phi + \sqrt{2} \lambda_s C_V \cos \phi$ $+ \lambda_p(P_P^p + P_V^p + PE_P + PE_V + PA_P + PA_V + (2S_{\eta'_q} + 2S_V^{\eta'_q}) \sin \phi + \sqrt{2} S_V^{\eta'_s} \cos \phi)]$
$\eta \phi$	$\lambda_s[\frac{1}{\sqrt{2}} C_P \cos \phi - (E_V^s + E_P^s) \sin \phi] - \lambda_p(PA_P + PA_V) \sin \phi + \frac{1}{\sqrt{2}} \lambda_p S_{\eta_q} \cos \phi$
$\eta \rho^0$	$\frac{1}{2} [\lambda_d(C_V - C_P - E_V^d - E_P^d) \cos \phi - \lambda_s \sqrt{2} C_V \sin \phi] + \frac{1}{2} \lambda_p[(P_P^p + P_V^p + PE_P + PE_V + 2S_V^{\eta_q}) \cos \phi - \sqrt{2} S_V^{\eta_s} \sin \phi]$
$\eta' \rho^0$	$\frac{1}{2} [\lambda_d(C_V - C_P - E_V^d - E_P^d) \sin \phi + \lambda_s \sqrt{2} C_V \cos \phi] + \frac{1}{2} \lambda_p[(P_P^p + P_V^p + PE_P + PE_V + 2S_V^{\eta'_q}) \sin \phi + \sqrt{2} S_V^{\eta'_s} \cos \phi]$
D^+	
$\pi^+ \rho^0$	$\frac{1}{\sqrt{2}} [\lambda_d(T_V + C_P - A_P + A_V) + \lambda_p(P_V^p - P_P^p + PE_V - PE_P)]$
$\pi^0 \rho^+$	$\frac{1}{\sqrt{2}} [\lambda_d(T_P + C_V + A_P - A_V) + \lambda_p(P_P^p - P_V^p + PE_P - PE_V)]$
$\pi^+ \omega$	$\frac{1}{\sqrt{2}} [\lambda_d(T_V + C_P + A_P + A_V) + \lambda_p(P_P^p + P_V^p + PE_P + PE_V + 2S_P)]$
$\pi^+ \phi$	$\lambda_s(1.23C_P) + \lambda_p S_P$
$\eta \rho^+$	$\frac{1}{\sqrt{2}} [\lambda_d(T_P + C_V + A_V + A_P) \cos \phi - \lambda_s \sqrt{2} C_V \sin \phi] + \frac{1}{\sqrt{2}} \lambda_p[(P_P^p + P_V^p + PE_P + PE_V + 2S_V^{\eta_q}) \cos \phi - \sqrt{2} S_V^{\eta_s} \sin \phi]$
$\eta' \rho^+$	$\frac{1}{\sqrt{2}} [\lambda_d(T_P + C_V + A_V + A_P) \sin \phi + \lambda_s \sqrt{2} C_V \cos \phi] + \frac{1}{\sqrt{2}} \lambda_p[(P_P^p + P_V^p + PE_P + PE_V + 2S_V^{\eta'_q}) \sin \phi + \sqrt{2} S_V^{\eta'_s} \cos \phi]$
$K^+ \bar{K}^{*0}$	$\lambda_d A_V + \lambda_s(1.22T_V) + \lambda_p(P_V^p + PE_V)$
$\bar{K}^0 K^{*+}$	$\lambda_d A_P + \lambda_s(1.25T_P) + \lambda_p(P_P^p + PE_P)$
D_s^+	
$\pi^+ K^{*0}$	$\lambda_d(0.74T_V) + \lambda_s A_V + \lambda_p(P_V^p + PE_V)$
$\pi^0 K^{*+}$	$\frac{1}{\sqrt{2}} [\lambda_d(0.97C_V) - \lambda_s A_V - \lambda_p(P_V^p + PE_V)]$
$K^+ \rho^0$	$\frac{1}{\sqrt{2}} [\lambda_d(0.77C_P) - \lambda_s A_P - \lambda_p(P_P^p + PE_P)]$
$K^0 \rho^+$	$\lambda_d(0.81T_P) + \lambda_s A_P + \lambda_p(P_P^p + PE_P)$
ηK^{*+}	$\frac{1}{\sqrt{2}} [\lambda_d C_V + \lambda_s A_V + \lambda_p(P_V^p + PE_V + 2S_V^{\eta_q})] \cos \phi - [\lambda_s(T_P + C_V + A_P) + \lambda_p(P_P^p + PE_P + S_V^{\eta_s})] \sin \phi$
$\eta' K^{*+}$	$\frac{1}{\sqrt{2}} [\lambda_d C_V + \lambda_s A_V + \lambda_p(P_V^p + PE_V + 2S_V^{\eta'_q})] \sin \phi + [\lambda_s(T_P + C_V + A_P) + \lambda_p(P_P^p + PE_P + S_V^{\eta'_s})] \cos \phi$
$K^+ \omega$	$\frac{1}{\sqrt{2}} [\lambda_d(0.71C_P) + \lambda_s A_P + \lambda_p(P_P^p + PE_P + 2S_P)]$
$K^+ \phi$	$\lambda_s(T_V + C_P + A_V) + \lambda_p(P_V^p + PE_V + S_P)$

$D^0 \rightarrow K^{*-} \pi^+, \bar{K}^{*0} \pi^0, K^- \rho^+$, and $\bar{K}^0 \rho^0$, respectively. SU(3)-breaking effects in $T_{V,P}$ and $C_{V,P}$ in the SCS modes can be estimated based on factorization. However, as explained in [9], we found that the consideration of SU(3) breaking in $T_{V,P}$ and $C_{V,P}$ alone would render even larger deviations from the data. For example, we obtained $T_V(\pi^+ \rho^-) \simeq 0.82T_V$, $T_P(\pi^- \rho^+) \simeq 0.92T_P$, $T_V(K^+ K^{*-}) \simeq 1.29T_V$ and $T_P(K^- K^{*+}) \simeq 1.28T_P$. They will lead to too-small branching fractions for $D^0 \rightarrow \pi^+ \rho^-$ and $D^0 \rightarrow \pi^- \rho^+$ and too-large $\mathcal{B}(D^0 \rightarrow K^+ K^{*-})$ and $\mathcal{B}(D^0 \rightarrow K^- K^{*+})$

compared to experiment. Hence, the discrepancy becomes even worse. To circumvent this dilemma, we instead focus on SU(3) breaking in the W -exchange amplitudes, as we did in [9]. By the same token, our predictions on the branching fractions of SCS VP decays of D^+ and D_s^+ in [9] were made without taking into account SU(3)-breaking amplitudes due to the absence of W -exchange contributions.

For the experimental branching fractions of the quasi-two-body $D \rightarrow VP$ decays listed in Table V, we have applied the narrow-width approximation (NWA)

TABLE V. Branching fractions (in units of 10^{-3}) of singly Cabibbo-suppressed $D \rightarrow VP$ decays. The predictions made using solution (S3') have taken into account SU(3)-breaking effects under solution (iv) (see Table III). Experimental branching fractions taken from [30] in conjunction with the narrow-width approximation are also quoted in units of 10^{-3} .

Mode	$\mathcal{B}_{\text{theory}}$	$\mathcal{B}_{\text{exp}}$	Mode	$\mathcal{B}_{\text{theory}}$	$\mathcal{B}_{\text{exp}}$
$D^0 \rightarrow \pi^+ \rho^-$	5.15 ± 0.21	5.15 ± 0.25	$D^0 \rightarrow \pi^0 \omega$	0.13 ± 0.02	0.117 ± 0.035
$D^0 \rightarrow \pi^- \rho^+$	10.10 ± 0.37	10.1 ± 0.4	$D^0 \rightarrow \pi^0 \phi$	0.95 ± 0.02	1.20 ± 0.04
$D^0 \rightarrow \pi^0 \rho^0$	3.79 ± 0.12	3.86 ± 0.23	$D^0 \rightarrow \eta \omega$	2.09 ± 0.09	1.98 ± 0.18
$D^0 \rightarrow K^+ K^{*-}$	1.65 ± 0.06	1.65 ± 0.11	$D^0 \rightarrow \eta' \omega$	0.02 ± 0.00	$\dots$
$D^0 \rightarrow K^- K^{*+}$	4.56 ± 0.15	4.56 ± 0.21	$D^0 \rightarrow \eta \phi$	0.19 ± 0.02	0.181 ± 0.034^c
$D^0 \rightarrow K^0 \bar{K}^{*0}$	0.25 ± 0.01	0.246 ± 0.048	$D^0 \rightarrow \eta \rho^0$	0.59 ± 0.06	$\dots$
$D^0 \rightarrow \bar{K}^0 K^{*0}$	0.34 ± 0.06	0.336 ± 0.063	$D^0 \rightarrow \eta' \rho^0$	0.06 ± 0.00	$\dots$
$D^+ \rightarrow \pi^+ \rho^0$	0.68 ± 0.09	0.83 ± 0.15	$D^+ \rightarrow \eta \rho^+$	0.94 ± 0.42	$\dots$
$D^+ \rightarrow \pi^0 \rho^+$	4.44 ± 0.59	$\dots$	$D^+ \rightarrow \eta' \rho^+$	1.23 ± 0.11	$\dots$
$D^+ \rightarrow \pi^+ \omega$	0.22 ± 0.05	0.28 ± 0.06	$D^+ \rightarrow K^+ \bar{K}^{*0}$	5.92 ± 0.18	3.71 ± 0.16^d
$D^+ \rightarrow \pi^+ \phi$	4.87 ± 0.10	5.59 ± 0.10	$D^+ \rightarrow \bar{K}^0 K^{*+}$	16.28 ± 0.61	17.6 ± 1.8^e
$D_s^+ \rightarrow \pi^+ K^{*0}$	2.06 ± 0.08	2.23 ± 0.33^a	$D_s^+ \rightarrow \eta K^{*+}$	0.46 ± 0.19	$\dots$
$D_s^+ \rightarrow \pi^0 K^{*+}$	0.92 ± 0.06	0.75 ± 0.25^b	$D_s^+ \rightarrow \eta' K^{*+}$	0.41 ± 0.02	$\dots$
$D_s^+ \rightarrow K^+ \rho^0$	1.22 ± 0.06	2.5 ± 0.4	$D_s^+ \rightarrow K^+ \omega$	0.99 ± 0.05	0.87 ± 0.25
$D_s^+ \rightarrow K^0 \rho^+$	7.64 ± 0.33	5.46 ± 0.95^b	$D_s^+ \rightarrow K^+ \phi$	0.12 ± 0.02	0.18 ± 0.04

^aThe new measurement of $\mathcal{B}(D_s^+ \rightarrow \pi^+ K^{*0}) = (2.71 \pm 0.72 \pm 0.30) \times 10^{-3}$ from BESIII [38] is taken into account in the world average.

^bData from BESIII [38].

^cApparently, the published result of the BESIII measurement of $D^0 \rightarrow \eta \phi$ [39] was not taken into account in the world average in the 2020 version of PDG [30].

^dThe new measurement of $\mathcal{B}(D^+ \rightarrow K^+ \bar{K}^{*0}) = (3.10 \pm 0.46 \pm 0.68 \pm 0.18) \times 10^{-3}$ from BESIII [40] is taken into account in the world average.

^eThe new measurement of $\mathcal{B}(D^+ \rightarrow K_S K^{*+}) = (8.69 \pm 0.40 \pm 0.64 \pm 0.51) \times 10^{-3}$ from BESIII [40] is taken into account in the world average.

$$\Gamma(D \rightarrow VP \rightarrow P_1 P_2 P)_{\Gamma_V \rightarrow 0} = \Gamma(D \rightarrow VP) \mathcal{B}(V \rightarrow P_1 P_2) \quad (2.13)$$

to extract $\mathcal{B}(D \rightarrow VP)$. Notice that this relation is valid only in the narrow-width limit, namely, $\Gamma_V \rightarrow 0$. Corrections to the NWA have recently been studied in [36,37] for B decays. We shall return to this issue in Sec. III E.

Very recently, BESIII [38] reported the analysis of $D_s^+ \rightarrow K_S \pi^+ \pi^0$ decay and obtained the branching fractions of various modes (see Table V). It turns out that our predictions of $\mathcal{B}(D_s^+ \rightarrow \pi^+ K^{*0}) = (3.65 \pm 0.24) \times 10^{-3}$ and $\mathcal{B}(D_s^+ \rightarrow K^0 \rho^+) = (11.47 \pm 0.48) \times 10^{-3}$ in [9] are too large compared to the BESIII new measurements listed in Table V. In particular, the latter is larger by a factor of 2. Also, it was already noticed in [9] that $\mathcal{B}(D_s^+ \rightarrow K^+ \omega) = (2.12 \pm 0.10) \times 10^{-3}$ was 2 to 3 times bigger than the BESIII result of $(0.87 \pm 0.25) \times 10^{-3}$ [41]. This calls for the necessity of incorporating the SU(3)-breaking effects in the tree amplitudes $T_{V,P}$ and $C_{V,P}$ of the SCS D_s^+ decays.

There was a poorly measured branching fraction in the D^+ sector, namely, $\mathcal{B}(D^+ \rightarrow \bar{K}^0 K^{*+}) = (34 \pm 16) \times 10^{-3}$ [30]. Thanks to BESIII, a new measurement of this mode with a

significantly improved precision is now available [40]. It is based on the amplitude analysis of $D^+ \rightarrow K^+ K_S \pi^0$. The new world average predominated by BESIII now becomes $\mathcal{B}(D^+ \rightarrow \bar{K}^0 K^{*+}) = (17.6 \pm 1.8) \times 10^{-3}$. Our prediction of $(9.80 \pm 0.41) \times 10^{-3}$ in [9] is too small by about 4 σ .

