Excited K meson, $K_c(4180)$, with hidden charm as a $D\bar{D}K$ bound stateTian-Wei Wu,¹ Ming-Zhu Liu,² and Li-Sheng Geng^{1,3,4,5,*}¹*School of Physics, Beihang University, Beijing 102206, China*²*School of Space and Environment, Beihang University, Beijing 102206, China*³*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*⁴*Beijing Advanced Innovation Center for Big Data-Based Precision Medicine, School of Medicine and Engineering, Beihang University, Beijing 100191, China*⁵*School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China*

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Motivated by the recent discovery of two new states in the $B^+ \rightarrow D^+ D^- K^+$ decay by the LHCb Collaboration, we study the $D\bar{D}K$ three-body system by solving the Schrödinger equation with the Gaussian Expansion Method. We show that the $D\bar{D}K$ system can bind with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$ and a binding energy of $B_3(D\bar{D}K) = 48.9^{+1.4}_{-2.4}$ MeV. It can decay into $J/\psi K$ and $D_s \bar{D}^*$ via triangle diagrams, yielding a partial decay width of about 1 MeV. As a result, if discovered, it will serve as a highly nontrivial check on the nature of the many exotic hadrons discovered so far and on nonperturbative QCD as well. Assuming heavy quark spin symmetry, the same formalism is applied to study the $D\bar{D}^*K$ system, which is shown to also bind with quantum numbers $I(J^P) = \frac{1}{2}(1^-)$ and a binding energy of $B_3(D\bar{D}^*K) \simeq 77.3^{+3.1}_{-6.6}$ MeV, consistent with the results of previous works.

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Starting from 2003 [1,2], many exotic hadronic states have been discovered experimentally. Most of them cannot easily fit into the conventional quark model picture, i.e., baryons consisting of qqq and mesons of $q\bar{q}$. As a result, they have attracted a lot of attention and inspired intensive discussions about their true nature. Among the many possible interpretations, the molecule picture has been widely employed to interpret some of these exotic states (for recent comprehensive reviews, see, e.g., Refs. [3–10]). Nonetheless, it is difficult if not impossible to distinguish different scenarios because there are always some free parameters in each model to fit the experimental data. In this paper, we propose to study the existence of a three-body $D\bar{D}K$ bound state to decisively confirm or repute the molecular picture, with the assumption that $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are predominantly DK and D^*K bound states.

Being about 160 and 70 MeV lower than the corresponding $c\bar{s}$ states predicted by the naive quark model,

$D_{s0}^*(2317)$ and $D_{s1}(2460)$ can be naturally interpreted as DK and D^*K bound states [11–15]. If this is the case, in the sense that the deuteron is a bound state of a proton and neutron, a naive but straight forward question to ask is whether DDK (and/or DD^*K) can form three-body bound states. A number of recent studies showed that they indeed bind [16–20]. Lately, the Belle Collaboration has performed the first experimental search for the existence of the DDK bound state and reported an upper limit for its production yield [21].

Leading-order chiral perturbation theory dictates that in the isospin-zero channel the $\bar{D}K$ and $\bar{D}^*K$ interactions are only half those of the DK and D^*K interactions [22]. In addition, both $X(3872)$ and $Z_c(3900)$ can be explained as $D\bar{D}^*$ molecules,¹ which implies that the $D\bar{D}^*$ interaction is attractive as well, but not as strong as the DK and D^*K interactions. A recent lattice QCD study shows that the $D\bar{D}$ interaction is attractive such that a shallow $D\bar{D}$ bound state exists, consistent with the early theoretical results [30] and more recent analysis of the $\gamma\gamma \rightarrow D\bar{D}$ reaction [31]. Motivated by these facts, the existence of a three-body $D\bar{D}^*K$ bound state has been studied. In Ref. [32], the authors studied the $D\bar{D}^*K$ system using the Born-Oppenheimer

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¹More specifically, $X(3872)$ is a shallow bound state of $D\bar{D}^*$ with isospin zero [23–26], but $Z_c(3900)$ is a resonant state of $D\bar{D}^*$ with isospin 1 [27–29].

approximation via a delocalized π bond. A bound state with the quantum numbers of K^* and a mass of $4317.92^{+6.13}_{-6.55}$ MeV was found. In Ref. [33], using the so-called fixed-center approximation in coupled channels, the authors solved the Faddeev equation and found a heavy hidden charm K^* meson with a mass about 4307 ± 2 MeV, consistent with Ref. [32].

