Complete solution to the strong CP problem: Supersymmetric extension of the Nelson-Barr model

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(Received 1 March 2020; accepted 22 April 2021; published 2 June 2021)

We present a supersymmetric solution to the strong CP problem based on spontaneous CP violation which simultaneously addresses the effects coming from supersymmetry breaking. The generated CP violating phase is communicated to the quark sector by interacting with a heavy quark a la Nelson-Barr. The Majorana mass of the right handed neutrinos is generated by interactions with the CP violating sector and so does not conserve CP . This gives the neutrino sector a nontrivial CP violating phase which can then generate the baryon asymmetry of the universe through leptogenesis. The problematic phase in the supersymmetry breaking parameters are suppressed by appealing to a particular gauge mediation model which naturally suppresses the phases of the tree-level gluino mass and Higgs bilinear masses. This suppression plus the fact that in gauge mediation all loop generated flavor and CP violation is of the minimal flavor violation variety allows for a complete and consistent solution to the strong CP problem.

DOI: [10.1103/PhysRevD.103.L111701](https://doi.org/10.1103/PhysRevD.103.L111701)

I. INTRODUCTION

Despite the fact that supersymmetry has evaded all observations, it remains a compelling model of nature which ameliorates the naturalness problem, provides a WIMP dark matter candidate, and leads to gauge coupling unification. However, the soft masses problematically contain multiple sources of flavor and CP -violation. Although flavor violation decouples from the standard model (SM) if the SUSY breaking scale is taken to be large, this is not necessarily the case for CP violating effects. For example, $\bar{\theta}$ of QCD gets a contribution from the tree-level gluino mass,

$$\bar{\theta} = \theta + \text{Arg}\{\text{Det}(M_u M_d)\} - 3\text{Arg}(M_{\tilde{g}}). \quad (1)$$

This places rather strong constraints on the phase of the gluino mass, since $\bar{\theta} \lesssim 10^{-10}$.¹ Even if this difficulty of the tree-level gluino mass is ignored, radiative corrections to

$M_u, M_d, M_{\tilde{g}}$ involving A -terms and sfermion mass matrices must not reintroduce large phases into any of these parameters. In fact, radiative corrections to $M_u, M_d, M_{\tilde{g}}$ restrict the phases of the squark mass matrices to be smaller than $\mathcal{O}(10^{-8})$. Thus, the strong CP problem places strong constraints on diverse aspects of supersymmetry breaking and its mediation to the visible sector. This is the essence of the supersymmetric strong CP -problem.

One elegant solution to the strong CP problem, and its supersymmetric extension, is to assume the existence of some global $U(1)$ symmetry [1,2]. Under this $U(1)$ symmetry, some set of fields charged under $SU(3)$ color transform chirally rendering $\bar{\theta}$ unphysical. However, it is believed that quantum gravity does not respect global symmetries and this clever solution is sullied by Planck suppressed operators unless these operators are somehow forbidden up to dimension 10. Although this can be engineered by appealing to discrete gauge symmetries, these models tend to be complicated with little motivation beyond removing these problematic Planck suppressed operators which break the $U(1)$ symmetry.

A more appealing approach to the strong CP problem is to assume that CP is an exact symmetry which is spontaneously broken. Although this may seem like another problematic global symmetry, it has been shown that the CP symmetry could be a gauge symmetry in extra dimensional models [3–5]. Importantly, since gravity respects

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¹This constraint comes from the measurement of the neutron EDM.

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gauge symmetries, we do not need to worry about the quality of the remaining CP symmetry. Since CP is spontaneously broken in these types of models, the phase of the CKM matrix must be generated. A mechanism for generating the phase in the CKM matrix, when CP is broken spontaneously, was first found by Nelson-Barr [6–8]. Their mechanism also applies in supersymmetry and will be discussed below in more details.

In this work, we will present a complete model which uses spontaneous CP violation to solve the strong CP problem. This model will also solve the CP problem associated with supersymmetry breaking and address how to generate the baryon asymmetry of the universe. The paper is organized as follows: we overview the Nelson-Barr mechanism in the framework of supersymmetry, present the potential problems of their merging, and then show a complete and consistent model that resolves these problems.