In light of the discrepancy between theory and several new data of VP decays of D^+ and D_s^+ , SU(3) breaking in SCS decays of both D^+ and D_s^+ is needed. It appears that we have a rule of thumb in the absence of W -exchange: It is necessary to consider its SU(3)-breaking effects if only one of the $T_{V,P}$ and $C_{V,P}$ topological amplitudes appears in the decay amplitude. According to this simple rule, we need to account for SU(3) breaking in the color-allowed or color-suppressed tree topological amplitude only in the following SCS modes:

$$\begin{aligned} D^0 \rightarrow \pi^0 \phi, \quad D^+ \rightarrow \pi^+ \phi, \quad K^+ \bar{K}^{*0}, \quad K_S K^{*+}, \\ D_s^+ \rightarrow \pi^+ K^{*0}, \quad \pi^0 K^{*+}, \quad K^+ \rho^0, \quad K^0 \rho^+, \quad K^+ \omega. \end{aligned} \quad (2.14)$$

The SU(3)-breaking effects in SCS $D_s \rightarrow VP$ decays can be estimated from Eq. (2.12) and the factorization

$$\begin{aligned} \frac{T_P^{(D_s^+ \rightarrow K^0 \rho^+)}}{T_P} &= \frac{F_1^{D_s K}(m_\rho^2)}{F_1^{DK}(m_\rho^2)}, & \frac{C_V^{(D_s^+ \rightarrow \pi^0 K^{*+})}}{C_V} &= \frac{f_\pi m_{K^*} A_0^{D_s K^*}(m_\pi^2)}{f_K m_\rho A_0^{D\rho}(m_K^2)}, \\ \frac{T_V^{(D_s^+ \rightarrow \pi^+ K^{*0})}}{T_V} &= \frac{A_0^{D_s K^*}(m_\pi^2)}{A_0^{DK^*}(m_\pi^2)}, & \frac{C_P^{(D_s^+ \rightarrow K^+ \rho^0)}}{C_P} &= \frac{f_\rho m_\rho F_1^{D_s K}(m_\rho^2)}{f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2)}, \end{aligned} \quad (2.15)$$

and

$$\frac{C_P^{(D_s^+ \rightarrow K^+ \omega)}}{C_P} = \frac{f_\omega m_\omega F_1^{D_s K}(m_\omega^2)}{f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2)}, \quad (2.16)$$

where we have assumed that a_1 and a_2 are process independent. Likewise, for SCS $D^+ \rightarrow VP$ decays, we have

$$\begin{aligned} \frac{T_P^{(D^+ \rightarrow K^+ \bar{K}^{*0})}}{T_P} &= \frac{f_{K^*} m_{K^*} F_1^{D_s K}(m_{K^*}^2)}{f_\rho m_\rho F_1^{DK}(m_\rho^2)}, \\ \frac{T_V^{(D^+ \rightarrow \bar{K}^0 K^{*+})}}{T_V} &= \frac{f_K A_0^{DK^*}(m_{K^*}^2)}{f_\pi A_0^{DK^*}(m_\pi^2)}, \end{aligned} \quad (2.17)$$

and

$$\frac{C_P^{(D^+ \rightarrow \pi^+ \phi)}}{C_P} = \frac{f_\phi m_\phi F_1^{D\pi}(m_\phi^2)}{f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2)}. \quad (2.18)$$

Using the decay constants and form factors together with their q^2 dependence evaluated in the covariant confining quark model [42], we show the SU(3)-breaking effects in the decay amplitudes in Table IV. Now, we see from Table V that the agreement with the new BESIII measurements of $D_s^+ \rightarrow \pi^+ K^{*0}$ and $K^0 \rho^+$ is substantially improved. Likewise, the issue with $\mathcal{B}(D_s^+ \rightarrow K^+ \omega)$ as noticed in [9] is also resolved. However, the predicted $\mathcal{B}(D_s^+ \rightarrow K^+ \rho^0)$ now becomes smaller than experiment. In the topological diagram approach, $D_s^+ \rightarrow K^+ \rho^0$ and $D_s^+ \rightarrow K^+ \omega$ should have similar rates, as their amplitudes are mainly governed by $\lambda_d C_P / \sqrt{2}$. Their SU(3)-breaking effects are also similar. It is thus desirable to have an improved measurement of $\mathcal{B}(D_s^+ \rightarrow K^+ \rho^0)$. For the $D_s^+ \rightarrow K^+ \phi$ mode, if we consider its SU(3) breaking in both T_V and C_P amplitudes, we have $T_V(K^+ \phi) = 1.45 T_V$ and $C_P(K^+ \phi) = 1.20 C_P$. This will lead to $\mathcal{B}(D_s^+ \rightarrow K^+ \phi) = (0.46 \pm 0.07) \times 10^{-3}$, which is too large compared to the current value of $(0.182 \pm 0.041) \times 10^{-3}$ [30]. Thus, in the rule of thumb, we note that if two of the $T_{V,P}$ and $C_{V,P}$ topologies or more appear in the decay amplitude, we should not consider their SU(3)-breaking effects.

In the D^+ sector, the predicted $\mathcal{B}(D^+ \rightarrow \pi^+ \phi) = (3.22 \pm 0.17) \times 10^{-3}$ in the absence of SU(3) breaking is smaller than the experimental value of $(5.59 \pm 0.10) \times 10^{-3}$ by a factor of 1.7. Thus, a SU(3) breaking in C_P is welcome, though it is not large enough in our calculation.

The new prediction of $\mathcal{B}(D^+ \rightarrow \bar{K}^0 K^{*+}) = (16.3 \pm 0.6) \times 10^{-3}$ is now in good agreement with the BESIII measurement. Unfortunately, the original nice agreement between theory and experiment for $\mathcal{B}(D^+ \rightarrow K^+ \bar{K}^{*0})$ (see Table VIII of [9]) is spoiled when the SU(3) breaking in T_V is taken into account. Our result of $\mathcal{B}(D^+ \rightarrow K^+ \bar{K}^{*0}) = (5.92 \pm 0.18) \times 10^{-3}$ is too large compared to the measured value of $(3.71 \pm 0.16) \times 10^{-3}$. Thus, we face a dilemma that both $D^+ \rightarrow \bar{K}^0 K^{*+}$, $K^+ \bar{K}^{*0}$ modes cannot be accounted for simultaneously in the same SU(3)-breaking scheme. For this, we need to wait for further experimental justification.

In Table III we show four sets of solutions for SU(3) breaking in the W -exchange amplitudes extracted from a fit to eight SCS channels. All the solutions are the same for E_V^d and E_P^d but differ for E_V^s and E_P^s . It turns out that $D^0 \rightarrow \eta \phi$ is the only channel which receives contributions from both $E_{V,P}^s$: $A(D^0 \rightarrow \eta \phi) \propto \frac{1}{\sqrt{2}} C_P \cos \phi - (E_V^s + E_P^s) \sin \phi$ (see Table IV). Hence, it can be used to discriminate these solutions. Explicitly, we find (in units of 10^{-3})

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow \eta \phi) &= 0.12 \pm 0.01, & 0.23 \pm 0.01, \\ &0.008 \pm 0.005, & 0.19 \pm 0.02 \end{aligned} \quad (2.19)$$

for solutions (i), (ii), (iii), and (iv), respectively. It is obvious that the calculated branching fraction based on solution (iv) agrees better with the measured value of 0.18 ± 0.03 than the other solutions. Therefore, we will stick to solution (iv) for SU(3) breaking in the W -exchange amplitudes hereafter.

D. Direct CP violation

We note that the QCD-penguin exchange diagrams receive sizable long-distance contributions from final-state rescattering [21]. We assume that the long-distance PE_V and PE_P are of the same order of magnitude as E_P and E_V , respectively.⁵ For concreteness, we follow [9] to assign the same magnitude and phase as the W -exchange amplitudes with 20% and 30° uncertainties, respectively, so that [in units of $10^{-6}(\epsilon \cdot p_D)$]

$$\begin{aligned} (PE_V)^{\text{LD}} &\approx (0.58 \pm 0.12) e^{i(283 \pm 30)^\circ}, \\ (PE_P)^{\text{LD}} &\approx (1.69 \pm 0.34) e^{-i(108 \pm 30)^\circ}. \end{aligned} \quad (2.20)$$

⁵As noted before, the notation of PE_P and PE_V in our previous publications [9,21] should be interchanged, $PE_P \leftrightarrow PE_V$.

For simplicity, we assume its flavor independence, that is, $(PE)_d^{\text{LD}} = (PE)_s^{\text{LD}}$. CP asymmetries of the SCS $D \rightarrow VP$ decays are updated in Table VI. The improvements over our previous work (see Table IX of [9]) are as follows: (i) In our previous work, the factor of $\epsilon \cdot p_D$ term in the QCD-penguin amplitudes has been replaced by $p_c(m_D/m_V)$, while the tree amplitudes are expressed in terms of $\epsilon \cdot p_D$. This inconsistency is corrected in this work. (ii) Typos in the code for $D^+ \rightarrow \pi^0 \rho^+$ and $D_s^+ \rightarrow K^+ \phi$ are corrected. The resultant CP asymmetries $a_{CP}^{\text{dir}}(D^+ \rightarrow \pi^0 \rho^+) = (-0.44 \pm 0.52) \times 10^{-3}$ and $a_{CP}^{\text{dir}}(D_s^+ \rightarrow K^+ \phi) = (-1.33 \pm 1.59) \times 10^{-3}$ are quite different from the previous values of $(0.08 \pm 0.11) \times 10^{-3}$ and 0, respectively. (iii) Singlet QCD-penguin contributions as well as weak penguin annihilation (PE and PA) effects are included under the column denoted by $a_{\text{dir}}^{(\text{t+pe+pa+s})}$. The $D^0 \rightarrow \pi^0 \phi$ and

$D^+ \rightarrow \pi^+ \phi$ decays proceed only through the tree diagram C_P . Nevertheless, they receive a small flavor-singlet QCD-penguin contribution S_P . Owing to the interference between the tree and singlet QCD-penguin, CP asymmetries no longer vanish, though they are very small, of order 10^{-6} .

From Table VI we identify several golden modes which have large branching fractions and sizeable CP asymmetries at the order of 10^{-3} :

$$\begin{aligned} D^0 &\rightarrow \pi^+ \rho^-, K^+ K^{*-}, & D^+ &\rightarrow \eta \rho^+, K^+ \bar{K}^{*0}, \\ D_s^+ &\rightarrow \pi^+ K^{*0}, \pi^0 K^{*+}. \end{aligned} \quad (2.21)$$

It is interesting to notice that the CP asymmetry difference defined by

TABLE VI. Direct CP asymmetries of singly Cabibbo-suppressed $D \rightarrow VP$ decays (in units of 10^{-3}) in this work, where $a_{\text{dir}}^{(\text{tree})}$ denotes CP asymmetry arising from purely tree amplitudes. The superscript (t + p) denotes tree plus QCD-penguin amplitudes, (t + pe + pa + s) for tree plus PE , PA , and S amplitudes, (t + pe^{LD}) for tree plus long-distance PE amplitude induced from final-state rescattering, and “tot” for the total amplitude. As explained in the text, we use solution (iv) for the SU(3)-breaking effect in the W -exchange amplitudes (see Table III). The predictions from [24] in the FAT approach with the $\rho - \omega$ mixing are listed in the last column for comparison.

Mode	$a_{\text{dir}}^{(\text{tree})}$	$a_{\text{dir}}^{(\text{t+p})}$	$a_{\text{dir}}^{(\text{t+pe+pa+s})}$	$a_{\text{dir}}^{(\text{t+pe}^{\text{LD}})}$	$a_{\text{dir}}^{(\text{tot})}$	$a_{\text{dir}}^{(\text{tot})}$ [24]
$D^0 \rightarrow \pi^+ \rho^-$	0	-0.00 ± 0.00	-0.011 ± 0.000	0.77 ± 0.22	0.76 ± 0.22	-0.03
$D^0 \rightarrow \pi^- \rho^+$	0	0.01 ± 0.00	0.008 ± 0.001	-0.13 ± 0.08	-0.11 ± 0.08	-0.01
$D^0 \rightarrow \pi^0 \rho^0$	0	-0.01 ± 0.00	-0.004 ± 0.000	0.28 ± 0.16	0.27 ± 0.16	-0.03
$D^0 \rightarrow K^+ K^{*-}$	0	-0.01 ± 0.01	0.011 ± 0.000	-0.85 ± 0.24	-0.85 ± 0.24	-0.01
$D^0 \rightarrow K^- K^{*+}$	0	-0.03 ± 0.00	-0.009 ± 0.000	0.08 ± 0.09	0.04 ± 0.09	0
$D^0 \rightarrow K^0 \bar{K}^{*0}$	-0.03 ± 0.02	-0.03 ± 0.02	-0.03 ± 0.02	-0.03 ± 0.02	-0.03 ± 0.02	-0.7
$D^0 \rightarrow \bar{K}^0 K^{*0}$	1.07 ± 0.12	1.07 ± 0.12	1.07 ± 0.12	1.07 ± 0.12	1.07 ± 0.12	-0.7
$D^0 \rightarrow \pi^0 \omega$	0	0.04 ± 0.00	0.04 ± 0.01	-1.51 ± 0.87	-1.43 ± 0.87	0.02
$D^0 \rightarrow \pi^0 \phi$	0	0	-0.004	0	-0.004	-0.0002
$D^0 \rightarrow \eta \omega$	-0.13 ± 0.01	-0.12 ± 0.01	-0.13 ± 0.01	-0.35 ± 0.10	-0.35 ± 0.10	-0.1
$D^0 \rightarrow \eta' \omega$	2.06 ± 0.11	1.93 ± 0.11	1.93 ± 0.11	1.48 ± 0.61	1.23 ± 0.60	2.2
$D^0 \rightarrow \eta \phi$	0	0	0.009	0	0.009	0.003
$D^0 \rightarrow \eta \rho^0$	0.45 ± 0.03	0.51 ± 0.03	0.49 ± 0.03	0.16 ± 0.30	0.26 ± 0.31	1.0
$D^0 \rightarrow \eta' \rho^0$	-0.65 ± 0.06	-0.63 ± 0.06	-0.62 ± 0.06	-0.17 ± 0.23	-0.13 ± 0.23	-0.1
$D^+ \rightarrow \pi^+ \rho^0$	0	0.33 ± 0.02	0.10 ± 0.01	0.83 ± 1.36	1.26 ± 1.34	0.5
$D^+ \rightarrow \pi^0 \rho^+$	0	0.10 ± 0.01	0.04 ± 0.00	-0.58 ± 0.52	-0.44 ± 0.52	0.2
$D^+ \rightarrow \pi^+ \omega$	0	0.01 ± 0.01	0.08 ± 0.01	0.93 ± 2.28	1.03 ± 2.28	-0.05
$D^+ \rightarrow \pi^+ \phi$	0	0	-0.004	0	-0.004	-0.0001
$D^+ \rightarrow \eta \rho^+$	-1.85 ± 0.51	-1.97 ± 0.54	-1.93 ± 0.55	-2.31 ± 0.92	-2.50 ± 0.98	-0.6
$D^+ \rightarrow \eta' \rho^+$	0.23 ± 0.05	0.20 ± 0.05	0.21 ± 0.05	0.39 ± 0.16	0.34 ± 0.15	0.5
$D^+ \rightarrow K^+ \bar{K}^{*0}$	-0.11 ± 0.01	-0.14 ± 0.01	-0.11 ± 0.01	-0.77 ± 0.24	-0.80 ± 0.24	0.2
$D^+ \rightarrow \bar{K}^0 K^{*+}$	-0.04 ± 0.01	-0.05 ± 0.01	-0.05 ± 0.01	-0.06 ± 0.06	0.04 ± 0.07	0.04
$D_s^+ \rightarrow \pi^+ K^{*0}$	0.18 ± 0.02	0.24 ± 0.02	0.19 ± 0.02	1.25 ± 0.41	1.32 ± 0.41	-0.1
$D_s^+ \rightarrow \pi^0 K^{*+}$	0.13 ± 0.02	0.12 ± 0.03	0.11 ± 0.03	1.35 ± 0.40	1.31 ± 0.40	-0.2
$D_s^+ \rightarrow K^+ \rho^0$	0.14 ± 0.03	0.11 ± 0.02	0.15 ± 0.03	-0.26 ± 0.12	-0.29 ± 0.12	0.3
$D_s^+ \rightarrow K^0 \rho^+$	0.06 ± 0.02	0.08 ± 0.02	0.08 ± 0.02	-0.10 ± 0.10	-0.07 ± 0.10	0.3
$D_s^+ \rightarrow \eta K^{*+}$	1.18 ± 0.23	0.86 ± 0.16	0.95 ± 0.18	0.95 ± 0.75	0.40 ± 0.70	1.1
$D_s^+ \rightarrow \eta' K^{*+}$	-0.19 ± 0.04	-0.16 ± 0.04	0.14 ± 0.04	-0.33 ± 0.19	-0.24 ± 0.19	-0.5
$D_s^+ \rightarrow K^+ \omega$	-0.15 ± 0.03	-0.14 ± 0.03	-0.16 ± 0.03	0.27 ± 0.14	0.28 ± 0.14	-2.3
$D_s^+ \rightarrow K^+ \phi$	0	-0.32 ± 0.02	-0.14 ± 0.01	-0.88 ± 1.61	-1.33 ± 1.59	-0.8

