New $X_{0,1}(2900)$ -like exotic states in b -baryon decays

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(Received 12 April 2021; accepted 20 July 2021; published 13 August 2021)

In the $B^+ \rightarrow D^+ D^- K^+$ decay, LHCb has reported the observation of the open-charm exotic states $X_{0,1}^0 \equiv X_{0,1}(2900)^0$ with four different quark flavors ($ud\bar{s}\bar{c}$), where the subscripts (0,1) denote the spins. To confirm the discovery, we propose $\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)}$ in the final state interaction, where $X_{0,1}^{0(--)}$ with $sud\bar{c}$ ($ds\bar{u}\bar{c}$) are the new $X_{0,1}$ -like exotic states. More specifically, $\Lambda_c^+ D_s^-$ in $\Lambda_b \rightarrow \Lambda_c^+ D_s^-$ are transformed as $\Sigma_c^{0(++)} X_{0,1}^{0(--)}$, by exchanging $\pi^{+(-)}$. As the order of magnitude estimates, we calculate $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)}) = (2.3 \pm 0.6, 4.3 \pm 0.8_{-2.5}^{+3.3}) \times 10^{-4}$. In addition, we estimate other b -baryon decays with the $X_{0,1}$ -like states, such as $\mathcal{B}(\Xi_b^{0(-)} \rightarrow \Xi_c^{0(+)}(2645) X_{0,1}^{0(--)})$, $\Lambda_b \rightarrow \Xi_c^0 X_{0,1}^0 \sim 10^{-5}$. While one needs $\Lambda_b \rightarrow \Sigma_c^{0(++)} M_c M$ to observe $X_{0,1}^{0(--) \rightarrow M_c M}$ with $M_c M = D_s^- \pi^+$, $\bar{D}^0 \bar{K}^0$ ($D_s^- \pi^-$, $D^- K^-$), $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)}, X_{0,1}^{0(--) \rightarrow M_c M}) \sim 10^{-4}$ are accessible to the LHCb experiment.

DOI: [10.1103/PhysRevD.104.034008](https://doi.org/10.1103/PhysRevD.104.034008)

I. INTRODUCTION

The LHCb Collaboration has recently observed $X_{0,1}^0 \equiv X_{0,1}(2900)^0$ from the $B^+ \rightarrow D^+ D^- K^+$ decay [1,2], where the subscript 0(1) denotes the spin. With the resonant strong decays $X_{0,1}^0 \rightarrow D^- K^+$ in the $D^- K^+$ invariant mass spectrum, one concludes that $X_{0,1}^0$ are the first open-charm exotic states, which consist of four quarks with different flavors, $\bar{c}d\bar{s}u$. Explicitly, the masses and decay widths of $X_{0,1}^0$ are given by [1,2]

$$\begin{aligned} (m_{X_0^0}, m_{X_1^0}) &= (2.866 \pm 0.007 \pm 0.002, 2.904 \\ &\quad \pm 0.005 \pm 0.001) \text{ GeV}, \\ (\Gamma_{X_0^0}, \Gamma_{X_1^0}) &= (57 \pm 12 \pm 4, 110 \pm 11 \pm 4) \text{ MeV}, \end{aligned} \quad (1)$$

where $m_{X_0^0} \simeq m_{X_1^0}$ and $\Gamma_{X_0^0} \simeq \Gamma_{X_1^0}/2$. From the strong decays $X(5568)^\pm \rightarrow B_s^0 \pi^\pm$, the “open-beauty” exotic state $X(5568)$ with four different quark flavors $\bar{b}s\bar{d}u$ ($\bar{b}s\bar{u}d$) was once reported to be observed by D0 Collaboration [3].

Unfortunately, the discovery was not confirmed by LHCb and other experiments [4–7]. Since it is possible that $X_{0,1}^0$ can be further examined as the tetraquarks, which definitely improves our knowledge of QCD and the quark model, one needs to provide the different decays to confirm the discovery.