II. SUPERSYMMETRIC NB MODELS

In this section, we will present the basics of the supersymmetric Nelson-Barr mechanism. The simplest version of this mechanism requires two fields to spontaneously break CP , $\eta_{1,2}$, with different phases and a pair of down-type chiral multiplets D' , $\bar{D}'$ which mix with the SM d-quarks. In order to forbid unwanted interactions, the $\eta_{1,2}$, D' , $\bar{D}'$ have charge -1 under some Z_2 . The superpotential of the Nelson-Barr sector takes the form,

$$W = Y_{\alpha,i} \eta_{\alpha} D' \bar{d}_i + M_{D'} D' \bar{D}' + y_{ij} H_d Q_i \bar{d}_j. \quad (2)$$

After CP is broken, i.e., $\langle \eta_i \rangle \neq 0$, the mass matrix of down-type quarks become

$$\mathcal{M} = \begin{pmatrix} m_d & B \\ 0 & M_{D'} \end{pmatrix}, \quad m_d \equiv y v_d, \quad B_i = Y_{\alpha,i} \langle \eta_{\alpha} \rangle. \quad (3)$$

Since the only source of CP violation at tree-level is B_i , $\text{Arg}[\det \mathcal{M}] = 0$ and $\bar{\theta} = 0$ is maintained. A large CKM phase is then generated when the D and $\bar{D}$ are integrated out, as long as $M_D \lesssim B_i$. The superpotential is protected from other problematic radiative corrections due to the effects of the nonrenormalization theorem.

Some complications arise from Planck suppressed operators, which are not forbidden by the global symmetries. The operators $\frac{1}{M_P} \eta_{\alpha} \eta_{\beta} D' \bar{D}'$ and $\frac{1}{M_P} \eta_{\alpha} H_d Q_i \bar{D}'$ in the superpotential, for example, would introduce complex contributions to m_d and M_D respectively. If either of these masses has a large complex component, it would lead to $\text{Arg}[\det \mathcal{M}] \neq 0$ and reintroduce the strong CP problem. Because these operators are suppressed by the Planck scale, they can be sufficiently suppressed if $\eta_{\alpha} \lesssim 10^8$ GeV [9]. This will have implications on how the baryon asymmetry can be generated as we will see below.

We now look at the problems of supersymmetry breaking as well as the radiative corrections induced after supersymmetry is broken. There are two relevant sources of supersymmetry breaking in the visible sector, the gravitino mass and the F-term of some field. If CP is broken above the scale of supersymmetry breaking, both of these will in general be complex. In gravity mediation, the F-term usually provides the dominate contribution to the gluino mass and so must be real. However, even the anomaly mediated piece proportional to $m_{3/2}$ is problematically large. Since this piece is only loop suppressed, if $m_{3/2}$ has a large phase, it will lead to $\bar{\theta} \sim 10^{-2}$. Thus, gravity mediated models require both $m_{3/2}$ and the F-term be real to a very high level of precision.

Models based on pure anomaly mediation do better than gravity mediated models when it comes to the gluino mass, since the F-term does not contribute to the gluino masses. However, since the gluino mass is proportional to the gravitino mass, the gravitino mass must still be real. Although a real gravitino may be a manageable feat, the tachyonic slepton problem must be solved.² The most obvious and natural solution to this problem is to allow the SUSY breaking F-term to interact with the MSSM fields in the Kähler potential via Planck suppressed operators [11]. In this case, the sleptons are no longer tachyonic and have masses that are much larger than the gluino masses. Generically, the F-term breaking SUSY is complex leading to large phases in the sfermion mass matrices. One might hope that since the sfermions are much heavier than the gluino, these phases would decouple and have little effect on $\bar{\theta}$. As we will see below, in Nelson-Barr models, decoupling the sfermions does not remove the effects of these phases.

