K^0 and K^+ -meson electromagnetic form factors: A nonperturbative relativistic quark model versus experimental, perturbative, and lattice quantum-chromodynamics results

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It has been previously shown that a particular nonperturbative constituent-quark model of hadrons describes experimental measurements of electromagnetic form factors of light charged mesons through a small number of common phenomenological parameters, matching at the same time the quantum-chromodynamics (QCD) asymptotics for the π -meson form factor at large momentum transfer. Here we start with the determination of the K^0 electromagnetic form factor in this approach. Precise measurement of the K^0 charge radius makes it possible to constrain model parameters with high accuracy. Then, with all parameters fixed, we revisit the K^+ form factor and find that it matches experimental measurements in the infrared, lattice results at moderate momentum transfer, and the perturbative QCD asymptotics in the ultraviolet. In this way we obtain a narrow constraint on the K^+ charge radius, $\langle r_{K^+}^2 \rangle = 0.403_{-0.006}^{+0.007} \text{ fm}^2$, and extend the successful infrared-ultraviolet connection from π to K mesons.

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I. INTRODUCTION AND OUTLINE

Quantitative description of strongly coupled composite systems remains one of the principal unsolved problems in particle physics. In quantum chromodynamics (QCD), perturbative methods fail at low energies, where this gauge theory of strong interactions becomes strongly coupled. The key questions regarding the structure of hadrons and the origin of their masses remain unanswered (see e.g., Ref. [1]). Beyond QCD, the same questions arise in numerous extensions of the Standard Model (SM) of particle physics which predict composite particles bound by new strong gauge forces. Testable first-principle quantitative predictions of these models can only be obtained through lattice calculations, which are however resource consuming. Most of detailed lattice calculations of phenomenological importance are concentrated on the QCD case and are not directly applicable to various beyond-SM theories. It is therefore important to develop novel, ready to use quantitative approaches to the description of strongly coupled particle

bound states, even those applicable to particular problems only. Predictions obtained within these approaches can be tested on QCD, where experimental results are available.

These methods are however not abundant since they need to be fully nonperturbative and have to link parameters of the fundamental theory, defined at weak coupling (ultraviolet), to observables defined at strong coupling (infrared). The present work aims to contribute to the development of one such method.

The approach we follow makes use of a constituent-quark model which keeps relativistic invariance at every step of calculations. This nonperturbative method [2–4] is based on Dirac relativistic Hamiltonian dynamics (see e.g., Ref. [5]) in its instant form. Relativistic invariance is guaranteed by the use of modified impulse approximation [2]. The model was used to calculate the electromagnetic form factor, F_π , of the π meson as a function of squared momentum transfer, Q^2 , with a small number of parameters as early as in 1998 [6]. Subsequent measurements [7] of $F_\pi(Q^2)$ matched the model predictions perfectly [8].

At large Q^2 , when the constituent-quark masses are switched off, $F_\pi(Q^2)$ calculated in the model agrees with the perturbative QCD prediction [9–11], both in the functional form [12] and numerically [13]. This is remarkable because the correct ultraviolet asymptotics is reached automatically for the same choice of parameters which describes infrared data.

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Further studies within this approach included calculations of electromagnetic properties of other light mesons [14,15], as well as preliminary studies of the π -meson gravitational form factor [16] and a generalization of the infrared-ultraviolet π -meson link to gauge theories beyond QCD [17], in general agreement with available lattice results. However, one of the parameters of the model for K mesons remained poorly constrained because of the low precision of the experimental determination of the K^+ charge radius, $\langle r_{K^+}^2 \rangle^{1/2}$. In Ref. [15], we presented the allowed range for the K^+ form factor, $F_{K^+}(Q^2)$, and pointed out that more data are necessary to obtain firm predictions. In particular, no conclusive study of the ultraviolet behavior of $F_{K^+}(Q^2)$ was possible with that level of precision.