$$\Delta a_{CP}^{VP} \equiv a_{CP}(K^+ K^{*-}) - a_{CP}(\pi^+ \rho^-), \quad (2.22)$$

in analogy to ΔA_{CP} defined in Eq. (1.1) for the corresponding PP final states, is predicted to be $(-1.61 \pm 0.33) \times 10^{-3}$, which is very similar to the recently observed CP asymmetry difference between $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$. This is an attractive and measurable observable in the near future. It is thus desirable to first search for CP violation in the aforementioned golden modes.

Since the quasi-two-body decays $D^0 \rightarrow \pi^+ \rho^-$ ($D^0 \rightarrow \pi^- \rho^+$) and $D^0 \rightarrow K^+ K^{*-}$ ($D^0 \rightarrow K^- K^{*+}$) are

connected by an interchange of all d and s quarks, there exists a general U -spin relation between corresponding CP -rate differences [43]

$$|A(D \rightarrow f)|^2 - |A(\bar{D} \rightarrow \bar{f})|^2 = -(|A(UD \rightarrow Uf)|^2 - |A(U\bar{D} \rightarrow U\bar{f})|^2), \quad (2.23)$$

where U denotes a U -spin transformation ($d \leftrightarrow s$) and the overall minus sign on the right-hand side comes from a change in the CKM factors. Hence,

$$\begin{aligned} \Gamma(D^0 \rightarrow \pi^+ \rho^-) - \Gamma(\bar{D}^0 \rightarrow \pi^- \rho^+) &= -(\Gamma(D^0 \rightarrow K^+ K^{*-}) - \Gamma(\bar{D}^0 \rightarrow K^- K^{*+})), \\ \Gamma(D^0 \rightarrow \pi^- \rho^+) - \Gamma(\bar{D}^0 \rightarrow \pi^+ \rho^-) &= -(\Gamma(D^0 \rightarrow K^- K^{*+}) - \Gamma(\bar{D}^0 \rightarrow K^+ K^{*-})). \end{aligned} \quad (2.24)$$

Since the CP -averaged rates of $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ are identical in the U -spin limit, we get the U -spin relations

$$\begin{aligned} a_{CP}^{\text{dir}}(D^0 \rightarrow K^+ K^{*-}) &= -a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^+ \rho^-), \\ a_{CP}^{\text{dir}}(D^0 \rightarrow K^- K^{*+}) &= -a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^- \rho^+). \end{aligned} \quad (2.25)$$

CP asymmetries for the two above-mentioned modes have equal magnitudes and opposite signs. It is evident from Table VI that the first U -spin relation in Eq. (2.25) is

approximately respected in our calculation, but this is not so for the second U -spin relation.

To see the effect of long-distance QCD-penguin exchange through final-state rescattering, we compare CP violation under the two columns $a_{\text{dir}}^{(t+pe\text{LD})}$ and $a_{\text{dir}}^{(\text{tot})}$ in Table VI. It is obvious that CP asymmetries of the following modes are dominated by the long-distance QCD-penguin exchange:

$$\begin{aligned} D^0 &\rightarrow \pi^+ \rho^-, \pi^- \rho^+, \pi^0 \rho^0, K^+ K^{*-}, K^- K^{*+}, \pi^0 \omega, \eta \omega, \\ D^+ &\rightarrow \pi^0 \rho^+, \pi^+ \omega, \eta \rho^+, K^+ \bar{K}^{*0}, \\ D_s^+ &\rightarrow \pi^+ K^{*0}, \pi^0 K^{*+}, K^+ \rho^0, K^0 \rho^+, K^+ \omega. \end{aligned} \quad (2.26)$$

The six golden modes listed in Eq. (2.21) all belong to this category. To see this, we consider the four decay modes $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$. From Table IV, their amplitudes read

$$\begin{aligned} \mathcal{M}(D^0 \rightarrow \pi^+ \rho^-) &= \lambda_d(T_\rho + E_\pi^d) + \lambda_d P_\rho^d + \lambda_s P_\rho^s - \lambda_b(PE_\rho + PA_\rho + PA_\pi), \\ \mathcal{M}(D^0 \rightarrow \rho^+ \pi^-) &= \lambda_d(T_\pi + E_\rho^d) + \lambda_d P_\pi^d + \lambda_s P_\pi^s - \lambda_b(PE_\pi + PA_\rho + PA_\pi), \\ \mathcal{M}(D^0 \rightarrow K^+ K^{*-}) &= \lambda_s(T_{K^*} + E_K^s) + \lambda_d P_{K^*}^d + \lambda_s P_{K^*}^s - \lambda_b(PE_{K^*} + PA_{K^*} + PA_K), \\ \mathcal{M}(D^0 \rightarrow K^{*+} K^-) &= \lambda_s(T_K + E_{K^*}^s) + \lambda_d P_K^d + \lambda_s P_K^s - \lambda_b(PE_K + PA_{K^*} + PA_K), \end{aligned} \quad (2.27)$$

with $\lambda_p \equiv V_{cp}^* V_{up}$ ($p = d, s, b$). Direct CP asymmetries are expressed by

$$\begin{aligned} a_{CP}^{\text{dir}}(\pi^+ \rho^-) &= 1.30 \times 10^{-3} \left| \frac{P_V^s + PE_V^s + PA_V^s + PA_P^d}{T_V + E_P^d + \Delta P} \right|_{\pi\rho} \sin \theta_{\pi\rho}, \\ a_{CP}^{\text{dir}}(\rho^+ \pi^-) &= 1.30 \times 10^{-3} \left| \frac{P_P^s + PE_P^s + PA_P^s + PA_V^d}{T_P + E_V^d + \Delta P} \right|_{\rho\pi} \sin \theta_{\rho\pi}, \\ a_{CP}^{\text{dir}}(K^+ K^{*-}) &= -1.30 \times 10^{-3} \left| \frac{P_V^d + PE_V^d + PA_V^s + PA_P^d}{T_V + E_P^s - \Delta P} \right|_{KK^*} \sin \theta_{KK^*}, \\ a_{CP}^{\text{dir}}(K^{*+} K^-) &= -1.30 \times 10^{-3} \left| \frac{P_P^d + PE_P^d + PA_P^s + PA_V^d}{T_P + E_V^s - \Delta P} \right|_{K^*K} \sin \theta_{K^*K}, \end{aligned} \quad (2.28)$$

TABLE VII. Numerical results of the topological tree and penguin amplitudes of $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ in units of $10^{-6}(\epsilon \cdot p_D)$. Only central values are presented here.

$D^0 \rightarrow \pi^+ \rho^-$	T_ρ	E_π^d	P_ρ^d	P_ρ^s	PE_ρ	$PA_\rho + PA_\pi$
	2.18	$-0.02 + 1.03i$	$(0.92 + 0.48i)10^{-1}$	$(0.96 + 0.44i)10^{-1}$	4.60×10^{-2}	4.96×10^{-4}
$D^0 \rightarrow \pi^- \rho^+$	T_π	E_ρ^d	P_π^d	P_π^s	PE_π	$PA_\rho + PA_\pi$
	$1.72 + 3.11i$	$0.00 - 0.20i$	$-(0.74 + 0.66i)10^{-1}$	$-(0.78 + 0.64i)10^{-1}$	-2.51×10^{-2}	4.96×10^{-4}
$D^0 \rightarrow K^+ K^{*-}$	T_{K^*}	E_K^s	$P_{K^*}^d$	$P_{K^*}^s$	PE_{K^*}	$PA_{K^*} + PA_K$
	2.18	$-0.08 + 0.67i$	$(1.18 + 0.61i)10^{-1}$	$(1.22 + 0.56i)10^{-1}$	6.06×10^{-2}	1.78×10^{-3}
$D^0 \rightarrow K^- K^{*+}$	T_K	$E_{K^*}^s$	P_K^d	P_K^s	PE_K	$PA_{K^*} + PA_K$
	$1.72 + 3.11i$	$-0.83 + 0.44i$	$-(1.12 + 1.04i)10^{-1}$	$-(1.18 + 1.00i)10^{-1}$	-2.77×10^{-2}	1.78×10^{-3}

where $\theta_{\pi\rho}$ is the phase of $(P_V^s + \dots + PA_P^d)$ relative to $(T_V + E_\rho^d + \Delta P)$ and likewise for the other phases.

In Table VII we show the numerical values of the topological tree and penguin amplitudes of $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$. The magnitudes of $T_{V,P}$ and $E_{V,P}^{d,s}$ are taken from Tables II and III, while QCD-penguins P , PE , and PA are calculated in QCDF through Eq. (2.6). It follows that

$$\left(\frac{P_V^s}{T_V}\right)_{\pi^+\rho^-} = 0.048e^{i24.7^\circ},$$

$$\left(\frac{P_V^s + PE_V^s + PA_V^d + PA_P^s}{T_V + E_\rho^d + \Delta P}\right)_{\pi^+\rho^-} = 0.062e^{-i8.9^\circ}, \quad (2.29)$$

for $D^0 \rightarrow \pi^+ \rho^-$ and

$$\left(\frac{P_V^d}{T_V}\right)_{K^+K^{*-}} = 0.061e^{i27.3^\circ},$$

$$\left(\frac{P_V^d + PE_V^d + PA_V^s + PA_P^d}{T_V + E_P^s - \Delta P}\right)_{K^+K^{*-}} = 0.086e^{i1.2^\circ}, \quad (2.30)$$

for $D^0 \rightarrow K^+ K^{*-}$. It is clear that the magnitude of P/T , which is of order 0.20 in the PP sector [see Eq. (1.7)], is further suppressed in the VP sector. Substituting Eqs. (2.29) and (2.30) into Eq. (2.28) yields

$$a_{\text{dir}}^{(t+p+pe+pa)}(\pi^+\rho^-) = -1.2 \times 10^{-5},$$

$$a_{\text{dir}}^{(t+p+pe+pa)}(K^+K^{*-}) = -2.4 \times 10^{-6}, \quad (2.31)$$

which are consistent with the results shown in Table VI.⁶ Obviously, the phase of $(P_V^d + PE_V^d + PA_V^s + PA_P^d)$ is almost compensated by that of $(T_V + E_P^s - \Delta P)$, that is, $\theta_{K^+K^{*-}} = 1.2^\circ$, so that the resulting CP asymmetry in $D^0 \rightarrow K^+ K^{*-}$ is vanishingly small, of order 10^{-6} . Likewise,

⁶In the absence of tree contributions to CP violation (i.e., $a_{\text{dir}}^t = 0$), $a_{\text{dir}}^{(t+p+pe+pa)}$ will be the sum of $a_{\text{dir}}^{(t+p)}$ and $a_{\text{dir}}^{(t+pe+pa)}$ shown in Table VI. Therefore, $a_{\text{dir}}^{(t+p+pe+pa)}(\pi^+\rho^-) = (-1.2 \pm 0.6) \times 10^{-5}$ and $a_{\text{dir}}^{(t+p+pe+pa)}(K^+K^{*-}) = (-2.2 \pm 0.4) \times 10^{-6}$.

