$P_c(4312)^+$, $P_c(4380)^+$, and $P_c(4457)^+$ as double triangle cuspsSatoshi X. Nakamura^{*}*University of Science and Technology of China, Hefei 230026, People's Republic of China
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Understanding the nature of the hidden charm pentaquark(like) signals in the LHCb data for $\Lambda_b^0 \rightarrow J/\psi p K^-$ is a central problem of hadron spectroscopy. We propose a scenario completely different from previous ones such as hadron molecules and compact pentaquarks. We identify relevant double triangle mechanisms with leading or lower-order singularities. The associated anomalous threshold cusps at the $\Sigma_c^{(*)} \bar{D}^{(*)}$ thresholds are significantly more singular than the ordinary ones. Then we demonstrate that the double triangle amplitudes reproduce the peak structures of $P_c(4312)^+$, $P_c(4380)^+$, and $P_c(4457)^+$ in the LHCb data, through an interference with other common mechanisms. Only the $P_c(4440)^+$ peak is due to a resonance with a width and strength significantly smaller than previously estimated. P_c^+ signals are expected in other processes, and the proposed model (partly) explains the current data, such as the GlueX J/ψ photoproduction data with no P_c^+ signals and the LHCb $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ data with a possible signal only from $P_c(4440)^+$. In general, the double triangle singularity is now a possible option to interpret resonancelike structures near thresholds.

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The hadron spectrum is a reflection of nonperturbative aspects of QCD. Exotic hadrons have structures different from the conventional quark-antiquark and three-quark structures [1], and are expected to provide a clue to assess a different aspect of QCD. Recent experimental discoveries of exotic hadron(like) signals are therefore exciting [2–9].

The LHCb Collaboration recently observed in $\Lambda_b^0 \rightarrow J/\psi p K^-$ three resonance(like) structures [10], interpreted as pentaquark states called $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$, by updating their previous analysis [11]. Understanding the nature of the P_c^+ 's is certainly a central problem of the hadron spectroscopy. Because of the proximity of the P_c^+ masses to the $\Sigma_c(2455)\bar{D}^{(*)}$ thresholds,¹ P_c^+ 's as $\Sigma_c(2455)\bar{D}^{(*)}$ molecules (bound states) may seem a natural interpretation of their identity [13–36].

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¹We follow the hadron naming scheme of [12]. For simplicity, however, we often denote $\Sigma_c(2455)^{++}$, $\Sigma_c(2520)^{++}$, $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, and J/ψ by Σ_c , Σ_c^* , Λ_c^* , Λ_c^{**} , and ψ , respectively. $\Lambda_c^{(*)}$ and $\Sigma_c^{(*)}$ are also collectively denoted by Y_c . Charge indices are often suppressed.

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Yet, constituent pentaquark pictures [37–44] and a hadro-charmonium [45] are also possible options. The LHCb data of the $J/\psi p$ invariant mass ($M_{J/\psi p}$) distribution have been analyzed with a K -matrix model that claimed a virtual state for $P_c(4312)^+$ [46]. Another analysis [34–36] based on one-pion-exchange plus contact interactions for the coupled $\Sigma_c^{(*)} \bar{D}^{(*)}$ system interpreted all the P_c^+ 's as $\Sigma_c^{(*)} \bar{D}^{(*)}$ bound states; they also claimed the existence of a narrow $P_c(4380)^+$.² So far, all the models assigned a pole to each of the P_c^+ peaks.³

In order to establish P_c^+ 's as hadronic states, it is important to confirm their signals in other processes such as J/ψ photoproduction off a nucleon [48–54]. The GlueX Collaboration conducted such an experiment, finding no evidence [55]. This may indicate that the P_c^+ states couple weakly with a photon and could be seen in higher statistics data. Another possibility is that the P_c^+ peaks in $\Lambda_b^0 \rightarrow J/\psi p K^-$ are caused by kinematical effects such as threshold cusp and triangle singularity (TS) [56], and do not appear in the photoproduction; however, no relevant mechanism has been found. TS [57,58] proposed for the

²In this paper, $P_c(4380)^+$ does not refer to a broad state [11].

³Although a K -matrix analysis [47] claimed $P_c(4457)^+$ as a threshold cusp, we put the conclusion on hold for insufficient quality of fitting the $P_c(4312)^+$ peak and $\Sigma_c(2520)\bar{D}$ threshold region.