The problematic contribution to $\bar{\theta}$ in gravity mediated models come from the loop corrections to the gluino mass and down quark mass matrix. The dangerous supersymmetry breaking terms are

$$V_{\text{soft}} \supset \bar{d}_i^{\dagger} m_{d_{ij}}^2 \bar{d}_j + Q_i^T m_{Q_{ij}}^2 Q_j^* + m_D^2 |D|^2 + m_{\bar{D}}^2 |\bar{D}|^2 + A_{d_{ij}} H_d Q_i \bar{d}_j + B_q \mu q \bar{q} + A_{Y_{\alpha j}} D \eta_{\alpha} \bar{d}_j. \quad (4)$$

Even if we assume a real tree level gluino mass, the right diagram found in Fig. 1 leads to a dangerous one loop correction to gluino mass which gives the constraint

$$\frac{\alpha_s}{4\pi} \frac{\text{Im}(B_i^{\dagger} A_{Y_{\alpha i}} \langle \eta_{\alpha} \rangle)}{(|B_j|^2 + M_{D'}^2) \text{Re}(m_{\tilde{g}})} \lesssim 10^{-10}, \quad (5)$$

where $m_{\tilde{g}}$ is the gluino mass. This constraint requires that the A-terms be universal or adhere to some kind of minimal

²This problem may be solved by allowing the supersymmetry breaking field to interact with the Higgs superfields in the Kähler [10].

FIG. 1. Loop diagrams contribution to $\bar{\theta}$.

flavor violation. In models based on gravity mediated supersymmetry breaking, this will not be the case when all Planck suppressed operators are included in the Kähler. Thus, even if the gluino mass is real at tree level, further conditions are needed to solve the strong CP problem. For heavy soft masses, the constraint in Eq. (5) on $\bar{\theta}$ is unaffected since this constraints is independent of the MSSM soft mass scale.

The other problematic radiative corrections come from loop corrections to the quark mass matrices. The Feynman diagram for these contributions are shown in the left of Fig. 1. These corrections to the quark mass matrix are even more dangerous since they only depends on the diagonal part of the A-terms. This diagram gives a constraint

$$\frac{\alpha_s A_d \text{Im}(B^\dagger m_d \delta m_Q^2 m_d^{-1} (\delta m_d^2 - \delta m_B^2) B)}{4\pi m_{\text{SUSY}}^5 (|B_j|^2 + M_{D'}^2)} \lesssim 10^{-11}, \quad (6)$$

where A_d is the diagonal part of the A-terms and δm_d^2 , δm_B^2 are the variation of these masses from universal, and m_{SUSY} is the scale of the sfermion masses. Again, this constraint is not made better by decoupling the soft masses. Thus, even models like pure gravity mediation have problematic radiative corrections to $\bar{\theta}$.

Gauge mediation models fare better [12,13]. In gauge mediation models, the SUSY breaking scale is much lower, scaling with the mediation scale. This reduction in the SUSY breaking scale also suppresses the gravitino mass since they are related through the cosmological constant. This helps the strong CP problem in three ways. First, the reduction in the SUSY breaking scale neutralize the effects of the Planck suppressed operators, since their contribution to the soft masses will be suppressed by a factor of $(M_*/M_P)^{(n-4)}$ where n is an integer corresponding to the dimensionality of the operator. Second, the reduction in the gravitino mass makes the anomaly mediated contribution to the gauginos benign. Third, since the A-terms are loop suppressed relative to the other soft masses and are proportional to the MSSM Yukawa couplings, the radiative corrections to $\bar{\theta}$ are also suppressed. The outstanding problems to be solved in gauge mediation are generating a real tree-level contribution to the gluino mass and solving the $B\mu$ problem with $B\mu$ being real. As we will discuss below, these can be done for a particular model of gauge mediation [14].

Although the dominant source of the soft masses is flavor universal in gauge mediation, the NB (Nelson-Barr) sector breaks this universality. Another contribution to the soft masses comes from inserting a loop of $\bar{d}_i$ and D' into the typical two-loop gauge mediated soft mass diagram using the interactions controlled by $Y_{i,\alpha}$.³ This generates a flavor nonuniversal contribution to the squark masses proportional to $Y_{\alpha,i} Y_{\alpha,j}^\dagger$. For $Y \sim 1$, this leads to the constraint $B_i \sim M_{D'} \gtrsim 10^2 M_*$, where M_* is the messenger scale.