Theoretical attempts have been given to understand the open-charm exotic states [8–10]. Considering the light quark $q = (u, d, s)$ as a triplet (3) under the $SU(3)$ flavor symmetry [$SU(3)_f$], the open-charm exotic states $q_1 q_2 \bar{q}_3 \bar{c}$ are in the irreducible forms of $(3 \times 3 \times \bar{3})\bar{c} = (3 + 3 + \bar{6} + 15)\bar{c}$, where $\bar{6}$ and 15 are able to include three different quark flavors. Consequently, one classifies X_0^0 and X_1^0 into $\bar{6}$ and 15, with $X_0^0 = (ud - du)\bar{s}\bar{c}$ and $X_1^0 = (ud + du)\bar{s}\bar{c}$, respectively [10]. Moreover, $X_{0(1)}^0$ is assigned the quantum numbers $J^P = 0^+(1^-)$, which is based on the first radially (orbitally) excited state $2S$ ($1P$) of the two-body Coulomb and chromomagnetic interaction models. In this classification, there can be $X_{0,1}$ -like exotic states,

$$\begin{aligned} X_{0,1}^{\prime - -} &= (ds \mp sd)\bar{u}\bar{c}, & X_{0,1}^{\prime 0} &= (su \mp us)\bar{d}\bar{c}, \\ Y_{0,1}^{\prime -} &= (sn - ns)\bar{n}\bar{c}, \end{aligned} \quad (2)$$

with $J^P = (0^+, 1^-)$, $n = u, d$, and $m_{X_{0(1)}^{\prime -}} \simeq m_{X_{0(1)}^0}$. In addition, $X_{0(1)}^0, X_{0(1)}^{\prime 0} \rightarrow DM$ are related with the same strong coupling constant. Therefore, the observation of $X_{0,1}^0$ can be regarded to confirm the discovery of $X_{0,1}$.

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As the discovery channel, $B^+ \rightarrow D^+ X_{0(1)}^0, X_{0(1)}^0 \rightarrow D^- K^+$ has been interpreted to proceed through the final state interaction [11–14]. Following the $B^+ \rightarrow D_s^{*+} \bar{D}^0$ weak decay, $D_s^{*+} \bar{D}^0$ in the rescattering effect are transformed as $D^+ X_{0,1}^0$, which is by exchanging K^0 . Moreover, the same mechanism has been applied to the production of the hidden-charm (strange) pentaquark $\mathcal{P}_{c(s)}$ with $c\bar{c}uud$ ($c\bar{c}uds$) in $\Lambda_b \rightarrow J/\Psi \mathcal{P}_{c(s)}$ [15–18]. Therefore, it is reasonable to consider the b -baryon decays with the new $X_{0,1}^0$ -like exotic states in the final state interaction. Specifically, we propose $\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)}$ as depicted in Fig. 1, with $X_{0,1}^0$ and $X_{0,1}'$ in Eq. (2).

To estimate $\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)}$, the couplings in the triangle loop should be crucial. One has observed $\mathcal{B}(\Lambda_b \rightarrow \Lambda_c^+ D_s^-) \simeq 10^{-2}$ and $\mathcal{B}(\Sigma_c^{0(++)} \rightarrow \Lambda_c^+ \pi^{-(+)}) \simeq 100\%$ [19], indicating the sizeable weak and strong couplings of the baryon decays. In addition, the coupling of $X_{0,1}' \rightarrow D_s^- \pi$ is not small, due to the $SU(3)$ flavor [$SU(3)_f$] symmetry that has been enabled to relate $X_{0,1}'$ and $X_{0,1}^0$ decays [10]. Hence, $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)})$ are anticipated to be as accessible as $\mathcal{B}(B^+ \rightarrow D^+ X_{0,1}^0)$ to the LHCb experiment. In this paper, $\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)}$ will be demonstrated as the promising decay channels to confirm the existence of the $X_{0,1}^0$ -like exotic states.

II. FORMALISM

The open-charm exotic states $X_{0,1}^0$ observed in $B^+ \rightarrow D^+ D^- K^+$ need to be confirmed by different decays. We think $\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(--)}$ may be promising. Like $X_{0,1}^0(\bar{c}d\bar{s}u)$, $X_{0,1}^0(\bar{c}s\bar{d}u)$ and $X_{0,1}'(\bar{c}s\bar{u}d)$ also consist of four different quark flavors. See Fig. 1, $\Lambda_b \rightarrow \Sigma_c X_{0,1}'$ are the triangle-rescattering decays, separated into two parts. The first part is the weak decay $\Lambda_b \rightarrow \Lambda_c^+ D_s^-$, which proceeds through the $\Lambda_b \rightarrow \Lambda_c^+$ transition and D_s^- meson production. By following Refs. [20–22], we derive the amplitude of $\Lambda_b \rightarrow \Lambda_c^+ D_s^-$ as

$$\mathcal{M}_b(\Lambda_b \rightarrow \Lambda_c^+ D_s^-) = \bar{u}_{\Lambda_c}(F_b^+ - F_b^- \gamma_5)u_{\Lambda_b}, \quad (3)$$

where

$$F_b^+ = C_w m_- \left[f_1 + \left(\frac{m_{D_s}^2}{m_+ m_-} \right) f_3 \right],$$

$$F_b^- = C_w m_+ \left[g_1 + \left(\frac{m_{D_s}^2}{m_+^2} \right) g_3 \right]. \quad (4)$$