 FIG. 1. $\Lambda_b^0 \rightarrow J/\psi p K^-$ mechanisms: (a) double triangle, (b) one loop, (c) $P_c(4440)^+$ excitation, (d) direct decay.

previous P_c^+ [11] no longer fit the updated P_c^+ signals unless a pole is included [10].

In this work, we point out that a double triangle (DT) diagram [Fig. 1(a)] creates a kinematical effect that has not been explored for interpreting resonancelike structures. The effect is caused by the fact that each loop hits a TS and that the two TS can even occur almost simultaneously (leading singularity [59]). The associated anomalous threshold cusps are significantly more singular than the ordinary ones and, thus, can be a new option to understand exotic hadronlike signals near thresholds. We then demonstrate that the DT amplitudes reproduce the peak structures of $P_c(4312)^+$, $P_c(4380)^+$, and $P_c(4457)^+$ in the LHCb data, through an interference with other common mechanisms [Figs. 1(b) and 1(d)]. The LHCb data require this proposed model to have only $P_c(4440)^+$ as a resonance with width and strength significantly smaller than previously estimated. The model can also (partly) explain P_c^+ signals in other data such as no P_c^+ signals in the J/ψ photo-production, and $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ data [60] suggesting only a $P_c(4440)^+$ signal. In this way, we bring a completely new understanding of the P_c^+ structures in the LHCb data.

II. MODEL

Our model for $\Lambda_b^0 \rightarrow J/\psi p K^-$ is diagrammatically represented in Fig. 1. For loop diagrams of Figs. 1(a) and 1(b), we assume that color-favored $\Lambda_b^0 \rightarrow \Lambda_c^{(*)} \bar{D}^{(*)} K^-$ decays dominate over color-suppressed ones. We do not include color-suppressed $\Lambda_b^0 \rightarrow \Sigma_c^{(*)} \bar{D}^{(*)} K^-$ vertices that previous models often used with a possible problem of explaining P_c^+ production rates [26]. We could include color-suppressed processes since their suppression is generally difficult to predict [35] and the DT amplitudes might be more suppressed. We still take this assumption because the DT amplitudes are actually not significantly suppressed and, for fitting only the $M_{J/\psi p}$ distribution data, color-suppressed contributions are redundant and would not significantly improve the fit. Regarding the parity, we consider only parity-conserving mechanisms; parity-violating mechanisms should exist but are redundant in the fits. Partial waves of $J^P = 1/2^-, 3/2^-, 1/2^+, \text{ and } 3/2^+$ are considered; J^P denotes the spin parity of $J/\psi p$.

We present amplitude formulas for representative diagrams in Fig. 1; see the Supplemental Material [61] for complete formulas and parameter values. We use the particle mass and width values from [12], and denote the energy, width, momentum, and polarization vector of a particle x by E_x , Γ_x , $\mathbf{p}_x$, and $\boldsymbol{\epsilon}_x$, respectively. We also denote a baryon (B)-meson (M) pair by $BM(J^P)$. A DT diagram [Fig. 1(a)] that includes $\Sigma_c \bar{D}(1/2^-)$ consists of four vertices such as $\Lambda_b^0 \rightarrow \Lambda_c^+ \bar{D} \bar{K}^*$, $\bar{K}^* \rightarrow \bar{K} \pi$, $\Lambda_c^+ \pi \rightarrow \Sigma_c$, and $\Sigma_c \bar{D}(1/2^-) \rightarrow J/\psi p$ given by

$$c_{\Lambda_c \bar{D} \bar{K}^* \Lambda_b} \left(\frac{1}{2} t_{\bar{D}} \frac{1}{2} t_{\bar{K}^*} |00 \right) \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_{\bar{K}^*} F_{\bar{K}^* \bar{D} \Lambda_c \Lambda_b}^{00}, \quad (1)$$

$$c_{\bar{K} \pi \bar{K}^*} \left(1 t_{\pi} \frac{1}{2} t_{\bar{K}} \left| \frac{1}{2} t_{\bar{K}^*} \right. \right) \boldsymbol{\epsilon}_{\bar{K}^*} \cdot (\mathbf{p}_{\bar{K}} - \mathbf{p}_{\pi}) f_{\bar{K} \pi \bar{K}^*}^1, \quad (2)$$

$$c_{\Lambda_c \pi \Sigma_c} \boldsymbol{\sigma} \cdot \mathbf{p}_{\pi} f_{\Lambda_c \pi \Sigma_c}^1, \quad (3)$$