Even though gauge mediation solves many of the problems associated with the supersymmetric strong CP problem, the model is far from complete. For example, the baryon asymmetry must be generated and requires an additional source of CP violation beyond that in the quark Yukawa couplings. The simplest way to generate the baryon asymmetry of the universe (BAU) is through leptogenesis [15]. This requires adding heavy right-handed neutrinos, N_i^c ($i = 1, 2, 3$), that have CP violating interactions. These heavy right handed neutrinos can also explain the smallness of the neutrino masses via the seesaw mechanism [16–20]. Since NB theories require CP be violated spontaneously, CP violation must be communicated to the neutrino sector as well. Because the right-handed neutrinos are singlets, it may seem trivial to generate CP violation in the neutrino sector by directly coupling the CP breaking fields, η_α , to the right handed-neutrinos through the interaction $Y_{\alpha}^{ij} \eta_\alpha N_i^c N_j^c$. However, this operator is problematic, since it violates the important Z_2 symmetry protecting the NB theory from other operators spoiling the mechanism. Fortunately, as we will see below, other discrete symmetries can be used to forbid the problematic operators from the NB theory and still allow for this CP violating interaction in the lepton sector. Thus, allowing CP violation in the neutrino sector.

Another difficulty of the NB theory is that breaking CP tends to generate a domain wall problem. Since the superpotential preserves CP , the potential is invariant under $\langle \eta_\alpha \rangle \rightarrow \langle \eta_\alpha \rangle^*$. The domain wall problem can be avoided if the reheating temperature is low enough that CP is not restored after inflation. Due to the nonrenormalizable operators $\langle \eta_\alpha \rangle \lesssim 10^8$ GeV and so $T_{rh} < 10^8$ GeV. However, thermal leptogenesis generally requires $T_{rh} \gtrsim 10^9$ GeV [21]. In the model we discuss below, the problematic Planck suppressed operators in the superpotential are forbidden and the upper bound on $\langle \eta_\alpha \rangle$ is removed, allowing for a viable thermal leptogenesis model.

III. A VIABLE SUPERSYMMETRIC NELSON-BARR MODEL

In the last section we highlighted the difficulties of the SUSY NB mechanism. In this section, we will present a

³For more discussion, see Appendix A.

TABLE I. Charge assignments for all the fields.

	Matter fields									R-breaking sector		SUSY breaking sectors		
Fields	$\bar{\mathbf{5}}$	$\mathbf{10}$	N^c	H_u	H_d	$5'$	$\bar{5}'$	$\eta_{1,2}$	$Y_{1,2}$	S	S'	S''	$\Psi_{1,2}/\bar{\Psi}_{1,2}$	
Z_4	i	i	i	$-$	$-$	i	$-i$	$-$	$+$	$+$	$+$	$+$	$+$	
Z_{4R}	0	0	0	2	2	0	0	2	2	2	2	2	0	
Z'_3					0					0	1	0	1	
Z''_3	1	2	0	2	0	2	1	0	0	0	0	1	0	

model that can alleviate all of these problems and show that the NB mechanism can be consistent with a complete supersymmetric model of nature.

First of all, let us describe the NB sector of the model. The matter content and the charge assignments are listed in Table I.⁴ The superpotential for the Nelson-Barr theory consistent with these symmetries is given by

$$W = y_{d_{ij}} H_d \bar{\mathbf{5}}_i \mathbf{10}_j + y_{u_{ij}} H_u \mathbf{10}_i \mathbf{10}_j + y_{\nu_{ij}} H_u N_i^c \bar{\mathbf{5}}_j + \lambda_D S \bar{5}' \bar{5}' + Y_{\alpha j} \eta_\alpha \bar{5}' \bar{\mathbf{5}}_j + y_{N_{ij}} \eta_\alpha N_i^c N_j^c, \quad (7)$$