In the above, we define $m_{\pm} = m_{\Lambda_b} \pm m_{\Lambda_c}$ and $C_w = i(G_F/\sqrt{2})a_1 V_{cb} V_{cs}^* f_{D_s}$, where G_F , V_{ij} , f_{D_s} , and $f_{1,3}(g_{1,3})$ are the Fermi constant, CKM matrix element, decay constant, and $\Lambda_b \rightarrow \Lambda_c^+$ transition form factors [23], respectively, while a_1 results from the factorization [24,25]. The second part as the rescattering effect proceeds through the strong decays, such that $\Lambda_c^+ D_s^-$ are turned into $\Sigma_c X_{0,1}'$ by emitting or receiving a pion. Accordingly, the amplitudes are given by [10,26,27]

$$\mathcal{M}_0(X_{0,1}' \rightarrow D_s^- \pi) = g_0,$$

$$\hat{\mathcal{M}}_1(X_{0,1}' \rightarrow D_s^- \pi) = g_1 \epsilon \cdot (p_{D_s} - p_\pi),$$

$$\mathcal{M}_c(\Sigma_c \rightarrow \Lambda_c^+ \pi) = g_c \bar{u}_{\Lambda_c} \gamma_5 u_{\Sigma_c}, \quad (5)$$

with $g_{c,0,1}$ the strong coupling constants and ϵ_μ the polarization four vector, where $\mathcal{M}_c(\Sigma_c \rightarrow \Lambda_c^+ \pi)$ is presented as that used in the quark model and lattice QCD.

To proceed, we assemble the two parts as the rescattering amplitudes, given by [28,29]

$$\mathcal{M}(\Lambda_b \rightarrow \Sigma_c X_{0,1}') = \int \frac{d^4 q_3}{(2\pi)^4} \frac{\mathcal{M}_b \mathcal{M}_c^\dagger \mathcal{M}_{0,1}}{(q_1^2 - m_{D_s}^2)(q_2^2 - m_\pi^2)(q_3^2 - m_{\Lambda_c}^2)}, \quad (6)$$

where $\mathcal{M}_1 \equiv \hat{\mathcal{M}}_1 F_\Lambda(q_2^2)$, and $F_\Lambda(q_2^2) \equiv (\Lambda^2 - m_\pi^2)/(\Lambda^2 - q_2^2)$ is the form factor that takes care of the divergence in $\Lambda_b \rightarrow \Sigma_c X_{0,1}'$. Besides, $q_1 = p_1 - q_3$ and $q_2 = p_3 - q_3$ correspond to the momentum flows in Fig. 1. In the general form, one expresses the amplitudes of $\Lambda_b \rightarrow \Sigma_c X_{0,1}'$ as

FIG. 1. Rescattering $\Lambda_b \rightarrow \Sigma_c X_{0,1}'$ decays: the left panel presents the quark lines and the right panel the momentum flows.

$$\begin{aligned}\mathcal{M}(\Lambda_b \rightarrow \Sigma_c X'_0) &= \bar{u}_{\Sigma_c} (F_0^+ - F_0^- \gamma_5) u_{\Lambda_b}, \\ \mathcal{M}(\Lambda_b \rightarrow \Sigma_c X'_1) &= \bar{u}_{\Sigma_c} [(F_1^+ \gamma^\mu - F_1^- \gamma^\mu \gamma^5) \\ &\quad + (G_1^+ p_3^\mu - G_1^- p_3^\mu \gamma^5)] u_{\Lambda_b} \epsilon_\mu. \quad (7)\end{aligned}$$

$$\{C_0; C^\mu; C^{\mu\nu}\} = \int \frac{d^4 q_3}{i\pi^2} \frac{\{1; q_3^\mu; q_3^\mu q_3^\nu\}}{(q_3^2 - m_{\Lambda_c}^2)[(q_3 - p_1)^2 - m_{D_s}^2][(q_3 - p_3)^2 - m_\pi^2]}, \quad (8)$$

