Reconciling resonant leptogenesis and baryogenesis via neutrino oscillations

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Right-handed neutrinos offer an elegant solution to two well-established phenomena beyond the Standard Model (SM)—masses and oscillations of neutrinos, as well as the baryon asymmetry of the Universe. It is also a minimalistic solution, since it requires only singlet Majorana fermions to be added to the SM particle content. If these fermions are nearly degenerate, the mass scale of right-handed neutrinos can be very low and accessible by the present and planned experiments. There are at least two well-studied mechanisms of low-scale leptogenesis: baryogenesis via oscillations and resonant leptogenesis. These two mechanisms were often considered separate, but they can, in fact, be understood as two different regimes of one and the same mechanism, described by a unique set of quantum kinetic equations. In this work, we show, using a unified description based on quantum kinetic equations, that the parameter spaces of these two regimes of low-scale leptogenesis significantly overlap. We present a comprehensive study of the parameter space of low-scale leptogenesis with the mass scale ranging from 0.1 to $\sim 10^6$ GeV. The unified perspective of this work reveals the synergy between intensity and energy frontiers in the quest for heavy Majorana neutrinos.

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I. INTRODUCTION

The origin of the light neutrino masses and the baryon asymmetry of the Universe remain some of the burning problems for physics beyond the Standard Model (SM). Adding heavy neutrinos to the SM offers an economical solution to both problems [throughout this work, we will use the terms heavy neutrinos and heavy neutral leptons (HNLs) interchangeably]. The masses of active neutrinos are coming from the type-I seesaw mechanism [1–6], and the baryon asymmetry of the Universe (BAU) is generated through the combined action of anomalous processes with fermion number nonconservation [7] and lepton-number- and -flavor-violating reactions involving heavy neutrinos. The scale of the heavy neutrino masses, however, remains an open question. The seesaw mechanism on its own does not imply any specific scale for the heavy neutrino masses. One may wonder if the answer can come from leptogenesis. Indeed, the specific patterns of Majorana masses of HNLs single out different scales. For example, if the spectrum

of HNLs is hierarchical, $M_I \ll M_J$, a lower bound of 10^9 GeV was found for the mass of the lightest heavy neutrino [8]. The mechanism leading to BAU in this case is known as *thermal leptogenesis*. If the spectrum is nearly degenerate, i.e., there is a pair of HNLs such that $\Delta M \ll M$, leptogenesis may take place for M as small as $\mathcal{O}(100)$ GeV or even a few MeV [9]. The corresponding leptogenesis mechanisms are called *resonant leptogenesis* [10–19] and *baryogenesis via oscillations* [20,21].

In recent years, these last scenarios have received significant attention, from both the theoretical (see, e.g., [9,22–67]) and experimental (see, e.g., [68–94]) perspectives. This interest is primarily related to the potential testability of the model in present and near-future experiments. HNLs are a canonical example of feebly interacting particles (see, e.g., [95,96]), which could have avoided discovery not because they are heavy but because their interactions are very weak.

For the observed BAU to be generated, the three Sakharov conditions need to be met [97]:

- (i) efficient baryon number violation,
- (ii) sizable C and CP violation, and
- (iii) substantial deviation from thermal equilibrium.

Heavy Majorana neutrinos can provide both CP violation via complex Yukawa couplings as well as a deviation from equilibrium. In leptogenesis scenarios, the asymmetry is generated in the lepton sector (hence the name) and transferred to the baryon sector by nonperturbative sphaleron processes which violate baryon number conservation [7].

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In thermal leptogenesis, the deviation from equilibrium takes place during the decays of HNLs, when the decay rate cannot catch up to the expansion rate of the Universe. The asymmetry between decays into leptons and antileptons comes about because of the interference between tree and one-loop processes like $N \rightarrow \ell H$. See, e.g., [98] for details. This *decay asymmetry* is further enhanced if the two HNLs are degenerate in mass, as is the case in resonant leptogenesis.¹ We will discuss this mechanism in greater detail in Sec. IV A. In baryogenesis via neutrino oscillations, the deviation from equilibrium happens during freeze-in. As a result of the seesaw mechanism, the Yukawa couplings of light HNLs must be tiny. Thus, the equilibration rate is much lower than the Hubble rate, so the HNLs remain out of equilibrium. The asymmetry generated in processes such as scatterings, decays, and inverse decays of the HNLs is further enhanced by their oscillations.

Several studies of resonant leptogenesis suggested that the scale of heavy neutrinos can be as low as ~ 100 GeV. At the same time, leptogenesis via oscillations was primarily studied in the few- GeV regime. In Ref. [99], it was suggested that the maximal HNL mass in this mechanism is around the W -boson mass, where decays into W and Z bosons become kinematically allowed and enhance the equilibration of the lepton asymmetries. The argument came about as follows. If the HNLs are kinematically allowed to decay into W (or W and Z), the rate of this process will exceed the Hubble rate well before the moment of sphaleron freeze-out. This means that the HNLs will be in thermal equilibrium and all asymmetries will be washed out. However, this is not the end of the story. For $T \lesssim M$, the heavy neutrinos begin to freeze out, and their abundance, as well as the washout of the lepton asymmetries, becomes Boltzmann suppressed.

Because of the different approximations that were applied to these mechanisms, the overlap between them remained an open question.

These mechanisms may appear quite different at first glance—but, as we show in this work, it turns out that the same equations can be used to describe both mechanisms.

In this work, we systematically study generation of the BAU in the model with two HNLs with masses in the range $10\text{--}10^4$ GeV. We find that the parameter space of *baryogenesis via neutrino oscillations* is seamlessly connected with the parameter space of *resonant leptogenesis*. So there is just one mechanism, with different regimes. These regimes are characterized by whether the majority of the asymmetry is produced during the freeze-in or freeze-out of the HNLs (cf. Fig. 1). We identify three main reasons why leptogenesis remains viable for $M \sim M_{W,Z}$.

- (i) Although the equilibration rate of the heavy neutrinos generically exceeds the Hubble rate at the

temperature of the sphaleron freeze-out, the lepton flavor washout rate can be suppressed, thereby preserving the BAU in one of the leptonic flavors. As was pointed out in Ref. [100], this allows for freeze-in leptogenesis with HNL masses above the electroweak scale.²

- (ii) Because of the finite mass of the heavy neutrinos, their equilibrium distribution changes with temperature. This effect acts as a source for a deviation from equilibrium. We find that this effect remains important even for heavy neutrinos with masses at the GeV scale. This confirms the possibility of GeV-scale freeze-out leptogenesis as suggested in Ref. [36].
- (iii) We find that another factor preventing washout of the lepton asymmetries is approximate generalized lepton number conservation.³ The processes that would erase the lepton asymmetry are suppressed by a factor of $\sim \Delta M/\Gamma$, as identified in Refs. [102,103].

A short account of the results was presented in Ref. [54]. The paper is organized as follows. We start from an overview of the evolution of the leptogenesis calculations in Sec. II. Then we define our notations and introduce the seesaw mechanism in Sec. III. In Sec. IV, we describe resonant leptogenesis and baryogenesis via neutrino oscillations. We present the quantum kinetic equations for matrices of densities which are at the core of our numerical study. We also show how the usual Boltzmann equations can be obtained as a limit of the quantum kinetic equations. Section V is dedicated to the determination of the heavy neutrino production rates entering the kinetic equations. With all ingredients at hand, we perform the study of the parameter space in Sec. VI. This section also contains a detailed discussion of the obtained results. Next, in Sec. VII, we discuss several other works which considered the HNLs with masses around M_W . We conclude in Sec. VIII. Technical details of the implementation, relation to the pseudo-Dirac basis, conserved lepton numbers, and fine-tuning are discussed in the appendixes.

II. CONVERGENCE TOWARD A UNIFIED PICTURE

In this section, we present a brief overview of the development of the calculations and methodology in low-scale leptogenesis.

The importance of a resonant enhancement for leptogenesis [10–15,18] was realized soon after leptogenesis was proposed as a baryogenesis mechanism.⁴ Such a

¹In fact, the resonant enhancement has been already noted in Ref. [7].

²A more detailed study of the effect of freeze-in on hierarchical leptogenesis followed in Ref. [101], which confirmed the importance of freeze-in in the conventional leptogenesis scenario.

³In principle, there are several ways of assigning a lepton number to the heavy neutrinos. We discuss them in Appendix C.

⁴It is worth noting that resonant enhancement in baryogenesis predates idea of leptogenesis itself [7,104].

FIG. 1. A sketch of the evolution of the HNL abundance in the early Universe. If we assume that the initial HNL abundance vanishes, there are two opportunities to generate the observed BAU. The first is during a period of freeze-in, while the first HNLs are being produced and they approach equilibrium. The second opportunity is when the Universe cools down to temperatures below the HNL mass, and the HNLs decay out of equilibrium simultaneously with a freeze-out of the SM lepton number caused by the Boltzmann-suppressed washout rates.

resonantly enhanced decay asymmetry offered an exciting opportunity—the mass scale of the HNLs could, in principle, be lowered to the electroweak scale [15,19,105] (it is worth noting that both the effects from a noninstantaneous freeze-out of sphalerons and spectator effects [106–108] were included in Ref. [105]).

However, the fact that the finite-order perturbation theory breaks down in the limit of degenerate HNL masses was already clear in the earliest papers [14–16], and different approaches of resolving these issues followed soon thereafter [18,109,110], with the general conclusion that the decay asymmetry is regulated by the width of the HNLs. This prompted further investigations into resonant leptogenesis using techniques of nonequilibrium quantum field theory (QFT) (see, e.g., [111–121]). One of the most successful formalisms in this framework is the closed-time-path (CTP) formalism, also known as the Schwinger-Keldysh formalism [111,112]. In this formulation of nonequilibrium QFT, one typically deals with n -point functions of the different particles and uses them to calculate observables, such as particle numbers and distributions, and their time evolution can be obtained by solving the Schwinger-Dyson equations on the CTP. One of the biggest differences compared to equilibrium QFT is that time translation invariance is explicitly broken (both by the boundary condition and by the expansion of the Universe), and each two-point function is a function of two time coordinates (and one momentum coordinate assuming translation invariance). Consequentially, the Schwinger-Dyson equations are integro-differential equations which include both derivatives and integration over the time variables.

The main difference between the existing approaches is in the strategies used to solve these equations. Perhaps the most straightforward way is to solve the Schwinger-Dyson equations directly. However, this is possible only numerically or in specific limits. Nonetheless, this can be used to cross-check the results of other methods [60,61]. Another approach is to perform a Wigner transformation of the equations (it is often used to describe transport phenomena [122]), which leads to density-matrix-like equations in Ref. [62]. This method was further

developed to estimate the resonant enhancement in resonant leptogenesis [65,123] and was also used in Ref. [40] to derive the quantum Boltzmann equations used in leptogenesis via oscillations.⁵ One of the shortcomings of this approach is that the off-diagonal matrix elements lie on an unphysical “average” energy shell $E = (E_1 + E_2)/2$ and cannot be used for arbitrarily large HNL mass differences.⁶ Recently, equations valid for arbitrary HNL mass spectra were derived in Ref. [52]. The formalism developed in Ref. [128] instead works in the so-called two-momentum picture and also leads to similar quantum-Boltzmann equations [63,129], however, with an important difference—a *mixing* term that acts as an additional source of lepton asymmetries. The differences between these results prompted further investigations in Refs. [130,131].

Meanwhile, another line of research started to develop. The idea of baryogenesis via oscillations was put forward in Ref. [20]. It was further developed in Ref. [21]. In particular, it was shown there that the lepton backreactions (missed in Ref. [20]) play an important role and the BAU generation is possible with two singlet fermions. This work introduced the kinetic equations for three lepton chemical potentials and two 2×2 HNL density matrices. These equations were used in all consequent works on baryogenesis via neutrino oscillations. Further clarifications of the mechanism were presented in Refs. [23,30]. In particular, the role of the process without helicity flip (we will refer to them as fermion number violating) was pointed out in Ref. [23]. It was noted that, once such processes are included, total asymmetry may be generated at the fourth order in Yukawa couplings. However, the relevant rates were not estimated correctly at that time. The neutrality of

⁵One should note that quantum Boltzmann equations are also used to describe decoherence effects in flavored leptogenesis [124–127].

⁶One should note that, as soon as $\Delta M \gg \Gamma$, the standard Boltzmann equations may be used again, ensuring overlap between the two approximations, since $M \gg \Gamma$ is typically satisfied in leptogenesis.

cosmic plasma has been accounted for in Ref. [31] by introducing the so-called susceptibility matrices relating the leptonic chemical potentials to the number densities. The temperature dependence of the susceptibility matrices and corrections coming from fermion masses were introduced in Ref. [35]. An important step has been performed in Refs. [35,44], where the role of the fermion-number-violating processes was systematically accounted for and the equations were extended to the Higgs phase. Effects related to the noninstantaneous freeze-out of sphalerons were discussed in Refs. [46,48]. The equations of Ref. [35] were further improved in Refs. [50,51,53].