$\theta_{\pi\rho} = -8.9^\circ$ also implies a very small direct CP violation in $D^0 \rightarrow \pi^+\rho^-$, of order 10^{-5} . For completeness, we give the results for the $\pi^-\rho^+$ and K^-K^{*+} modes,

$$a_{\text{dir}}^{(t+p+pe+pa)}(\pi^-\rho^+) = 2.1 \times 10^{-5},$$

$$a_{\text{dir}}^{(t+p+pe+pa)}(K^-K^{*+}) = -3.8 \times 10^{-5}. \quad (2.32)$$

After including the long-distance contributions to PE via final-state rescattering [see Eq. (2.20)], we have

$$\left(\frac{P_V^s + PE_V^s + PA_V^d + PA_P^s + PE_V^{\text{LD}}}{T_V + E_\rho^d + \Delta P}\right)_{\pi^+\rho^-} = 0.704e^{i77^\circ},$$

$$\left(\frac{P_V^d + PE_V^d + PA_V^s + PA_P^d + PE_V^{\text{LD}}}{T_V + E_P^s - \Delta P}\right)_{K^+K^{*-}} = 0.771e^{i84^\circ}. \quad (2.33)$$

It follows that

$$a_{CP}^{\text{dir}}(\pi^+\rho^-) = 0.89 \times 10^{-3},$$

$$a_{CP}^{\text{dir}}(K^+K^{*-}) = -1.00 \times 10^{-3}, \quad (2.34)$$

consistent with the predictions of $a_{CP}^{\text{dir}}(\pi^+\rho^-) = (0.76 \pm 0.22) \times 10^{-3}$ and $a_{CP}^{\text{dir}}(K^+K^{*-}) = (-0.85 \pm 0.24) \times 10^{-3}$ shown in Table VI.⁷

Two remarks are in order: (i) In the topological amplitude approach, the magnitude and the relative strong phase of each individual topological tree amplitude in charm decays can be extracted from the data. Consequently, direct CP asymmetries in charmed meson decays induced at tree level can be reliably estimated. Since the contribution from PA to $D^0 \rightarrow K^0 \bar{K}^{*0}$ and $D^0 \rightarrow \bar{K}^0 K^{*0}$ are very small compared to the W -exchange amplitudes, CP asymmetries

⁷As stated in [9], predictions in Tables V and VI are made by sampling 10^4 points in the parameter space, assuming that each of the parameters has a Gaussian distribution with the corresponding central value and symmetrized standard deviation. Then, the predicted values are the mean and standard deviation of data computed using the 10^4 points. This explains the difference between Eq. (2.34) and the central values presented in Table VI.

in these two modes are induced at tree level, in analogy to $D^0 \rightarrow K_S K_S$. We find a large CP asymmetry at the per mille level for $D^0 \rightarrow K_S K^{*0}$ but not for $D^0 \rightarrow K_S \bar{K}^{*0}$:

$$a_{CP}^{\text{dir}}(D^0 \rightarrow K_S K^{*0}) = (1.07 \pm 0.12) \times 10^{-3}. \quad (2.35)$$

This is consistent with the upper limit of 0.3% found in [44]. In view of the current efforts in search of CP asymmetry in $D^0 \rightarrow K_S K_S$, it is worthwhile to pursue that in the $D^0 \rightarrow K_S K^{*0}$ decay as well. (ii) Although the magnitude of PE and PA is smaller than P (see, e.g., Table VII), we see from Table VI that $a_{\text{dir}}^{(t+pe+pa)}$ is not necessarily smaller than $a_{\text{dir}}^{(t+p)}$ in some channels. It depends on the relative phases between T and P as well as between T and PE .

E. Comparison with the FAT approach

From Table VI it is evident that the predicted CP asymmetries given in [24] based on the FAT approach are generally smaller than ours by 1 to 2 orders of magnitude. To see the underlying reason, we consider QCD-penguin exchange PE and annihilation PA amplitudes. It follows from Eqs. (2.6) and (2.10) that

$$\begin{aligned} PE &= \frac{G_F}{\sqrt{2}} (f_D f_P f_V) \frac{C_F}{N_c^2} [c_3 A_1^i + c_5 A_3^i + (c_5 + N_c c_6) A_3^f], \\ PA &= \frac{G_F}{\sqrt{2}} (f_D f_P f_V) \frac{C_F}{N_c^2} [c_4 A_1^i + c_6 A_2^i], \end{aligned} \quad (2.36)$$

where the superscripts i and f refer to gluon emission from the initial- and final-state quarks, respectively. The subscript k on $A_k^{i,f}$ refers to one of the three possible Dirac structures: $k=1$ for $(V-A) \otimes (V-A)$, $k=2$ for $(V-A) \otimes (V+A)$, and $k=3$ for $-2(S-P) \otimes (S+P)$ [32]. The amplitudes A_k have the expressions

$$\begin{aligned} A_1 &\propto \langle M_1 M_2 | (\bar{u}q)_{V-A} \otimes (\bar{q}c)_{V-A} | D \rangle, \\ A_2 &\propto \langle M_1 M_2 | (\bar{u}q)_{V-A} \otimes (\bar{q}c)_{V+A} | D \rangle, \\ A_3 &\propto -2 \langle M_1 M_2 | (\bar{u}q)_{S+P} \otimes (\bar{q}c)_{S-P} | D \rangle, \end{aligned} \quad (2.37)$$

with $(\bar{q}_1 q_2)_{S\pm P} \equiv \bar{q}_1 (1 \pm \gamma_5) q_2$ and $(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$. Now A_k^f corresponds to the factorizable contribution, while A_k^i corresponds to the nonfactorizable

contribution of A_k . It turns out that $A_1^f = A_2^f = 0$ because of helicity suppression, but this is not so for A_3^f owing to its $(S-P)(S+P)$ structure for the four-quark operator; that is,

$$A_3^f \propto -2 \langle M_1 M_2 | (\bar{u}q)_{S+P} | 0 \rangle \langle 0 | (\bar{q}c)_{S-P} | D \rangle. \quad (2.38)$$

In [24] the factorizable $PE_{V(P)}^f$ amplitude has the expression

$$\begin{aligned} PE_{V(P)}^f &= \frac{G_F}{\sqrt{2}} a_6(\mu) (-2) \\ &\times \langle VP(PV) | (\bar{u}q)_{S+P} | 0 \rangle \langle 0 | (\bar{q}c)_{S-P} | D \rangle. \end{aligned} \quad (2.39)$$

It was evaluated in the pole model by assuming its dominance by resonant pseudoscalars. Explicitly, it reads⁸

$$\begin{aligned} &\langle VP(PV) | (\bar{u}q)_{S+P} | 0 \rangle \\ &\rightarrow \langle VP(PV) | H_s | P^* \rangle \frac{1}{m_D^2 - m_{P^*}^2} \langle P^* | (\bar{u}q)_{S+P} | 0 \rangle, \end{aligned} \quad (2.40)$$

where P^* represents the pole resonant pseudoscalar meson and H_s is the corresponding strong Hamiltonian. Since $\langle PV | H_s | P^* \rangle = -\langle VP | H_s | P^* \rangle$ [24], this leads to the relation $A_3^f(PV) + A_3^f(VP) = 0$. Take $D^0 \rightarrow \pi^+ \rho^-$ as an example, where P_V and PE_V^f are given by [24]

$$\begin{aligned} P_V &= \frac{G_F}{\sqrt{2}} [a_4(\mu) - r_\chi^\pi a_6(\mu)] 2f_\pi m_\rho A_0^{D\rho}(m_\pi^2), \\ PE_V^f &= \frac{G_F}{\sqrt{2}} a_6(\mu) g_{PPV} r_\chi^\pi f_{P^*} f_D m_D^2 \frac{1}{m_D^2 - m_{P^*}^2}. \end{aligned} \quad (2.41)$$

It was claimed in [24] that there was a numerical coincidence that P_V and PE_V^f canceled each other. As a consequence, CP asymmetries in $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ decays are very small, of order 10^{-5} .

In QCDF, A_3^f is expressed in terms of the twist-2 light-cone distribution amplitude (LCDA) Φ_M and the twist-3 one Φ_m . A direct evaluation of the weak annihilation diagram with the four-quark operator $(\bar{u}q)_{S+P} \otimes (\bar{q}c)_{S-P}$ yields [32]

$$\begin{aligned} A_3^f(VP) &= \pi \alpha_s \int_0^1 dx dy \left\{ r_\chi^V \Phi_P(x) \Phi_V(y) \frac{2(1+\bar{x})}{\bar{x}^2 y} - r_\chi^P \Phi_V(y) \Phi_P(x) \frac{2(1+y)}{\bar{x} y^2} \right\}, \\ A_3^f(PV) &= \pi \alpha_s \int_0^1 dx dy \left\{ r_\chi^P \Phi_V(x) \Phi_P(y) \frac{2(1+\bar{x})}{\bar{x}^2 y} + r_\chi^V \Phi_P(y) \Phi_V(x) \frac{2(1+y)}{\bar{x} y^2} \right\}, \end{aligned} \quad (2.42)$$

⁸For the PP case, it was assumed in [23] that $\langle P_1 P_2 | (\bar{q}_1 q_2) | 0 \rangle$ was dominated by the lowest scalar resonance, so $\langle P_1 P_2 | (\bar{q}_1 q_2) | 0 \rangle = \langle P_1 P_2 | S \rangle \langle S | \bar{q}_1 q_2 | 0 \rangle = g_S T_S^{\text{BW}}(q^2) m_S \tilde{f}_S$, with T_S^{BW} being the Breit-Wigner line shape for the scalar resonance S . However, the light scalars such as $\sigma(500)$, $f_0(980)$, and $a_0(980)$ were not taken into account in [23]. In QCDF, A_3^f is expressed in terms of the twist-2 LCDA Φ_M and the twist-3 one Φ_m .

with $\bar{x} = 1 - x$ and $\bar{y} = 1 - y$. It is obvious that $A_3^f(PV) \neq -A_3^f(VP)$ in the QCDF approach, contrary to the aforementioned pole model assumption. It is known that the integrals in Eq. (2.42) involve endpoint divergences. We follow [31] to model the endpoint divergence $X \equiv \int_0^1 dx/\bar{x}$ in the penguin annihilation diagram as

$$X_A = \ln\left(\frac{m_D}{\Lambda_h}\right)(1 + \rho_A e^{i\phi_A}), \quad (2.43)$$

with Λ_h being a typical hadronic scale of 0.5 GeV. The expressions of A_3^f can be further simplified by using the asymptotic distribution amplitudes $\Phi_{P,V}(x) = 6x\bar{x}$, $\Phi_p(x) = 1$ and $\Phi_v(x) = 3(x - \bar{x})$ [32]:

$$\begin{aligned} A_3^f(VP) &\approx 6\pi\alpha_s[3r_\chi^V(2X_A - 1)(2 - X_A) - r_\chi^P(2X_A^2 - X_A)], \\ A_3^f(PV) &\approx 6\pi\alpha_s[-3r_\chi^P(2X_A - 1)(2 - X_A) + r_\chi^V(2X_A^2 - X_A)]. \end{aligned} \quad (2.44)$$

In principle, one can also add the superscripts “VP” and “PV” to distinguish penguin annihilation effects in $D \rightarrow VP$ and $D \rightarrow PV$ decays. Unfortunately, unlike B decays we do not have any knowledge about the parameters ρ_A and ϕ_A for charm decays. Therefore, we consider the “default” value $X_A = \ln(m_D/\Lambda_h)$ in this work. The numerical values of PE and PA shown in Table VII are obtained using the default value of X_A . We see from this table that $-\lambda_b PE_V$ is smaller than $\lambda_d P_d + \lambda_s P_s$, and they are of the same sign. Therefore, there is no cancellation between P_V and PE_V in QCDF.

In short, before the consideration of the long-distance contribution to PE through final-state rescattering, the predicted CP asymmetries in $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ in QCDF are also very small, of order 10^{-5} [$K^+ K^{*-}$ is further suppressed; see Eqs. (2.31) and (2.32)]. However, they are very small in QCDF for a reason that is quite different from FAT: The phase angles $\theta_{\pi\rho}$, $\theta_{\rho\pi}$, θ_{KK^*} , and θ_{K^*K} in Eq. (2.28) become smaller or even close to zero after including the contributions from PE , PA , and W -exchange to the ratio of P/T . It is the long-distance QCD-penguin exchange that explains why our predictions of CP asymmetries in $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ are much bigger than those in the FAT approach.

III. CP VIOLATION IN THREE-BODY D DECAYS THROUGH VECTOR RESONANCES

In this section we study the Dalitz plots of CP asymmetry distributions in some of the SCS $D \rightarrow P_1 P_2 P_3$ decays. In particular, we focus on the three-body $D^0 \rightarrow K^+ K^- \pi^0$, $D^0 \rightarrow \pi^+ \pi^- \pi^0$, $D^+ \rightarrow K^+ K_S \pi^0$, and $D_s^+ \rightarrow K^0 \pi^+ \pi^0$ decays as they receive contributions from quasi-two-body $D^0 \rightarrow K^\pm K^{*\mp}$, $D^0 \rightarrow \pi^\pm \rho^\mp$, $\pi^0 \rho^0$, $D^+ \rightarrow K_S K^{*+}$, $K^+ \bar{K}^{*0}$, and $D_s^+ \rightarrow \pi^{+0} K^{*0,+}$ decays, respectively. Due to the interference between various vector resonances carrying the same mass but different electric charges, the magnitude and sign of local CP asymmetry vary from region to region. Large CP asymmetries can occur in some localized regions of phase space. For previous studies of CP violation in three-body D decays, see [17,45]. For a study of three-body hadronic D decay amplitudes within the framework of QCDF, the reader is referred to [46]. For the proper three-body approaches using more complex theoretical background in Fadeev techniques, and Khuri-Treiman and triangle singularities, see [47–50].

In principle, CP asymmetry may also arise from the interference between P - and S -wave contributions. For example, a significant CP asymmetry in the $\rho(770)$ region coming from the interference between the $\rho(770)$ and S -wave resonances has been observed by the LHCb [51,52]. See also the discussions in [46]. Owing to the lack of knowledge on scalar resonances, we will not pursue this direction in this work.