such that we obtain

$$\begin{aligned}C^\mu &= -p_1^\mu C_1 - p_3^\mu C_2, \\ C^{\mu\nu} &= g^{\mu\nu} C_{00} + p_1^\mu p_1^\nu C_{11} + p_3^\mu p_3^\nu C_{22} \\ &\quad + (p_1^\mu p_3^\nu + p_1^\nu p_3^\mu) C_{12}, \quad (9)\end{aligned}$$

with the parameters $C_0, C_1, C_2, C_{00}, C_{11}, C_{12}, C_{22}$ to be given in the numerical analysis. By replacing m_π in Eq. (8) with Λ , we obtain $\{C'_0; C'^\mu; C'^{\mu\nu}\}$ and then define another set of parameters: $\tilde{C}_{i(ij)} = C_{i(ij)} - C'_{i(ij)}$, where $i = (0, 1, 2)$ and $ij = (00, 11, 22, 12)$. We hence derive that

$$\begin{aligned}F_0^\pm &= \frac{\mp i g_c g_0 F_b^\mp}{16\pi^2} (m_{\Lambda_c} C_0 \pm m_{\Lambda_b} C_1 + m_{\Sigma_c} C_2), \\ F_1^\pm &= \frac{i g_c g_1 F_b^\mp}{8\pi^2} \tilde{C}_{00}, \\ G_1^\pm &= \frac{-i g_c g_1 F_b^\mp}{8\pi^2} [m_{\Lambda_c} (\tilde{C}_0 + \tilde{C}_1 + \tilde{C}_2) \\ &\quad \pm m_{\Lambda_b} (\tilde{C}_1 + \tilde{C}_{11} + \tilde{C}_{12}) + m_{\Sigma_c} (\tilde{C}_2 + \tilde{C}_{12} + \tilde{C}_{22})]. \quad (10)\end{aligned}$$

III. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical analysis, we adopt $(V_{cb}, V_{cs}) = (A\lambda^2, 1 - \lambda^2/2)$ and $f_{D_s} = (249.9 \pm 0.5)$ MeV, with $A =$

To obtain $F_{0,1}^\pm$ and $G_1^\pm$, we need to integrate over the variables of the triangle loop in Eq. (6), for which the equations are given by [30–32]

0.790 ± 0.017 and $\lambda = 0.22650 \pm 0.00048$ [19]. We get $(f_1, g_1) = (0.59, 0.53)$ and $(f_3, g_3) = (-0.02, -0.03)$ from the lattice QCD calculation [33]. Using Eqs. (3) and (4) and $\mathcal{B}(\Lambda_b \rightarrow \Lambda_c^+ D_s^-) = (1.10 \pm 0.10)\%$ [19], we extract $a_1 = 0.93 \pm 0.04$, where a_1 of $\mathcal{O}(1.0)$ demonstrates the feasibility of the generalized factorization [20–22]. From $B(\Sigma_c \rightarrow \Lambda_c \pi) \simeq 100\%$ [19], we determine $g_c = 19.1$ GeV. The $SU(3)_f$ symmetry has been enabled to relate $X_{0,1}^{0(-)} \rightarrow D_s^- \pi^{+(-)}$ and $X_{0,1}^0 \rightarrow D^- K^+$, such that we obtain $g_0 = (2.85 \pm 0.32)$ GeV and $g_1 = 4.63 \pm 0.25$ from $\Gamma_{X_{0,1}^0}$ in Eq. (1); besides, we take $m_{X'_{0,1}} \simeq m_{X_{0,1}^0}$.

The integration of the triangle loop in $\Lambda_b \rightarrow \Sigma_c X'_0$ gives

$$\begin{aligned}(C_0, C_1, C_2) &= (0.37 - 0.32i, -1.50 - 2.30i, \\ &\quad -0.91 - 0.55i) \text{ GeV}^{-2}. \quad (11)\end{aligned}$$