It is interesting that the oscillations found in Ref. [63] were initially assumed to be a genuinely different source of asymmetry than the one in the scenario of leptogenesis via oscillations, since they already appeared at fourth order in the Yukawa couplings $\mathcal{O}(F^4)$, instead of $\mathcal{O}(F^6)$ as reported in the initial papers [20,21]. As discussed above, this apparent discrepancy was a result of the relativistic approximations, where fermion-number-violating (FNV) terms suppressed by a factor $(M/T)^2$ were neglected. Early investigations [36] into nonrelativistic corrections suggested that these terms are already important for GeV-scale HNLs and may allow for freeze-out leptogenesis with GeV-scale HNLs.

In spite of the gradual convergence of the methods and results, a conclusive study showing that the parameter spaces of the two regimes of leptogenesis are connected was missing. Strong hints of such a possibility were already provided in Ref. [100], where it was shown that freeze-in leptogenesis remains important for HNL masses above the TeV scale. Similarly, the study in Ref. [41] suggests that leptogenesis via HNL oscillations extends to HNL masses as large as 100 GeV. On the other hand, in Ref. [36], it was suggested that already moderate decay asymmetries allow for freeze-out leptogenesis at the GeV scale. More recently, this was confirmed in a similar study including flavor effects [132]. In this work, we perform a unified study including all the effects relevant for mass scales up to a few TeV.

III. THE SEESAW FORMULA AND THE LIGHT NEUTRINO MASSES

For completeness of the paper, in this section, we briefly review the type-I seesaw mechanism and how it constrains the properties of the heavy neutrinos. The Lagrangian of the model reads

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{\nu}_{R_I}\not{\partial}\nu_{R_I} - F_{\alpha I}\bar{L}_\alpha\tilde{\Phi}\nu_{R_I} - \frac{1}{2}M_{MIJ}\bar{\nu}_{R_I}^c\nu_{R_J}, \quad (1)$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian, ν_{R_I} are right-handed neutrinos labeled with the generation indices I and J , L_α are the left-handed lepton doublets labeled with the flavor index $\alpha = e, \mu, \tau$, and $\tilde{\Phi} = i\sigma_2\Phi$, where Φ is the Higgs doublet. $F_{\alpha I}$ is the matrix of Yukawa couplings in the basis where

charged lepton Yukawa couplings and the Majorana mass term of the right-handed neutrinos M_M are diagonal. After electroweak symmetry breaking, the Higgs field in the Lagrangian (1) obtains a vacuum expectation value $\langle\Phi\rangle = (0, v)^T$, $v = 174.1$ GeV, at zero temperature. The interaction terms in Eq. (1) effectively become Dirac mass terms coupling the left and right chiral components of the neutrinos. However, since the right-handed neutrinos also have a Majorana mass, the spectrum of the theory can be obtained only once we diagonalize the full mass matrix. Introducing $m_D \equiv vF$ and assuming that the elements of m_D are much smaller than the elements of M_M , we can approximately diagonalize the neutrino mass matrix

$$m_\nu = m_D M_M^{-1} m_D^T + \mathcal{O}(m_D^4), \quad (2)$$

which gives us three light mass eigenstates and the heavy eigenstates with a mass matrix

$$M_N = M_M + m_D^T m_D M_M^{-1} + M_M^{-1} m_D^T m_D + \mathcal{O}(m_D^4). \quad (3)$$

The number of heavy eigenstates is the same as the number of right-handed fields ν_{R_I} . At least two HNLs are need to explain the two observed mass splittings in the active neutrino sector. In this work, we focus on the minimal scenario with two heavy neutrinos. We label the physical states of heavy neutrinos as N_2 and N_3 and denote their masses M_2 and M_3 . Throughout this work, we will be interested in the case when $N_{2,3}$ have close masses, i.e., $|M_2 + M_3| \gg |M_2 - M_3|$. Therefore, it will be convenient to use the average mass M and the mass splitting ΔM . In order to match the notations of Ref. [49], we define them through

$$\begin{aligned} M_2 &= M - \Delta M, \\ M_3 &= M + \Delta M. \end{aligned} \quad (4)$$

So strictly speaking, ΔM is half of the mass splitting.

A. Parametrization of the Yukawa couplings

The masses of the light neutrinos m_ν are constrained by the neutrino oscillations experiments (we use the global fit [133]). Out of the nine parameters in the light neutrino mass matrix, five are already measured: two mass differences and three mixing angles. The remaining unknown parameters are the mass of the lightest neutrino, two Majorana phases, and the CP -violating phase δ .⁸ In the model with two HNLs,

⁷We leave the label N_1 for a potential sterile neutrino dark matter candidate of the νMSM [21].

⁸It is exciting that these parameters may be probed in the not so distant future; for inverted hierarchy, the next generation of neutrinoless double beta decay experiments may provide information on the Majorana phases [134], and the CP -violating phase δ is already constrained by T2K [135], with further improvements expected from the DUNE experiment [136].

the lightest neutrino is massless (up to tiny loop corrections [137]). Therefore, it makes sense to speak about the neutrino mass hierarchy rather than ordering. In what follows, we refer to normal (inverted) mass hierarchy as NH (IH).

The measured low-energy parameters mean that the choice of heavy neutrino masses M and the Yukawa couplings F is not completely free. To take this into account, we can parametrize the neutrino Yukawa couplings using the Casas-Ibarra parametrization [138]:

$$F = \frac{i}{v} U_\nu \sqrt{m_\nu^{\text{diag}}} \mathcal{R} \sqrt{M_M}, \quad (5)$$

where the matrix m_ν^{diag} is the diagonal neutrino mass matrix (M_M is already diagonal in our basis), U_ν is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, and $\mathcal{R}$ is a complex orthogonal matrix $\mathcal{R}\mathcal{R}^T = 1$. For the PMNS matrix, we use the standard parametrization [139]:

$$U_\nu = V^{(23)} U_\delta V^{(13)} U_{-\delta} V^{(12)} \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}), \quad (6)$$

where $U_{\pm\delta} = \text{diag}(1, e^{\mp i\delta/2}, e^{\pm i\delta/2})$ and the nonvanishing entries of $V^{(\alpha\beta)}$ for $\alpha = e, \mu, \tau$ are

$$V_{aa}^{(\alpha\beta)} = V_{bb}^{(\alpha\beta)} = \cos \theta_{\alpha\beta}, \quad V_{a\beta}^{(\alpha\beta)} = -V_{\beta a}^{(\alpha\beta)} = \sin \theta_{\alpha\beta}, \\ V_{\gamma\gamma}^{(\alpha\beta)}|_{\gamma \neq \alpha, \beta} = 1. \quad (7)$$

In the case of two heavy neutrinos, there is only one relevant Majorana phase in the PMNS matrix. We parametrize it as $\eta = \frac{1}{2}(\alpha_{21} - \alpha_{31})$ for normal and $\eta = \frac{1}{2}\alpha_{21}$ for inverted neutrino mass hierarchy with $\eta \in [0, 2\pi]$. The light neutrino mass matrix $m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ with $m_1 = 0$ for NH and $m_2 = 0$ for IH.

In the model with two right-handed neutrinos, the matrices $\mathcal{R}$ that depend on the neutrino mass hierarchy are given by

$$\mathcal{R}^{\text{NH}} = \begin{pmatrix} 0 & 0 \\ \cos \omega & \sin \omega \\ -\xi \sin \omega & \xi \cos \omega \end{pmatrix}, \\ \mathcal{R}^{\text{IH}} = \begin{pmatrix} \cos \omega & \sin \omega \\ -\xi \sin \omega & \xi \cos \omega \\ 0 & 0 \end{pmatrix} \quad (8)$$

with a complex angle $\omega = \text{Re}\omega + i\text{Im}\omega$ and the discrete parameter $\xi = \pm 1$. The change of the sign of ξ can be compensated by $\omega \rightarrow -\omega$ along with $N_3 \rightarrow -N_3$ [125], so we fix $\xi = +1$. It is sufficient to constrain $\text{Re}\omega \in [0, \pi]$, as larger angles change only the overall sign of the Yukawa couplings.

In summary, there are six free parameters of the theory. They are listed in Table I along with their ranges considered

TABLE I. Parameters of the theory: average mass M ; mass splitting ΔM ; $\text{Im}\omega$; $\text{Re}\omega$; Dirac δ and Majorana η phases. In the second row, we indicate the ranges of these parameters which were considered in this work.

M , GeV	$\log_{10}(\Delta M/M)$	$\text{Im}\omega$	$\text{Re}\omega$	δ	η
$[0.1 - 7000]$	$[-19, -0.5]$	$[-7, 7]$	$[0, \pi]$	$[0, 2\pi]$	$[0, 2\pi]$

in this work. The upper boundary of the mass range in Table I is somewhat arbitrary. In Sec. VIE, we show that a specific scaling law exists above $M \sim 2$ TeV, so our results are applicable for heavier HNLs as well.

B. Heavy neutrino mixing

As a consequence of the seesaw mechanism, the heavy neutrino mass eigenstates are mixed with the doublet neutrinos and can interact with the rest of the Standard Model, and the interaction states $\nu_{L\alpha}$ are superpositions of the mass eigenstates ν_i and N_I :

$$\nu_{L\alpha} = [U_\nu]_{\alpha I} \nu_i + \theta_{\alpha I} N_I^c. \quad (9)$$

The mixing angle between a heavy neutrino N_I and the active neutrinos $\nu_{L\alpha}$ is expected to be small and is approximately given by

$$\theta_{\alpha I} \approx \frac{v F_{\alpha I}}{M_I}. \quad (10)$$

This mixing angle appears in amplitudes for heavy neutrino production, which means that the heavy neutrino production rate is suppressed by

$$U_{\alpha I}^2 \equiv |\theta_{\alpha I}|^2. \quad (11)$$

For phenomenological applications, it is convenient to introduce the quantities

$$U_I^2 \equiv \sum_\alpha U_{\alpha I}^2, \quad U_\alpha^2 \equiv \sum_I U_{\alpha I}^2, \quad \text{and} \quad U^2 \equiv \sum_{\alpha I} U_{\alpha I}^2, \quad (12)$$

which quantify, respectively, the total mixing angle of a particular heavy neutrino, the total mixing to a particular flavor, and the overall mixing between the heavy and light neutrinos:

$$U^2 = \frac{\sum_i m_i}{M} \cosh(2\text{Im}\omega). \quad (13)$$

This total mixing is useful to characterize the overall suppression of the interactions of the HNLs.

IV. BARYOGENESIS THROUGH LEPTOGENESIS

As stated in the introduction, any leptogenesis mechanism relies on HNLs to satisfy two of the Sakharov conditions which are not met in the SM— CP violation and a sufficient deviation from thermal equilibrium. In thermal leptogenesis, the CP violation arises via the loop corrections to the HNL decay rates:

$$\Gamma \sim \left| \text{---} + \text{---} + \text{---} \right|^2 \quad (14)$$

where the thick lines represent the HNLs, the thin and dashed lines represent the lepton and Higgs doublets respectively. These decay rates can differ for decays into leptons and antileptons, which leads to an overall lepton asymmetry. This difference is often parametrized by the *decay asymmetry*:

$$\epsilon_I^\alpha = \frac{\Gamma(N_I \rightarrow \bar{\phi} \ell_\alpha) - \Gamma(N_I \rightarrow \phi \bar{\ell}_\alpha)}{\Gamma(N_I \rightarrow \bar{\phi} \ell_\alpha) + \Gamma(N_I \rightarrow \phi \bar{\ell}_\alpha)}. \quad (15)$$

The evolution of the lepton asymmetries and the HNL number densities is an out-of-equilibrium process which can be described by the following Boltzmann equations (see, e.g., [108,124,140]):

$$T_{\text{ref}} \frac{dY_{N_I}}{dz} = -\Gamma_{N_I} (Y_{N_I} - Y_{N_I}^{\text{eq}}), \quad (16)$$

$$T_{\text{ref}} \frac{dY_{\Delta_\alpha}}{dz} = 2\epsilon_I^\alpha \Gamma_{N_I} (Y_{N_I} - Y_{N_I}^{\text{eq}}) - W_\alpha \mu_\alpha. \quad (17)$$

We used the notation $z = T_{\text{ref}}/T$ (where T_{ref} is some reference temperature—either the HNL mass M or the sphaleron temperature T_{sph}), Γ_{N_I} is the HNL decay rate, W_α is the lepton number washout rate, μ_α are the chemical potentials in the lepton flavors, and $Y_X = n_X/s$ are the yields—the ratios of the number density to the entropy density. In the case of hierarchical HNL masses, the decay asymmetry ϵ_I^α is bounded from above [8], which translates to a lower bound on the mass of the lightest HNL:

$$M_I \gtrsim 10^9 \text{ GeV}, \quad (18)$$

known as the Davidson-Ibarra bound.