Lately, the LHCb Collaboration found two new states in the $B^+ \rightarrow D^+ D^- K^+$ decay [34,35], namely, $X_0(2866)$ and $X_1(2900)$. The former has been interpreted as a $\bar{D}^* K^*$ bound state of spin zero [36–43]. Although a $D\bar{D}K$ bound state cannot be found in the $D^+ D^- K^+$ invariant mass spectrum, the recent experimental discovery indicates that a three-body $D\bar{D}K$ bound state, if it exists, could have been formed already and remains to be discovered at the current facilities. Motivated by these theoretical and experiment works, we study the strange hidden charm $D\bar{D}K$ system using the Gaussian Expansion Method (GEM) [44]. We indeed find a $D\bar{D}K$ bound state as well as a $D\bar{D}^* K$ bound state utilizing heavy quark spin symmetry.

II. THEORETICAL FRAMEWORK

There exist several widely used approaches to solve three-body problems, such as the Faddeev equation [45], the Gaussian expansion method [44], the stochastic variational method [46], and the hyperspherical harmonic expansion method [47]. With the same inputs, the results of all these methods agree very well with each other as shown in the benchmark study [48]. In this work, we utilize the Gaussian expansion method to study the $D\bar{D}^{(*)}K$ three-body systems, which has been widely used to solve three-, four-, and even five-body problems [49] because of its high precision and rapid convergence. Namely, we study the three-body $D\bar{D}^{(*)}K$ systems by solving the following Schrödinger equation

$$\hat{H}\Psi = E\Psi, \quad (1)$$

where the Hamiltonian $\hat{H}$ includes the kinetic term and three two-body interaction terms

$$\hat{H} = T + V_{DK} + V_{\bar{D}^{(*)}K} + V_{D\bar{D}^{(*)}}. \quad (2)$$

To solve the Schrödinger equation, we have to first specify the two-body interactions.

For the DK interaction, we refer to chiral perturbation theory, in which the most important contribution is the leading-order Weinberg-Tomozawa (WT) term [22]

$$V_{DK}(\mathbf{q}) = -\frac{C_W(I)}{2f_\pi^2}, \quad (3)$$

where the pion mass decay constant $f_\pi = 130.7$ MeV and $C_W(I)$ represents the strength of the WT interaction with

$C_W(0) = 2$ for the isospin 0 and $C_W(1) = 0$ for the isospin 1 configurations, respectively. This DK potential can be rewritten in coordinate space by Fourier transformation, and we use the same form of the DK interaction as that adopted in Ref. [18], which explicitly reads

$$V_{DK}(r; R_c) = C(R_c)e^{-(r/R_c)^2}. \quad (4)$$

Here, R_c is a coordinate space cutoff representing the effective interaction range. In this work, we choose R_c ranging from 0.5 to 2.0 fm to study the related uncertainties. The $C(R_c)$ is a running constant related to R_c , which can be determined by reproducing the $D_{s0}^*(2317)$ state.

The $\bar{D}K$ interaction can be related to the DK interaction in chiral perturbation theory, where the leading-order $\bar{D}K$ potential is half of the DK interaction in the isospin-zero channel [22]. Thus, we take the $\bar{D}K$ potential to have the same form as that of Eq. (4) but multiplied with 1/2. Assuming heavy quark spin symmetry, the $\bar{D}^*K$ interaction is the same as the $\bar{D}K$ interaction.

For the $D\bar{D}$ interaction, there are no concrete experimental data, and we have to resort to phenomenological models, e.g., the one boson exchange (OBE) model of Ref. [50]. In Ref. [18], the DD OBE potential has been derived with the exchange of σ , ρ , and ω mesons. According to G parity, the only difference between the $D\bar{D}$ potential and the DD potential in the OBE model is the sign of the ω exchange potential. The explicit form of the DD interaction can be found in Ref. [18]. For the $D\bar{D}^*$ interaction, one can also exchange a π in addition to the σ , ρ , and ω exchanges. We use the $D\bar{D}^*$ OBE potential of Ref. [50], which reproduces the well-known $X(3872)$ state as a molecular state. We choose a cutoff $\Lambda = 1.01$ GeV to reproduce the binding energy 4.0 MeV of $X(3872)$ with respect to the $D\bar{D}^*$ threshold. It should be mentioned that whether or not the $D\bar{D}$ system can form a bound state is still under discussion [30,31,51]. In Ref. [30], the authors found a very narrow heavy scalar with mass around 3700 MeV [$\text{Re}(\sqrt{s}) = 3698 \pm 35$ MeV, $\text{Im}(\sqrt{s}) = -0.10 \pm 0.06$ MeV]. In Ref. [51], the authors found a shallow $D\bar{D}$ bound state with a binding energy $4.0^{+3.7}_{-5.0}$ MeV in lattice QCD. In Ref. [31], the authors claimed that the S -wave $D\bar{D}$ final-state interaction can produce a bound state around 3720 MeV with $I = 0$ by investigating the $\gamma\gamma \rightarrow D\bar{D}$ reaction. In our model, with a cutoff $\Lambda = 1.01$ GeV, the $D\bar{D}$ system cannot form a bound state with the OBE potential but will do so if a larger cutoff is adopted.² In the following, consistent with the $D\bar{D}^*$ case, we choose $\Lambda = 1.01$ GeV for the $D\bar{D}$ OBE potential.