$$c_{\psi p \Sigma_c \bar{D}}^{1/2-} \left(1 t_{\Sigma_c} \frac{1}{2} t_{\bar{D}} \left| \frac{1}{2} t_p \right. \right) \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_{\psi} f_{\psi p \Sigma_c \bar{D}}^0 f_{\Sigma_c \bar{D}}^0, \quad (4)$$

respectively. Dipole form factors with a cutoff Λ are denoted by $F_{ijk,l}^{LL'}$, f_{ij}^L , and $f_{ij,k}^L$. The parentheses are isospin Clebsch-Gordan coefficients (CGCs) with t_x being the isospin z component of x . In Eq. (1) and later in Eq. (6), the isospin of the $\bar{D}^{(*)} \bar{K}^{(*)}$ pair is 0 for assuming the color-favored Λ_b^0 decays. The couplings $c_{\bar{K} \pi \bar{K}^*}$ and $c_{\Lambda_c \pi \Sigma_c}$ are determined by the K^* and $\Sigma_c(2455)$ decay widths. The DT amplitude is

$$\begin{aligned} A_{\Sigma_c \bar{D}(1/2^-)}^{\text{DT}} &= \sum_{\text{charge}} c_{\psi p \Sigma_c \bar{D}}^{1/2-} c_{\Lambda_c \pi \Sigma_c} c_{\bar{K} \pi \bar{K}^*} c_{\Lambda_c \bar{D} \bar{K}^* \Lambda_b} \\ &\times (\text{isospin CGCs}) \int d^3 p_{\pi} \int d^3 p_{\bar{D}} \\ &\times \frac{\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}_{\psi} \boldsymbol{\sigma} \cdot \mathbf{p}_{\pi}}{E - E_{\bar{K}} - E_{\Sigma_c} - E_{\bar{D}} + \frac{i}{2} \Gamma_{\Sigma_c}} \\ &\times \frac{\boldsymbol{\sigma} \cdot (\mathbf{p}_{\bar{K}} - \mathbf{p}_{\pi})}{E - E_{\bar{K}} - E_{\pi} - E_{\Lambda_c} - E_{\bar{D}} + i\epsilon} \\ &\times \frac{f_{\psi p \Sigma_c \bar{D}}^0 f_{\Sigma_c \bar{D}}^0 f_{\Lambda_c \pi \Sigma_c}^1 f_{\bar{K} \pi \bar{K}^*}^1 F_{\bar{K}^* \bar{D} \Lambda_c \Lambda_b}^{00}}{E - E_{\bar{K}^*} - E_{\Lambda_c} - E_{\bar{D}} + \frac{i}{2} \Gamma_{\bar{K}^*}}, \end{aligned} \quad (5)$$

where different charge states in the loops are summed with charge dependent particle masses; E is the total energy. The amplitude is implicitly sandwiched by initial Λ_b^0 and final p spin states.

The $\Lambda_c^+ \bar{D}^{*0}(1/2^-)$ one-loop amplitude [Fig. 1(b)] is composed by an initial $\Lambda_b^0 \rightarrow \Lambda_c^+ \bar{D}^{*0} K^-$ vertex,

$$c_{\Lambda_c \bar{D}^* \bar{K} \Lambda_b} \left(\frac{1}{2} t_{\bar{D}^*} \frac{1}{2} t_{\bar{K}} |00 \right) \sigma \cdot \epsilon_{\bar{D}^*} F_{\bar{K} \bar{D}^* \Lambda_c \Lambda_b}^{00}, \quad (6)$$

and the subsequent $\Lambda_c^+ \bar{D}^{*0}(1/2^-) \rightarrow J/\psi p$ interaction,

$$c_{\psi p \Lambda_c \bar{D}^*}^{1/2^-} \sigma \cdot \epsilon_{\psi} \sigma \cdot \epsilon_{\bar{D}^*} f_{\psi p \Lambda_c \bar{D}^*}^0. \quad (7)$$

The one-loop amplitude is

$$A_{\Lambda_c \bar{D}^*}^{1L} = 3c_{\psi p \Lambda_c \bar{D}^*}^{1/2^-} c_{\Lambda_c \bar{D}^* \bar{K} \Lambda_b} \left(\frac{1}{2} t_{\bar{D}^*} \frac{1}{2} t_{\bar{K}} |00 \right) \sigma \cdot \epsilon_{\psi} \times \int d^3 p_{\bar{D}^*} \frac{f_{\psi p \Lambda_c \bar{D}^*}^0 F_{\bar{K} \bar{D}^* \Lambda_c \Lambda_b}^{00}}{E - E_{\bar{K}} - E_{\Lambda_c} - E_{\bar{D}^*} + \frac{i}{2} \Gamma_{\bar{D}^*}}, \quad (8)$$

with $\Gamma_{\bar{D}^{*0}} = 55$ keV [63].