The present work fills this gap and makes use of much more precise experimental measurements of the K^0 charge radius, $\langle r_{K^0}^2 \rangle^{1/2}$. The calculation of the K^0 electromagnetic form factor is a straightforward generalization of that for K^+ , and $F_{K^0}(Q^2)$ is determined by the same set of parameters. In Sec. II, we use the measurement of $\langle r_{K^0}^2 \rangle^{1/2}$ to remove the freedom in the only unconstrained parameter and make firm predictions for $F_{K^0}(Q^2)$. With all parameters fixed in this way, in Sec. III we calculate and explore $F_{K^+}(Q^2)$. Its behavior at $Q^2 \rightarrow 0$ determines the K^+ charge radius, which we constrain to a very narrow range. The precision is now sufficient to demonstrate the agreement with recent lattice calculations of $F_{K^+}(Q^2)$ at $Q^2 \sim \text{a few GeV}^2$ and to determine unambiguously the asymptotics $F_{K^+}(Q^2 \rightarrow \infty)$, which we find to agree precisely with the perturbative QCD prediction. Section IV summarizes the main results of this work, stressing in particular, the importance of the obtained extension of the quantitative infrared-ultraviolet connection from π to K mesons.

II. K^0 FORM FACTOR AND PARAMETER FIXING

The general formalism and particular formulas for calculation of meson electromagnetic form factors have been presented in previous publications [2,6,8,15,18], (see, in particular, Appendix A of Ref. [15] for the K^+ meson); expressions for K^0 are obtained by a simple substitution of u to d quark charges and magnetic moments. To calculate the electromagnetic form factor of a meson in our approach, one has to fix the following parameters (see also Table I in Sec. IV): (i) masses M_{q_1, q_2} of two constituent quarks q_1, q_2 composing the meson, (ii) wave-function confinement scale b , and (iii) values κ_{q_1, q_2} of anomalous magnetic moments of the two quarks.

The values of $\kappa_{u, d, s}$ are determined *a priori* from model-independent sum rules [19] (see Ref. [15] for more details) so they are not treated as free parameters. Here, we use the

most recent data from Ref. [20] to reevaluate κ_s and its uncertainties. Also, the value of the light-quark constituent mass, $M_u = M_d = 0.22$ GeV, was determined with high accuracy in previous studies of the π -meson form factor (this value was also motivated in Ref. [21] on different grounds). Given the precision of measurement of π -meson related quantities, their contribution to the error balance is expected to be small compared to the K -meson ones. We are left with two parameters for the K mesons; M_s and b . One combination of them is determined from the requirement of reproducing the correct experimental value of the meson decay constant f_K (cf. Refs. [15,22]) and the only remaining parameter is subject to fit. Previous studies of the π and K^+ mesons used lower-energy measurements of the meson charge radius to fix it. While for the π meson this was sufficient to determine the behavior of $F_\pi(Q^2)$ for all Q^2 and even to prove that the correct asymptotics are reached at $Q^2 \rightarrow 0$ if M_u is switched off [13], precision of available measurements of $\langle r_{K^+}^2 \rangle^{1/2}$ is too low to make firm conclusions even at moderate Q^2 . In Ref. [15], we determined the range of available values of $F_{K^+}(Q^2)$ and noted that future experiments, even at moderate Q^2 , will constrain the K^+ charge radius in this approach.

The confinement scale b deserves some comments. First, its definition depends on the particular choice of the meson wave function, but it has been shown [6] (see also Ref. [23]) that the result for the form factor does not depend on this choice, provided the parameters are fixed in such a way that the correct decay constant is reproduced. Here we use the same parametrization of the wave function as in Ref. [15]. Second, though the direct derivation of b in terms of underlying theory is not available, one expects that its value is determined by masses and strong-interaction properties of the two quarks composing the meson. Therefore, the values of b are different for π and K but are expected to be the same for K^+ and K^0 . This makes it possible to use f_K and either $\langle r_{K^0}^2 \rangle$ or $\langle r_{K^+}^2 \rangle$ to constrain M_s and b required for derivation of both $F_{K^+}(Q^2)$ and $F_{K^0}(Q^2)$. Self-consistency of this approach will be checked with the asymptotics of $F_{K^+}(Q^2)$ in Sec. III.