²More concretely, with a cutoff of 1.415 GeV, one can obtain a binding energy of 4 MeV for the $D\bar{D}$ system. The resulting $D\bar{D}K$ three-body bound state will have a binding energy of 20 MeV larger.

FIG. 1. Three permutations of the Jacobi coordinates for the $D\bar{D}^{(*)}K$ system.

As all the two-body interactions have been specified, we use the GEM to solve the Schrödinger equation. The three-body wave functions can be constructed in Jacobi coordinates as

$$\Psi = \sum_{c=1}^3 \Psi(\mathbf{r}_c, \mathbf{R}_c), \quad (5)$$

where $c = 1-3$ is the label of the Jacobi channels shown in Fig. 1. In each Jacobi channel, the wave function $\Psi(\mathbf{r}_c, \mathbf{R}_c)$ reads

$$\Psi(\mathbf{r}_c, \mathbf{R}_c) = C_{c,\alpha} H_{t,T}^c \Phi_{IL,\lambda}(\mathbf{r}_c, \mathbf{R}_c) \quad (6)$$

where $C_{c,\alpha}$ is the expansion coefficient and the $\alpha = \{nN, tT, IL\lambda\}$ labels the basis number with the configuration sets of the Jacobi channels. $H_{t,T}^c$ is the three-body isospin wave function where t is the isospin of the subsystem in Jacobi channel c and T is the total isospin. Considering that the isospin 1 DK interaction is 0 in our model, the isospin t_{DK} of the DK subsystem should be 0, and thus the total isospin T of the $D\bar{D}K(D\bar{D}^*K)$ system is $1/2$.

The three-body spatial wave function $\Phi(\mathbf{r}_c, \mathbf{R}_c)$ is constructed by two two-body wave functions as

$$\begin{aligned} \Phi_{IL,\lambda}(\mathbf{r}_c, \mathbf{R}_c) &= [\phi_{n_l c}^G(\mathbf{r}_c) \psi_{N_{l,c}}^G(\mathbf{R}_c)]_{\lambda}, \\ \phi_{nlm}^G(\mathbf{r}_c) &= N_{nl} r_c^L e^{-\nu_n r_c^2} Y_{lm}(\hat{r}_c), \\ \psi_{NLM}^G(\mathbf{R}_c) &= N_{NL} R_c^L e^{-\lambda_n R_c^2} Y_{LM}(\hat{R}_c). \end{aligned} \quad (7)$$

Here, $N_{nl}(N_{NL})$ is the normalization constant of the Gaussian basis, $n(N)$ is the number of Gaussian basis used, and $l(L)$ is the orbital angular momentum corresponding to the Jacobi coordinates $r(R)$. Since only S -wave

interactions are considered, the total orbital angular momentum is $\lambda = 0$, and thus the quantum numbers $I(J^P)$ of this $D\bar{D}K$ three-body system are $\frac{1}{2}(0^-)$, while those for $D\bar{D}^*K$ are $\frac{1}{2}(1^-)$.

With the constructed wave functions, the Schrödinger equation can be transformed into a generalized matrix eigenvalue problem with the Gaussian basis functions

$$[T_{\alpha\alpha'}^{ab} + V_{\alpha\alpha'}^{ab} - EN_{\alpha\alpha'}^{ab}] C_{b,\alpha'} = 0, \quad (8)$$

where $T_{\alpha\alpha'}^{ab}$ is the matrix element of kinetic energy, $V_{\alpha\alpha'}^{ab}$ is the matrix element of potential energy, and $N_{\alpha\alpha'}^{ab}$ is the normalization matrix element.