On the other hand, $\Lambda_b \rightarrow \Sigma_c X'_1$ encounters the logarithmic divergence, for which the cutoff Λ needs to be introduced. Since Λ of $\mathcal{O}(1.0)$ GeV has been commonly used as a phenomenological parameter [34–36], we take $\Lambda = (1.00, 1.25, 1.50)$ GeV for the demonstration, which results in

$$\begin{aligned}\tilde{C}_0 &= (0.26 + 0.17i, 0.36 + 0.10i, 0.41 + 0.05i) \text{ GeV}^{-2}, \quad \tilde{C}_1 = (0.95 - 2.15i, 0.35 - 2.75i, -0.11 - 2.94i) \text{ GeV}^{-2}, \\ \tilde{C}_2 &= (0.06 - 0.24i, 0.08 - 0.35i, 0.05 - 0.45i) \text{ GeV}^{-2}, \quad \tilde{C}_{00} = (-1.02 - 0.51i, -1.62 - 0.45i, -2.17 - 0.25i), \\ \tilde{C}_{11} &= (-0.52 + 1.85i, -0.01 + 2.19i, 0.33 + 2.23i) \text{ GeV}^{-2}, \quad \tilde{C}_{12} = (-0.02 + 0.19i, 0.00 + 0.26i, 0.03 + 0.31i) \text{ GeV}^{-2}, \\ \tilde{C}_{22} &= (0.01 + 0.06i, 0.02 + 0.08i, 0.02 + 0.11i) \text{ GeV}^{-2}. \quad (12)\end{aligned}$$

As a consequence, we obtain

$$\begin{aligned}\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0,1}^{0(-)}) &= (2.3 \pm 0.6) \times 10^{-4}, \\ \mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_1^{0(-)}) &= (4.3 \pm 0.8_{-2.5}^{+3.3}) \times 10^{-4}, \quad (13)\end{aligned}$$

where $\mathcal{B}_{0,1}$ as large as 10^{-4} are not caused by the triangle singularity [14]. Indeed, the sizeable weak and strong

coupling constants play the key role. However, the calculations are at most as accurate as the order of magnitude estimations due to the large uncertainties in Eq. (13). Explicitly, the first uncertainties combine the errors from $V_{cb(cs)}$, f_{D_s} , $g_{0,1}$, a_1 , and the second ones from Λ , where we simply take $\Lambda = (1.25 \pm 0.25)$ GeV for a naive estimate of $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c X'_1)$, in accordance with $\tilde{C}_i$ and $\tilde{C}_{ij}$ in Eq. (12).

In the heavy-baryon chiral perturbation theory [37,38], the amplitude of $\Sigma_c \rightarrow \Lambda_c^+ \pi$ in Eq. (5) has another form: $\mathcal{M}'_c(\Sigma_c \rightarrow \Lambda_c^+ \pi) = g'_c \bar{u}_{\Lambda_c} (\gamma_\mu \gamma_5 p_\pi^\mu) u_{\Sigma_c}$, where $g'_c = 4.0$. Subsequently, p_π^μ added to the triangle loop causes the logarithmic (linear) divergence for $\Lambda_b \rightarrow \Sigma_c^{0(++)} X_{0(1)}^{00(-)}$, which corresponds to the integration with $C^{\mu\nu}$ ($C^{\mu\nu\rho}$). Note that $C^{\mu\nu\rho}$ has an extra q_3^ρ compared to $C^{\mu\nu}$ in Eq. (8), whose detailed form can be found in [30–32]. We hence obtain different results, given by

$$\begin{aligned} \mathcal{B}'_0(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_0^{00(-)}) &= (3.1 \pm 0.9_{-0.9}^{+0.5}) \times 10^{-4}, \\ \mathcal{B}'_1(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_1^{00(-)}) &= (4.5 \pm 0.8_{-2.5}^{+3.1}) \times 10^{-3}, \end{aligned} \quad (14)$$

where $\mathcal{B}'_0(\Lambda_b \rightarrow \Sigma_c X_0)$ is slightly deviated from $\mathcal{B}_0(\Lambda_b \rightarrow \Sigma_c X_0)$; however, due to the cutoff, it is more uncertain. On the other hand, $\mathcal{B}'_1(\Lambda_b \rightarrow \Sigma_c X_1)$ is 10 times larger than $\mathcal{B}_1(\Lambda_b \rightarrow \Sigma_c X_1)$, presenting a sensitivity to p_π^μ from the linear divergence. To distinguish between the two different strong couplings, $\mathcal{B}_0/\mathcal{B}_1 \simeq 0.5$ and $\mathcal{B}'_0/\mathcal{B}'_1 \simeq 0.07$ can be used for the future experimental examination.