The $P_c(4440)^+$ amplitude [Fig. 1(c)] is given in the Breit-Wigner form with an adjustable $P_c(4440)^+$ mass, width, and coupling. For J^P of $P_c(4440)^+$, we examine $1/2^\pm$ and $3/2^\pm$ cases. The internal structure of $P_c(4440)^+$ is beyond the scope of this work. We also consider a direct decay mechanism [Fig. 1(d)] in each partial wave with a real coupling strength. This simulates all mechanisms not belonging to the diagrams of Figs. 1(a)–1(c).

The $Y_c \bar{D}^{(*)}$ pairs in the DT and one-loop diagrams require a nonperturbative treatment. Meanwhile, the lattice QCD [64] found a weak $J/\psi p \rightarrow J/\psi p$ interaction. Thus, a reasonable approach is to develop a $Y_c \bar{D}^{(*)}$ coupled-channel model incorporating the heavy quark spin symmetry [28,34–36]. However, we take a simpler approach and use a single-channel contact interaction model [65] to calculate a $Y_c \bar{D}^{(*)}$ scattering amplitude that maintains only the elastic unitarity. Then, a perturbative transition to $J/\psi p$ follows. The resulting $Y_c \bar{D}^{(*)} \rightarrow J/\psi p$ amplitude is implemented in Eqs. (5) and (8); see the Supplemental Material [61]. Other possible coupled-channel effects are assumed to be absorbed by complex couplings fitted to the data. This simplification can be justified for the limited purpose of proposing a new scenario for the P_c^+ structures in the LHCb data. Within our model, the P_c^+ structures [except for $P_c(4440)^+$] are mostly described by the kinematical effects and not directly by poles from the $Y_c \bar{D}^{(*)}$ scattering. Therefore, the data can only loosely constrain our $Y_c \bar{D}^{(*)}$ interactions. Conversely, the details of the $Y_c \bar{D}^{(*)}$ scattering do not play a major role. This is in contrast with hadron-molecule models for which the $Y_c \bar{D}^{(*)}$ interactions

must be fine-tuned to get poles at right positions; the details do matter.

III. RESULTS

A. Double triangle amplitudes and their singular behaviors

The DT amplitudes cause the leading and lower-order singularities (or anomalous thresholds) [59] when Σ_c and Σ_c^* propagates in Fig. 1(a), respectively. This is analogous to a single triangle diagram causing TS as its leading singularity. According to the Coleman-Norton theorem [66], the leading singularity occurs when a given loop process is kinematically allowed at the classical level. The lower-order singularity occurs when one (or more) of the intermediate states is necessarily off shell. Details on the kinematical conditions are given in the Supplemental Material [61]. As a TS-enhanced amplitude shows a logarithmically singular behavior, the DT leading and lower-order singularities generate their own singular behaviors. While the singular behaviors can, in principle, be examined by analyzing the corresponding Landau equations [59,67], we study them numerically below.

We plot in Figs. 2(a) and 2(b), the DT amplitude that includes $\Sigma_c^+ \bar{D}^0(1/2^-)$ [red solid curves]. A singular

FIG. 2. Double triangle amplitudes. The panel (a) [(b)] shows the real [imaginary] part. The red solid curves are for $\Sigma_c^{(*)} \bar{D}^{(*)} = \Sigma_c^+ \bar{D}^0$ in Fig. 1(a) [$\Lambda = 1$ GeV; perturbative $\Sigma_c^{(*)} \bar{D}^{(*)} \rightarrow J/\psi p$]. They reduce to the blue dashed ones by using $m_{\Lambda_c^+} = 3$ GeV. The green dotted curves are the $\Sigma_c^+ \bar{D}^0$ one-loop amplitude. All the amplitudes are normalized so that the real parts have the same peak height. The dotted vertical lines indicate the $\Sigma_c^{(*)} \bar{D}^{(*)}$ thresholds. The amplitudes shown in the panel (c) [(d)] are obtained from those in (a) [(b)] by replacing Σ_c^+ with Σ_c^{*+} .