where $10 \ni \{Q, \bar{U}, \bar{E}\}$, $\bar{5} \ni \{\bar{D}, L\}$, N_i are the right-handed neutrinos, $5', \bar{5}'$ contains the NB quark superfield, S is a singlet under the SM gauge symmetries and is necessary to break the Z_{4R} allowing for a mass for $5', \bar{5}'$, and η_α is the field responsible for breaking CP .⁵ Our superpotential has been significantly modified from that in Eq. (2). The most obvious difference is we have coupled the CP breaking field to the right handed neutrinos. This is possible because we have replaced the Z_2 symmetry with an anomaly free Z_4 . The Z_{4R} symmetry is used to forbid the problematic operators $\frac{1}{M_P} \eta_\alpha \eta_\beta D' \bar{D}'$ and $\frac{1}{M_P} \eta_\alpha H_d Q_i \bar{D}'$. With these operators forbidden, the scale of CP breaking can be pushed up and in turn the reheat temperature.⁶ This higher reheat temperature then salvages leptogenesis. To make the Z_{4R} symmetry shown in Table I anomaly free requires us to extend the NB quark superfields to a $5'$ and $\bar{5}'$ [22]. This inclusion of a vector pair of lepton doublets, L' and $\bar{L}'$, leads to additional CP violation in the lepton sector at the NB mechanism. Although interesting for explaining CP violation in the neutrino sector, we leave this to future work.

One challenge coming from the Z_{4R} symmetry is that it forbids a supersymmetric mass term for both the $H_u H_d$ and $5' \bar{5}'$. The $H_u H_d$ mass term we will discuss later, since CP violation in this mass term can be problematic. The $5', \bar{5}'$ mass term problem was overcome in Eq. (7) by the

introduction of a new singlet field, S , which was charged under the R symmetry. This symmetry is broken supersymmetrically by the superpotential

$$W_S = \mu^2 S - \frac{1}{3} \lambda S^3. \quad (8)$$

Because $\langle W \rangle \neq 0$ at the minimum of the potential, these interactions will generate a gravitino mass and play a role in cancelling the cosmological constant.⁷

Problematically, the interaction $S \eta_\alpha \eta_\beta$ is still allowed by all the symmetries in Table I. If present, this operator would result in mixing between S and η_α giving a phase to M_D . However, this operator can be suppressed by adding a Z_3 symmetry under which S has charge 1, $\bar{5}'$ has charge 2, and all other fields are neutral. The $\mu^2 S$ term in Eq. (8), which is charged under this Z_3 , we assume is generated at the Planck scale by Planck scale dynamics. Since the theory preserves CP at the Planck scale, μ and λ are real. The parameter μ acts as a soft breaking of the Z_3 symmetry. In this case, all interactions must preserve the Z_3 with μ transforming as a 1 under the Z_3 . In this case, the only allowed operator leading to mass mixing between S and η_α is $\mu^2 S \eta_\alpha \eta_\beta / M_P^2$. As long as $\mu \lesssim 10^{-13}$, this operator will not reintroduce the strong CP problem.⁸

Next, we discuss the spontaneous breaking of the CP symmetry. The superpotential for this breaking is

$$W_\eta = \lambda_1 Y_1 (\eta_1^2 + M_1^2) + \lambda_2 Y_2 (\eta_2^2 - M_2^2), \quad (9)$$

where Y_i are singlets introduced to assist in breaking the CP symmetry. We have assumed that some of the couplings between Y_i and η_α are small to show that η_1 and η_2 can indeed have different phases. However, it is not difficult to find a potential with more general couplings that gives different phases for η_1 and η_2 .

As we discussed above, the soft masses must be quite diagonal and real to avoid generating a large $\bar{\theta}$. These problems are solved somewhat generically for any gauge mediation model. The outstanding problems are to generate a real gluino mass and solve the $B\mu$ problem without

⁴For a table explaining the operators forbidden by each symmetry, see Appendix B.

⁵We use $SU(5)$ notation for presentational simplicity. We do not consider a full $SU(5)$ theory.

⁶The vev of S , which breaks the Z_{4R} can be as large as 10^{12} GeV. As long as the reheat temperature is less than this there will be no domain wall problem. Since the vev of S and η_i are similar in size, the same reasoning holds for the Z_4 symmetry.

⁷Since this superpotential has two degrees of freedom, μ and λ , one can be used to cancel the cosmological constant.