Leptogenesis with HNLs below this mass scale can be realized either through resonant leptogenesis [19] or through leptogenesis via neutrino oscillations [20,21]. These two mechanisms were discovered independently for the different possible masses of the heavy neutrinos, leptogenesis via oscillations for GeV-scale HNLs and resonant leptogenesis for HNLs with masses from the TeV scale to the Davidson-Ibarra bound. In the following sections, we briefly overview the main features of the two

mechanisms and discuss the similarities and differences between them.

A. Resonant leptogenesis

The Davidson-Ibarra bound on the decay asymmetry from Ref. [8] relaxes (as does the bound on the HNL mass) if the HNL masses are degenerate [10–15,18]. This is a consequence of the resonant enhancement in the CP -violating decays, which was discussed even before the idea of leptogenesis in Ref. [104].

This *resonant* contribution to the decay asymmetry comes from the interference of the tree-level and wave-function amplitudes [the first and the last diagram in Eq. (14), respectively]

$$\epsilon_I^{\text{wf}} = \frac{\text{Im}[(F^\dagger F)_{32}]^2}{16\pi(F^\dagger F)_{II}} \frac{M_2 M_3}{M_3^2 - M_2^2} \quad (19)$$

which is enhanced when $M_3 \rightarrow M_2$. In the exactly degenerate limit, the decay asymmetry diverges in the one-loop approximation. This apparent divergence is as an artifact of applying the S -matrix theory to unstable HNLs, which cannot be asymptotic states. To find the correct size of such a decay asymmetry, in the past two decades there were several studies of leptogenesis using nonequilibrium QFT methods [57–66,128,141–148].

The decay asymmetry then obtains a finite regulator of order of the decay width of the heavy neutrinos $A \sim M_I \Gamma_I$, and

$$\epsilon^{\text{wf}} = \frac{\text{Im}[(F^\dagger F)_{32}]^2 [(F^\dagger F)_{22} + (F^\dagger F)_{33}] M_2 M_3 (M_2^2 - M_3^2)}{16\pi(F^\dagger F)_{22}(F^\dagger F)_{33} (M_2^2 - M_3^2)^2 + A^2}. \quad (20)$$

The different approaches of deriving the Boltzmann equations and the decay asymmetries from first principles do not always agree and can lead to $\mathcal{O}(1)$ different results in the degenerate limit, i.e., when $\Delta M \sim \Gamma_I$.⁹ In practice, the decay asymmetries ϵ combined with the Boltzmann equations (even if derived using nonequilibrium QFT methods) do not accurately describe the HNL dynamics. Instead, one has to rely on quantum kinetic equations that include the HNL coherence terms as dynamical degrees of freedom as we discuss in the following section.

B. Leptogenesis via neutrino oscillations

In leptogenesis via HNL oscillations [20,21], the asymmetry is not produced in the decays of the HNLs but instead

⁹One should, however, note that the *two-time* formulation [60,61,149] (which relies on very few approximations) and the so-called *Wigner space* approach [62,65] (where all interaction rates are evaluated on the same HNL average mass shell) lead to excellent agreement when the masses are not hierarchical [150].

while the HNLs are produced and approach equilibrium in the early Universe. The HNLs generated via the SM particle interactions are produced in their interaction basis, which does not necessarily coincide with their mass basis. Because of this misalignment, the HNLs begin to oscillate and through these CP -violating oscillations they generate the lepton asymmetry.

Since the GeV-scale HNLs in this mechanism remain relativistic at temperatures relevant for baryogenesis $T \geq T_{sph} \approx 130$ GeV, the calculation of the BAU does not rely on calculating their decay rates as in resonant leptogenesis. Instead, a more appropriate picture is that of neutrino oscillations. To correctly take these processes into account, in Refs. [20,21], the equations used to describe the kinetic evolution of neutrinos due to Raffelt and Sigl [151] were modified to include oscillations between HNLs.

1. Evolution equations for baryogenesis via neutrino oscillations

Compared to the initial developments [20,21], there have been several systematic improvements to the kinetic equations for HNLs. One of the most significant improvements in recent years is the inclusion of corrections caused by the finite HNL mass [44,45]. On the other hand, it was also found that the same equations arise in the nonequilibrium formulation of QFT [40,47]. The key difference compared to the Boltzmann equations (17) is that, besides the HNL number density Y_{N_i} , one also has to keep track of the HNL correlations. This information is encoded in the *density matrix* $(\rho_N)_{IJ}$, where $(\rho_N)_{II} \sim f_{N_i}$ in the mass basis. As we will show in Sec. IV C, the off-diagonal correlations $(\rho_N)_{IJ}$ become negligible in the limit of fast oscillations, and we recover the usual Boltzmann equations. The equations governing the HNL densities¹⁰ (modified from Refs. [40,44,45,47,62] to be valid in both relativistic and nonrelativistic limits; cf. Refs. [51–53]) including both the positive (negative) HNL ρ_N ($\bar{\rho}_N$) helicities and the leptonic asymmetries n_{Δ_α} are given by

$$i \frac{dn_{\Delta_\alpha}}{dt} = -2i \frac{\mu_\alpha}{T} \int \frac{d^3k}{(2\pi)^3} \text{Tr}[\Gamma_\alpha] f_N(1 - f_N) + i \int \frac{d^3k}{(2\pi)^3} \text{Tr}[\tilde{\Gamma}_\alpha(\delta\bar{\rho}_N - \delta\rho_N)], \quad (21a)$$

¹⁰The equations in the current form—with lepton chemical potentials—are valid as long as leptons stay in equilibrium. In particular, for temperatures above $T > 85$ TeV, the right-handed electrons are not in equilibrium [152], and susceptibility matrices relating chemical potentials with number densities need to be modified accordingly.

$$i \frac{d\delta\rho_N}{dt} = -i \frac{d\rho_N^{eq}}{dt} + [H_N, \rho_N] - \frac{i}{2} \{\Gamma, \delta\rho_N\} - \frac{i}{2} \sum_\alpha \tilde{\Gamma}_\alpha \left[2 \frac{\mu_\alpha}{T} f_N(1 - f_N) \right], \quad (21b)$$

$$i \frac{d\delta\bar{\rho}_N}{dt} = -i \frac{d\bar{\rho}_N^{eq}}{dt} - [H_N, \bar{\rho}_N] - \frac{i}{2} \{\Gamma, \delta\bar{\rho}_N\} + \frac{i}{2} \sum_\alpha \tilde{\Gamma}_\alpha \left[2 \frac{\mu_\alpha}{T} f_N(1 - f_N) \right], \quad (21c)$$

where we introduced $\delta\rho_N = \rho_N - \rho_N^{eq}$ and $\rho_N^{eq} \approx 1_{2 \times 2} f_N$. The function $f_N = 1/(e^{\omega_k/T} + 1)$ is the equilibrium distribution function of the massive fermions, $\omega_k = \sqrt{M^2 + k^2}$. This distribution function is temperature dependent. Its time derivative acts as a source of the deviation from equilibrium; therefore, in what follows, we will refer to $d\rho^{eq}/dt$ as the *source term*. Note that we omit any Hubble expansion terms, as we implicitly consider the *comoving densities*. The effective Hamiltonian describing the coherent oscillations of the HNLs is

$$H_N = H_0 + H_I, \quad H_0 = \frac{M^2}{2E_N}, \quad H_I = h_+ Y_+ + h_- Y_-, \quad (22)$$

where $E_N = \sqrt{k_N^2 + M^2}$. The quadratic combinations of Yukawa couplings $Y_\pm$ appearing in Eq. (22) and in the rates are defined below. The damping rates are

$$\Gamma = \gamma_+ Y_+ + \gamma_- Y_-, \quad \Gamma_\alpha = \gamma_+ Y_+^\alpha + \gamma_- Y_-^\alpha. \quad (23)$$

The communication terms, describing the transitions from HNLs to active neutrinos, are

$$\tilde{\Gamma} = -\gamma_+ Y_+ + \gamma_- Y_-, \quad \tilde{\Gamma}_\alpha = -\gamma_+ Y_+^\alpha + \gamma_- Y_-^\alpha. \quad (24)$$

In the expressions above, the subscripts $+$ and $-$ refer to the fermion-number-conserving and -violating quantities correspondingly. The functions $h_\pm$ and $\gamma_\pm$ depend only on kinematics (i.e., on the common mass of HNLs). These functions have to be determined over the whole temperature region of the interest, which includes both symmetric and Higgs phases.

The dependence on the Yukawa coupling constants factorizes out from the rates (22)–(24), and we have

$$Y_+ = F^\dagger F, \quad Y_- = G F^T F^* G^*, \quad (25)$$

$$(Y_+^\alpha)_{IJ} = F_{I\alpha}^\dagger F_{\alpha J}, \quad (Y_-^\alpha)_{IJ} = (G F^T)_{I\alpha} (F^* G^*)_{\alpha J}, \quad (26)$$

where the matrix G encodes the generalized Majorana condition between the Majorana fields $N_I \equiv \nu_{R_I} + \nu_{R_I}^c$, with $N_I = G_{IJ} N_J^c$. If we transform N_I from the basis where the

Majorana condition is given by $\tilde{N}_I = \tilde{N}_I^C$ by a unitary transformation $N_I = U_{IJ}\tilde{N}_J$, this matrix is given by $G = U^\dagger U^*$. The relation between the number densities and the chemical potentials to leptons in Eq. (21) has to take into account the neutrality of plasma. When the system is in equilibrium with respect to sphaleron processes, this relation reads

$$\mu_\alpha = \omega_{\alpha\beta}(T)n_{\Delta_\beta}, \quad (27)$$

where $\omega_{\alpha\beta}(T)$ is the so-called susceptibility matrix; see, e.g., [35,46].

C. Boltzmann equations as a limit of the quantum kinetic equations

As mentioned above, one of the key quantities in thermal leptogenesis are the decay asymmetries ϵ . These quantities are typically calculated via Feynman diagrams as shown in Eq. (14) but can also be obtained as a limit of the full density-matrix equations. In this section, we show how these quantities arise from the density-matrix approach and discuss when such approximations can be applied. To find approximate expressions, it is instructive to explicitly write

the equations governing the HNL correlations ρ_{IJ} for $I \neq J$ (note that we omitted the subscript N for brevity):

$$\begin{aligned} \frac{d\delta\rho_{IJ}}{dt} = & -i(H_{II} - H_{JJ})\delta\rho_{IJ} - \frac{1}{2}(\Gamma_{II} + \Gamma_{JJ})\delta\rho_{IJ} \\ & - \frac{1}{2}\Gamma_{IJ}(\delta\rho_{II} - \delta\rho_{JJ}) - iH_{IJ}(\delta\rho_{II} - \delta\rho_{JJ}). \end{aligned} \quad (28)$$

If the diagonal elements $\delta\rho_{II}$ and $\delta\rho_{JJ}$ change on a timescale much slower than the oscillation period, we can approximate the off-diagonal correlations from Eq. (21) by the steady state limit (obtained by setting $\frac{d\delta\rho_{IJ}}{dt} \rightarrow 0$):

$$\text{for } I \neq J: \delta\rho_{IJ} \approx \frac{\frac{i}{2}\Gamma_{IJ}(\delta\rho_{II} + \delta\rho_{JJ}) + H_{IJ}(\delta\rho_{II} - \delta\rho_{JJ})}{(H_{II} - H_{JJ}) - \frac{i}{2}(\Gamma_{II} + \Gamma_{JJ})}, \quad (29)$$

where the analogous solutions for $\delta\bar{\rho}$ are obtained by replacing $H \rightarrow -H$ and $\rho \rightarrow \bar{\rho}$. We may insert this solution into the source term for the lepton asymmetry from Eq. (21a):

$$\begin{aligned} \text{Tr}[\tilde{\Gamma}_\alpha(\bar{\rho}_N - \rho_N)] \approx & \sum_I \delta\rho_{II} \frac{H_{22} - H_{33}}{(H_{22} - H_{33})^2 + (\frac{\Gamma_{22} + \Gamma_{33}}{2})^2} \\ & \times \{2(\gamma_+^2 + \gamma_-^2)\text{Im}[(F^\dagger F)_{23}F_{\alpha 2}F_{\alpha 3}^*] + 4\gamma_+\gamma_-\text{Im}[(F^\dagger F)_{23}F_{\alpha 2}^*F_{\alpha 3}]\}, \end{aligned} \quad (30)$$

where we neglect higher-order terms in the Yukawa couplings, including the helicity asymmetry $\bar{\rho}_{II} - \rho_{II}$. The two terms can be identified with the *lepton-flavor-violating* and *lepton-number-violating* source terms for the asymmetry.