behavior clearly shows up near the $\Sigma_c^+ \bar{D}^0$ threshold. We also plot a threshold cusp, due to the square-root singularity, from a $\Sigma_c^+ \bar{D}^0$ one-loop amplitude similar to Fig. 1(b) [green dotted curves]; the imaginary part does not vanish below the threshold for $\Gamma_{\Sigma_c^+} \neq 0$. The DT cusp due to the leading singularity is clearly more singular. For an illustration, we replace the Λ_c^+ mass in the DT amplitude with a hypothetically heavy value (3 GeV) so that the loop integral can hit only the $\Sigma_c^+ \bar{D}^0$ threshold singularity. The resultant amplitude [blue dashed curves] behaves like an ordinary threshold cusp. The amplitudes shown in Figs. 2(a) and 2(b) include $\Sigma_c^+ \bar{D}^0 \rightarrow J/\psi p$ treated as the first-order perturbation. A nonperturbative attractive $\Sigma_c^+ \bar{D}^0$ interaction significantly enhances the singular behaviors [68]. Similar plots in Figs. 2(c) and 2(d) are obtained by replacing Σ_c^+ in the amplitudes shown in Figs. 2(a) and 2(b) with Σ_c^{*+} . While this DT amplitude with $\Sigma_c^{*+} \bar{D}^0$ has the lower-order singularity and is more singular than the ordinary threshold cusp, it is less singular than the DT amplitude with $\Sigma_c^+ \bar{D}^0$ having the leading singularity.

To clarify that the DT amplitudes and ordinary threshold cusps have different singular behaviors, we divide the former by the latter and show the ratios in Fig. 3. The singular behaviors still remain in the real part of the ratios. The first [second] derivative of the ratio in Figs. 3(a) [3(b)] seems divergent at the $\Sigma_c^+ \bar{D}^0$ [$\Sigma_c^{*+} \bar{D}^0$] threshold, showing a qualitative difference between the leading and lower-order singularities. The results in Figs. 2–3 indicate that, in general, the DT cusp might be an option to understand resonancelike structures near thresholds.

As expected from Fig. 2, the DT amplitude alone creates a peak at the $\Sigma_c^{(*)} \bar{D}^{(*)}$ threshold in the $M_{J/\psi p}$ distribution of $\Lambda_b^0 \rightarrow J/\psi p K^-$. This peak cannot be identified with a P_c^+ peak, which is located slightly below the threshold. However, suppose there exists a smooth amplitude that interferes with the real (imaginary) part of the DT amplitude

FIG. 3. Ratio of double triangle (A_{DT}) to one-loop (A_{1L}) amplitudes. The ratio in the panel (a) [(b)] is from the amplitudes including $\Sigma_c^+ \bar{D}^0$ [$\Sigma_c^{*+} \bar{D}^0$] intermediate states. The upper red (lower blue) curves show $\text{Re}[A_{DT}/A_{1L}]$ ($\text{Im}[A_{DT}/A_{1L}]$). The dotted vertical lines in the panel (a) [(b)] indicate the $\Sigma_c^+ \bar{D}^0$ [$\Sigma_c^{*+} \bar{D}^0$] threshold.

of Fig. 2 destructively (constructively) near the threshold. This coherent sum generates a peak slightly below the $\Sigma_c^{(*)} \bar{D}^{(*)}$ threshold. This is how the P_c^+ -like structures [other than $P_c(4440)^+$] show up from the DT mechanisms.

B. Analysis of the LHCb data

For describing $\Lambda_b^0 \rightarrow J/\psi p K^-$, our full model includes (i) DT mechanisms with $\Sigma_c(2455)\bar{D}(1/2^-)$, $\Sigma_c(2520)\bar{D}(3/2^-)$, $\Sigma_c(2455)\bar{D}^*(1/2^-)$, $\Sigma_c(2455)\bar{D}^*(3/2^-)$, $\Sigma_c(2520)\bar{D}^*(1/2^-)$, and $\Sigma_c(2520)\bar{D}^*(3/2^-)$; (ii) one-loop mechanisms with $\Lambda_c^+ \bar{D}^{*0}(1/2^-)$, $\Lambda_c(2595)^+ \bar{D}^0(1/2^+)$, and $\Lambda_c(2625)^+ \bar{D}^0(3/2^+)$; (iii) $P_c(4440)^+$ mechanism; and (iv) direct decay mechanisms. Regarding the number of fitting parameters, each mechanism in the items (i)–(iii) has an adjustable complex overall factor to fit the data, 2×10 parameters. Each direct decay mechanism (iv) has a real coupling strength, four parameters. The $P_c(4440)^+$ mass and width are adjustable. We also adjust a repulsive $\Lambda_c^+ \bar{D}^{*0}$ interaction strength. Because the overall absolute normalization of the full amplitude is arbitrary, we have in total 26 parameters. Although the heavy quark spin symmetry may constrain the parameters, we adjust them rather freely in the fits. This can be justified because the fitting parameters can effectively absorb effects from mechanisms not explicitly considered such as parity-violating amplitudes; more discussions in the Supplemental Material [61].