The exotic $X_{0,1}^0$ are observed in $B^+ \rightarrow D^+ D^- K^+$. Likewise, we expect $X_{0,1}^{00(-)}$ to be observed in $\Lambda_b \rightarrow \Sigma_c^{0(++)} M_c M$, which receives the resonant contributions from $\Lambda_b \rightarrow \Sigma_c^{0(++)} X_0^{00(-)}, X_0^{00(-)} \rightarrow M_c M$ with $M_c M = D_s^- \pi^+, \bar{D}^0 \bar{K}^0$ ($D_s^- \pi^-, D^- K^-$). Approximately,

we present the resonant branching fractions as

$$\begin{aligned} \mathcal{B}(\Lambda_b \rightarrow \Sigma_c X_{0,1}', X_{0,1}' \rightarrow M_c M) &\simeq \mathcal{B}(\Lambda_b \rightarrow \Sigma_c X_{0,1}') \\ \mathcal{B}(X_{0,1}' \rightarrow M_c M), \end{aligned} \quad (15)$$

with $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c X_{0,1}')$ in Eq. (13). In the $SU(3)_f$ symmetry, $\mathcal{B}(X_{0,1}' \rightarrow M_c M)$ are given by [10]

$$\begin{aligned} \mathcal{B}(X_{0,1}^{00(-)} \rightarrow D_s^- \pi^\pm) &= (51, 53)\%, \\ \mathcal{B}(X_{0,1}^{00(-)} \rightarrow \bar{D}^0 \bar{K}^0 (D^- K^-)) &= (49, 47)\%. \end{aligned} \quad (16)$$

We can hence estimate $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{++} X_0^{--} X_{0,1}' \rightarrow D^- K^-)$ as large as 10^{-4} , which seems to be very accessible to the LHCb experiment. However, there might exist other resonant decays to interfere with $\Lambda_b \rightarrow \Sigma_c^{++} X_0^{--}, X_{0,1}' \rightarrow D^- K^-$, which would cause a complicated amplitude analysis. For example, the amplitude analysis of $B^+ \rightarrow D^+ X_{0,1}^0, X_{0,1}^0 \rightarrow D^- K^+$ is complicated, which is due to the resonant decays $B^+ \rightarrow M_{c\bar{c}} K^+, M_{c\bar{c}} \rightarrow D^+ D^-$ with $M_{c\bar{c}} = \psi(3770), \chi_{c0(c2)}(3770)$, and $\psi(4040, 4160, 4415)$ to interfere with $B^+ \rightarrow D^+ X_{0,1}^0, X_{0,1}^0 \rightarrow D^- K^+$ [1,2].

Apart from $\Lambda_b \rightarrow \Lambda_c^+ D_s^- \rightarrow \Sigma_c X_{0,1}'$ with π exchange, other b -baryon decays can also produce the $X_{0,1}$ -like states of $Y_1^{*-}(\bar{c}s\bar{n}n), Y_1^{*-}(\bar{c}n\bar{s}n)$, and $\bar{X}_{0,1}^{00}(c\bar{s}\bar{d}\bar{u})$ in Eq. (2). Taking the quark level $b \rightarrow c\bar{c}s, c\bar{c}d$, and $c\bar{u}d$ weak decays as the examples, the possible rescattering decays are given by

$$\begin{aligned} b &\rightarrow c\bar{c}s: \\ \Lambda_b &\rightarrow \Lambda_c^+ D_s^- \rightarrow \Sigma_c^+ Y_1^{*-}, \Xi_b^{0(-)} \rightarrow \Xi_c^{+(0)} D_s^- \rightarrow \Xi_c^{+(0)}(2645) Y_1^{*-} \text{ (with } \pi^0 \text{ exchange),} \\ \Xi_b^{0(-)} &\rightarrow \Xi_c^+ D_s^- \rightarrow \Xi_c^0(2645) X_{0,1}^{00}, \Xi_b^- \rightarrow \Xi_c^0 D_s^- \rightarrow \Xi_c^+(2645) X_{0,1}' \text{ (with } \pi^\pm \text{ exchange),} \\ b &\rightarrow c\bar{c}d: \\ \Lambda_b &\rightarrow \Lambda_c^+ D^- \rightarrow \Xi_c^{00} X_{0,1}^0 \text{ (with } K^\pm \text{ exchange),} \\ b &\rightarrow c\bar{u}d: \\ \Lambda_b &\rightarrow \Lambda_c^+ \pi^- \rightarrow \Lambda^{(*)} \bar{X}_{0,1}^{00}, \Lambda_b \rightarrow \Lambda_c^+ \pi^- \rightarrow \Sigma^{(*)0} \bar{X}_{0,1}^{00}, \\ \Xi_b^{0(-)} &\rightarrow \Xi_c^{+(0)} \pi^- \rightarrow \Xi^{(*)0(-)} \bar{X}_{0,1}^{00} \text{ (with } D_s^\pm \text{ exchange),} \end{aligned} \quad (17)$$