Substituting this solution into Eq. (21a) and inserting the vacuum expressions for H gives us an approximate expression for the two decay asymmetries:

$$\epsilon_I^{\alpha\text{LNV}} = -\frac{(M_2^2 - M_3^2)\text{Im}[(F^\dagger F)_{23}F_{\alpha 2}^*F_{\alpha 3}]}{(M_2^2 - M_3^2)^2 + E_N^2(\gamma_+ + \gamma_-)^2([F^\dagger F]_{22} + [F^\dagger F]_{33})^2} \frac{4E_N\gamma_+\gamma_-}{(\gamma_+ + \gamma_-)[F^\dagger F]_{II}} \quad (31)$$

and

$$\epsilon_I^{\alpha\text{LFV}} = \frac{(M_2^2 - M_3^2)\text{Im}[(F^\dagger F)_{23}F_{\alpha 2}F_{\alpha 3}^*]}{(M_2^2 - M_3^2)^2 + E_N^2(\gamma_+ + \gamma_-)^2([F^\dagger F]_{22} + [F^\dagger F]_{33})^2} \frac{2E_N(\gamma_+^2 + \gamma_-^2)}{(\gamma_+ + \gamma_-)[F^\dagger F]_{II}}. \quad (32)$$

Notice that $\epsilon_I^{\alpha\text{LFV}}$ violates only the SM lepton flavor (hence the name LFV), as the sum over α leads to a vanishing lepton asymmetry, whereas $\epsilon_I^{\alpha\text{LNV}}$ directly violates the SM lepton number. The SM lepton number discussed here is related to, but not identical to, the fermion number, which also includes the helicity asymmetry in the heavy neutrino sector as discussed in Appendix C.

A few comments on these expressions are in order:

- (i) In contrast to the asymmetries used in Refs. [36,132], the decay rates entering Eqs. (31) and (32) include

explicitly the helicity dependence of the rates and lead to a result similar to Ref. [101] in the hierarchical limit;

- (ii) in the nonrelativistic limit $\gamma_\pm \rightarrow M/(16\pi)$ and $|k_0| \rightarrow M$, we recover the relation (20);
- (iii) in the same limit, these expressions qualitatively agree with the regulators from Ref. [60], as well as those from Ref. [65] if $\text{Re}[F^\dagger F]_{23} \ll [F^\dagger F]_{II}$;
- (iv) although the decay asymmetry from Eqs. (31) and (32) remains finite when $\Delta M_N^2 \ll |k_0|\gamma_\pm F^2$, the approximation of fast oscillations which we used here breaks down, and one has to rely on quantum kinetic equations (21).

1. Applicability of the decay asymmetries

We conclude this section with a short comment on the different ways of regulating the decay asymmetry ϵ^{LFV} . To explore the breakdown of Boltzmann equations in the GeV-mass regime, we recover the result from Refs. [20,21], following the procedure described in Ref. [29]. Taken at face value, Eqs. (31) and (32) appear well behaved in the limit of $\Delta M_N \rightarrow 0$, but once we take the full picture of heavy neutrino oscillations into account we find that this is not the whole story. In the relativistic limit $T \gg M$, the rate $\gamma_+ \sim T$, and $\gamma_- \approx 0$, Eq. (32) simplifies to

$$\epsilon_I^{\alpha\text{LFV}} \approx \frac{(M_2^2 - M_3^2) \text{Im}[(F^\dagger F)_{23} F_{a2} F_{a3}^*]}{(M_2^2 - M_3^2)^2 + E_N^2 \gamma_+^2 ([F^\dagger F]_{22} + [F^\dagger F]_{33})^2 [F^\dagger F]_{II}} \frac{2E_N \gamma_+}{[F^\dagger F]_{II}}. \quad (33)$$

In contrast to the temperature-independent decay asymmetries often used for resonant leptogenesis, this decay asymmetry has a dramatic temperature dependence and becomes enhanced when $M/T \rightarrow 0$:

$$\epsilon_I^{\alpha\text{LFV}} \sim \frac{1}{z^2} \frac{\text{Im}[(F^\dagger F)_{23} F_{a2} F_{a3}^*]}{M_2^2 - M_3^2} \frac{2E_N \gamma_+}{[F^\dagger F]_{II} T^2}, \quad (34)$$

where $z \equiv M/T$. The question is what prevents the initial lepton asymmetries from diverging. We might expect that the asymmetry is already made finite by the width in Eq. (32). On the other hand, if we solve the kinetic equations, we find that another effect suppresses the decay asymmetries much sooner: the finite time required for HNL oscillations. As shown in Fig. 2, the approximate solution obtained in the limit of fast oscillations misses the size of the off-diagonal correlations by several orders of magnitude, since the requirement of fast oscillations is not satisfied until several oscillations occurred. The fact that the oscillation timescale regulates the divergences of Eq. (34) is exactly what leads to the exponent $(M_2 - M_3)^{3/2}$ in the expression for the total lepton asymmetry [21]. As we have shown in this section, a systematic treatment of the low-scale leptogenesis in all corners of the parameter space is possible only within the framework of quantum kinetic equations (21). The crucial ingredient of these equations are the rate coefficients entering Eqs. (22)–(24). The next section is dedicated to the determination of these coefficients in all relevant regimes.

V. THE HEAVY NEUTRINO PRODUCTION RATES

In recent years, significant progress has been made toward determining the production rate of the heavy neutrinos in the early Universe [25,27,101,153–155]. In particular, the production rate of GeV-scale heavy neutrinos at temperatures $T \gg M$ has been studied in great detail [35,45,48]. For GeV-scale neutrinos, important effects arise at temperatures below the electroweak crossover, where the

FIG. 2. An example time evolution of the off-diagonal correlations which enters the decay asymmetry (blue, solid line). In the limit of fast oscillations, this highly oscillatory behavior is well described by the static limit (green, solid line). The fast oscillations lead to a numerically stiff system. In red (dashed line), we show how the full solution approaches the static limit once we average it over some a finite time interval. This example shows that using the static solution before a single oscillation period can lead to a significant overestimate of the BAU, even when it is regulated by a finite width.

mixing between the heavy and light neutrinos can significantly affect the production rate [44,45]. The active neutrino interaction rate—which affects the heavy neutrino production at these regimes—has even been calculated at next-to-leading order [156].

At present, there are no readily available estimates of the full heavy neutrino production rate for $T \sim M$. In Ref. [101], the rate was calculated only including the *naive* leading order $1 \leftrightarrow 2$ and $2 \leftrightarrow 2$ processes, thereby neglecting the $1 + n \rightarrow 2 + n$ processes enhanced by multiple soft scatterings, also known as the Landau-Pomeranchuk-Migdal (LPM) effect. On the other hand, earlier calculations [25,27,153] provide only the helicity-averaged rate, which is not sufficient to track the full evolution of heavy neutrinos as the Universe cools down from $T \gg M$ to $T \sim M$. Below, we describe our approach to the calculation of the heavy neutrino production rate.

The production rate of the heavy neutrinos can be expressed through the spectral (*anti-Hermitian*¹¹) part of their self-energy Σ_N^A . To simplify the calculation, we factor out the Yukawa couplings from the self-energies and introduce

$$\tilde{\Sigma}_N = g_w (\hat{\Sigma}_P F^\dagger F + \hat{\Sigma}_L G F^T Y^* G^*), \quad (35)$$

where $g_w = 2$ counts the $SU(2)$ degrees of freedom and the symmetric matrix G encodes the generalized Majorana condition $N = G N^c$ and must be included to ensure flavor covariance of the equations.

¹¹In the literature, this is also known as the *imaginary* part of the self-energy, which is strictly speaking not accurate for heavy neutrinos whose self-energy is a complex matrix in Dirac and flavor spaces.

The rate coefficients $\gamma_{\pm}$ are related to the self-energy as

$$\gamma_{\pm} = \frac{g_w}{|k_0|} (\hat{\Sigma}_{N0}^A \pm \hat{\Sigma}_{N\pm}^A \hat{k}_i) (k_0 \pm |k|) \Big|_{k_0=\omega_k} = \frac{g_w}{|k_0|} \hat{\Sigma}_{N\mp}^A \cdot k_{\pm} \Big|_{k_0=\omega_k}, \quad (36)$$

where we define $\Sigma_{\pm}$ and $k_{\pm}$ as $a_{\pm} = a_0 \pm a_i k_i / |k|$. The results of Ref. [153], as well as Ref. [35], provide a calculation of the spin-averaged heavy neutrino production rate, which is related to the rates $\gamma_{\pm}$ as

$$\text{Im}\Pi = (\gamma_+ + \gamma_-) |k_0|, \quad (37)$$

where $\text{Im}\Pi$ —in the notations of Ref. [35]—is the anti-Hermitian part of self-energy.

For $M \ll T$, when neutrinos are relativistic, and $k_- \sim \frac{M^2}{2k}$, the γ_- rate is suppressed by M^2/T^2 , and $\text{Im}\Pi \approx \gamma_+$ corresponds to the total heavy neutrino production rate. In Ref. [45], these rates were expressed in terms of $Q_{\pm}$ with

$$\gamma_+(k) \approx Q_+(k)T, \quad \gamma_-(k) \approx Q_-(k) \frac{M^2}{T}. \quad (38)$$

On the other hand, for $M \gg T$, the two rates $\gamma_{\pm}$ become equal, as the heavy neutrinos become nonrelativistic and both $k_+ \approx k_- \approx M$ and $\hat{\Sigma}_{N\pm} \approx \frac{M}{32\pi}$.

In the intermediate regime, for temperatures $T \sim M$, we estimate the full rate by extrapolating the relativistic rate from Ref. [45] and combining it with the rate of heavy neutrino decays above the “thresholds”¹² corresponding to decays into the Higgs, $W^{\pm}$, and Z bosons for $M_N > M_{H,Z,W}$.

We perform the extrapolation as follows; we note that the heavy neutrino production rate $\Gamma_N = \gamma_+ + \gamma_-$ appears to have a rather weak dependence on the heavy neutrino mass in the results from Refs. [27,153] for masses $M \lesssim T$. The fermion-number-conserving (FNC) rate has the same behavior for $M \ll T$, where it becomes independent of M , whereas for $M \gg T$ the heavy neutrino decays dominate. In contrast to this, the FNV rate γ_- has a strong dependence on the heavy neutrino mass M in the relativistic regime. However, we note that the dependence does not directly appear in the self-energy $\hat{\Sigma}$ but mainly through the prefactor $k_-/|k_0| \sim M^2/k^2$.

In summary, we estimate the self-energy of the heavy neutrino as the sum of an M -independent term and an M -dependent term for masses above the heavy neutrino decay threshold:

¹²We note that this threshold is not completely accurate, as there are other channels allowing the heavy neutrino to decay; furthermore, the thermal masses should be taken with other processes at the same order in the couplings—namely, the Landau-Pomeranchuk-Migdal contributions. Nonetheless, we use it to distinguish the regimes where the production rate is dominated by decays $M_N > T$ and the regime where the scatterings dominate $M_N < T$.

$$\hat{\Sigma}(k, M) \approx \hat{\Sigma}(k, 0) + \theta(M - m_{\phi}) \hat{\Sigma}^{1 \leftrightarrow 2}(k, M). \quad (39)$$

If we compare this approximation to the naive $1 \leftrightarrow 2$ decay rate, we find that there is a clear M dependence in the $1 \leftrightarrow 2$ rate even for $M < m_{\phi}$. This dependence is caused by the finite size of the effective lepton mass m_{ℓ} and the fact that the decay of the heavy neutrino would be forbidden for $m_{\phi} - m_{\ell} < M < m_{\phi} + m_{\ell}$. Note, however, that this kinematically forbidden region is an artifact of our approximation when we treat the effective mass of the lepton as a physical mass term. Such kinematically forbidden regions in reality disappear once the $2 \leftrightarrow 2$ scatterings, as well as the LPM effect, are included.

The contributions from the $1 \leftrightarrow 2$ processes to the anti-Hermitian part of the heavy neutrino self-energy (cf. Fig. 3) are given by

$$\hat{\Sigma}_N(k) = \int \frac{d^4 p}{(2\pi)^4} 2\Delta_{\phi}^A(p-k) \hat{S}_{\ell}^A(p) [1 - f_F(p) + f_B(p-k)], \quad (40)$$

where $\Delta_{\phi}^A(k)$ and $\hat{S}_{\ell}^A(k)$ are the spectral functions. To obtain the naive result for the $1 \leftrightarrow 2$ heavy neutrino production rate, we replace them by the tree-level approximation

$$\Delta_{\phi}^A(k) \approx \pi \delta(k^2 - m_{\phi}^2) \text{sgn}(k^0), \\ \hat{S}_{\ell}^A(k) \approx \pi \delta(k^2 - m_{\ell}^2) \text{sgn}(k^0) (\not{k} - m_{\ell}), \quad (41)$$

where $m_{\ell}^2 = T^2(g_1^2 + 3g_2^2)/16$ and $m_{\phi}^2 = T^2(g_1^2 + 3g_2^2 + 4h_t^2 + 8\lambda)/16$. Note that we omit the chiral projector in $\hat{S}_{\ell}$, as we factored it out in the definition of $\hat{\Sigma}_N$.