Regarding the elastic $Y_c \bar{D}^{(*)}$ interaction strengths, we examine if the fit favors an attractive or repulsive interaction for a given BM (J^P). The fit favors attractions for $\Sigma_c(2455)\bar{D}(1/2^-)$, $\Sigma_c(2520)\bar{D}(3/2^-)$, $\Sigma_c(2455)\bar{D}^*(1/2^-)$, $\Sigma_c(2455)\bar{D}^*(3/2^-)$, $\Lambda_c(2595)^+ \bar{D}^0(1/2^+)$, $\Lambda_c(2625)^+ \bar{D}^0(3/2^+)$, and repulsions for $\Sigma_c(2520)\bar{D}^*(1/2^-)$, $\Sigma_c(2520)\bar{D}^*(3/2^-)$, $\Lambda_c^+ \bar{D}^{*0}(1/2^-)$. Then we fix the coupling strength so that the scattering length is $a \sim 0.5 \text{ fm}^4$ for the attraction. The repulsive $\Lambda_c^+ \bar{D}^{*0}(1/2^-)$ interaction strength is fitted to the data because the fit quality is rather sensitive; $a \sim -0.4, -0.2$, and -0.05 fm for $\Lambda \sim 0.8, 1$, and $1.5\text{--}2 \text{ GeV}$, respectively. The other repulsive interactions have the same strength as $\Lambda_c^+ \bar{D}^{*0}$. It is noted that, within our model, spectrum peak positions are not very sensitive to the a values. The cutoff in the form factors is fixed at $\Lambda = 1 \text{ GeV}$ unless otherwise stated. Only the direct decay amplitudes include different cutoffs on $p_{\bar{K}}$ so that their $M_{J/\psi p}$ distribution is similar to the phase-space shape.

We compare our calculation with the LHCb data [10] in Fig. 4(a). The experimental resolution is considered in the calculation. Our full model (red solid curve) well fits the data. The $P_c(4312)^+$, $P_c(4380)^+$, and $P_c(4457)^+$ peaks are well described by the kinematical effects from the

⁴The scattering length (a) is related to the phase shift (δ) by $p \cot \delta = 1/a + \mathcal{O}(p^2)$.

FIG. 4. $J/\psi p$ invariant mass ($M_{J/\psi p}$) distribution for $\Lambda_b^0 \rightarrow J/\psi p K^-$. (a) Comparison with the LHCb data ($\cos \theta_{p_c}$ -weighted samples) [10]. The red solid (blue dashed) curve is from the full (simplified) model; see the text for details on the models. The dotted vertical lines indicate thresholds for, from left to right, $\Lambda_c^+ \bar{D}^{*0}$, $\Sigma_c(2455)^{++} D^-$, $\Sigma_c(2520)^{++} D^-$, $\Lambda_c(2595)^+ \bar{D}^0$, $\Sigma_c(2455)^{++} D^{*-}$, $\Lambda_c(2625)^+ \bar{D}^0$, and $\Sigma_c(2520)^{++} D^{*-}$, respectively. (b) Partial wave contributions specified by J^P of the $J/\psi p$ pair. The magenta dashed, blue dotted, green dash-dotted, and brown dash-two-dotted curves are for $J^P = 1/2^-, 3/2^-, 1/2^+, \text{ and } 3/2^+$, respectively. The solid violet peak is the $P_c(4440)^+(3/2^-)$ contribution without interference. The thin black dotted curve is the sum of the direct decay mechanisms. The red solid curve is from the full model. In the inset, the green solid, orange dotted, and cyan dashed curves are contributions from the double triangle mechanism with $\Sigma_c \bar{D}(1/2^-)$, that with $\Sigma_c \bar{D}^*(1/2^-)$, and the $\Lambda_c^+ \bar{D}^{*0}(1/2^-)$ one-loop mechanism, respectively, without the interference.

