Higher-spin gauge models with (1, 1) supersymmetry in AdS_3 : Reduction to (1, 0) superspace

Daniel Hutchings^{*,} Jessica Hutomo^{†,} and Sergei M. Kuzenko[‡]

*Department of Physics M013, The University of Western Australia,
35 Stirling Highway, Crawley, Western Australia 6009, Australia*

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In three dimensions, there are two types of $\mathcal{N} = 2$ anti-de Sitter (AdS) supersymmetry, which are denoted (1, 1) and (2, 0). They are characterized by different supercurrents and support different families of higher-spin gauge models (massless and massive), which were constructed in Hutomo *et al.* [$\mathcal{N} = 2$ supersymmetric higher spin gauge theories and current multiplets in three dimensions, *Phys. Rev. D* **98**, 125004 (2018); Higher spin supermultiplets in three dimensions: (2, 0) AdS supersymmetry, *Phys. Lett. B* **787**, 175 (2018)] for the (1, 1) and (2, 0) cases, respectively, using superspace techniques. It turns out that the precise difference between the (1, 1) and (2, 0) higher-spin supermultiplets can be pinned down by reducing these gauge theories to (1, 0) AdS superspace. The present paper is devoted to the (1, 1) $\rightarrow$ (1, 0) AdS superspace reduction. In conjunction with the outcomes of the (2, 0) $\rightarrow$ (1, 0) AdS superspace reduction carried out in Hutomo and Kuzenko [Field theories with (2, 0) AdS supersymmetry in $\mathcal{N} = 1$ AdS superspace, *Phys. Rev. D* **100**, 045010 (2019)], we demonstrate that every known higher-spin theory with (1, 1) or (2, 0) AdS supersymmetry decomposes into a sum of two off-shell (1, 0) supermultiplets that belong to four series of inequivalent higher-spin gauge models. The latter are reduced to components.

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I. INTRODUCTION

To study $\mathcal{N}$ -extended supersymmetric theories in d dimensions, it is advantageous to deal with a formulation that permits some amount of supersymmetry to be realized manifestly. In general, there exist two superspace settings to achieve this. One of them makes use of the standard $\mathcal{N}$ -extended superspace (sometimes endowed with additional commuting variables [1–3]) and provides a manifestly supersymmetric formulation. The other employs a smaller $\tilde{\mathcal{N}}$ -extended superspace, with $\tilde{\mathcal{N}} < \mathcal{N}$, to keep manifest only $\tilde{\mathcal{N}}$ supersymmetries. Both approaches have found numerous applications in the literature. For instance, it is known that the geometric properties of general $\mathcal{N} = 2$ supersymmetric nonlinear σ models in Minkowski space $\mathbb{M}^4$ [4] are remarkably transparent if these theories are realized in $\mathcal{N} = 1$ superspace [5,6]. One of the two supersymmetries is manifest and off-shell in this setting, while the second supersymmetry is hidden and on-shell

(the commutator of the first and the second supersymmetries closes *only* on-shell). On the other hand, in order to construct general off-shell $\mathcal{N} = 2$ supersymmetric nonlinear σ models, manifestly supersymmetric techniques are indispensable, and there exist two powerful $\mathcal{N} = 2$ superspace approaches: (i) the harmonic superspace [2,7]; and (ii) the projective superspace [3,8,9]. One of the conceptual virtues of these manifestly supersymmetric formulations is the possibility to generate $\mathcal{N} = 2$ nonlinear σ -model actions (and thus hyperkähler metrics) from Lagrangians of *arbitrary* functional form.¹ Analogous results exist for general $\mathcal{N} = 2$ supersymmetric nonlinear σ models in four-dimensional anti-de Sitter space (AdS_4), which were originally formulated as off-shell theories in $\mathcal{N} = 2$ AdS superspace [11]. This approach makes $\mathcal{N} = 2$ supersymmetry manifest, but the hyperkähler geometry of the σ -model target space is hidden. Some time later, the most general $\mathcal{N} = 2$ supersymmetric σ models in AdS_4 were constructed using a formulation in terms of $\mathcal{N} = 1$ covariantly chiral superfields [12,13]. One of the two supersymmetries is hidden in this approach, but the geometric properties of the $\mathcal{N} = 2$ supersymmetric σ models in AdS_4