We use the values of $f_K = 0.1562 \pm 0.0010$ GeV (same as in Ref. [15] for consistency; more recent estimates [20] coincide with it within error bars) and of $\langle r_{K^0}^2 \rangle = -0.077 \pm 0.010$ fm² [20] to find $M_s = 0.3177 \pm 0.0040$ GeV and $b = 0.7118 \mp 0.0050$ GeV (error bars of b and M_s are correlated). Figure 1 presents $F_{K^0}(Q^2)$ for these parameters in comparison with the lattice result [24] and with theoretical calculations of Ref. [25]. A certain disagreement with the lattice result is explained by the fact that the lattice calculation [24] does not reproduce the experimental value of the K^0 charge radius to which we tuned our parameters; as it has been pointed out in Ref. [1], the lattice value is $\langle r_{K^0}^2 \rangle \approx -0.026$ fm², far beyond the experimental error bars.

FIG. 1. Results of the present work for the neutral kaon form factor (full red line with light-red shadow uncertainty region) compared to the lattice results [24] (dashed blue line with light-blue uncertainty region) and the calculation of Ref. [25] (long-dashed black line).

III. K^+ FORM FACTOR FROM INFRARED TO ULTRAVIOLET

With all parameters fixed in Sec. II we proceed to calculate the form factor of K^+ . This repeats the calculation of Ref. [15] but with all remaining freedom restricted to tiny error bars in M_s and b specified in Sec. II. The resulting $F_{K^+}(Q^2)$ is compared to our previous work [15] in Fig. 2. The change in the precision is dramatic. Figure 3 compares our result for $F_{K^+}(Q^2)$ with other theoretical calculations.

Since the function $F_{K^+}(Q^2)$ is predicted without any freedom and with small uncertainties, it is interesting to study its behavior in more detail. First, we note that at

FIG. 2. Results of the present work for the charged kaon form factor (full red line with light-red shadow uncertainty region), with parameters fixed from the K^0 charge radius, compared to our previous work [15] (light-green shading), where the parameters were constrained from the K^+ charge radius. Experimental measurements: green crosses at $Q^2 \rightarrow 0$, NA7 [26]; squares, E93-018 [27]; circles, E98-108 [27].

FIG. 3. Results of the present work for the charged kaon form factor (full red line with light-red shadow uncertainty region) compared to the lattice results [24] (dashed blue line with light-blue uncertainty region) and other theoretical studies—1 (full black): Ref. [28]; 2 (dashed black): Ref. [29]; 3 (dot-dashed black): Ref. [30]; 4 (full gray): Ref. [31]; 5 (dot-dashed gray): Ref. [32]; 6 (dashed gray): Ref. [33]; 7 (dotted gray): Ref. [34]; 8 (dotted black): Ref. [35]; 9 (long-dashed black): Ref. [25]; 10 (long-dashed gray): Ref. [36].

$Q^2 \rightarrow 0$ it fits experimental measurements [26]. To be more precise in this statement, we obtain the value of the charged kaon charge radius predicted in our approach, $\langle r_{K^+}^2 \rangle = 0.403^{+0.007}_{-0.006} \text{ fm}^2$ (68% CL). Figure 4 presents a graphical comparison of this value with the experimental

FIG. 4. K^+ charge radius as determined by the NA7 experiment in CERN [26] (error bars scaled from reported 50% CL to 68% CL assuming Gaussian distribution), by lattice calculations [37], by a fit of Ref. [26] data in our model (Krutov *et al.* 2017, Ref. [15]) and by making use of constraints from $\langle r_{K^0}^2 \rangle$ in the present work.

determination of $\langle r_{K^+}^2 \rangle$ [26], our previous constraints [15] and lattice results [37]. We note that the value of $\langle r_{K^+}^2 \rangle$ obtained in the present work agrees well with all these previous results, while the precision is improved considerably. It also agrees with less precise predictions of chiral perturbation theory [38] and of a holographic light-front model [39].

Let us move now to moderate Q^2 of order of a few GeV^2 . Presently, there are no precise experimental measurements of $F_{K^+}(Q^2)$ in this range, so we compare our result to those of the lattice QCD calculation [24]; Fig. 3 demonstrates a good agreement (we will return to the lattice comparison in Sec. IV). Measurements of $F_{K^+}(Q^2)$ at the Jefferson Laboratory [40,41] and the Electron-Ion Collider [42] will make it possible to test our predictions experimentally.