considered mechanisms and not by poles near the peak positions. Only the $P_c(4440)^+$ peak requires a resonance pole for which we choose $J^P = 3/2^-$ in the figure. The fit quality does not significantly change when varying the cutoff over $\Lambda = 0.8\text{--}2$ GeV and when choosing $J^P = 1/2^\pm$ and $3/2^\pm$ for $P_c(4440)^+$; the fits thus do not favor a particular J^P of $P_c(4440)^+$. We simplify the model by omitting the $1/2^+$ and $3/2^+$ amplitudes, and treating $Y_c \bar{D}^{(*)} \rightarrow J/\psi p$ perturbatively. With 19 parameters to refit, main features of the data are fairly well captured (blue dashed curve). Still, the structures are less sharp near the thresholds because of lacking the strong $Y_c \bar{D}^{(*)}$ scattering. The $1/2^+$ and $3/2^+$ amplitudes are also needed for a more precise fit.

In Fig. 4(b), we show the full model's partial wave contributions that do not interfere with each other in the $M_{J/\psi p}$ distribution. In the $1/2^-$ (magenta dashed curve) and $3/2^-$ (blue dotted curve) contributions, the interference between the DT and direct decay mechanisms creates bumps slightly below the $\Sigma_c^{(*)} \bar{D}^{(*)}$ thresholds where the $P_c(4312)^+$, $P_c(4380)^+$, and $P_c(4457)^+$ peaks are located. The $1/2^-$ contribution also shows a bent due to the $\Lambda_c^+ \bar{D}^{*0}$ threshold cusp, being consistent with the data. The $1/2^+$ (green dash-dotted curve) and $3/2^+$ (brown dash-two-dotted curve) contributions exhibit $\Lambda_c(2595)^+ \bar{D}^0$ and $\Lambda_c(2625)^+ \bar{D}^0$ threshold cusps, respectively, capturing the characteristic structures in the data. Although the

$1/2^+$ peak seems a large contribution, this is due to a constructive interference between the $\Lambda_c(2595)^+ \bar{D}^0$ one-loop and direct decay amplitudes; the $\Lambda_c(2595)^+ \bar{D}^0$ one-loop amplitude itself is significantly smaller than the $\Lambda_c^+ \bar{D}^{*0}$ one-loop amplitude in magnitude. The sum of the direct decay mechanisms shows a phase-space-like distribution (thin black dotted curve). This partial wave decomposition would involve uncertainty due to the limited experimental information.

The $P_c(4440)^+(3/2^-)$ contribution without interference is shown in Fig. 4(b) [violet solid peak]. The mass and width from our fit is 4443.1 ± 1.4 MeV and 2.7 ± 2.4 MeV, respectively; the statistical errors are estimated just for a reference by varying only the $P_c(4440)^+$ mass, width, and coupling. The cutoff dependence is safely within the errors. Comparing with the LHCb analysis [10], $4440.3 \pm 1.3^{+4.1}_{-4.7}$ MeV and $20.6 \pm 4.9^{+8.7}_{-10.1}$ MeV, the width is significantly narrower. Also, the $P_c(4440)^+$ contribution is only $\sim 1/22$ of the LHCb's estimate: $\mathcal{R} \equiv \mathcal{B}(\Lambda_b^0 \rightarrow P_c^+ K^-) \mathcal{B}(P_c^+ \rightarrow J/\psi p) / \mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p K^-) = 1.11 \pm 0.33^{+0.22}_{-0.10} \%$. This difference arises because the LHCb fitted the structure at $M_{J/\psi p} \sim 4450$ MeV with incoherent $P_c(4440)^+$ and $P_c(4457)^+$ contributions, while we describe a large portion of the structure with the kinematical effects and attribute only the small spike to the $P_c(4440)^+$ and its interference.