When the on-shell condition is imposed through the delta functions from Eq. (41), the integral in Eq. (40) reduces to a one-dimensional integral with a well-known result:

FIG. 3. Heavy neutrino production before electroweak symmetry breaking. The double lines represent the resummed propagators for the Higgs and lepton doublets. Note that both the $1 \leftrightarrow 2$ and $2 \leftrightarrow 2$ processes can be described by this diagram. At first approximation, we can understand the resummation of the propagators as the Higgs and the lepton doublet, obtaining effective masses m_{ϕ} and m_{ℓ} , respectively. An important soft process not included in this diagram is the Landau-Pomeranchuk-Migdal effect, which corresponds to soft gauge boson exchanges between the lepton and Higgs.

$$\begin{aligned}\hat{\Sigma}_0^A &= \frac{T^2}{16\pi|\mathbf{k}|} [I_1(-\omega_+) - I_1(-\omega_-)], \\ \hat{\Sigma}_i^A &= \frac{T^2 \hat{\mathbf{k}}_i}{16\pi|\mathbf{k}|} \left[\frac{k^0}{|\mathbf{k}|} [I_1(-\omega_+) - I_1(-\omega_-)] \right. \\ &\quad \left. - \frac{M^2 + m_\ell^2 - m_\phi^2}{2|\mathbf{k}|T} [I_0(-\omega_+) - I_0(-\omega_-)] \right],\end{aligned}\quad (42)$$

$$\omega_\pm = \frac{-k_0(m_\phi^2 - m_\ell^2 - M^2) \pm k\sqrt{[M^2 - (m_\phi + m_\ell)^2][M^2 - (m_\phi - m_\ell)^2]}}{2M^2}.\quad (44)$$

A. Production rates after electroweak symmetry breaking

Following Ref. [35], we can identify two classes of processes that lead to the production of the heavy neutrinos: *direct* processes, where the heavy neutrino interacts with Higgs and lepton doublets, as well as *indirect* processes, in which the heavy neutrino interacts through the mixing with the light neutrinos.

One of the biggest differences compared to the symmetric phase arises in the decay of the heavy neutrino, as the Higgs doublet is replaced by a real scalar field. At the same time, decay channels into the Z and $W^\pm$ bosons open up. There is a gauge dependence in how we separate the direct and indirect processes, but the total rate in their sum remains gauge invariant. This gauge invariance can be lost if we do not keep all the direct and indirect diagrams at the same order in perturbation theory. As the resonant mixing between the heavy and light neutrinos is best described by resumming the lepton propagators, we implicitly include diagrams that are not matched by the direct rate and, therefore, introduce gauge dependence to our estimate of the heavy neutrino production rate. We resolve this issue by choosing a specific gauge, in this case the 't Hooft–Feynman gauge, due to the fact that it does not introduce any new scales.

1. Direct heavy neutrino production

In this section, we perform a simple estimate of the direct heavy neutrino production rate. As discussed at the beginning of this section, there are various contributions to the self-energy of the heavy neutrinos.

The situation slightly changes in the broken phase of the standard model. In the 't Hooft–Feynman gauge, the Goldstone bosons also contribute to the heavy neutrino production rate. We can approximate this rate by the same $1 \leftrightarrow 2$ integrals from the previous section, however, with m_ϕ replaced by the mass of the appropriate Goldstone boson (cf. Fig. 6), i.e., the gauge boson to which the Goldstone mode corresponds:

where the functions I_n correspond to the integrals

$$I_n(y) = \int^y dx x^n [1 - f_F(x) + f_B(y - x)],\quad (43)$$

with the integration boundaries

$$\begin{aligned}\hat{\Sigma}_N^{(1 \leftrightarrow 2)} &= \frac{1}{2g_w} \hat{\Sigma}_N^{(1 \leftrightarrow 2)} \Big|_{m_\phi=m_H} + \frac{1}{2g_w} \hat{\Sigma}_N^{(1 \leftrightarrow 2)} \Big|_{m_\phi=m_Z} \\ &\quad + \frac{1}{g_w} \hat{\Sigma}_N^{(1 \leftrightarrow 2)} \Big|_{m_\phi=m_W}.\end{aligned}\quad (45)$$

The result of this extrapolation in the symmetric phase is shown in Fig. 4 for the FNV and FNC rates. For comparison in Fig. 5 we show the excellent agreement of our extrapolation with the helicity-averaged rate from [153]. The

FIG. 4. The extrapolation used to estimate the FNV and FNC rates. The full result (blue, solid line) is obtained by adding the extrapolated relativistic self-energy (green, dot-dashed line) and the $1 \leftrightarrow 2$ self-energy (red, dashed line) for $M > M_H$. The upper panel shows the self-energies $\hat{\Sigma}_\pm$, which have a weaker dependence on the HNL mass M , compared to the interaction rates $\gamma_\pm$, which include a part of the HNL phase space suppression $k_\pm/|k_0|$.

FIG. 5. Comparison of the helicity-averaged HNL production rate from this work (green, dot-dashed line) with a previous calculation from Ref. [153]. In spite of using an extrapolation, we manage to reproduce the main features of the full calculation.

FIG. 6. Direct heavy neutrino production in the broken phase of the electroweak theory. The dashed lines represent the Higgs field h and the Goldstone modes corresponding to the $W^\pm$ and Z boson. The light (heavy) neutrinos are represented by the thin (thick) solid lines.

individual rates were divided by factors of $2g_w$, to take into account that only one isospin runs in the loop, as well as the $1/\sqrt{2}$ factor that appears in the coupling to the field h compared to ϕ .

Despite the fact that our calculation of the $1 \leftrightarrow 2$ rate misses the $1 + n \leftrightarrow 2 + n$ processes which are included in the LPM resummation, our results—after averaging over helicities—show a nice agreement with the full helicity averaged computation; see Fig. 5.

The remaining direct processes, such as $2 \leftrightarrow 2$ scatterings, can also be included in such diagrams if we replace the tree-level spectral functions from Eq. (41) by their resummed counterparts.

Production in decays and inverse decays.—The heavy neutrinos can be produced in the decays of the Higgs particle. During the electroweak crossover, the heavy neutrino mixes with the light neutrinos and can, therefore, also interact with the W and Z bosons. In the unitary gauge, this mixing would appear only as an indirect contribution. We use the 't Hooft–Feynman gauge, where the longitudinal modes of the massive gauge bosons manifest themselves as the Goldstone modes w , z , and q .

An accurate estimate of the nonperturbative contribution to masses of the Higgs and vector bosons typically requires a lattice calculation [157,158]. To ensure consistency with the rate calculation from Ref. [50], we follow their approach and rely on a one-loop calculation.

FIG. 7. Indirect heavy neutrino production. The double lines represent the resummed propagator for the lepton doublet.

2. Indirect heavy neutrino production

The indirect HNL production happens through the mixing of the HNL with the active neutrinos (c.f. Fig. 7). At tree level, the sum of the direct and indirect rates can be shown to be gauge invariant. However, if we resum the propagator of the doublet leptons, we implicitly include a whole set of gauge-dependent diagrams, which do not have their match in the tree-level diagrams of the direct processes. The gauge dependence of resummed propagators is well documented in the literature (see, e.g., [159]) and can be alleviated by using, e.g., higher-order effective actions. Note that in R_ξ gauges the mass of the Goldstones is given as $\xi \times m_{\text{gb}}$, where m_{gb} is the mass of the corresponding gauge boson. Therefore, if we choose $\xi = 1$, i.e., the 't Hooft–Feynman gauge, we avoid introducing new scales into the problem.

The resummed lepton propagators are given by (see, e.g., [160])

$$S_\ell^{\mathcal{A}} = P_L \frac{2(k - \Sigma_\ell^H) \Sigma_\ell^{\mathcal{A}} \cdot (k - \Sigma_\ell^H) - \Sigma_\ell^{\mathcal{A}} (k - \Sigma_\ell^H)^2 + \Sigma_\ell^{\mathcal{A}3}}{[(k - \Sigma_\ell^H)^2 - \Sigma_\ell^{\mathcal{A}2}]^2 + 4[\Sigma_\ell^{\mathcal{A}} \cdot (k - \Sigma_\ell^H)]^2} P_R, \quad (46)$$

$$S_\ell^H = P_L \frac{2\Sigma_\ell^{\mathcal{A}} \Sigma_\ell^{\mathcal{A}} \cdot (k - \Sigma_\ell^H) + (k - \Sigma_\ell^H)[(k - \Sigma_\ell^H)^2 - \Sigma_\ell^{\mathcal{A}2}]}{[(k - \Sigma_\ell^H)^2 - \Sigma_\ell^{\mathcal{A}2}]^2 + 4[\Sigma_\ell^{\mathcal{A}} \cdot (k - \Sigma_\ell^H)]^2} P_R, \quad (47)$$

where the superscripts H and $\mathcal{A}$ stand for the Hermitian (dispersive) and anti-Hermitian (dissipative) parts of the lepton self-energy. These expressions significantly simplify if we use light-cone coordinates, where the denominator factorizes, which effectively gives us two propagators, one for the particles and the other for the holes in the plasma. For the retarded propagator, this gives us

$$S_\ell^R = P_L \left(\frac{\gamma_-}{k_+ - \Sigma_+^R} + \frac{\gamma_+}{k_- - \Sigma_-^R} \right) P_R, \quad (48)$$

where the indices $\pm$ indicate the light-cone coordinates $a_\pm = (a_0 \pm a_i k_i / |k|)$. The indirect FNV and FNC rates are then given by [44,45]

$$\gamma_{\pm}^{\text{ID}} = g_w \frac{k_{\pm} \hat{\Sigma}_{\pm}^{\text{ID}}}{|k_0|} = v^2 \frac{k_{\pm} \Sigma_{\ell\pm}^A}{\Sigma_{\ell\pm}^A{}^2 + (k_{\pm} - \Sigma_{\ell\pm}^H)^2}, \quad (49)$$

where Σ_{ℓ} are the self-energies of the neutrinos. Interestingly, for large HNL masses, these self-energies have to be evaluated on the HNL mass shell, which leads to further uncertainties, as the active neutrino interaction rates have been calculated only for relativistic neutrinos. To estimate this rate, we use the same approach as for the rates in the symmetric phase; namely, we add the $1 \rightarrow 2$ contribution to the relativistic estimate when the HNL mass exceeds the decay thresholds. Fortunately, any such uncertainties are not important for larger HNL masses, since the off-shell active neutrino width does not grow faster than the HNL mass in the denominator itself, and the indirect contribution becomes negligible for $M \gg T$, as shown in Fig. 8.

By combining the estimates of the rates we have a consistent set of rates in both the broken and symmetric phases for any HNL mass. In spite of the uncertainties related to our extrapolation, our rates remain consistent with the spin-averaged rates from the literature as shown in Fig. 5.

FIG. 8. The FNV and FNC rates in the broken phase of the SM. The full rate (blue, solid line) is a sum of the relativistic direct contribution (red, dot-dashed line), the direct N decay (green, dashed line), and the indirect contribution from the HNL mixing with the active neutrinos (purple, dotted line). For completeness, we show both the self-energies (upper panels) and the interaction rates (lower panels).

VI. THE PARAMETER SPACE OF LEPTOGENESIS

A. Baryogenesis through freeze-in and freeze-out

As we have seen in the previous sections, the same equations can be used to describe both leptogenesis through neutrino oscillations and resonant leptogenesis. Because there is no clear cut between the two mechanisms, we may instead look at whether the majority of the asymmetry is produced during freeze-in (i.e., during the production of HNLs, when they are underabundant compared to their equilibrium density) or freeze-out (during the overabundance of HNLs caused by the expansion of the Universe) of the heavy neutrinos.

For a clear separation between the two regimes, we perform three parameter scans:

- (1) Parameter scan with vanishing initial abundance of the heavy neutrinos—*both freeze-in and freeze-out contribute* to the BAU generation.
- (2) Parameter scan with thermal initial conditions—*only freeze-out contributes*. Indeed, by freeze-in, we understand the period during which HNLs reach equilibrium. By artificially setting the thermal initial conditions, we eliminate this period.
- (3) Parameter scan with vanishing initial abundance of the heavy neutrinos and without the deviation from equilibrium caused by the expansion of the Universe—*only freeze-in contributes*. Technically, this is achieved by putting to zero the source term $d\rho_N^{\text{eq}}/dt$ from Eqs. (21).