III. PREDICTIONS AND DISCUSSIONS

We first study whether the three-body $D\bar{D}K$ and $D\bar{D}^*K$ systems bind in the theoretical framework and with the two-body interactions specified above.

The $D\bar{D}K$ system is found to bind with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$ and a three-body binding energy

$$B_3(D\bar{D}K) \simeq 48.9_{-2.4}^{+1.4} \text{ MeV}. \quad (9)$$

The results are weakly cutoff dependent, and therefore we vary the cutoff R_c from 0.5 to 2.0 fm to estimate the uncertainties originating from the DK and $\bar{D}K$ interactions. More concretely, the central value of the binding energy is obtained with $R_c = 1.0$ fm, while the uncertainties are taken from $R_c = 0.5$ and 2.0 fm in the specific numerical calculations.

Assuming heavy quark spin symmetry, the same formalism is applied to study the $D\bar{D}^*K$ system, where a bound state is found as well, with quantum numbers $I(J^P) = \frac{1}{2}(1^-)$ and a binding energy

TABLE I. Masses and binding energies (in units of MeV) of the $D\bar{D}K$ and $D\bar{D}^*K$ bound states, in comparison with the results of other works.

	This work	Ref. [32]	Ref. [33]
$\frac{1}{2}(0^-) D\bar{D}K$	$4181.2_{-1.4}^{+2.4} (48.9_{-2.4}^{+1.4})$	...	...
$\frac{1}{2}(1^-) D\bar{D}^*K$	$4294.1_{-3.1}^{+6.6} (77.3_{-6.6}^{+3.1})$	$4317.92_{-6.55}^{+6.13} (53.52_{-6.13}^{+6.55})$	$4307 \pm 2 (64 \pm 2)$

FIG. 2. RMS radii of subsystems in the $D\bar{D}K$ (left), $D\bar{D}^*K$ (middle), and DDK [18] bound states with a cutoff $R_c = 1.0$ fm.

$$B_3(D\bar{D}^*K) \simeq 77.3_{-6.6}^{+3.1} \text{ MeV}. \quad (10)$$

The binding energy of the $D\bar{D}^*K$ bound state is larger than the one of $D\bar{D}K$ mainly due to the more attractive $D\bar{D}^*$ interaction.

As mentioned earlier, the $D\bar{D}^*K$ state has been studied using other methods. In Table I, we compare our predictions with those of two earlier studies. It is clear that, although there are some differences in detail, the existence of a $D\bar{D}^*K$ bound state seems to be a robust prediction.

In Fig. 2, we show the root-mean-square (rms) radii of the three subsystems in the $D\bar{D}K$ and $D\bar{D}^*K$ bound states. With a cutoff of $R_c = 1.0$ fm, the rms radius of the DK subsystem in the $D\bar{D}K$ bound state is predicted to be 1.26 fm, while those of the $\bar{D}K$ and $D\bar{D}$ subsystems are much larger, yielding 2.27 and 2.10 fm, respectively. For the $D\bar{D}^*K$ bound state, the rms radii for DK , $\bar{D}^*K$, and $D\bar{D}^*$ are 1.15, 1.27, and 0.93 fm, respectively.³ The latter two are much smaller compared to the ones of $\bar{D}K$ and $D\bar{D}$ in the $D\bar{D}K$ bound state. It can be easily understood because the $D\bar{D}^*K$ bound state has a much larger binding energy. In Fig. 2, we also show the rms radii of the DDK bound state taken from Ref. [18], of which the DK one is about 1.32 fm and the DD one is about 1.36 fm.

In Table II, we present the weights of the three Jacobian channels for the $D\bar{D}K$ and $D\bar{D}^*K$ systems. One can see that channel 1 [corresponding to the $(DK) - \bar{D}^{(*)}$ structure] is dominant in both systems, such that they can be regarded as bound states mainly composed of a DK cluster with a $\bar{D}/\bar{D}^*$ meson. In this sense, these two bound states are analogous to the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ states. In Table III, the expectation values of the $D\bar{D}K$ and $D\bar{D}^*K$ bound states are shown. It is interesting to note that in both systems $\langle V_{DK} \rangle$ remains almost the same, while $\langle V_{D\bar{D}^*} \rangle$ contributes much more to the three-body binding energy compared with $\langle V_{D\bar{D}} \rangle$. This implies that the different binding energies and different sizes of these two bound states mainly result from the different $D\bar{D}$ and $D\bar{D}^*$ interactions.

³One should not compare the radii of $DK(\bar{D}K)$ with that of $D\bar{D}^*$ as their interactions are described using different methods, while the DK radii are known to depend on the details of the interactions [18].