⁸For additional discussion concerning this point, see Appendix C.

generating $\bar{\theta}$. Fortunately, these problems have already been solved in the Evans-Sudano-Yanagida (ESY) model [14] of gauge mediation. In this model, a singlet field, S' , charged under an additional symmetry Z'_3 is coupled to two pairs of messengers $\Psi_{1,2}, \bar{\Psi}_{1,2}$, which are effectively a $5, \bar{5}$. The allowed interactions are then

$$W_{S'} = -\frac{\lambda'}{3} S'^3 + \kappa S' \Psi_i \bar{\Psi}_i, \quad (10)$$

where λ' and κ are taken to be real and positive by a field redefinition of S' . In order for this mechanism to work, S' must also have a tachyonic soft masses⁹

$$V_{\text{soft}} \supset -m_S^2 |S'|^2, \quad (11)$$

where we have written the soft mass so that m_S^2 is positive. The Kähler also has a term of the form

$$\Delta K = -\frac{h}{\Lambda'^2} |S'|^4, \quad (12)$$

where h is a dimensionless coupling and is assumed to be positive, and Λ' is some scale lower than the Planck scale. The resulting potential, to leading order in SUSY breaking, is

$$V \supset -m_S^2 |S'|^2 + |\lambda' S'|^2 + 4\lambda' m_{3/2} \frac{h}{\Lambda'^2} |S'|^5 \cos 3\delta_{S'}, \quad (13)$$

where $\delta_{S'}$ is the phase of S' . As is clear from this potential, there are three minimum $\delta_{S'} = 0, \frac{2}{3}\pi, \frac{4}{3}\pi$. Since the gaugino masses for this model are

$$m_i = -\frac{\alpha_i}{4\pi} \lambda' |S'| e^{-i3\delta_{S'}}, \quad (14)$$

they will be real for all three minima. However, these three degenerate minima, due to the Z_3 , have introduced a domain wall problem. Fortunately, this Z_3 degeneracy is lifted by Planck suppressed operators leaving $\delta_{S'} = 0$ as the true vacuum of the theory.¹⁰

The Higgs bilinear masses are generated in this theory [14] by the addition of a field S'' , which couples to $H_u H_d$. S'' has a SUSY scale vev and F-term which are generated from dynamics which are similar to those generating the F-term and vev of S' . For the Z'_3 charge assignments listed in Table I, the vacuum structure of this theory is such that the phases of B and $m_{\tilde{g}}$ are correlated and so both are real. This mechanism solves the B, μ problem while suppressing any phases which would have reintroduce the strong CP problem.

⁹For a way to dynamically generate this tachyonic mass, see the appendix of [14].

¹⁰The vacuum degeneracy is broken by $\Delta W = \kappa S'^5 / M_P^2$. For appropriate signs of κ, λ' , and h the true minimum is $\delta_{S'} = 0$. Furthermore, this term is sufficient to ensure that the domain walls never dominate the energy of the universe [23].

IV. CONCLUSIONS

In this letter we have examined the Nelson-Barr solution to the strong CP problem in the context of supersymmetry. By generalizing the global symmetries of past supersymmetric Nelson-Barr models and including gauged R-symmetries, the problematic operators, which either tightly constrain or forbade past models, can be alleviated. Furthermore, this generalization allows for a Nelson-Barr like mechanism to generate a CP violating phase in the lepton sector which could explain the baryon asymmetry of the universe. It also requires the right-handed neutrino masses be generated by the field that spontaneously breaks CP . This not only gives another source of CP violation to assist in leptogenesis, but it also correlates these scales in a unique way. To complete our model, we have shown that if nature relies on a particular form of gauge mediation to generate the MSSM soft masses, no additional flavor or CP violation is generated in the low-scale. Thus, this is a consistent model of SUSY which can explain the baryon asymmetry of the universe, the strong CP problem, and has a completely consistent LHC spectrum.

ACKNOWLEDGMENTS

C. C. H, N. Y., and T. T. Y. would like to thanks Peter Cox for useful discussion during the early stages of this work. J. L. E and T. T. Y. would like to thank the people of IPMU for their hospitality, which facilitated the completion of this work. This work is supported by Grants-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science, and Technology (MEXT), Japan, No. 26104001 (T. T. Y.), No. 26104009 (T. T. Y.), No. 16H02176 (T. T. Y.), No. 17H02878 (T. T. Y.), No. 15H05889(N. Y.), No. 15K21733(N. Y.), and No. 17H02875 (N. Y.), and by the World Premier International Research Center Initiative (WPI), MEXT, Japan (T. T. Y.). T. T. Y. is a Hamamatsu Professor.