Because of the approximate linearity of the evolution equations, the three parameter scans are not independent, and the baryon asymmetry from both freeze-in and freeze-out can be obtained by summing the other two parameter scans. However, to avoid numerical errors due to cancellations of large numbers, we perform all three parameter scans independently.

B. The range of allowed masses and mixing angles

The parameter space of leptogenesis is quite large, as we have discussed in Sec. III A; in total, we have six unknown parameters. These parameters are the average mass M ; the mass splitting ΔM ; one Dirac and one Majorana phase of the PMNS matrix; and the real and imaginary parts of the complex angle ω . Instead of constraining these parameters directly, it is instructive to see how they are related to the experimentally observable quantities. Among these, the most relevant are the masses of the heavy neutrinos and their mixing angles. This is particularly important for direct searches, as the size of the mixing angle determines the number of heavy neutrinos that can be produced.

The condition of reproducing both the baryon asymmetry of the Universe and the light neutrino masses imposes constraints on the allowed mixing angles of the heavy neutrinos. The lower bound on the *total* mixing angle U^2 is mainly coming from the requirement of reproducing the

FIG. 9. The range of the total mixing angle U^2 consistent with both the seesaw mechanism and leptogenesis as a function of HNL mass M_N . The black solid lines show the results obtained with the full kinetic equations and vanishing initial conditions for HNLs. The blue, dashed lines correspond to thermal initial conditions. In this regime, the freeze-in does not contribute to the asymmetry generation. The red, dotted line corresponds to neglecting the effect of the expansion of the Universe on the distribution of the heavy neutrinos. In this case, the freeze-out cannot contribute, and the asymmetry is generated during freeze-in. The color contours represent the largest allowed value of the mass splitting $\Delta M/M$. Within the white regions, the mass splitting is smaller than 10^{-6} . The left (right) panel shows the case of normal (inverted) hierarchy.

light neutrino masses, while the upper bound comes from leptogenesis.

The mixing angle of the heavy neutrinos is tied to the size of their Yukawa couplings; see Eq. (10). For large values of the Yukawa couplings, the mixing angle itself will be large. The main issue for leptogenesis is that a large value of Yukawa couplings at the same time leads to a large washout strength.

To perform the study of the parameter space, we choose several specific benchmark points which extremize the ratios of the Yukawa couplings

$$\frac{\sum_i |F_{ai}|^2}{\sum_{i\beta} |F_{\beta i}|^2}. \quad (50)$$

It is interesting that these points correspond to special values of the PMNS phases, with¹³

$$\delta = n\pi/2, \quad \eta = m\pi/2, \quad \text{Re}\omega = l\pi/4 \quad (51)$$

with $m, n, l \in \mathbb{Z}$.

The same phases have been shown to maximize the total mixing U^2 in Refs. [42,49]. These choices of parameters are particularly favorable for leptogenesis, as they allow for a hierarchy in the washout strengths. This means that, even in the presence of a large overall washout parameter, the asymmetry may survive hidden in a particular lepton flavor.

Once the phases δ , η , and $\text{Re}\omega$ are fixed, we scan over the remaining parameters. The details of the numerical procedure are specified in Appendix A. We limit the HNL

¹³We should note, however, that these are only effective low-energy parameters, which do not quantify the full amount of CP violation in the theory.

mass to the range $[0.1, 7 \times 10^3]$ GeV. The results are shown in Fig. 9. As one can see from Fig. 9, the leptogenesis is efficient for the whole range of masses which we have considered. Moreover, it extends beyond the considered mass range. In particular, as we show in Sec. VI E, there exists a simple scaling which allows us to extend our results all the way to the Davidson-Ibarra bound.

C. Constraints on the heavy neutrino mass splitting

The mass splitting between the heavy neutrinos is one of the most important parameters for both leptogenesis scenarios. The main reason why leptogenesis is so sensitive to the mass splitting ΔM between the heavy neutrinos is that this parameter sets the scale for the oscillations that violate CP and lead to a lepton asymmetry. The temperature corresponding to the onset of oscillations depends on the Hubble rate and is given as [21]

$$T_{\text{osc}} \approx (M_0 M \Delta M)^{1/3} \quad \text{if } T_{\text{osc}} \gg M_N, \quad (52)$$

where $M_0 = \sqrt{90/(8\pi^3 g_*)} M_{\text{Pl}}$ and g_* is the effective number of relativistic degrees of freedom. For heavier neutrinos, it is possible that the oscillations begin when they are already nonrelativistic, which gives us a different temperature, since the typical HNL energy is M instead of T :

$$T_{\text{osc}} \approx (M_0 \Delta M)^{1/2} \quad \text{if } T_{\text{osc}} \lesssim M_N. \quad (53)$$

This oscillation temperature scale can be lowered once we take the Yukawa-induced thermal masses into account, as the thermal masses are aligned with the HNL interaction basis and do not lead to oscillations themselves. To understand the parameter space, it is instructive to consider

“slices,” where we vary the mass splitting and the magnitude of the Yukawa couplings (i.e., the parameter $\text{Im}\omega$) and keep the remaining parameters fixed.

The results of these parameter scans are shown in Fig. 10. For each mass, we show three lines: one corresponding to leptogenesis via freeze-in, one to leptogenesis via freeze-out, and one containing both contributions.

This helps us better understand the transition between the two regimes.

D. Main reasons why freeze-in and freeze-out leptogenesis parameter spaces are connected

For heavier HNLs which start to decay before the sphaleron freeze-out, two opposite effects take place.

FIG. 10. The allowed range of mass splittings and mixing angles for fixed phases. Left column: normal hierarchy, $\text{Re}\omega = \pi/4$, $\delta = \pi$, $\eta = 3\pi/2$. Right column: inverted hierarchy, $\text{Re}\omega = \pi/4$, $\delta = 0$, $\eta = \pi/2$. The different contours correspond to different initial conditions for the heavy neutrinos. The area inside the regions corresponds to a BAU greater than the observed asymmetry. The dark blue, solid curve includes both the contributions from freeze-in and freeze-out. The light blue, dashed curve corresponds to freeze-out only, and the red, dotted curve corresponds to baryogenesis via freeze-in. It is interesting that the largest mass splitting is realized exactly during freeze-in, as the HNL oscillations happen at high temperatures and, therefore, before the HNLs begin to decay.

Their decays are driving the system toward equilibrium, whereas the expansion of the Universe is driving it out of equilibrium. For heavy enough HNLs, decay channels into W and Z bosons open up, which leads to an enhancement of the total heavy neutrino equilibration rate. This increased washout can, in principle, erase all asymmetries produced during the freeze-in and oscillations of the heavy neutrinos [99]. However, the lepton number washout does not remain large indefinitely, as it also depends on the equilibrium density of the heavy neutrinos. This gives us the usual Boltzmann suppression factor [98] for the lepton number washout:

$$\bar{\Gamma}_\nu = \int \frac{d^3k}{(2\pi)^3} \Gamma f_N^{\text{eq}}(1 - f_N^{\text{eq}}) \sim (FF^\dagger)_{\alpha\alpha} M \exp(-M/T) \quad (54)$$

for $M \gtrsim T$.

The lepton number washout rate, therefore, reaches its maximum when $T \sim M$, and the lepton number typically freezes out when $M/T \sim 10$ [98]. To estimate the strength of the lepton number washout, we compute the rate close to its maximum, at $T = M$, and compare it to the Hubble rate $H = T^2/M_0$, $M_0 = \sqrt{90/(8\pi^3 g_*)} M_{\text{Pl}}$. This gives us the decay parameter [98]:

$$\bar{K} = \frac{\text{Tr} \Gamma_N}{2H} \Big|_{T=M} \approx \begin{cases} 30 \cosh(2\text{Im}\omega) & \text{for NH,} \\ 50 \cosh(2\text{Im}\omega) & \text{for IH,} \end{cases} \quad (55)$$

which we average over the two (almost degenerate) heavy neutrinos $\bar{K} = (K_1 + K_2)/2$. The value of $\bar{K}$ indicates whether the washout is strong $\gg 1$ or weak $\ll 1$. It is interesting that the washout rate exceeds the Hubble rate by at least a factor of $\mathcal{O}(30)$, which implies that the washout is already strong for small values of $\text{Im}\omega$ and becomes even larger when $\text{Im}\omega = \mathcal{O}(5)$.

At the same time, the expansion of the Universe drives the heavy neutrinos out of equilibrium. When the Universe cools down to $T \sim M$, the heavy neutrinos are no longer relativistic, and they become overabundant compared to their Boltzmann-suppressed equilibrium density. This late-time deviation from equilibrium is exactly what leads to freeze-out leptogenesis. It can typically be neglected in studies of leptogenesis with GeV-scale HNLs, since it is suppressed by a factor of $(M/T)^2$ for $M \lesssim T$ and reaches its maximum only when $M \approx T$.

The results from the parameter scan in Sec. VIB imply that freeze-in and freeze-out leptogenesis parameter spaces have significant overlap: freeze-in leptogenesis remains viable for masses much higher than what one could have expected, and freeze-out leptogenesis is already possible for masses as light as $M \sim 5$ GeV. We find that the main reason why freeze-in leptogenesis remains possible is the difference of the flavored washout compared to the average washout strength $\bar{K}$, which we further discuss in Sec. VID 1. We show how freeze-out leptogenesis can be realized in a scenario with GeV-scale heavy neutrinos in Sec. VID 2. Finally, we discuss why large mixing angles

do not necessarily imply a large lepton number washout in Appendix C.

1. Freeze-in leptogenesis for TeV-scale HNLs

For GeV-scale heavy neutrinos, the washout of the baryon number stops when the sphaleron processes freeze out, i.e., at $T_{\text{sph}} \approx 130$ GeV. The washout of the asymmetry in a particular lepton flavor depends not only on the washout parameter $\bar{K}$, but also on how strongly a particular flavor couples to the heavy neutrinos. In a realistic computation, we also need to take into account the so-called *spectator* effects (see, e.g., [35]), which redistribute the asymmetries among the remaining SM species. These effects are typically encoded in the susceptibility matrices $\omega_{\alpha\beta}$, as included in Eq. (21a).

The SM lepton number washout is determined by the smallest flavored washout W_α from Eq. (17). To determine the size of the lepton number washout, we have to take the spectator effects into account via the susceptibility matrix $\omega_{\alpha\beta}$. The susceptibility matrix is close to diagonal, and we can estimate the SM lepton number washout strength as

$$K_{\min} \approx \frac{\omega_{\alpha\alpha} T^2}{6} 2\bar{K} \cdot \frac{\min_\alpha \sum_i |F_{\alpha i}|^2}{\sum_{\beta,j} |F_{\beta j}|^2}. \quad (56)$$

The key quantity governing the size of the lepton number washout compared to the heavy neutrino washout rate is, therefore, the minimal branching ratio

$$\frac{\min_\alpha \sum_i |F_{\alpha i}|^2}{\sum_{\beta,j} |F_{\beta j}|^2} \in \begin{cases} [5.2 \times 10^{-3}, 0.12] & \text{for NH,} \\ [3.8 \times 10^{-4}, 1/3] & \text{for IH.} \end{cases} \quad (57)$$

This means that the effective lepton number washout strength is given by

$$K_{\min} \in \begin{cases} [0.1, 3] \times \cosh(2\text{Im}\omega) & \text{for NH,} \\ [0.01, 10] \times \cosh(2\text{Im}\omega) & \text{for IH.} \end{cases} \quad (58)$$

For normal ordering, this implies that the washout can be weak to moderate when $\text{Im}\omega \approx 0$, whereas for inverted ordering it can also be strong—depending on the values of the CP phases η and δ . In Fig. 11, we show an example of the parameter space in the IH case with a choice of PMNS phases that leads to strong washout, where the freeze-in parameter space completely closes around 500 GeV, and the BAU for larger masses becomes independent of the initial conditions.

2. Freeze-out leptogenesis for GeV-scale HNLs

Another reason why the parameter space of resonant leptogenesis remains connected with leptogenesis through neutrino oscillations is that HNL freeze-out provides a sufficient deviation from equilibrium even if their masses are as low as a few GeV. To obtain a qualitative estimate of

FIG. 11. The parameter space of leptogenesis in the case of inverted hierarchy (IH) and phases $\delta = \pi$ and $\eta = 0$, which do not lead to a *flavor hierarchical washout*. In contrast to Fig. 9, the freeze-in parameter space (red) stops at $M \sim 500$ GeV, where the flavored washout becomes too large for any prior asymmetry to survive. This choice of parameters corresponds to the *strong washout* regime, where all of the BAU is generated in the HNL decays. Note that the phase $\text{Re}\omega = \pi/4$ is the same as in Fig. 9, as it does not affect the washout of the lepton number.

the BAU, we can use the decay asymmetries (31) from Sec. IV C. The deviation from equilibrium caused by the expansion of the Universe is suppressed by a factor M^2/T^2 :

$$Y_{N_i} - Y_{N_i}^{\text{eq}} \approx \frac{M^2 T \log 2}{4\pi^2 s(T)}, \quad (59)$$

where $s(T)$ is the entropy density of the Universe.