where $\mathbf{B}^*$ stands for the higher-wave baryon state, and $\Xi_c(2645)$ is able to decay into $\Xi_c \pi$. These decays are worthy of the future explorations; particularly, we estimate that $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^+ Y_1^{*-}) \sim 10^{-4}$, $\mathcal{B}(\Xi_b^{0(-)} \rightarrow \Xi_c^{+(0)}(2645) X_0^{00(-)}) \sim 10^{-5}$, and $\mathcal{B}(\Lambda_b \rightarrow \Xi_c^0 X_{0,1}^0) \sim 10^{-5}$, which suggest interesting measurements. As the final remark, we emphasize that our estimations rely on the observed weak decays in the triangle loop, such as the color-allowed processes $\Lambda_b \rightarrow \Lambda_c^+ D_s^-, \Xi_b^{0(-)} \rightarrow \Xi_c^{+(0)} D_s^-$, and $\Lambda_b \rightarrow \Lambda_c^+ D^-(\pi^-)$. However, there should be other triangle rescatterings to be discovered in the future work. For example, those proceed through the similar color-allowed

Λ_b decay processes but with $\Lambda_c^+ D_s^-$ or $\Lambda_c^+ D^-(\pi^-)$ replaced by other particles of the same quark contents or those through the color-suppressed ones, which are not necessarily suppressed in the final state interaction. Moreover, when the open-charm exotic particles are interpreted as the bound states [8], it allows for the rescatterings with the exchanges of the vector mesons, which also deserve future investigation.

IV. CONCLUSIONS

In summary, we have proposed the rescattering decays $\Lambda_b \rightarrow \Lambda_c^+ D_s^- \rightarrow \Sigma_c X_{0,1}'$ with the π exchange, in order to

provide the different decays to confirm the $X_{0,1}$ -like exotic states observed in $B^+ \rightarrow D^+ D^- K^+$. As the order of magnitude estimates, we have calculated that $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{(++)} X_0^{0(--)}) = (2.3 \pm 0.6) \times 10^{-4}$ and $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^{0(++)} X_1^{0(--)}) = (4.3 \pm 0.8_{-2.5}^{+3.3}) \times 10^{-4}$. Other possible b -baryon decays with the $X_{0,1}$ -like exotic states have also been estimated, such as $\mathcal{B}(\Lambda_b \rightarrow \Sigma_c^+ Y_1'^-) \sim 10^{-4}$, $\mathcal{B}(\Xi_b^{0(-)} \rightarrow \Xi_c^{0(+)}(2645) X_0^{0(--)}) \sim 10^{-5}$, and $\mathcal{B}(\Lambda_b \rightarrow \Xi_c' 0 X_{0,1}^0) \sim 10^{-5}$. To measure $X_{0,1}^{0(--) \rightarrow M_c M}$ in the $M_c M$ invariant mass spectrum, where $M_c M$ can be $D_s^- \pi^+$, $\bar{D}^0 \bar{K}^0$ ($D_s^- \pi^-$, $D^- K^-$), we have introduced the three-body decay

channel $\Lambda_b \rightarrow \Sigma_c^{0(++)} M_c M$. The branching fraction estimated at the level of 10^{-4} can be accessible to the near future measurements.

ACKNOWLEDGMENTS

Y. Y. would like to thank Dr. Xu-Chang Zheng for useful discussions. Y. K. H. was supported in part by NSFC (Grant No. 11675030). Y. Y. was supported in part by NSFC (Grant No. 11905023) and CQCSTC (Grants No. cstc2020jcyj-msxmX0555, No. cstc2020jcyj-msxmX0810).