We turn now to the asymptotic regime, $Q^2 \rightarrow \infty$. A way to merge the regime of relativistic constituent-quark models, $Q^2 \rightarrow 0$, and that of the perturbative QCD, $Q^2 \rightarrow \infty$, is to allow the quark mass to run from the constituent-quark mass M in the infrared to the current-quark mass $m \ll M$ in the ultraviolet (see e.g., Ref. [43]). This implies switching off constituent-quark masses in the asymptotic regime. Within our approach, switching the mass off resulted [12] in the correct functional form of $F_\pi(Q^2 \rightarrow \infty)$ in agreement with QCD predictions [44,45]. Moreover, we have shown [13] that no matter how the masses are switched off, the correct coefficient of the perturbative QCD asymptotics of F_π [9–11] is properly reproduced. Since all parameters of the model are fixed in the low- Q^2 limit, this provides a nontrivial infrared-ultraviolet link yet to be understood in a fundamental theory. Extension of this approach to other gauge theories beyond QCD [17] resulted in a reasonable agreement with scarce available lattice calculations. Here we make use of the precise parameter-free prediction of $F_{K^+}(Q^2)$ to test whether this unusual result holds for the mesons other than π .

We follow the approach discussed in detail in Ref. [13]. We choose the same family of functions $M(Q^2)$ (see Sec. II. B of Ref. [13], motivated by Refs. [43,46]) which describe switching constituent-quark masses off at high Q^2 . The functional shapes for $M_{u,d}(Q^2)$ and $M_s(Q^2)$ are the same, while their values at $Q^2 = 0$ are fixed as described above. This implies in particular that $M(Q^2) \simeq M(0)$ for $Q^2 \lesssim 8 \text{ GeV}^2$; the range to be tested in coming experiments. We vary two parameters of the $M(Q^2)$ shape, which describe where and how smoothly the mass is turned off. In a complete analogy with what we observe for $F_\pi(Q^2)$ in Ref. [13] (cf. Fig. 4 of that work) we find that $F_{K^+}(Q^2 \rightarrow \infty)$ does not depend on these two parameters. Note that the behavior of $F_{K^+}(Q^2)$ in the transition region, before the asymptotics is reached, depends on the choice of the $M(Q^2)$ function.

We are now ready to answer the most intriguing question; whether our approach reproduces the correct

QCD asymptotics for $F_{K^+}(Q^2)$ like it does for $F_\pi(Q^2)$. The asymptotics, determined in Ref. [11], reads

$$F_{K^+}(Q^2 \rightarrow \infty) \sim F_{K^+}^{\text{as}} = \frac{8\pi}{Q^2} f_K^2 \alpha_S^{1\text{-loop}}(Q^2), \quad (1)$$

where $\alpha_S^{1\text{-loop}}(Q^2)$ is the one-loop running strong gauge coupling constant.

The correct Q^2 dependence of the form factor is guaranteed, for $M = 0$, by analytical calculations of Ref. [12]. We need to calculate the coefficient numerically to compare it with that given by Eq. (1). We start with the strong coupling at the Z -boson mass scale, $\alpha_S(Q^2 = M_Z^2) = 0.1187 \pm 0.0018$ [20], and evolve it down with the one-loop renormalization-group equation for QCD with $N_f = 5$ flavors. The uncertainty in Eq. (1) comes from a combination of experimental error bars of $\alpha_S(M_Z^2)$ and the intrinsic theoretical uncertainty behind the one-loop expression (1). The latter is not straightforward to estimate since the generalization of Eq. (1) to the second loop is nontrivial. To estimate the theoretical uncertainty in Eq. (1), we loosely assume that it is of order of the ratio of the second and first loop contributions to the beta function—summed up in quadratures with the uncertainty in $\alpha_S(M_Z^2)$ —this gives about 8% estimated QCD uncertainty at $Q_0^2 \sim 200 \text{ GeV}^2$. The ratio of $F_{K^+}(Q_0^2)$, calculated in our approach, to the asymptotics (1) for $\alpha_S(M_Z^2) = 0.1187$ gives