APPENDIX A: SOFT MASSES OF GAUGE MEDIATION

Although gauge mediation tends to do much better than other models of SUSY breaking, it still has a supersymmetric strong CP problem. Generically, the F-term of supersymmetry breaking will be complex. The leading order contribution to the gluino mass in gauge mediation is

$$m_{\tilde{g}} \simeq \frac{g_3^2}{16\pi^2} \frac{F}{M} n(\psi) \quad (A1)$$

where

$$W_M = (M + F\theta^2)\psi\bar{\psi} \quad (A2)$$

and $n(\psi)$ is the Dynkin index of $\psi + \bar{\psi}$ [24,25]. If F is complex, then we get

$$\bar{\theta} = \text{Arg}(m_{\tilde{g}}) = \text{Arg}(F), \quad (\text{A3})$$

which should be generically order one. This problem is solved by the Evans-Sudano-Yanagida (ESY) model [14] discussed in the main text.

The other soft masses of gauge mediated models of supersymmetry breaking are real. As mentioned in the draft, however, when combined with the Nelson-Barr mechanism, there are dangerous corrections that could lead to a large $\bar{\theta}$. In gauge mediation with the messenger sector in Eq. (A2), the soft masses are generated at two-loops from a diagram like that found on the left of Fig. 1 with a correction from the Nelson-Barr sector giving the contribution on the left. Comparing these contributions, we find

$$\frac{\delta \tilde{m}_{ij}^2}{\tilde{m}_{ij}^2} \sim \left(\frac{M}{M_{CP}} \right)^2 \frac{1}{4\pi^2} \lesssim 10^{-6} \quad (\text{A4})$$

This then leads to the constraint we sight in the draft.

APPENDIX B: THE DISCRETE SYMMETRIES

Since we have many different discrete symmetries which forbid various operators, Table II list our discrete symmetries and the operators they forbid. The Z_4 forbids direct coupling of the Nelson-Barr quarks to CP violation or the MSSM fields. Z_{4R} forbids problematic Planck suppressed operator. This allows us to push up the CP breaking scale beyond 10^8 GeV. The Z_3 's purpose is discussed in the previous section. Z'_3 and Z''_3 are introduced to suppress operators that would ruin the dynamics that generate a real $m_{\tilde{g}}$ and μB .

TABLE II. Discrete symmetries and the operators they suppress.

Symmetry	Operators Forbidden
Z_4	$\eta_\alpha \tilde{S}' \tilde{S}', H_d Q \tilde{D}'$
Z_{4R}	$\frac{\eta_\alpha \eta_\beta}{M_P} D' \tilde{D}', \frac{\eta_\alpha}{M_P} H_d Q \tilde{D}'$
Z_3	$S \eta_\alpha \eta_\beta$
Z'_3	$S', S'^2, S' \mathcal{O}$
Z''_3	S'', S''^2

APPENDIX C: GENERATING M_D WITHOUT CP VIOLATION

As discussed in the draft, our initial model for generating M_D is unable to forbid the problematic operator

$$W \supset \lambda_{\alpha\beta} S \eta_\alpha \eta_\beta \quad (\text{C1})$$

When η_α gets a vev, this will lead to a mixing mass term of the form

$$W \supset 2\lambda_{\alpha\beta} \langle \eta_\alpha \rangle S \eta_\beta \quad (\text{C2})$$

This operator was forbid by an additional discrete symmetry Z_3 under which S has charge 1. The μ has Z_3 charge 1 and so softly breaks this symmetry. This means that the operator

$$W \supset \frac{\mu^2}{M_P^2} S \eta_\alpha \eta_\beta \quad (\text{C3})$$

is still allowed. For $m_{3/2}$ of order MeV, $\mu \sim 10^{11}$ GeV which means this contribution is adequately suppressed.

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