A. $D^0 \rightarrow K^+ K^- \pi^0$

Consider the Dalitz plot analysis of $D^0 \rightarrow K^+ K^- \pi^0$ in the overlapped region of the vector resonances $K^{*+}(892)$ and $K^{*-}(892)$ so that

$$A(D^0 \rightarrow K^+(p_1) K^-(p_2) \pi^0(p_3)) \approx A_{K^{*+}} + A_{K^{*-}}, \quad (3.1)$$

with

$$A_{K^{*\pm}} \equiv A(D^0 \rightarrow K^\mp K^{*\pm} \rightarrow K^+ K^- \pi^0). \quad (3.2)$$

The explicit expression of $A_{K^{*+}}$ reads

$$\begin{aligned} A_{K^{*+}} &= \langle K^+(p_1) \pi^0(p_3) | \mathcal{L} | K^{*+} \rangle T_{K^*}^{\text{BW}}(s_{13}) \langle K^-(p_2) K^{*+} | H_W | D^0 \rangle \\ &= g^{K^{*+} \rightarrow K^+ \pi^0} F(s_{13}, m_{K^*}) \epsilon^* \cdot (p_3 - p_1) T_{K^*}^{\text{BW}}(s_{13}) \mathcal{M}(D^0 \rightarrow K^- K^{*+}), \end{aligned} \quad (3.3)$$

where $\mathcal{M}(D^0 \rightarrow K^- K^{*+})$ has been given in Eq. (2.27) and we have considered the relativistic Breit-Wigner line shape for $K^*(892)$,

$$T_{K^*}^{\text{BW}}(s) = \frac{1}{s - m_{K^*}^2 + im_{K^*} \Gamma_{K^*}(s)}, \quad (3.4)$$

and a mass-dependent width

$$\Gamma_{K^*}(s) = \Gamma_{K^*}^0 \left(\frac{q}{q_0} \right)^3 \frac{m_{K^*}}{\sqrt{s}} \frac{X_1^2(q)}{X_1^2(q_0)}, \quad (3.5)$$

with $q = |\vec{p}_1| = |\vec{p}_3|$ being the c.m. momentum in the rest frame of K^{*+} , q_0 the value of q when s_{13} is equal to $m_{K^*}^2$, and X_1 a Blatt-Weisskopf barrier factor given by

$$X_1(z) = \sqrt{\frac{1}{(z r_{\text{BW}})^2 + 1}}, \quad (3.6)$$

with $r_{\text{BW}} \approx 4.0 \text{ GeV}^{-1}$. In Eq. (3.5), $\Gamma_{K^*}^0$ is the nominal total width of K^* with $\Gamma_{K^*}^0 = \Gamma_{K^*}(m_{K^*}^2)$. Likewise, the amplitude $A_{K^{*-}}$ is given by

$$A_{K^{*-}} = g^{K^{*-} \rightarrow K^- \pi^0} F(s_{23}, m_{K^*}) \epsilon^* \cdot (p_2 - p_3) T_{K^*}^{\text{BW}}(s_{23}) \mathcal{M}(D^0 \rightarrow K^- K^{*+}). \quad (3.7)$$

To derive Eq. (3.3), we have taken the Lagrangian

$$\mathcal{L} = -ig^{K^* \rightarrow K\pi} \left(\frac{1}{\sqrt{2}} K^{*+\mu} K^- \overset{\leftrightarrow}{\partial}_\mu \pi^0 + \frac{1}{\sqrt{2}} K^{*-\mu} \pi^0 \overset{\leftrightarrow}{\partial}_\mu K^+ - \frac{1}{\sqrt{2}} \bar{K}^{*0\mu} \pi^0 \overset{\leftrightarrow}{\partial}_\mu K^0 + K^{*+\mu} \bar{K}^0 \overset{\leftrightarrow}{\partial}_\mu \pi^- - \frac{1}{\sqrt{2}} K^{*0\mu} \bar{K}^0 \overset{\leftrightarrow}{\partial}_\mu \pi^0 \right) \quad (3.8)$$

to obtain $\langle K^+ \pi^0 | K^{*+\mu} K^- \overset{\leftrightarrow}{\partial}_\mu \pi^0 | K^{*+} \rangle = i\epsilon^* \cdot (p_3 - p_1)$. The form factor $F(s, m_{K^*})$ in the amplitudes $A_{K^{*\pm}}$ is introduced for the following reason. The coupling $|g^{K^{*\pm} \rightarrow K^\pm \pi^0}| = 3.15$ is extracted from the measured $K^*(892)$ width through the relation

$$\Gamma_{K^{*\pm} \rightarrow K^\pm \pi^0} = \frac{q_0^3}{6\pi m_{K^*}^2} g_{K^{*\pm} \rightarrow K^\pm \pi^0}^2. \quad (3.9)$$

When $K^*(892)$ is off the mass shell, especially when s_{13} is approaching the upper bound of $(m_D - m_K)^2$, it is necessary to account for the off-shell effect. For this purpose, we follow [37] to introduce a form factor $F(s, m_R)$ parametrized as

$$F(s, m_R) = \left(\frac{\Lambda^2 + m_R^2}{\Lambda^2 + s} \right)^n, \quad (3.10)$$

with the cutoff Λ not far from the resonance,

$$\Lambda = m_R + \beta \Lambda_{\text{QCD}}, \quad (3.11)$$

where the parameter β is expected to be of order unity. We use $n = 1$, $\Lambda_{\text{QCD}} = 250 \text{ MeV}$, and $\beta = 1.0 \pm 0.2$ in subsequent calculations.

The sum over the polarizations of K^* yields the familiar relation

$$\sum_\lambda \epsilon^{*\mu}(k, \lambda) \epsilon^\nu(k, \lambda) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_{K^*}^2}. \quad (3.12)$$

However, when the transversality condition $\epsilon^\mu k_\mu = 0$ is imposed, the mass in the denominator of the second term should be replaced by the invariant mass $m_{13} = \sqrt{s_{13}}$ or $m_{23} = \sqrt{s_{23}}$ [53], where $s_{13} \equiv (p_1 + p_3)^2 = s_{K^+ \pi^0}$ and $s_{23} \equiv (p_2 + p_3)^2 = s_{K^- \pi^0}$. Replacing $\epsilon \cdot p_D$ in $A_{K^{*+}}$ by $\frac{1}{2} \epsilon \cdot$

$(p_D + p_2)$ and $\epsilon \cdot p_D$ in $A_{K^{*-}}$ by $\frac{1}{2} \epsilon \cdot (p_D + p_1)$, it is straightforward to show that

$$\begin{aligned} A_{K^{*+}} &= -\frac{1}{2} \left(s_{23} - s_{12} + \frac{(m_D^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{13}} \right) g^{K^{*+} \rightarrow K^+ \pi^0} \\ &\quad \times F(s_{13}, m_{K^*}) T_{K^*}^{\text{BW}}(s_{13}) \tilde{\mathcal{M}}(D^0 \rightarrow K^+ K^{*-}), \\ A_{K^{*-}} &= \frac{1}{2} \left(s_{13} - s_{12} + \frac{(m_D^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{23}} \right) g^{K^{*-} \rightarrow K^- \pi^0} \\ &\quad \times F(s_{23}, m_{K^*}) T_{K^*}^{\text{BW}}(s_{23}) \tilde{\mathcal{M}}(D^0 \rightarrow K^- K^{*+}), \end{aligned} \quad (3.13)$$

where $\mathcal{M} = \tilde{\mathcal{M}}(\epsilon \cdot p_D)$. Notice that the term in the big parentheses can be recast as

$$\begin{aligned} s_{23} - s_{12} + \frac{(m_D^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{13}} &= 4\vec{p}_1 \cdot \vec{p}_2 \\ &= 4|\vec{p}_1||\vec{p}_2| \cos \theta, \end{aligned} \quad (3.14)$$

where $|\vec{p}_1|$ and $|\vec{p}_2|$ are the momenta of K^+ and K^- , respectively, in the rest frame of K^+ and π^0 . Hence, the angular distribution is proportional to the Legendre polynomial $P_1(\cos \theta)$.

From Eqs. (3.32) and (2.27), we obtain

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow K^- K^{*+} \rightarrow K^+ K^- \pi^0) &= 1.39 \times 10^{-3}, \\ \mathcal{B}(D^0 \rightarrow K^+ K^{*-} \rightarrow K^+ K^- \pi^0) &= 0.51 \times 10^{-3}, \end{aligned} \quad (3.15)$$

and

$$\begin{aligned} a_{CP}(D^0 \rightarrow K^- K^{*+} \rightarrow K^+ K^- \pi^0) &= 0.06 \times 10^{-3}, \\ a_{CP}(D^0 \rightarrow K^+ K^{*-} \rightarrow K^+ K^- \pi^0) &= -0.99 \times 10^{-3}. \end{aligned} \quad (3.16)$$

FIG. 2. CP asymmetry distribution of $D^0 \rightarrow K^+ K^- \pi^0$ in the overlapped regions of $K^*(892)^+$ and $K^*(892)^-$.

Note that the branching fractions given above are consistent with that in Table V in conjunction with the narrow-width approximation. Likewise, the calculated CP asymmetries are consistent with Table VI.

We show in Fig. 2 the Dalitz plot of the CP asymmetry distribution in the overlapped regions of $K^*(892)^+$ and $K^*(892)^-$. Owing to the interference between $D^0 \rightarrow K^+ K^{*-} \rightarrow K^+ K^- \pi^0$ and $D^0 \rightarrow K^- K^{*+} \rightarrow K^+ K^- \pi^0$, local CP violation varies from region to region. For example, a_{CP} is of the percent level in the region around $(s_{K^+ \pi^0}, s_{K^- \pi^0}) = (0.75, 0.80)$ GeV^2 (the pink area in Fig. 2), and it becomes negative, of order -2.0×10^{-3} , in the vicinity of $(s_{K^+ \pi^0}, s_{K^- \pi^0}) = (0.70, 0.77)$ GeV^2 .

Note that an additional relative phase δ is introduced to Eq. (3.30) in [45] so that

$$\begin{aligned} A(D^0 \rightarrow K^+(p_1)K^-(p_2)\pi^0(p_3)) \\ = A_{K^{*+}} + e^{i\delta}A_{K^{*-}} = |A_{K^{*+}}|e^{i\delta_{K^{*+}}} + e^{i\delta}|A_{K^{*-}}|e^{i\delta_{K^{*-}}}. \end{aligned} \quad (3.17)$$

Experimentally, the phase δ_{exp} defined by

$$\begin{aligned} A(D^0 \rightarrow K^+(p_1)K^-(p_2)\pi^0(p_3)) \\ = (|A_{K^{*+}}| + e^{i\delta_{\text{exp}}}|A_{K^{*-}}|)e^{i\delta_{K^{*+}}} \end{aligned} \quad (3.18)$$

was measured to be $-35.5^\circ \pm 1.9^\circ \pm 2.2^\circ$ by *BABAR* [54]. Neglecting the small penguin contributions to $D^0 \rightarrow K^\pm K^{*\mp}$, we have

$$\delta = \delta_{\text{exp}} + \delta_{K^{*+}} - \delta_{K^{*-}}. \quad (3.19)$$

We find $\delta_{K^{*+}} = 8.82^\circ$ and $\delta_{K^{*-}} = -29.74^\circ$ from Eq. (2.27) and Table VII. Our $\delta = (3.06 \pm 2.91)^\circ$ is to be compared with $\delta = (-51.85 \pm 2.91)^\circ$ obtained in [45].

B. $D^0 \rightarrow \pi^+ \pi^- \pi^0$

The Dalitz analysis of the three-body $D^0 \rightarrow \pi^+ \pi^- \pi^0$ decay by *BABAR* [55] shows fit fractions of $(68.1 \pm 0.6)\%$, $(34.6 \pm 0.8)\%$ and $(25.9 \pm 1.1)\%$ from $\rho(770)^+ \pi^-$, $\rho(770)^- \pi^+$ and $\rho(770)^0 \pi^0$, respectively. Therefore, the quasi-two-body $D^0 \rightarrow \pi \rho(770)$ decay gives the dominant contributions to $D^0 \rightarrow \pi^+ \pi^- \pi^0$. Contributions from $\rho(1450)\pi$ and $\rho(1700)\pi$ are small. Scalar meson contributions are further suppressed.

Consider the Dalitz plot of $D^0 \rightarrow \pi^+ \pi^- \pi^0$ in the overlapped region of the vector resonances $\rho(770)^\pm$ and $\rho(770)^0$ so that

$$A(D^0 \rightarrow \pi^+(p_1)\pi^-(p_2)\pi^0(p_3)) = A_{\rho^+} + A_{\rho^-} + A_{\rho^0}, \quad (3.20)$$

with

$$\begin{aligned} A_{\rho^+} &\equiv A(D^0 \rightarrow \rho^+ \pi^- \rightarrow \pi^+ \pi^- \pi^0) = g^{\rho \rightarrow \pi \pi} \epsilon^* \cdot (p_1 - p_3) F(s_{13}, m_\rho) T_\rho(s_{13}) \mathcal{M}(D^0 \rightarrow \rho^+ \pi^-), \\ A_{\rho^-} &\equiv A(D^0 \rightarrow \rho^- \pi^+ \rightarrow \pi^+ \pi^- \pi^0) = g^{\rho \rightarrow \pi \pi} \epsilon^* \cdot (p_3 - p_2) F(s_{23}, m_\rho) T_\rho(s_{23}) \mathcal{M}(D^0 \rightarrow \rho^- \pi^+), \\ A_{\rho^0} &\equiv A(D^0 \rightarrow \rho^0 \pi^0 \rightarrow \pi^+ \pi^- \pi^0) = g^{\rho \rightarrow \pi \pi} \epsilon^* \cdot (p_2 - p_1) F(s_{12}, m_\rho) T_\rho(s_{12}) \mathcal{M}(D^0 \rightarrow \rho^0 \pi^0), \end{aligned} \quad (3.21)$$

where we have used the $\rho\pi\pi$ interaction Lagrangian

$$\mathcal{L} = -ig^{\rho \rightarrow \pi \pi} (\rho^{+\mu} \pi^0 \overleftrightarrow{\partial}_\mu \pi^- + \rho^{-\mu} \pi^+ \overleftrightarrow{\partial}_\mu \pi^0 + \rho^{0\mu} \pi^- \overleftrightarrow{\partial}_\mu \pi^+). \quad (3.22)$$