Following Ref. [19] in which the decay of the DDK bound state via triangle diagrams has been estimated, in the following, we compute the decay of the $D\bar{D}K$ bound state, which for convenience will be denoted by $K_c(4180)$. Regarding the decay of the $D\bar{D}^*K$ bound state, we refer to Ref. [52]. The decay of K_c can proceed via triangle diagrams shown in Fig. 3, where it first decays into $\bar{D}D_{s0}^*$, $\bar{D}_{s0}^*$ then decays into either DK or $D_s\eta$, and finally the $\bar{D}D$ and $\bar{D}\eta$ pairs fuse into either J/ψ or $\bar{D}^*$. We employ the effective Lagrangian approach to calculate the amplitudes of these hadronic loops.

The interaction between $K_c(4180)$ and its components is described by the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{K_c(x)} = & g_{K_c D_{s0} \bar{D}} K_c^T(x) \int dy \Phi_{K_c}(y^2) D_{s0}(x + \omega_{\bar{D}} y) \\ & \times \bar{D}(x - \omega_{D_{s0}} y) + \text{H.c.}, \end{aligned} \quad (11)$$

where $\omega_i = m_i/(m_i + m_j)$ is a kinematical parameter with m_i and m_j being the masses of the components of K_c , $\Phi(y)$ is a form factor for which we take a Gaussian form, and $g_{K_c D_{s0} \bar{D}}$ is the coupling constant between K_c and its components. We can determine the value of $g_{K_c D_{s0} \bar{D}}$ by the compositeness condition; see, e.g., Refs. [19, 53, 54]. In studies of hadronic molecules, the cutoff in the form factor is often chosen to be about 1 GeV, and the corresponding coupling is found to be 5.38 GeV. The Lagrangians describing the interaction between $D_{s0}^*(2317)$ and $DK/D_s\eta$ have the form

TABLE II. Weights of the $D\bar{D}K$ and $D\bar{D}^*K$ subsystems calculated with $R_c = 1$ fm.

Channels	$(DK) - \bar{D}^{(*)}$	$(\bar{D}^{(*)}K) - D$	$(D\bar{D}^{(*)}) - K$	Total
$D\bar{D}K$	86.8%	12.7%	0.5%	100.0%
$D\bar{D}^*K$	68.9%	17.4%	13.7%	100.0%

TABLE III. Expectation values (in units of MeV) of the Hamiltonian of the $D\bar{D}K$ and $D\bar{D}^*K$ systems calculated with $R_c = 1$ fm.

	$\langle T \rangle$	$\langle V_{DK} \rangle$	$\langle V_{\bar{D}^{(*)}K} \rangle$	$\langle V_{D\bar{D}^{(*)}} \rangle$
$D\bar{D}K$	99.08	-122.45	-15.46	-10.02
$D\bar{D}^*K$	156.94	-121.74	-35.01	-77.49

FIG. 3. Strong decays of $K_c(4180)$ to $J/\psi K$ and $D_s \bar{D}^*$ via triangle diagrams.

$$\begin{aligned}\mathcal{L}_{D_{s0}DK} &= g_{D_{s0}DK} D_{s0} DK, \\ \mathcal{L}_{D_{s0}D_s\eta} &= g_{D_{s0}D_s\eta} D_{s0} D_s \eta,\end{aligned}\quad (12)$$

where $g_{D_{s0}DK}$ and $g_{D_{s0}D_s\eta}$ denote the $D_{s0}^*(2317)$ coupling to DK and $D_s\eta$ with $g_{D_{s0}DK} = 10.21$ GeV and $g_{D_{s0}D_s\eta} = 6.40$ GeV, respectively [30]. The effective Lagrangian between J/ψ and $\bar{D}D$ and that between η and $\bar{D}\bar{D}^*$ can be written as

$$\begin{aligned}\mathcal{L}_{\psi\bar{D}D} &= -ig_{\psi\bar{D}D}\psi_\mu(\partial^\mu D D^\dagger - D\partial^\mu D^\dagger), \\ \mathcal{L}_{\bar{D}\bar{D}^*\eta} &= -ig_{\bar{D}\bar{D}^*\eta}(\bar{D}^\dagger\partial^\mu\eta\bar{D}^* - \bar{D}\partial^\mu\eta\bar{D}^{*\dagger}),\end{aligned}\quad (13)$$

where $g_{\psi\bar{D}D} = m_\psi/f_\psi$ with $f_\psi = 0.426$ GeV and $g_{\bar{D}\bar{D}^*\eta} = \sqrt{2}g/(\sqrt{3}f_\pi)$ with $g = 1.097$ GeV and $f_\pi = 0.1307$ GeV [54].