$$F_{K^+}/F_{K^+}^{\text{as}} \simeq 1.016_{-0.018}^{+0.020}, \quad (2)$$

FIG. 5. Ratio of the asymptotics of $F_{K^+}(Q^2 \rightarrow \infty)$, obtained in our model after switching off quark masses, to the perturbative QCD asymptotics [47]. Note that the correct functional form is obtained automatically in our approach, and this plot presents the ratio of the coefficients constrained in the previous study (Krutov et al. 2017, Ref. [15]) and by making use of constraints from $\langle r_{K^0}^2 \rangle$ in the present work. See the text for the estimate of the QCD uncertainty.

TABLE I. Inputs and outputs of the present work. See Ref. [15] for details and the text for references concerning the fixed parameters.

Fixed parameters	Light-quark constituent mass Quark anomalous magnetic moments	$\left\{ \begin{array}{l} M_u \equiv M_d = 0.22 \text{ GeV} \\ \kappa_u = -0.01055 \\ \kappa_{\bar{d}} = 0.03735 \\ \kappa_{\bar{s}} = -0.084 \pm 0.019 \end{array} \right\}$	Fixed from previous π -meson studies Fixed from Ref. [19] using data of Ref. [20]
Fitted parameters	Strange-quark constituent mass K -meson wave-function scale	$M_s = 0.3177 \pm 0.0040 \text{ GeV}$ $b = 0.7118 \mp 0.0050 \text{ GeV}$	Fitted here from measured f_K and $\langle r_{K^0}^2 \rangle$
Outputs	Form factors $F_{K^0}(Q^2)$, $F_{K^+}(Q^2)$	$Q^2 \rightarrow 0$: refined $\langle r_{K^+}^2 \rangle$, matches measurements $Q^2 \sim \text{few GeV}^2$: matches lattice calculations $Q^2 \rightarrow \infty$: matches the perturbative QCD asymptotics	

well within the 1.00 ± 0.08 QCD uncertainty (see Fig. 5). This remarkable agreement extends our successful infrared-ultraviolet link of F_π [13,17] to the K -meson form factors.

IV. DISCUSSION AND CONCLUSIONS

We have calculated the neutral kaon electromagnetic form factor within the same model and through the same parameters as it was previously done for the charged kaon [15]. This has allowed us to fix the remaining uncertain parameter of the model through the K^0 charge radius, which is measured with much better precision than that of K^+ . As a result, we also calculated $F_{K^+}(Q^2)$ with very small uncertainty (see Figs. 2 and 3) and without any remaining freedom in parameters (cf. Table I). At $Q^2 \rightarrow 0$, this calculation matches experimental and lattice results for the K^+ charge radius, at the same time predicting its value with much higher precision (cf. Fig. 4). The values of the form factors calculated in this work are given in Table II in Appendix together with their uncertainties.

At moderate $Q^2 \lesssim 4 \text{ GeV}^2$, we compare our results for $F_{K^0, K^+}(Q^2)$ with the lattice QCD calculations of Ref. [24] (see Figs. 1 and 3). Our $F_{K^+}(Q^2)$ agrees very well with the lattice result, while the agreement is much worse for $F_{K^0}(Q^2)$. This is not surprising since the lattice result does not reproduce the measured K^0 charge radius, which we used to fit the key parameter of our model. In a deeper context, this disagreement may be related to the fact that the lattice K^0 form factor was obtained as a difference between contributions from two quarks, weighted by their charges. We note that in our approach, electromagnetic properties of the quarks differ not only by their charges but also by their anomalous magnetic moments; form factors of K^0 and K^+ are therefore not decomposed into charge-weighted linear combinations of contributions from each quark. The contribution of the anomalous magnetic moments is $\sim 6\%$ for K^+ , while for K^0 it exceeds $\sim 50\%$. This explains in particular the fact that $F_{K^+}(Q^2)$ agrees with the lattice results while $F_{K^0}(Q^2)$ does not agree, which is also the case for the results of Refs. [1,25].