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- [1] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **125**, 242001 (2020).
 - [2] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **102**, 112003 (2020).
 - [3] V. M. Abazov *et al.* (D0 Collaboration), *Phys. Rev. Lett.* **117**, 022003 (2016).
 - [4] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **117**, 152003 (2016); **118**, 109904(A) (2017).
 - [5] A. M. Sirunyan *et al.* (CMS Collaboration), *Phys. Rev. Lett.* **120**, 202005 (2018).
 - [6] T. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **120**, 202006 (2018).
 - [7] M. Aaboud *et al.* (ATLAS Collaboration), *Phys. Rev. Lett.* **120**, 202007 (2018).
 - [8] R. Molina, T. Branz, and E. Oset, *Phys. Rev. D* **82**, 014010 (2010).
 - [9] M. Karliner and J. L. Rosner, *Phys. Rev. D* **102**, 094016 (2020).
 - [10] X. G. He, W. Wang, and R. Zhu, *Eur. Phys. J. C* **80**, 1026 (2020).
 - [11] T. J. Burns and E. S. Swanson, *Phys. Rev. D* **103**, 014004 (2021).
 - [12] T. J. Burns and E. S. Swanson, *Phys. Lett. B* **813**, 136057 (2021).
 - [13] Y. K. Chen, J. J. Han, Q. F. Lu, J. P. Wang, and F. S. Yu, *Eur. Phys. J. C* **81**, 71 (2021).
 - [14] X. H. Liu, M. J. Yan, H. W. Ke, G. Li, and J. J. Xie, *Eur. Phys. J. C* **80**, 1178 (2020).
 - [15] R. Aaij *et al.* (LHCb Collaboration), *Sci. Bull.* **66**, 1278 (2021).
 - [16] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **115**, 072001 (2015).
 - [17] Q. Wu, D. Y. Chen, and R. Ji, *Chin. Phys. Lett.* **38**, 071301 (2021).
 - [18] X. H. Liu, Q. Wang, and Q. Zhao, *Phys. Lett. B* **757**, 231 (2016).
 - [19] P. A. Zyla *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
 - [20] Y. K. Hsiao and C. Q. Geng, *Eur. Phys. J. C* **77**, 714 (2017).
 - [21] Y. K. Hsiao and C. Q. Geng, *Phys. Lett. B* **782**, 728 (2018).
 - [22] A. K. Leibovich, Z. Ligeti, I. W. Stewart, and M. B. Wise, *Phys. Lett. B* **586**, 337 (2004).
 - [23] T. Feldmann and M. W. Y. Yip, *Phys. Rev. D* **85**, 014035 (2012); **86**, 079901(E) (2012).
 - [24] Y. K. Hsiao, L. Yang, C. C. Lih, and S. Y. Tsai, *Eur. Phys. J. C* **80**, 1066 (2020).
 - [25] Y. K. Hsiao and C. Q. Geng, *Phys. Rev. D* **91**, 116007 (2015).
 - [26] C. Albertus, E. Hernandez, J. Nieves, and J. M. Verde-Velasco, *Phys. Rev. D* **72**, 094022 (2005).
 - [27] K. U. Can, G. Erkol, M. Oka, and T. T. Takahashi, *Phys. Lett. B* **768**, 309 (2017).
 - [28] Y. Yu and Y. K. Hsiao, arXiv:2012.14575.
 - [29] Y. K. Hsiao, Y. Yu, and B. C. Ke, *Eur. Phys. J. C* **80**, 895 (2020).
 - [30] G. 't Hooft and M. J. G. Veltman, *Nucl. Phys.* **B153**, 365 (1979).
 - [31] T. Hahn and M. Perez-Victoria, *Comput. Phys. Commun.* **118**, 153 (1999).
 - [32] A. Denner and S. Dittmaier, *Nucl. Phys.* **B734**, 62 (2006).
 - [33] W. Detmold, C. Lehner, and S. Meinel, *Phys. Rev. D* **92**, 034503 (2015).
 - [34] N. A. Tornqvist, *Z. Phys. C* **61**, 525 (1994).
 - [35] X. Q. Li, D. V. Bugg, and B. S. Zou, *Phys. Rev. D* **55**, 1421 (1997).
 - [36] Q. Wu, D. Y. Chen, X. J. Fan, and G. Li, *Eur. Phys. J. C* **79**, 265 (2019).
 - [37] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, *Phys. Rev. D* **46**, 1148 (1992); **55**, 5851(E) (1997).
 - [38] H. Y. Cheng and C. K. Chua, *Phys. Rev. D* **92**, 074014 (2015).