Numerically, the on-shell coupling is $|g^{\rho \rightarrow \pi \pi}| = 6.0$. Hence,

$$\begin{aligned} A_{\rho^+} &= -\frac{1}{2}(s_{12} - s_{23}) g^{\rho \rightarrow \pi \pi} F(s_{13}, m_\rho) T_\rho(s_{13}) \tilde{\mathcal{M}}(D^0 \rightarrow \rho^+ \pi^-), \\ A_{\rho^-} &= \frac{1}{2}(s_{12} - s_{13}) g^{\rho \rightarrow \pi \pi} F(s_{23}, m_\rho) T_\rho(s_{23}) \tilde{\mathcal{M}}(D^0 \rightarrow \rho^- \pi^+), \\ A_{\rho^0} &= \frac{1}{2}(s_{13} - s_{23}) g^{\rho \rightarrow \pi \pi} F(s_{12}, m_\rho) T_\rho(s_{12}) \tilde{\mathcal{M}}(D^0 \rightarrow \rho^0 \pi^0). \end{aligned} \quad (3.23)$$

Since the $\rho(770)$ resonance is broad, a popular choice for describing its line shape is the Gounaris-Sakurai (GS) model [56] given by

$$T_\rho^{\text{GS}}(s) = \frac{1 + D\Gamma_\rho^0/m_\rho}{s - m_\rho^2 - f(s) + im_\rho\Gamma_\rho(s)}. \quad (3.24)$$

In this model, the real part of the pion-pion scattering amplitude with an intermediate ρ exchange calculated from the dispersion relation is taken into account by the $f(s)$ term in the propagator of $T_\rho^{\text{GS}}(s)$. Unitarity far from the pole mass is thus ensured. Explicitly,

$$f(s) = \Gamma_\rho^0 \frac{m_\rho^2}{q_0^3} \left[q^2 [h(\sqrt{s}) - h(m_\rho)] + (m_\rho^2 - s) q_0^2 \frac{dh}{ds} \Big|_{m_\rho} \right], \quad (3.25)$$

and

$$h(s) = \frac{2}{\pi} \frac{q}{\sqrt{s}} \ln \left(\frac{\sqrt{s} + 2q}{2m_\pi} \right),$$

$$\frac{dh}{ds} \Big|_{m_\rho} = h(m_\rho) \left[\frac{1}{8q_0^2} - \frac{1}{2m_\rho^2} \right] + \frac{1}{2\pi m_\rho^2}. \quad (3.26)$$

The constant parameter D is given by

$$D = \frac{3}{\pi} \frac{m_\pi^2}{q_0^2} \ln \left(\frac{m_\rho + 2q_0}{2m_\pi} \right) + \frac{m_\rho}{2\pi q_0} - \frac{m_\pi^2 m_\rho}{\pi q_0^3}. \quad (3.27)$$

We find

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow \pi^+\rho^- \rightarrow \pi^+\pi^-\pi^0) &= 5.59 \times 10^{-3}, \\ \mathcal{B}(D^0 \rightarrow \pi^-\rho^+ \rightarrow \pi^+\pi^-\pi^0) &= 13.11 \times 10^{-3}, \\ \mathcal{B}(D^0 \rightarrow \pi^0\rho^0 \rightarrow \pi^+\pi^-\pi^0) &= 5.07 \times 10^{-3}, \end{aligned} \quad (3.28)$$

and

$$\begin{aligned} a_{CP}(D^0 \rightarrow \pi^+\rho^- \rightarrow \pi^+\pi^-\pi^0) &= 0.886 \times 10^{-3}, \\ a_{CP}(D^0 \rightarrow \pi^-\rho^+ \rightarrow \pi^+\pi^-\pi^0) &= -0.150 \times 10^{-3}, \\ a_{CP}(D^0 \rightarrow \pi^0\rho^0 \rightarrow \pi^+\pi^-\pi^0) &= 0.295 \times 10^{-3}. \end{aligned} \quad (3.29)$$

CP asymmetry distributions in the Dalitz plot are illustrated in Fig. 3. The interference between the vector resonances $\rho^\pm$ and ρ^0 yields rich information on the local CP asymmetries in the Dalitz plot. It can reach the 4×10^{-3} level in the regions of $(s_{\pi^+\pi^0}, s_{\pi^-\pi^0}) = (0.31, 0.60)$ GeV^2 and $(0.38, 0.54)$ GeV^2 and becomes very negative, of order -5×10^{-3} , at $(s_{\pi^+\pi^0}, s_{\pi^-\pi^0}) = (0.25, 0.67)$ GeV^2 .

C. $D^+ \rightarrow K^+ K_S \pi^0$

For the SCS $D^+ \rightarrow VP$ decays, we see from Table VI that the quasi-two-body decays $\pi^+\rho^0$, $\pi^0\rho^+$, $\eta\rho^+$, and $K^+ \bar{K}^{*0}$ are predicted to have large CP asymmetries at the per mille level, of order $(1.0\text{--}2.5) \times 10^{-3}$. The first two modes $\pi^+\rho^0$ and $\pi^0\rho^+$ manifest in the decays $D^+ \rightarrow \pi^+\pi^+\pi^-$ and $\pi^+\pi^0\pi^0$, respectively. Due to the presence of two neutral pions, it is naively expected that the former three-body decay has a rate larger than the latter. Experimentally, it is the other way around: $\mathcal{B}(D^+ \rightarrow \pi^+\pi^+\pi^-) = (3.27 \pm 0.18) \times 10^{-3} < \mathcal{B}(D^+ \rightarrow \pi^+\pi^0\pi^0) = (4.7 \pm 0.4) \times 10^{-3}$ [30]. The $\pi^+\pi^+\pi^-$ mode is dominated by the S -wave contribution to $\pi^+\pi^-$, while the fit fraction of $\pi^+\rho^0$ is about 25%, yielding $\mathcal{B}(D^+ \rightarrow \pi^+\rho^0) = (0.83 \pm 0.15) \times 10^{-3}$ [30]. An amplitude analysis of $D^+ \rightarrow \pi^+\pi^0\pi^0$ is still not available. Our prediction of $\mathcal{B}(D^+ \rightarrow \pi^0\rho^+) = (4.44 \pm 0.59) \times 10^{-3}$ (see Table V) implies that this mode is dominated by $\pi^0\rho^+$.

The $D^+ \rightarrow \eta\pi^+\pi^0$ decay receives resonant contributions from ρ^+ and $a_0(980)^\pm$. To study the ρ^+ contribution experimentally, the contribution from $a_0(980)$ can be

FIG. 3. CP asymmetry distribution of $D^0 \rightarrow \pi^+\pi^-\pi^0$ in the overlapped region of $\rho(770)^\pm$ and $\rho(770)^0$ with $s_{\pi^+\pi^0}$ versus $s_{\pi^-\pi^0}$ ($s_{\pi^+\pi^-}$) in the left (right) panel.

eliminated with a simple cut by demanding $M(\pi^+\pi^0) < 1 \text{ GeV}$.⁹ This will allow us to extract the branching fraction of $D^+ \rightarrow \eta\rho^+$, which is predicted to be $(0.94 \pm 0.42) \times 10^{-3}$ in this work (see Table V). The branching fraction of $D^+ \rightarrow \eta\pi^+\pi^0$ was measured by CLEO to be $(1.38 \pm 0.35) \times 10^{-3}$. An improved measurement by BESIII [57] recently yielded $(2.23 \pm 0.15 \pm 0.10) \times 10^{-3}$.

The $D^+ \rightarrow K^+K^-\pi^+$ mode is dominated by the quasi-two-body decays $\pi^+\phi$, $K^+\bar{K}^{*0}(892)$, and $K^+\bar{K}^{*0}(1430)$ with the fit fractions of order 28%, 26%, and 19%, respectively [58]. In this work we find CP asymmetries with the values of -3.7×10^{-6} and $(-0.80 \pm 0.24) \times 10^{-3}$, respectively, for $\pi^+\phi$ and $K^+\bar{K}^{*0}(892)$. In [6] it was claimed that a sizable CP violation could occur in $D^+ \rightarrow K^+\bar{K}^{*0}(1430)$ within the FAT approach. A search of CP violation in $D^+ \rightarrow K^+K^-\pi^+$ has been carried out by LHCb without any evidence [59].

In short, among the aforementioned four $D^+ \rightarrow VP$ modes, analysis of $\pi^0\rho^+$ and $\eta\rho^+$ is not available yet, and the branching fraction of $\pi^+\rho^0$ is too small. Finally, we turn to the decay $D^+ \rightarrow K^+\bar{K}^0\pi^0$ whose amplitude analysis was recently performed by BESIII [40]. It is dominated by

the quasi-two-body decays $K_S K^{*+}$ and $K^+\bar{K}^{*0}$ with the fit fractions of order 57% and 10%, respectively,

$$A(D^+ \rightarrow K^+(p_1)\bar{K}^0(p_2)\pi^0(p_3)) \approx A_{K^{*+}} + A_{\bar{K}^{*0}}, \quad (3.30)$$

with

$$\begin{aligned} A_{K^{*+}} &\equiv A(D^+ \rightarrow K^{*+}\bar{K}^0 \rightarrow K^+\bar{K}^0\pi^0), \\ A_{\bar{K}^{*0}} &\equiv A(D^+ \rightarrow K^+\bar{K}^{*0} \rightarrow K^+\bar{K}^0\pi^0). \end{aligned} \quad (3.31)$$

Following a similar analysis for $D^0 \rightarrow K^+K^-\pi^0$, we obtain

$$\begin{aligned} A_{K^{*+}} &= -\frac{1}{2} \left(s_{23} - s_{12} + \frac{(m_D^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{13}} \right) g^{K^{*+} \rightarrow K^+\pi^0} \\ &\quad \times F(s_{13}, m_{K^*}) T_{K^*}^{\text{BW}}(s_{13}) \tilde{\mathcal{M}}(D^+ \rightarrow \bar{K}^0 K^{*+}), \\ A_{\bar{K}^{*0}} &= -\frac{1}{2} \left(s_{13} - s_{12} + \frac{(m_D^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{23}} \right) g^{\bar{K}^{*0} \rightarrow \bar{K}^0\pi^0} \\ &\quad \times F(s_{23}, m_{K^*}) T_{K^*}^{\text{BW}}(s_{23}) \tilde{\mathcal{M}}(D^+ \rightarrow K^+ \bar{K}^{*0}), \end{aligned} \quad (3.32)$$

where

$$\begin{aligned} \mathcal{M}(D^+ \rightarrow \bar{K}^0 K^{*+}) &= \lambda_d(A_P + P_P^d) + \lambda_s(1.25T_P + P_P^s) - \lambda_b P E_P, \\ \mathcal{M}(D^+ \rightarrow K^+ \bar{K}^{*0}) &= \lambda_d(A_V + P_V^d) + \lambda_s(1.22T_V + P_V^s) - \lambda_b P E_V, \end{aligned} \quad (3.33)$$

with $\mathcal{M} = \tilde{\mathcal{M}}(\epsilon \cdot p_D)$.

Numerically, we obtain

$$\begin{aligned} \mathcal{B}(D^+ \rightarrow \bar{K}^0 K^{*+} \rightarrow K^+\bar{K}^0\pi^0) &= 4.92 \times 10^{-3}, \\ \mathcal{B}(D^+ \rightarrow K^+ \bar{K}^{*0} \rightarrow K^+\bar{K}^0\pi^0) &= 1.81 \times 10^{-3}, \end{aligned} \quad (3.34)$$

and

$$\begin{aligned} a_{CP}(D^+ \rightarrow \bar{K}^0 K^{*+} \rightarrow K^+\bar{K}^0\pi^0) &= 0.06 \times 10^{-3}, \\ a_{CP}(D^+ \rightarrow K^+ \bar{K}^{*0} \rightarrow K^+\bar{K}^0\pi^0) &= -0.90 \times 10^{-3}. \end{aligned} \quad (3.35)$$

The CP asymmetry distribution of $D^+ \rightarrow K^+\bar{K}^0\pi^0$ in phase space is exhibited in Fig. 4. Regions with positive

and negative a_{CP} at the percent level are clearly seen in the plot.

D. $D_s^+ \rightarrow K^0\pi^+\pi^0$

Considering the decay amplitude of $D_s^+ \rightarrow K^0\pi^+\pi^0$ in the overlapped region of the K^{*+} and K^{*0} resonances, we have

$$A(D_s^+ \rightarrow K^0(p_1)\pi^+(p_2)\pi^0(p_3)) \approx A_{K^{*+}} + A_{K^{*0}}, \quad (3.36)$$

with

$$\begin{aligned} A_{K^{*+}} &\equiv A(D_s^+ \rightarrow \pi^0 K^{*+} \rightarrow K^0\pi^+\pi^0) = g^{K^{*+} \rightarrow K^0\pi^+} F(s_{12}, m_{K^*}) \frac{\epsilon^* \cdot (p_2 - p_1)}{s_{12} - m_{K^*}^2 + im_{K^*}\Gamma_{K^*}(s_{12})} \mathcal{M}(D_s^+ \rightarrow \pi^0 K^{*+}), \\ A_{K^{*0}} &\equiv A(D_s^+ \rightarrow \pi^+ K^{*0} \rightarrow K^0\pi^+\pi^0) = g^{K^{*0} \rightarrow K^0\pi^0} F(s_{13}, m_{K^*}) \frac{\epsilon^* \cdot (p_1 - p_3)}{s_{13} - m_{K^*}^2 + im_{K^*}\Gamma_{K^*}(s_{13})} \mathcal{M}(D_s^+ \rightarrow \pi^+ K^{*0}), \end{aligned} \quad (3.37)$$

⁹It is known, though, that $a_0(980)$ has a width of 50–100 MeV [30].