At $Q^2 \rightarrow \infty$, upon switching constituent-quark masses off, $F_{K^+}(Q^2)$ reaches precisely the perturbative QCD asymptotics (cf. Fig. 5). This is achieved without any parameter tuning; all free parameters of the model are fixed in the infrared. This is similar to the behavior of $F_\pi(Q^2)$ found previously in our approach; the asymptotics are different but both are achieved automatically. Our calculation does not predict the scale of the momentum transfer at which the transition to the perturbative regime happens—it is related to the scale at which $M(Q^2)$ goes to zero—but the correct asymptotics are reached independently of that scale. This scale, predicted to be as low as $Q^2 \simeq 3.5 \text{ GeV}^2$ for F_π [48], is probably actually higher, as is suggested by measurements of $F_\pi(Q^2)$ [7] and lattice calculations of $F_{K^+}(Q^2)$ [24]. To determine this scale in our model, further experimental input is required [49]. However, the relations between infrared and ultraviolet properties of the form factors are valid independently of their behavior at moderate Q^2 . This makes it possible to apply these relations in new physics models with strongly-coupled gauge theories [17]. In particular, these include models with composite Higgs and dark-matter particles (see e.g., Refs. [50,51]); there, the low- Q^2 form factor of a neutral meson-like state, similar to K^0 , determines observable properties relevant for the dark-matter search. This avenue will be explored elsewhere.

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APPENDIX: NUMERICAL RESULTS FOR THE K^0 AND K^+ FORM FACTORS

Table II presents the form factors $F_{K^0}(Q^2)$, $F_{K^+}(Q^2)$, determined in this work, in the numerical form.

TABLE II. Values of K^0 and K^+ electromagnetic form factors obtained in this work.

Q^2 , GeV ²	F_{K^0}	F_{K^+}
0.0	0.0000 ± 0.0000	$1.0000 + 0.0000 - 0.0000$
0.1	0.0262 ± 0.0029	$0.8509 + 0.0006 - 0.0006$
0.2	0.0412 ± 0.0041	$0.7444 + 0.0009 - 0.0009$
0.3	0.0497 ± 0.0046	$0.6628 + 0.0012 - 0.0012$
0.4	0.0544 ± 0.0048	$0.5978 + 0.0014 - 0.0013$
0.5	0.0567 ± 0.0047	$0.5447 + 0.0015 - 0.0015$
0.6	0.0576 ± 0.0047	$0.5003 + 0.0016 - 0.0015$
0.7	0.0576 ± 0.0045	$0.4626 + 0.0017 - 0.0016$
0.8	0.0571 ± 0.0044	$0.4302 + 0.0018 - 0.0017$
0.9	0.0562 ± 0.0042	$0.4019 + 0.0018 - 0.0017$
1.0	0.0550 ± 0.0041	$0.3770 + 0.0019 - 0.0018$
1.5	0.0481 ± 0.0034	$0.2871 + 0.0021 - 0.0019$
2.0	0.0416 ± 0.0028	$0.2305 + 0.0021 - 0.0020$
2.5	0.0361 ± 0.0024	$0.1916 + 0.0022 - 0.0020$
3.0	0.0316 ± 0.0021	$0.1631 + 0.0022 - 0.0020$
3.5	0.0279 ± 0.0019	$0.1415 + 0.0022 - 0.0020$
4.0	0.0248 ± 0.0017	$0.1244 + 0.0021 - 0.0020$
4.5	0.0222 ± 0.0015	$0.1107 + 0.0021 - 0.0019$
5.0	0.0200 ± 0.0014	$0.0994 + 0.0021 - 0.0019$
5.5	0.0181 ± 0.0013	$0.0900 + 0.0020 - 0.0018$
6.0	0.0165 ± 0.0012	$0.0820 + 0.0020 - 0.0018$
6.5	0.0151 ± 0.0011	$0.0751 + 0.0019 - 0.0017$
7.0	0.0139 ± 0.0010	$0.0692 + 0.0017 - 0.0016$
7.5	0.0128 ± 0.0010	$0.0640 + 0.0017 - 0.0015$
8.0	0.0118 ± 0.0009	$0.0594 + 0.0016 - 0.0014$

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