If we neglect the lepton number washout, in Eq. (17) only the LNV part of the decay asymmetry survives, which gives us an estimate of the BAU

$$Y_B \approx \sum_I \int dz \Gamma_I \epsilon_I^{\text{LNV}} (Y_{N_i} - Y_{N_i}^{\text{eq}}) \sim 10^{-10} \left(\frac{M}{3 \text{ GeV}} \right)^5 \times \frac{\text{Im}[F^\dagger F]_{23}^2 [F^\dagger F]_{II}^2 v^2}{[F^\dagger F]_{II}^2 m_\nu M}, \quad (60)$$

where we used the mass splitting that saturates the resonant enhancement $\Delta M \sim \Gamma_I$ and included only the leading term in γ_- .

This approximation gives us an estimate similar to the results from Refs. [36,132], but the origin of this M^4 suppression is in reality quite different. In Ref. [36], the additional M^2/T^2 factor arises through the interplay of the thermal HNL masses and the thermal decay rates, while here it is a direct result of the helicity-dependent rates $\gamma_\pm$, where the FNV rate is suppressed by a factor $\gamma_- \sim M^2/T^2$.

We should, however, note that this approximation has very limited applicability, and an accurate lower bound can be obtained only by using the full density-matrix equations for the following reasons:

- (i) The mass splitting that is needed to saturate the decay asymmetry $\Delta M \sim \Gamma_I$ corresponds to an oscillation temperature very close to the sphaleron freeze-out temperature, which means that the approximation of fast oscillations does not hold;
- (ii) all the rates in the quantum Boltzmann equations change significantly during the electroweak cross-over—in particular, the FNV rate γ_- changes by 2 orders of magnitude between the temperature $T > 150$ GeV and $T \sim 130$ GeV (cf. Figs. 4 and 8).

E. Approximate scaling between the TeV scale and the Davidson-Ibarra bound

For heavy neutrino masses above $\mathcal{O}(2)$ TeV, the freeze-out of the lepton asymmetry happens before the sphaleron processes freeze out. As a consequence of this, the only scales in the problem are the masses of the heavy neutrinos M_N and the temperature T .

In this section, we show how the solutions to the kinetic equations (21) change for different values of the heavy neutrino masses M_N .

The coefficients H_N and Γ in Eqs. (21) are all dimensionful quantities. When solving leptogenesis equations in an expanding universe, it is convenient to choose a dimensionless time variable $x = T_{\text{ref}}/T$; for $M_N \gtrsim 2$ TeV, this is typically chosen as the average mass of the two heavy neutrinos $T_{\text{ref}} = M$. The freeze-out of the lepton asymmetry then occurs for $x = \mathcal{O}(10)$.

If we change the heavy neutrino masses by a factor $M_N \rightarrow \zeta M_N$, the coefficients H_0/M and Γ/M then scale as

$$H_0/M = \frac{z^2 T_{\text{com}}^2}{E_N} \frac{\Delta M}{M^2} \rightarrow \zeta^{-1} H_0, \quad \Gamma/M = \gamma(x) \cdot Y/M \rightarrow \Gamma/M, \quad (61)$$

where Y denotes either Y_+ or Y_- from Eq. (26) and we used $Y \sim F^2 \propto M$ for a fixed value of m_ν , as can be seen from the parametrization (5). We note that the equilibration rate remains the same, while the bare Hamiltonian term scales inversely with the mass of the heavy neutrinos. Fortunately, the mass splitting ΔM in resonant leptogenesis is typically much smaller than the mass M . This effectively allows us to scale ΔM independently of M . In order to cancel the ζ scaling of H_0 , we replace $\Delta M \rightarrow \Delta M \zeta^2$. This induces only a small correction to the Yukawa couplings $F \rightarrow F + \mathcal{O}(\zeta^2 \Delta M^2/M^2)$. Finally, this means that we do not need to repeat the parameter scan for every mass above $M > 2$ TeV, as the results are related by an appropriate rescaling of the mass splitting ΔM . This conclusion has several caveats:

- (i) For mass splittings of $\Delta M \sim M$, the scaling relation does not hold and the correction to F becomes significant.

- (ii) The up quark reaches equilibrium only at temperatures below 10^6 GeV; therefore, if a significant portion of the lepton asymmetry is produced above these temperatures, the simple scaling solution will not be sufficient, as the susceptibility matrices change, and the distribution of the lepton asymmetries among the flavors may be modified. However, we note that this is a small change in the number of degrees of freedom entering the equations, and we expect it to lead to no more than an $\mathcal{O}(1)$ change in the baryon asymmetry.
- (iii) For temperatures above 10^9 GeV, the “flavored” approximation breaks down, and we have to keep track of the lepton doublet correlation matrices.
- (iv) The typical energy scale of the interactions changes with temperature, which means that the running of the couplings has to be taken into account, which leads to a modification of the rates entering the kinetic equations.
- (v) Finally, when solving the equations for the heavy neutrino oscillations, we implicitly neglect the vertex diagrams which are important for thermal leptogenesis. However, the contribution of this diagram is subleading compared to the wave-function contribution below the Davidson-Ibarra bound.

We may conclude that the biggest differences occur when the majority of the BAU is produced at temperatures above 10^9 GeV.

VII. COMPARISON WITH OTHER STUDIES OF LEPTOGENESIS IN THE DOMAIN $M \sim M_W$

In this section, we compare our results with several other works that discuss leptogenesis in the regime where resonant leptogenesis and leptogenesis via oscillations overlap. We summarize the comparison with these works below.

- (i) *Hernández, Kekic, López-Pavón, Racker, and Salvado [41].*—The authors consider the HNL masses between 0.1 and 100 GeV. This work considers only the symmetric phase of the SM and does not include the FNV processes (which have not yet been included in the quantum kinetic equations prior to Refs. [44,45]) nor the HNL decays to W and Z bosons. While this approach is valid in a large part of parameter space for GeV-scale HNLs, it leads to an underestimate of HNL equilibration rates when $M \sim 100$ GeV.
- (ii) *Antusch, Cazzato, Drewes, Fischer, Garbrecht, Gueter, and Klaric [47].*—The authors consider the HNL masses between 0.1 and 50 GeV. For heavier HNLs, the M/T corrections become significant. Both FNV and FNC processes are included, albeit only in the symmetric phase. The enhancement of the FNV rate in the broken phase was not

included, and the resulting FNV rate is underestimated for $T \leq 160$ GeV. Nonetheless, it is interesting to note that even these underestimated FNV rates exceed the Hubble rate when $M \sim 50$ GeV, without preventing successful baryogenesis.

- (iii) *Hambye and Teresi [36,37].*—The authors study resonant leptogenesis in the GeV regime. They apply the language of decay asymmetries to GeV-scale heavy neutrinos and indicate that leptogenesis in both freeze-in and freeze-out are possible for GeV-scale HNLs. One should note that, although the decay asymmetry obtained in this work qualitatively differs from the expressions in Eq. (31), the authors find the same parametric suppression of the LNV decay asymmetry of the order of $\mathcal{O}(M^2/T^2)$. In Ref. [37], the authors used the density-matrix approach and found semiquantitative agreement with the approach using Boltzmann equations. The authors also find that the LNV asymmetry scales with the product of the helicity-dependent rates $\gamma_+\gamma_-$, similar to the LNV source term in Eq. (31). This study of parameter space is limited to $M < 10$ GeV. As we summarize in Sec. IV C, the Boltzmann equations can be understood as an approximate limit of the density-matrix approach, however, in a limited range of validity. A conclusive study of the leptogenesis parameter space in this regime requires a unified treatment that can reproduce both baryogenesis via oscillations and resonant leptogenesis in the appropriate limits (as we present in this work).

- (iv) *Granelli, Moffat, and Petcov [132].*—The authors consider resonant and flavored leptogenesis in the GeV regime. They find that the flavored decay asymmetry can lead to a BAU several orders of magnitude larger than what the LNV contribution alone would imply. Because of the reliance on the Boltzmann equations, this approach also has limited applicability (in the same sense as Ref. [36]). One should also note that the flavored decay asymmetry studied here can be understood as a limit of fast oscillations from Eq. (32). If interpreted this way, these results can be understood as an alternative approach to leptogenesis via neutrino oscillations using the language of Boltzmann equations.

One should note that Refs. [36,37,132] often discuss two separate decay asymmetries—one coming from mixing and the other one coming from the oscillations [63,66,129,130] of the HNLs. We do not make such a separation, as no separate sources of asymmetry appear in the derivation of the equations using either the canonical Raffelt-Sigl formalism of neutrino oscillations (see, e.g., [20,21,45,52]) or the CTP approach in Wigner space [40,47,62]. Because of this, we do not introduce any such additional terms by

hand, as this can lead to double counting and, therefore, violate the symmetries that are present in the equations in the relativistic limit (for a detailed account of the conserved charges, see Appendix C).

VIII. DISCUSSION AND CONCLUSIONS

In this work, we investigated the similarities and differences between resonant leptogenesis and baryogenesis via neutrino oscillations in the minimal extension of the Standard Model by two heavy neutrinos. We found that the two mechanisms are closely related and that the equations used to describe the two mechanisms are virtually the same. Since the defining feature of resonant leptogenesis, namely, the resonant production of baryon asymmetry, is also present in baryogenesis via neutrino oscillations, we focus on the major difference between the two mechanisms, namely, the question of whether the majority of the BAU is produced during the *freeze-in* or *freeze-out* of the heavy neutrinos.

We found significant overlap between the two regimes; namely, freeze-in leptogenesis turns out to play a major role in generating the BAU even for TeV and heavier Majorana neutrinos. This regime mainly coincides with relatively large $\Delta M/M^2 \sim 10^{-7} \text{ GeV}^{-1}$ mass splitting, compared to the one optimal for a resonant enhancement $\Delta M/M^2 \sim 10^{-13} \text{ GeV}^{-1}$. Furthermore, this also implies a strong dependence on the initial condition which is absent in freeze-out leptogenesis.

On the other hand, we also find viable freeze-out leptogenesis with masses as low as $M = 5 \text{ GeV}$. This can qualitatively be understood via the relatively large decay asymmetries, which lead to a BAU only suppressed by $(M/T)^2$.

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APPENDIX A: NUMERICAL IMPLEMENTATION OF THE LEPTOGENESIS EQUATIONS

The equations governing the evolution of the heavy neutrino densities, as well as the lepton asymmetries, depend on multiple scales and can, in principle, be stiff and, therefore, numerically expensive. Since the parameter space spans many orders of magnitude, there is no single analytical approximation that can be applied everywhere in the parameter space. Nonetheless, there are several simplifications of the equations that can lead to improvements in accuracy and speed of the numerical calculations. In the

following, we summarize the main approximations and simplifications that we use.

1. Momentum averaging

Equations (21) are integro-differential equations. In principle, one has to solve a differential equation for each of the HNL momentum modes and integrate over them at each time step to accurately describe the evolution of the lepton asymmetries. Such a calculation was performed in Refs. [26,48,50]; however, the numerical intensity of such calculations makes it unfeasible for large-scale parameter scans. In this work, we use a momentum-averaging procedure, whereby we consider only the time evolution of the integrated HNL matrix of densities:

$$n_N \equiv \int \frac{d^3k}{(2\pi)^3} \rho_N. \quad (\text{A1})$$

We then assume that the density matrices remain proportional to the equilibrium distribution but differ by a total normalization factor:

$$\rho_N(k) \approx \frac{n_N}{n_N^{\text{eq}}} f_N^{\text{eq}}, \quad (\text{A2})$$

which allows us to replace the momentum-dependent matrices ρ_N in Eq. (21) by the HNL number n_N . This also gives us a procedure to calculate the momentum average of the HNL rate coefficients:

$$\langle X \rangle = \frac{1}{n_N^{\text{eq}}} \int \frac{d^3k}{(2\pi)^3} X(k) f_N^{\text{eq}}, \quad (\text{A3})$$

for the coefficient in Eqs. (22)–(24): $\gamma_{\pm}$, $h_{\pm}$, and $1/E_N$.

This approximation gives rise to some artifacts, like the oscillatory behavior of the lepton asymmetries which are not present in the full system, where the oscillations of the different momentum modes do not happen simultaneously. However, in Refs. [26,48,50], it was shown that the momentum-dependent treatment leads to $\mathcal{O}(1)$ corrections, which justifies the approach taken in this work.