FIG. 4. CP asymmetry distribution of $D^+ \rightarrow K^+ \bar{K}^0 \pi^0$ in the overlapped regions of $K^*(892)^+$ and $\bar{K}^*(892)^0$.

where use of the $K^* K \pi$ interaction Lagrangian (3.8) has been made and

$$\begin{aligned} \mathcal{M}(D_s^+ \rightarrow \pi^0 K^{*+}) &= \frac{1}{\sqrt{2}} [\lambda_d(0.97C_V - P_V^d) - \lambda_s(A_V + P_V^s) \\ &\quad + \lambda_b P E_V], \\ \mathcal{M}(D_s^+ \rightarrow \pi^+ K^{*0}) &= \lambda_d(0.74T_V + P_V^d) \\ &\quad + \lambda_s(A_V + P_V^s) - \lambda_b P E_V, \end{aligned} \quad (3.38)$$

with the SU(3)-breaking effects in T_V and C_V included (cf. Table IV). Hence,

$$\begin{aligned} A_{K^{*+}} &= -\frac{1}{2} \left(s_{23} - s_{13} + \frac{(m_{D_s}^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{s_{12}} \right) g^{K^{*+} \rightarrow K^0 \pi^+} \\ &\quad \times F(s_{12}, m_{K^*}) T_{K^*}^{\text{BW}}(s_{12}) \tilde{\mathcal{M}}(D_s^+ \rightarrow \pi^0 K^{*+}), \\ A_{K^{*0}} &= \frac{1}{2} \left(s_{23} - s_{12} + \frac{(m_{D_s}^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{s_{13}} \right) g^{K^{*0} \rightarrow K^0 \pi^0} \\ &\quad \times F(s_{13}, m_{K^*}) T_{K^*}^{\text{BW}}(s_{13}) \tilde{\mathcal{M}}(D_s^+ \rightarrow \pi^+ K^{*0}). \end{aligned} \quad (3.39)$$

We obtain

$$\begin{aligned} \mathcal{B}(D_s^+ \rightarrow \pi^0 K^{*+} \rightarrow K^0 \pi^+ \pi^0) &= 0.55 \times 10^{-3}, \\ \mathcal{B}(D_s^+ \rightarrow \pi^+ K^{*0} \rightarrow K^0 \pi^+ \pi^0) &= 0.64 \times 10^{-3}, \end{aligned} \quad (3.40)$$

and

$$\begin{aligned} a_{CP}(D_s^+ \rightarrow \pi^0 K^{*+} \rightarrow K^0 \pi^+ \pi^0) &= 1.53 \times 10^{-3}, \\ a_{CP}(D_s^+ \rightarrow \pi^+ K^{*0} \rightarrow K^0 \pi^+ \pi^0) &= 1.47 \times 10^{-3}. \end{aligned} \quad (3.41)$$

FIG. 5. CP asymmetry distribution of $D_s^+ \rightarrow K^0 \pi^+ \pi^0$ in the overlapped regions of $K^*(892)^+$ and $K^*(892)^0$.

Figure 5 shows the Dalitz plot of the CP asymmetry distribution of $D_s^+ \rightarrow K^0 \pi^+ \pi^0$. Since CP violation in $D_s^+ \rightarrow \pi^0 K^{*+}$ and $D_s^+ \rightarrow \pi^+ K^{*0}$ is large and positive, CP asymmetry in most of the phase space is positive, though it can become negative in some regions. For example, it is of order -0.35×10^{-3} at $(s_{K^0 \pi^0}, s_{K^0 \pi^+}) = (1.97, 1.10) \text{ GeV}^2$.

E. Finite-width effects

Since all the framework has been set up, we are ready to discuss the finite-width effects. First of all, we need to show that the relation Eq. (2.13) is valid in the narrow-width limit. For illustration we take $D^0 \rightarrow \pi^+ \rho^- \rightarrow \pi^+(p_1) \pi^-(p_2) \pi^0(p_3)$ as an example. Since

$$s_{12} - s_{13} = 4\vec{p}_1 \cdot \vec{p}_2 = 4|\vec{p}_1||\vec{p}_2| \cos \theta, \quad (3.42)$$

where $|\vec{p}_1|$ and $|\vec{p}_2|$ are the momenta of π^+ and π^- , respectively, in the rest frame of π^- and π^0 , and

$$\begin{aligned} q &= |\vec{p}_2| = |\vec{p}_3| = \frac{\sqrt{s_{23} - 4m_\pi^2}}{2}, \\ |\vec{p}_1| &= \left(\frac{(m_D^2 - m_\pi^2 - s_{23})^2}{4s_{23}} - m_\pi^2 \right)^{1/2} = \frac{m_D}{\sqrt{s_{23}}} \tilde{p}_c, \end{aligned} \quad (3.43)$$

with $\tilde{p}_c$ being the c.m. momentum of $\pi^+(p_1)$ in the D^0 rest frame, the amplitude A_{ρ^-} in Eq. (3.23) can be recast to

$$\begin{aligned} A_{\rho^-} &= -g^{\rho \rightarrow \pi\pi} F(s_{23}, m_\rho) T_\rho^{\text{GS}}(s_{23}) \\ &\quad \times 2q \cos \theta \tilde{\mathcal{M}}(D^0 \rightarrow \pi^+ \rho^-) \frac{m_D}{\sqrt{s_{23}}} \tilde{p}_c. \end{aligned} \quad (3.44)$$

The decay rate is given by

$$\Gamma(D^0 \rightarrow \pi^+ \rho^- \rightarrow \pi^+ \pi^- \pi^0) = \frac{1}{2} \frac{|g^{\rho \rightarrow \pi^+ \pi^-}|^2}{(2\pi)^3 32 m_D^3} \times \int ds_{23} ds_{12} \frac{F(s_{23}, m_\rho)^2 (1 + D\Gamma_\rho^0/m_\rho)^2}{(s_{23} - m_{f_2}^2 - f(s_{23}))^2 + m_\rho^2 \Gamma_\rho^2(s_{23})} \times 4q^2 \cos^2 \theta |\tilde{\mathcal{M}}(D^0 \rightarrow \pi^+ \rho^-)|^2 \frac{m_D^2}{s_{23}} \tilde{p}_c^2. \quad (3.45)$$

The angular distribution part is given by [see Eq. (4.64) of [37]]

$$\int_{(s_{12})_{\min}}^{(s_{12})_{\max}} ds_{12} \cos^2 \theta = \frac{4}{3} \frac{m_D}{\sqrt{s_{23}}} q \tilde{p}_c. \quad (3.46)$$

In the narrow-width limit, we have [37]

$$\frac{m_\rho \Gamma_\rho(s) (1 + D\Gamma_\rho^0/m_\rho)^2}{(s - m_\rho^2 - f(s))^2 + m_\rho^2 \Gamma_\rho^2(s)} \xrightarrow{\Gamma_\rho \rightarrow 0} \pi \delta(s - m_\rho^2 - f(s)) \rightarrow \pi \delta(s - m_\rho^2). \quad (3.47)$$

As a result of the δ -function, $s_{23} \rightarrow m_\rho^2$. We then obtain the desired NWA:

$$\Gamma(D^0 \rightarrow \pi^+ \rho^- \rightarrow \pi^+ \pi^- \pi^0) \xrightarrow{\Gamma_\rho \rightarrow 0} \Gamma(D^0 \rightarrow \pi^+ \rho^-) \mathcal{B}(\rho^- \rightarrow \pi^- \pi^0), \quad (3.48)$$

where use of the relations

$$\Gamma_{\rho \rightarrow \pi^+ \pi^-} = \frac{q_0^3}{6\pi m_\rho^2} g_{\rho \rightarrow \pi^+ \pi^-}^2, \quad \Gamma_{D^0 \rightarrow \pi^+ \rho^-} = \frac{p_c^3}{8\pi m_\rho^2} |\tilde{\mathcal{M}}(D^0 \rightarrow \pi^+ \rho^-)|^2, \quad (3.49)$$

has been made, with q_0 and p_c being q and $\tilde{p}_c$, respectively, except for the replacement of s_{23} by m_ρ^2 . Numerically, we have also verified the NWA (3.48).

The finite-width effect is accounted for by the quantity η_R defined by [36,37]

$$\eta_R \equiv \frac{\Gamma(D \rightarrow RP_3 \rightarrow P_1 P_2 P_3)_{\Gamma_R \rightarrow 0}}{\Gamma(D \rightarrow RP_3 \rightarrow P_1 P_2 P_3)} = \frac{\Gamma(D \rightarrow RP_3) \mathcal{B}(R \rightarrow P_1 P_2)}{\Gamma(D \rightarrow RP_3 \rightarrow P_1 P_2 P_3)} = 1 + \delta, \quad (3.50)$$

so that the deviation of η_R from unity measures the degree of departure from the NWA when the resonance width is finite. It is naively expected that the correction δ will be of order Γ_R/m_R . The calculated η_V parameters for vector resonances ρ and K^* produced in the three-body D decays are summarized in Table VIII. Since ρ is 3 times broader than that of $K^*(892)$, it is naively expected that the deviation of η_ρ from unity should be larger than that of η_{K^*} . However, our calculation shows that they are close to each other. Our results are to be compared with $\eta_\rho = 0.93$ in the GS line-shape model and 1.11 in the BW line shape for $B^+ \rightarrow \rho^0 \pi^+ \rightarrow \pi^+ \pi^- \pi^+$, and $\eta_{K^*} = 1.067 \pm 0.002$ for $B^+ \rightarrow K^{*0} \pi^+ \rightarrow K^+ \pi^+ \pi^-$ [36,37].

When the resonance is sufficiently broad, it is necessary to take into account the finite-width effects characterized by the parameter η_R . Explicitly,

$$\mathcal{B}(D \rightarrow VP) = \eta_V \mathcal{B}(D \rightarrow VP)_{\text{NWA}}, \quad (3.51)$$

where $\mathcal{B}(D \rightarrow RP)_{\text{NWA}}$ denotes the branching fraction obtained from Eq. (2.13) valid in the NWA. Take $D^0 \rightarrow \pi \rho$ as an example. The branching fractions have been extracted from $D^0 \rightarrow \pi^+ \pi^- \pi^0$ by BABAR using the Breit-Wigner line shape [55]. According to our calculation, $\eta_\rho^{\text{BW}} = 1.116$. Hence, the PDG values of (5.15 ± 0.25) , (10.1 ± 0.4) , and (3.86 ± 0.23) (in units of 10^{-3}) [30] for the branching fractions of $D^0 \rightarrow \pi^+ \rho^-$, $\pi^- \rho^+$, and $\pi^0 \rho^0$, respectively, should be corrected to (5.75 ± 0.28) , (11.3 ± 0.5) , and (4.31 ± 0.26) .

TABLE VIII. Summary of the η_R parameter for vector resonances produced in the three-body D decays. The parameter η_{ρ^+} extracted from $D^0 \rightarrow \pi^- \rho^+ \rightarrow \pi^+ \pi^- \pi^0$ is the same as η_{ρ^-} and likewise for η_{ρ^0} .

Resonance	$D \rightarrow Rh_3 \rightarrow h_1 h_2 h_3$	Γ_R (MeV) [30]	Γ_R/m_R	η_R
$\rho(770)$	$D^0 \rightarrow \pi^+ \rho^- \rightarrow \pi^+ \pi^- \pi^0$	149.1 ± 0.8	0.192	0.944 (GS) 1.116 (BW)
$K^*(892)$	$D^0 \rightarrow K^+ K^{*-} \rightarrow K^+ K^- \pi^0$	47.3 ± 0.5	0.053	1.112 (BW)
$K^*(892)$	$D^+ \rightarrow K^+ \bar{K}^{*0} \rightarrow K^+ K^- \pi^0$	47.3 ± 0.5	0.053	1.112 (BW)
$K^*(892)$	$D_s^+ \rightarrow K^{*+} \pi^0 \rightarrow K^0 \pi^+ \pi^0$	47.3 ± 0.5	0.053	1.099 (BW)

IV. CONCLUSIONS

In this work we have reexamined direct CP violation in the quasi-two-body $D \rightarrow VP$ decays and studied CP asymmetries in three-body D decays proceeding through intermediate vector resonances within the framework of the topological amplitude approach for tree amplitudes and the QCD factorization approach for penguin amplitudes. As we pointed out in 2012, the long-distance penguin exchange through final-state rescattering gives the major direct CP violation to both $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$. It nicely accounts for the CP asymmetry difference between the aforementioned two modes observed by the LHCb in 2019. The same mechanism implies that CP asymmetry can occur at the per mille level in many of the $D \rightarrow VP$ channels.

Our main results are as follows:

- (i) In light of several new measurements of Cabibbo-favored modes, we have performed a refit to the data to extract topological tree amplitudes. The topological W -exchange E_V is twice as large as the old value, and its phase is significantly different from the old one.
- (ii) There are six golden modes with sufficiently large branching fractions and CP asymmetries of order 10^{-3} : $D^0 \rightarrow \pi^+\rho^-, K^+K^{*-}, D^+ \rightarrow \eta\rho^+, K^+\bar{K}^{*0}$, and $D_s^+ \rightarrow \pi^+K^{*0}, \pi^0K^{*+}$. In particular, we predict $a_{CP}(K^+K^{*-}) - a_{CP}(\pi^+\rho^-) = (-1.61 \pm 0.33) \times 10^{-3}$, which is very similar to the observed CP asymmetry difference between $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ by the LHCb.
- (iii) In the U -spin limit, we have the relations $a_{CP}^{\text{dir}}(D^0 \rightarrow K^\pm K^{*\mp}) = -a_{CP}^{\text{dir}}(D^0 \rightarrow \pi^\pm \rho^\mp)$. We have found that the U -spin relation is approximately respected for $D^0 \rightarrow K^+K^{*-}$ but not for $D^0 \rightarrow K^-K^{*+}$.
- (iv) Analogous to the PP sector, $SU(3)$ breaking in $E_{V,P}$ is obtained from a fit to eight singly Cabibbo-suppressed D^0 decays. There are four different solutions, which can be discriminated by the decay mode $D^0 \rightarrow \eta\phi$ to favor solution (iv). An important consequence is that $D^0 \rightarrow K_S K^{*0}$ is predicted to have a positive CP asymmetry at the per mille level. It is an example that the asymmetry is induced at tree level.
- (v) The consideration of $SU(3)$ -breaking effects in the VP sector is more subtle than the PP case. It appears that $SU(3)$ breaking can only occur in one of the topological tree amplitudes in each decay channel. We are able to account for the recent new measurements from BESIII except for $D^+ \rightarrow K^+\bar{K}^{*0}$. For this, we need to wait for further experimental justification.
- (vi) We have included the flavor-singlet QCD-penguin contributions, which were missing in our previous studies, to calculate their effects on modes such as $D \rightarrow \pi\phi$ and $D \rightarrow \eta\phi$. As shown in Table VI, the inclusion of the singlet penguins in these decay modes results in nonzero a_{dir} , albeit not to a level that can be easily measured experimentally.
- (vii) We compare our approach with the FAT approach in detail. The main difference lies in the treatment of the factorizable part of the penguin-exchange amplitude PE characterized by A_3^f and the consideration of the long-distance contribution to PE through final-state rescattering. In QCDF, A_3^f is evaluated in terms of twist-2 and -3 light-cone distribution amplitudes, and it does not have an almost complete cancellation with PE as claimed by the FAT analysis.
- (viii) For three-body D decays, we show the Dalitz plot of the CP asymmetry distribution of $D^0 \rightarrow K^+K^-\pi^0$ in the overlapped regions of K^{*+} and K^{*-} resonances and likewise for $D^0 \rightarrow \pi^+\pi^-\pi^0, D^+ \rightarrow K^+K_S\pi^0$, and $D_s^+ \rightarrow K^0\pi^+\pi^0$. Regional asymmetry varies in magnitude and sign from region to region and can reach the percent level in certain invariant mass regions.
- (ix) We have considered finite-width corrections to the narrow-width approximation for the $\rho(770)$ and $K^*(890)$ resonances. As an example of the implications, the PDG value of $(5.15 \pm 0.25) \times 10^{-3}$ for the branching fraction of $D^0 \rightarrow \pi^+\rho^-$ should be corrected to $(5.75 \pm 0.28) \times 10^{-3}$.