The quality of the fit could be slightly improved with more attractive $\Sigma_c^{(*)} \bar{D}^{(*)}$ interactions. However, such

interactions generate virtual poles near the thresholds and, as a result, $\gamma p \rightarrow J/\psi p$ cross section would have sharp peaks at the $\Sigma_c^{(*)}\bar{D}^{(*)}$ thresholds. Since the GlueX did not find such peaks [55], we fix the $\Sigma_c^{(*)}\bar{D}^{(*)}$ interactions at the moderate strength so that no poles are generated close to the thresholds.⁵

To examine whether the DT amplitudes are significantly suppressed compared to other mechanisms, we show the DT and one-loop contributions to the $1/2^-$ partial wave in the insert of Fig. 4(b). The peak height of the DT contributions is comparable to that of the one-loop contribution. We can further compare the coupling strength of the DT amplitude [Eq. (5)] to that of the $\Lambda_c^+\bar{D}^{*0}$ one-loop amplitude [Eq. (8)] by introducing a ratio,

$$R = \frac{\left| \frac{c_{\psi p, \Sigma_c \bar{D}}^{1/2-} c_{\Lambda_c \bar{D}^* \bar{K}^*, \Lambda_b}}{c_{\psi p, \Lambda_c \bar{D}^*}^{1/2-} c_{\Lambda_c \bar{D}^* \bar{K}^*, \Lambda_b}} \right|}{}, \quad (9)$$

where well-controlled parameters ($c_{\bar{K}\pi, \bar{K}^*}$, $c_{\Lambda_c \pi, \Sigma_c}$) are excluded. Fitting the LHCb data gives $R = 7.2\text{--}3.2$ for $\Lambda = 0.8\text{--}2$ GeV. Thus, unreasonably large parameter values ($R \gg 1$) are not necessary, indicating that the comparable strengths of the DT and one-loop amplitudes are not an artifact.

Our model partly explains the absence of P_c^+ signals in J/ψ photoproduction data [55]. All P_c^+ peaks, except for $P_c(4440)^+$, in $\Lambda_b^0 \rightarrow J/\psi p K^-$ are caused by the kinematical effect (DT mechanisms and its interference) that requires an accompanying K^- in the final state. The $\gamma p \rightarrow J/\psi p$ process cannot accommodate this kinematical effect and thus, has no P_c^+ signals. For $P_c(4440)^+$, its width and fit fraction from our analysis are significantly smaller than those from the LHCb's. In this case, observing a $P_c(4440)^+$ signal in J/ψ photoproduction would be more challenging than expected based on the LHCb result.

⁵Our $\Sigma_c \bar{D}$ interaction model with $a \sim 0.5$ fm generates a virtual pole at ~ 20 MeV below the threshold.

An evidence for P_c^+ was also found in $\Lambda_b^0 \rightarrow J/\psi p \pi^-$. The $M_{J/\psi p}$ distribution [Fig. 3(b) of [60]] seems that the $M_{J/\psi p}$ bin of $P_c(4440)^+$ is enhanced, whereas this is not the case for the other P_c^+ 's. This observation may conflict with some P_c^+ models but not ours because the $P_c(4440)^+$ [DT] mechanisms in $\Lambda_b^0 \rightarrow J/\psi p K^-$ can [cannot] be shared by $\Lambda_b^0 \rightarrow J/\psi p \pi^-$, and, thus, only $P_c(4440)^+$ appears in the latter. For the limited statistics, P_c^+ signals in $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ are still inconclusive. Higher statistics data can seriously test the models.

IV. SUMMARY AND FUTURE PERSPECTIVE

We analyzed the LHCb data for the $M_{J/\psi p}$ distribution of $\Lambda_b^0 \rightarrow J/\psi p K^-$. We found that the P_c^+ structures are well described by the double triangle cusps and their interference with the common mechanisms. Only $P_c(4440)^+$ is interpreted as a resonance but its width and strength are significantly smaller than the LHCb result. The analysis thus proposes an understanding of the P_c^+ peaks completely different from the previous ones such as hadron molecules and compact pentaquarks.

An interesting next step is to study $\Lambda_b^0 \rightarrow \Sigma_c^{(*)}\bar{D}^{(*)}K^-$. An immediate prediction is TS peaks at $M_{\Sigma_c K^-} \sim 3.40$ GeV and $M_{\Sigma_c^* K^-} \sim 3.21$ GeV due to a single triangle diagram obtained by removing $\Sigma_c^{(*)}\bar{D}^{(*)} \rightarrow J/\psi p$ vertex from Fig. 1(a). To examine the P_c^+ structures, a coupled-channel treatment of the $Y_c \bar{D}^{(*)}$ scattering is necessary. It is also interesting to understand other resonancelike structures near thresholds with DT cusps that should now be in the list of possible interpretations.

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