With the above Lagrangians, the amplitudes of the two triangle diagrams of Fig. 3 can be straightforwardly calculated,

$$\begin{aligned}i\mathcal{M}_{K_c \rightarrow J/\psi K} &= g_{\psi\bar{D}D} g_{D_{s0}DK} g_{K_c D_{s0}\bar{D}} \\ &\times \int \frac{d^4 q}{(2\pi)^4} \Phi[(k_1\omega_{\bar{D}} - k_2\omega_{D_{s0}})^2] \\ &\times \frac{1}{q^2 - m_D^2} \frac{1}{k_1^2 - m_{D_{s0}}^2} \frac{1}{k_2^2 - m_{\bar{D}}^2} (q^\mu - k_2^\mu) \cdot \varepsilon_\mu^\psi, \\ i\mathcal{M}_{K_c \rightarrow D_s \bar{D}^*} &= g_{\bar{D}\bar{D}^*\eta} g_{D_{s0}D_s\eta} g_{K_c D_{s0}\bar{D}} \\ &\times \int \frac{d^4 q}{(2\pi)^4} \Phi[(k_1\omega_D - k_2\omega_{D_{s0}})^2] \\ &\times \frac{1}{q^2 - m_\eta^2} \frac{1}{k_1^2 - m_{D_{s0}}^2} \frac{1}{k_2^2 - m_{\bar{D}}^2} q^\mu \cdot \varepsilon_\mu^{\bar{D}^*},\end{aligned}\quad (14)$$

where k_2 and k_1 are the momenta of $\bar{D}$ and $D_{s0}^*(2317)$, q is the intermediate momentum, and ε_μ^ψ and $\varepsilon_\mu^{\bar{D}^*}$ are the polarization vectors of ψ and $\bar{D}^*$.

With these amplitudes, the decay width can be easily obtained,

$$\Gamma[K_c \rightarrow] = \frac{1}{2J+1} \frac{1}{32\pi^2} \frac{|\vec{p}_1|}{m_{K_c}^2} |\mathcal{M}|^2 d\Omega, \quad (15)$$

where $\vec{p}_1$ is the 3-momenta of either final state in the rest frame of K_c and m_{K_c} is the mass of the $D\bar{D}K$ bound state.

With a cutoff of 1 GeV for the form factor $\Phi_{K_c}(y^2)$, the decay widths of $K_c(4180)$ to $J/\psi K$ and $D_s \bar{D}^*$ are found to be 0.5 and 0.2 MeV, respectively, which indicates that this state is very narrow. Compared to the DDK bound state $R(4140)$, for which $\Gamma \simeq 2-3$ MeV, the relative smallness of the partial decay width of $K_c(4180)$ into $J/\psi K$ is mainly due to the suppression of the D exchange contribution in comparison with the corresponding K exchange contribution.

IV. CONCLUSION

We employed the Gaussian expansion method to study the $D\bar{D}K$ system with the leading-order DK and $\bar{D}K$ potentials obtained in chiral perturbation theory and the $D\bar{D}$ potential from the OBE model. We found the existence of a $D\bar{D}K$ bound state with a binding energy about 49 MeV. It is interesting to note that the predicted $D\bar{D}K$ three-body bound state is only 4 MeV below the respective $\bar{D}D_{s0}^*(2317)$ threshold, though its binding could increase by 20 MeV if the $D\bar{D}$ interaction is strong enough to generate a bound state as claimed by the recent lattice QCD study. Utilizing heavy quark spin symmetry, we also studied the $D\bar{D}^*K$ system in the same framework, and we found a $D\bar{D}^*K$ bound state with a binding energy of about 77 MeV, consistent with the results of earlier works.

We studied the decays of the $D\bar{D}K$ bound state via triangle diagrams and found that its partial decay widths to $J/\psi K$ and $D_s \bar{D}^*$ are about 0.5 and 0.2 MeV, respectively. Different from the DDK bound state, these two exotic states are more likely to be discovered at the current facilities because of their hidden charm nature. It is interesting to note that a recent study in QCD sum rules does not find a $\bar{D}D_{s0}^*(2317)$ bound state [55], consistent with the current picture that the $K_c(4180)$ state is a three-body molecule. As a result, we strongly encourage our experimental colleagues to search for it.

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