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- [1] R. Aaij *et al.* (LHCb Collaboration), Observation of CP Violation in Charm Decays, *Phys. Rev. Lett.* **122**, 211803 (2019).
- [2] R. Aaij *et al.* (LHCb Collaboration), Measurement of CP asymmetry in $D^0 \rightarrow K^-K^+$ and $D^0 \rightarrow \pi^-\pi^+$ decays, *J. High Energy Phys.* **07** (2014) 041.
- [3] R. Aaij *et al.* (LHCb Collaboration), Measurement of the Difference of Time-Integrated CP Asymmetries in $D^0 \rightarrow K^-K^+$ and $D^0 \rightarrow \pi^-\pi^+$ Decays, *Phys. Rev. Lett.* **116**, 191601 (2016).
- [4] Z. Z. Xing, A U-spin prediction for the CP -forbidden transition $e^+e^- \rightarrow D^0\bar{D}^0 \rightarrow (K^+K^-)_D(\pi^+\pi^-)_D$, *Mod. Phys. Lett. A* **34**, 1950238 (2019).
- [5] M. Chala, A. Lenz, A. V. Rusov, and J. Scholtz, ΔA_{CP} within the Standard Model and beyond, *J. High Energy Phys.* **07** (2019) 161.
- [6] H. N. Li, C. D. Lü, and F. S. Yu, Implications on the first observation of charm CPV at LHCb, *arXiv:1903.10638*.
- [7] Y. Grossman and S. Schacht, The emergence of the $\Delta U = 0$ rule in charm physics, *J. High Energy Phys.* **07** (2019) 020.
- [8] A. Soni, Resonance enhancement of charm CP , *arXiv:1905.00907*.
- [9] H. Y. Cheng and C. W. Chiang, Revisiting CP violation in $D \rightarrow PP$ and VP decays, *Phys. Rev. D* **100**, 093002 (2019).
- [10] A. Dery and Y. Nir, Implications of the LHCb discovery of CP violation in charm decays, *J. High Energy Phys.* **12** (2019) 104.
- [11] L. Calibbi, T. Li, Y. Li, and B. Zhu, Simple model for large CP violation in charm decays, B -physics anomalies, muon $g-2$ and dark matter, *J. High Energy Phys.* **10** (2020) 070.
- [12] A. Soni, Charm CP : ΔA_{CP} and radiative decays, *Proc. Sci., LATTICE2019* (2020) 111 [*arXiv:2001.10014*].
- [13] S. Bianco and I. I. Bigi, 2019/20 lessons from $\tau(\Omega_c^0)$ and $\tau(\Xi_c^0)$ and CP asymmetry in charm decays, *Int. J. Mod. Phys. A* **35**, 2030013 (2020).
- [14] M. Saur and F. S. Yu, Charm CPV: Observation and prospects, *Sci. Bull.* **65**, 1428 (2020).
- [15] I. Bediaga and C. Göbel, Direct CP violation in beauty and charm hadron decays, *Prog. Part. Nucl. Phys.* **114**, 103808 (2020).
- [16] A. Lenz and G. Wilkinson, Mixing and CP violation in the charm system, *arXiv:2011.04443*.
- [17] A. Dery, Y. Grossman, S. Schacht, and A. Soffer, Probing the $\Delta U = 0$ rule in three body charm decays, *J. High Energy Phys.* **05** (2021) 179.
- [18] R. Aaij *et al.* (LHCb Collaboration), Evidence for CP Violation in Time-Integrated $D^0 \rightarrow h^-h^+$ Decay Rates, *Phys. Rev. Lett.* **108**, 111602 (2012).
- [19] R. Aaij *et al.* (LHCb Collaboration), Search for direct CP violation in $D^0 \rightarrow h^-h^+$ modes using semileptonic B decays, *Phys. Lett. B* **723**, 33 (2013).
- [20] A. Khodjamirian and A. A. Petrov, Direct CP asymmetry in $D \rightarrow \pi^-\pi^+$ and $D \rightarrow K^-K^+$ in QCD-based approach, *Phys. Lett. B* **774**, 235 (2017).
- [21] H. Y. Cheng and C. W. Chiang, Direct CP violation in two-body hadronic charmed meson decays, *Phys. Rev. D* **85**, 034036 (2012); Erratum, *Phys. Rev. D* **85**, 079903 (2012).
- [22] H. Y. Cheng and C. W. Chiang, $SU(3)$ symmetry breaking and CP violation in $D \rightarrow PP$ decays, *Phys. Rev. D* **86**, 014014 (2012).
- [23] H.-n. Li, C. D. Lu, and F. S. Yu, Branching ratios and direct CP asymmetries in $D \rightarrow PP$ decays, *Phys. Rev. D* **86**, 036012 (2012).
- [24] Q. Qin, H. n. Li, C. D. Lü, and F. S. Yu, Branching ratios and direct CP asymmetries in $D \rightarrow PV$ decays, *Phys. Rev. D* **89**, 054006 (2014).
- [25] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, Global analysis of two-body $D \rightarrow VP$ decays within the framework of flavor symmetry, *Phys. Rev. D* **93**, 114010 (2016).
- [26] Y. Q. Chen *et al.* (Belle Collaboration), Dalitz analysis of $D^0 \rightarrow K^-\pi^+\eta$ decays at Belle, *Phys. Rev. D* **102**, 012002 (2020).
- [27] M. Ablikim *et al.* (BESIII Collaboration), Analysis of the decay $D^0 \rightarrow K_S^0 K^+ K^-$, *arXiv:2006.02800*.
- [28] M. Ablikim *et al.* (BESIII Collaboration), Amplitude analysis and branching fraction measurement of $D_s^+ \rightarrow K^+ K^- \pi^+$, *Phys. Rev. D* **104**, 012016 (2021).
- [29] R. Aaij *et al.* (LHCb Collaboration), Study of $\eta - \eta'$ mixing from measurement of $B_{(s)}^0 \rightarrow J/\psi \eta^{(\prime)}$ decay rates, *J. High Energy Phys.* **01** (2015) 024.
- [30] P. A. Zyla *et al.* (Particle Data Group), Review of particle physics, *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
- [31] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, QCD factorization for $B \rightarrow \pi\pi$ decays: Strong phases and CP violation in the heavy quark limit, *Phys. Rev. Lett.* **83**, 1914 (1999); QCD factorization for exclusive, non-leptonic B meson decays: General arguments and the case of heavy-light final states, *Nucl. Phys.* **B591**, 313(E) (2000).
- [32] M. Beneke and M. Neubert, QCD factorization for $B \rightarrow PP$ and $B \rightarrow PV$ decays, *Nucl. Phys.* **B675**, 333 (2003).
- [33] Y. Y. Keum, H. N. Li, and A. I. Sanda, Penguin enhancement and $B \rightarrow K\pi$ decays in perturbative QCD, *Phys. Rev. D* **63**, 054008 (2001).
- [34] H. Y. Cheng and S. Oh, Flavor $SU(3)$ symmetry and QCD factorization in $B \rightarrow PP$ and PV decays, *J. High Energy Phys.* **09** (2011) 024.
- [35] M. Wirbel, B. Stech, and M. Bauer, Exclusive semileptonic decays of heavy mesons, *Z. Phys. C* **29**, 637 (1985); M. Bauer, B. Stech, and M. Wirbel, Exclusive non-leptonic decays of D^- , D_s^- and B -mesons, *Z. Phys. C* **34**, 103 (1987).
- [36] H. Y. Cheng, C. W. Chiang, and C. K. Chua, Width effects in resonant three-body decays: B decay as an example, *Phys. Lett. B* **813**, 136058 (2021).
- [37] H. Y. Cheng, C. W. Chiang, and C. K. Chua, Finite-width effects in three-body B decays, *Phys. Rev. D* **103**, 036017 (2021).
- [38] M. Ablikim *et al.* (BESIII Collaboration), Amplitude analysis and branching-fraction measurement of $D_s^+ \rightarrow K_S^0 \pi^+ \pi^0$, *J. High Energy Phys.* **06** (2021) 181.
- [39] M. Ablikim *et al.* (BESIII Collaboration), Measurement of branching fractions for D meson decaying into ϕ meson and a pseudoscalar meson, *Phys. Lett. B* **798**, 135017 (2019).
- [40] M. Ablikim *et al.* (BESIII Collaboration), Study of the decay $D^+ \rightarrow K^*(892)^+ K_S^0$ in $D^+ \rightarrow K^+ K_S^0 \pi^0$, *Phys. Rev. D* **104**, 012006 (2021).
- [41] M. Ablikim *et al.* (BESIII Collaboration), Observation of the W -annihilation decay $D_s^+ \rightarrow \omega \pi^+$ and evidence for $D_s^+ \rightarrow \omega K^+$, *Phys. Rev. D* **99**, 091101 (2019).
- [42] M. A. Ivanov, J. G. Körner, J. N. Pandya, P. Santorelli, N. R. Soni, and C. T. Tran, Exclusive semileptonic decays of D

- and D_s mesons in the covariant confining quark model, *Front. Phys.* **14**, 64401 (2019).
- [43] M. Gronau, U spin symmetry in charmless B decays, *Phys. Lett. B* **492**, 297 (2000).
- [44] U. Nierste and S. Schacht, Neutral $D \rightarrow KK^*$ Decays as Discovery Channels for Charm CP Violation, *Phys. Rev. Lett.* **119**, 251801 (2017).
- [45] H. Zhou, B. Zheng, and Z. H. Zhang, Analysis of CP violation in $D^0 \rightarrow K^+K^-\pi^0$, *Adv. High Energy Phys.* **2018**, 7627308 (2018).
- [46] D. Boito, J. P. Dedonder, B. El-Bennich, R. Escribano, R. Kaminski, L. Lesniak, and B. Loiseau, Parametrizations of three-body hadronic B - and D -decay amplitudes in terms of analytic and unitary meson-meson form factors, *Phys. Rev. D* **96**, 113003 (2017).
- [47] F. Niecknig and B. Kubis, Dispersion-theoretical analysis of the $D^+ \rightarrow K^-\pi^+\pi^+$ Dalitz plot, *J. High Energy Phys.* **10** (2015) 142.
- [48] S. X. Nakamura, Coupled-channel analysis of $D^+ \rightarrow K^-\pi^+\pi^+$ decay, *Phys. Rev. D* **93**, 014005 (2016).
- [49] P. C. Magalhaes, M. R. Robilotta, K. S. F. F. Guimaraes, T. Frederico, W. de Paula, I. Bediaga, A. C. d. Reis, C. M. Maekawa, and G. R. S. Zarnauskas, Towards three-body unitarity in $D^+ \rightarrow K^-\pi^+\pi^+$, *Phys. Rev. D* **84**, 094001 (2011).
- [50] R. T. Aoude, P. C. Magalhães, A. C. Dos Reis, and M. R. Robilotta, Multimeson model for the $D^+ \rightarrow K^+K^-K^+$ decay amplitude, *Phys. Rev. D* **98**, 056021 (2018).
- [51] R. Aaij *et al.* (LHCb Collaboration), Amplitude analysis of the $B^+ \rightarrow \pi^+\pi^+\pi^-$ decay, *Phys. Rev. D* **101**, 012006 (2020).
- [52] R. Aaij *et al.* (LHCb Collaboration), Measurement of CP Violation in the Phase Space of $B^\pm \rightarrow K^+K^-\pi^\pm$ and $B^\pm \rightarrow \pi^+\pi^-\pi^\pm$ Decays, *Phys. Rev. Lett.* **112**, 011801 (2014).
- [53] D. Asner, Charm Dalitz plot analysis formalism and results: Expanded RPP-2004 version, *Phys. Lett. B* **592**, 1 (2004).
- [54] B. Aubert *et al.* (BABAR Collaboration), Amplitude analysis of the decay $D^0 \rightarrow K^+K^-\pi^0$, *Phys. Rev. D* **76**, 011102 (2007).
- [55] B. Aubert *et al.* (BABAR Collaboration), Measurement of CP Violation Parameters with a Dalitz Plot Analysis of $B^\pm \rightarrow D_{\pi^+\pi^-\pi^0}K^\pm$, *Phys. Rev. Lett.* **99**, 251801 (2007).
- [56] G. J. Gounaris and J. J. Sakurai, Finite-Width Corrections to the Vector Meson Dominance Prediction for $\rho \rightarrow e^+e^-$, *Phys. Rev. Lett.* **21**, 244 (1968).
- [57] M. Ablikim *et al.* (BESIII Collaboration), Observation of $D^+ \rightarrow \eta\eta\pi^+$ and improved measurement of $D^{0(+)} \rightarrow \eta\pi^+\pi^{-(0)}$, *Phys. Rev. D* **101**, 052009 (2020).
- [58] P. Rubin *et al.* (CLEO Collaboration), Search for CP violation in the Dalitz-plot analysis of $D^\pm \rightarrow K^+K^-\pi^\pm$, *Phys. Rev. D* **78**, 072003 (2008).
- [59] R. Aaij *et al.* (LHCb Collaboration), Search for CP violation in $D^+ \rightarrow K^-K^+\pi^+$ decays, *Phys. Rev. D* **84**, 112008 (2011).