2. Transformation of the equations into vector form

The variables in the system of Eqs. (21) are a combination of different objects—numerically, the lepton asymmetries form a real vector, while the HNL densities $\delta\rho_N$ and $\delta\bar{\rho}_N$ are Hermitian matrices with complex elements. In general, matrices with complex elements will have $n_s^2 \times 2$ degrees of freedom, while a Hermitian matrix has only n_s^2 degrees of freedom (where n_s is the number of HNLs). Any numerical ordinary differential equation solver will treat these matrices as general complex matrices and introduce more degrees of freedom than are strictly necessary to solve the system. To avoid

this, we may choose to parametrize all Hermitian matrices as real four-component vectors:

$$\rho_N = \sum_{i=0}^3 \sigma_i \rho_N^i, \quad \bar{\rho}_N = \sum_{i=0}^3 \sigma_i \bar{\rho}_N^i. \quad (\text{A4})$$

Once we transform all the equations accordingly, the full system (21) can be written in matrix form:

$$\dot{q}_i = -A_{ij} q_j, \quad (\text{A5})$$

where all degrees of freedom can be encoded in an 11-dimensional vector:

$$q = (\rho_N^i, \bar{\rho}_N^i, n_{\Delta_a})^T. \quad (\text{A6})$$

3. Approximate integration of fast modes

In general, the system of equations (21) contains many different timescales. The presence of significantly different timescales can make the system stiff, which limits the speed of the numerical computations.

Several strategies of reducing the stiffness of the system have been laid out in Ref. [40], where the fast equilibration modes can be integrated out in the case of large mixing angles and small mass splittings, and the equations can even be solved semianalytically.

Let us briefly overview the main idea of this calculation. Let us assume that we can separate the fast and slow degrees of freedom as $q = (q_F, q_S)^T$, with

$$\dot{q}_F = -A_{FF} q_F - A_{FS} q_S, \quad (\text{A7})$$

$$\dot{q}_S = -A_{SF} q_F - A_{SS} q_S, \quad (\text{A8})$$

where A_{AB} are block matrices and the eigenvalues of A_{FF} are much bigger than the eigenvalues of A_{SS} . Because of the large eigenvalues of A_{FF} , the fast modes reach quasistatic equilibrium:

$$q_F \rightarrow q_F^{\text{QS}} \approx -A_{FF}^{-1} A_{FS} q_S. \quad (\text{A9})$$

We can reinsert this solution into the equation for the slow modes to track the evolution of the rest of the system, which gives us a differential equation for the slow modes:

$$\dot{q}_S \approx -(A_{SS} - A_{SF} A_{FF}^{-1} A_{FS}) q_S. \quad (\text{A10})$$

In practice, it is, however, impractical to keep track of the fast and slow modes, as these change with temperature (see, e.g., [56]). What we can do instead is to replace the matrix A with one that approaches the same quasistatic limit for the fast modes but leaves the slow modes intact.

One such ansatz is

$$A \rightarrow A(A/\Lambda + 1)^{-1}, \quad (\text{A11})$$

where we choose the cutoff scale Λ between the fast and the slow scales $A_{FF} \gg \Lambda \gg A_{SS}$. This choice ensures that the fast modes approach the same quasistatic limit as in Eq. (A9), but at the timescale Λ , i.e., for $A_{FF} \gg \Lambda$, we have

$$\dot{q}_F \approx -\Lambda(q_F - A_{FF}^{-1} A_{FS} q_S), \quad (\text{A12})$$

whereas for the slow modes we recover Eq. (A10).

What is perhaps most important is that we do not need to block-diagonalize the system into fast and slow modes at each point in time as the matrix A evolves. Instead, the fast and slow modes are automatically separated based on how they compare to the scale Λ . Another useful feature of this approach is that it equally applies when the HNL oscillations become fast, since it does not depend on A_{FF} being real. This ensures that the quasistatic limit of the off-diagonal source term is the same as in Eq. (29) but also prevents the system from becoming too stiff.

APPENDIX B: PSEUDO-DIRAC LIMIT

In this work, we are interested in resonant leptogenesis and baryogenesis via oscillations which both require that the two HNLs have nearly equal masses. In this limit, there is an approximate $U(1)$ symmetry in the theory [two exactly degenerate Majorana particles form a Dirac one with the associated $U(1)$]. This symmetry allows us to assign a lepton number to different flavors of the heavy neutrinos and to effectively combine the two Majorana neutrinos into a single Dirac neutrino:

$$P_R \Psi = \frac{\nu_{R_2} + i\nu_{R_3}}{\sqrt{2}}, \quad (\text{B1})$$

and the Lagrangian takes the form

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{\Psi} i \partial_\mu \gamma^\mu \Psi - M \bar{\Psi} \Psi + \mathcal{L}_{\text{int}}, \quad (\text{B2})$$

$$\begin{aligned} \mathcal{L}_{\text{int}} = & -\frac{\Delta M}{2} (\bar{\Psi} \Psi^c + \bar{\Psi}^c \Psi) - (h_{\alpha 2} \langle \Phi \rangle \bar{\nu}_{L\alpha} \Psi \\ & + h_{\alpha 3} \langle \Phi \rangle \bar{\nu}_{L\alpha} \Psi^c + \text{H.c.}). \end{aligned} \quad (\text{B3})$$

This limit is also realized in the Yukawa couplings. For large values of $|\text{Im}\omega|$, they are much bigger than the naive estimate:

$$|F_{\alpha l}|^2 \gg m_\nu M_M / v^2. \quad (\text{B4})$$

In such a scenario, it is convenient to introduce the matrix of Yukawa couplings $h_{\alpha l}$ related to the matrix $F_{\alpha l}$ defined in Eq. (1) as follows:

$$F_{\alpha l} = h_{\alpha l} [U_N^*]_{Jl}, \quad U_N = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & 1 \\ i & 1 \end{pmatrix}, \quad (\text{B5})$$

where the couplings h_{aI} are manifestly hierarchical with

$$\frac{\sum_{\alpha} |h_{a2}|^2}{\sum_{\alpha} |h_{a3}|^2} = e^{4\text{Im}\omega} + \mathcal{O}(M_3 - M_2). \quad (\text{B6})$$

APPENDIX C: CONSERVED *LEPTON* NUMBERS

One of the main effects that lead to large values of the mixing angle even for $U^2 \gg m_{\nu}/M$ is the approximate conservation of the lepton number.

There are several ways of assigning a lepton number to heavy neutrinos so that it is approximately conserved. In the following, we will discuss three that are important for leptogenesis.

1. Helicity as a lepton number

The first, and most commonly discussed in low-scale leptogenesis mechanisms, is the association of a lepton number with the helicity of the heavy neutrinos. This regime effectively corresponds to heavy neutrinos being relativistic so that their masses can be neglected. It was discussed already in Ref. [20] and is the main reason why three heavy neutrinos were required for successful baryogenesis via oscillations. When this was further developed in Ref. [21], it was noted that this conservation is not necessarily a barrier for production of total lepton asymmetry, as part of the total (conserved) lepton asymmetry is hidden in the heavy neutrino sector. Furthermore, it is interesting that this lepton number remains conserved regardless of the number of the heavy neutrinos, as long as they are all relativistic.

This lepton number conservation is a direct consequence of one of the helicity-dependent rates dominating; in the case when $\gamma_+ \gg \gamma_-$, we can verify that $\sum_{\alpha} \tilde{\Gamma}_{\alpha} \approx -\Gamma$. Summing over the lepton asymmetries and the HNL helicity asymmetry, we find the conserved combination:

$$\frac{d}{dt} \left[\sum_{\alpha} n_{\Delta_{\alpha}} - \int \frac{d^3k}{(2\pi)^3} \text{Tr}[\delta\rho_N - \delta\bar{\rho}_N] \right] \approx 0, \quad (\text{C1})$$

which can be verified by inserting the expressions from Eq. (21) into the derivatives of the components.

On the other hand, in the broken phase, the FNV rate γ_- can be orders of magnitude larger than γ_+ , giving us $\sum_{\alpha} \tilde{\Gamma}_{\alpha} \approx \Gamma$. This gives us a different assignment of lepton number [44]:

$$\frac{d}{dt} \left[\sum_{\alpha} n_{\Delta_{\alpha}} + \int \frac{d^3k}{(2\pi)^3} \text{Tr}[\delta\rho_N - \delta\bar{\rho}_N] \right] \approx 0. \quad (\text{C2})$$

2. Approximate Dirac lepton number

There is another type of lepton number that is typically considered in low-scale seesaw mechanisms. Instead of assigning a lepton number to all right-handed neutrinos, we may assign different lepton numbers to the superpositions of the heavy neutrino flavor eigenstates. We effectively combine the two Majorana spinors into a single Dirac spinor. There are, however, terms that violate this symmetry, namely, the mass splitting between the two heavy neutrinos and the particular structure of the Yukawa couplings.

The conserved combination is most evident in the nonrelativistic limit (when $\gamma_+ = \gamma_-$, as well as $h_+ = h_-$). In the pseudo-Dirac limit, we can neglect the mass difference between the HNLs, as well as the terms suppressed by $e^{-4|\text{Im}\omega|}$ in the Yukawa couplings. This makes the equilibration matrix proportional to the identity ($\Gamma \sim 1_{2 \times 2}$), and all of the $\tilde{\Gamma}_{\alpha} \sim \tilde{\Gamma}$ are proportional to each other. We can introduce a matrix $\kappa \equiv \tilde{\Gamma}/\Gamma$, which selects the combination of HNL flavors that carries the lepton number.

We can then verify that the combination

$$\frac{d}{dt} \left[\sum_{\alpha} n_{\Delta_{\alpha}} + \int \frac{d^3k}{(2\pi)^3} \text{Tr}[\kappa(\delta\rho_N - \delta\bar{\rho}_N)] \right] \approx 0, \quad (\text{C3})$$

as long as the superpositions of the HNL flavors remain coherent.

This effect was taken into account in Refs. [102,103] and leads to a suppression of the flavored washout parameter [which enters the washout term in the Boltzmann equation (17)]:

$$K_{\alpha}^{\text{effective}} = K_{\alpha} \left[1 + 4 \frac{\text{Re}[F_{2\alpha}^{\dagger} F_{a3}] \text{Re}[(F^{\dagger} F)_{23}] - \text{Im}[F_{2\alpha}^{\dagger} F_{a3}] \text{Im}[(F^{\dagger} F)_{23}]}{(F F^{\dagger})_{\alpha\alpha} \text{Tr}(F^{\dagger} F)} \times \frac{(\Gamma_{22} + \Gamma_{33})^2}{(\Gamma_{22} + \Gamma_{33})^2 + 4(M_2^2 - M_3^2)^2} \right].$$

It is important to note that this effect is already included in the density-matrix equations used here, which do not need to be modified further.

APPENDIX D: FINE-TUNING

It is often argued that the mass splittings required for low-scale leptogenesis are fine-tuned (see, e.g., [31,161] for definitions of the fine-tuning). As we see in Fig. 10, the range of mass splittings can, in fact, be quite large, as leptogenesis is possible for $0 \leq \Delta M/M \lesssim 10^{-2}$. Compared to thermal leptogenesis (where the HNL masses can be

hierarchical), a mass splitting of $\Delta M/M \lesssim 10^{-2}$ can appear too small and hard to justify. However, from the technical naturalness standpoint, the model with two hierarchical HNLs and large values of Yukawas appears to be tuned. If there are no cancellations between the tree-level and loop corrections to the light neutrino masses, the mass splitting may not exceed [162]

FIG. 12. The same as Fig. 10, but including the tuning limits on the mass splitting from Eqs. (D1) and (D2). The allowed range of mass splittings and mixing angles for a benchmark point with normal hierarchy and parameters $\text{Re}\omega = \pi/4$, $\delta = \pi$, and $\eta = 3\pi/2$ for NH and $\text{Re}\omega = \pi/4$, $\delta = 0$, and $\eta = \pi/2$ for IH. The yellow, solid line shows the upper limit on the mass splitting (D1), and the green, dashed line shows the lower bound on the mass splitting (D2). For comparison, we also show the magnitude of the mass splitting that is induced by the Higgs vacuum expectation value corrections to the HNL mass matrix. The Lagrangian mass splittings smaller than this value are completely obscured by this contribution and, because of this, cannot be measured in any experiment.

$$\frac{\Delta M}{M} \leq \text{sech}(2\text{Im}\omega) \frac{v^2}{M^2} \frac{(4\pi)^2}{4l'(M^2)}. \quad (\text{D1})$$

On the other hand, the small mass differences between the heavy neutrinos receive radiative corrections (see, e.g., [163–165]). If we consider the correction from the Planck scale M_{Pl} , the lower bound on the mass splitting is given by [166]

$$\frac{\Delta M}{M} \gtrsim \frac{M\Delta m_\nu}{(4\pi v)^2} \log \frac{M_{\text{Pl}}}{\mu_0}, \quad (\text{D2})$$

where a choice of $\mu_0 \sim M$ gives us a logarithm of $\mathcal{O}(50)$. In Fig. 12, we confront the range of mass splittings from leptogenesis with the tuning bounds discussed above.

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