*daniel.hutchings@research.uwa.edu.au

†jessica.hutomo@research.uwa.edu.au

‡sergei.kuzenko@uwa.edu.au

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¹This possibility was realized for the first time in an important paper [10], where it was demonstrated that the most general interacting Lagrangian of the q^+ hypermultiplet superfields can be treated as the hyperkähler potential parametrizing the complete set of hyperkähler metrics.

become transparent. Specifically, the target space must be a noncompact hyperkähler manifold endowed with a Killing vector field that generates an $\text{SO}(2)$ group of rotations on the two-sphere of complex structures. The two σ -model formulations developed in [11] and [12,13] are related via the $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ AdS_4 superspace reduction worked out in [14].

Extended supersymmetric theories in AdS_3 possess exceptionally many superspace realizations. We recall that the (connected) AdS group in three dimensions is not a simple group,

$$\text{SO}_0(2, 2) \cong (\text{SL}(2, \mathbb{R}) \times \text{SL}(2, \mathbb{R}))/\mathbb{Z}_2, \quad (1.1)$$

and so are its supersymmetric extensions $\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})$. This means that there are several species of $\mathcal{N}$ -extended AdS supersymmetry [15], which are known as the (p, q) AdS supersymmetry types, where the integers $p \geq q \geq 0$ are such that $\mathcal{N} = p + q$. In principle, field theories possessing the (p, q) AdS supersymmetry may be realized in the so-called (p, q) AdS superspace [16], $\text{AdS}^{(3|p,q)}$, which may be viewed as a maximally supersymmetric solution of the (p, q) AdS supergravity [15], with anti-de Sitter space AdS_3 being the bosonic body of the superspace.² More specifically, within the supergravity framework of [16,17] the superspace $\text{AdS}^{(3|p,q)}$ originates as a maximally symmetric supergeometry with covariantly constant torsion and curvature generated by a symmetric real torsion $S^{IJ} = S^{JI}$, with the structure-group indices I, J taking values from 1 to $\mathcal{N}$. It turns out that S^{IJ} is nonsingular, and the parameters p and $q = \mathcal{N} - p$ determine its signature. Since the isometry group of $\text{AdS}^{(3|p,q)}$ is $\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})$ and since S^{IJ} is invariant under its compact subgroup $\text{SO}(p) \times \text{SO}(q)$, the global realization of the superspace is

$$\text{AdS}^{(3|p,q)} = \frac{\text{OSp}(p|2; \mathbb{R}) \times \text{OSp}(q|2; \mathbb{R})}{\text{SL}(2, \mathbb{R}) \times \text{SO}(p) \times \text{SO}(q)}. \quad (1.2)$$

In fact, starting from the superspace geometry of $\mathcal{N}$ -extended conformal supergravity [17] and restricting the torsion to be covariantly constant and Lorentz invariant, a general AdS superspace solution for $\mathcal{N} \geq 4$ includes not only the torsion S^{IJ} but also a completely antisymmetric curvature $X^{IJKL} = X^{[IJKL]}$. It turns out that the latter may be nonzero only if $S^{IJ} = S\delta^{IJ}$, which means $p = \mathcal{N}$ and $q = 0$. Such solutions correspond to exotic AdS superspaces, $\text{AdS}_{S,X}^{3|2\mathcal{N}}$, for which the isometry group is, in general, a subgroup of $\text{OSp}(\mathcal{N}|2; \mathbb{R}) \times \text{SL}(2, \mathbb{R})$. The superspace $\text{AdS}_{S,X}^{3|2\mathcal{N}}$ is not conformally flat. All the

superspaces (1.2) are conformally flat; see [16] for the technical details.³ It should be mentioned that $(1, 0)$ AdS superspace (or simply $\mathcal{N} = 1$ AdS superspace, $\text{AdS}^{(3|1,0)} \equiv \text{AdS}^{3|2}$) was introduced in [21].

Consider a (p, q) supersymmetric field theory formulated in $\text{AdS}^{(3|p,q)}$, with $p + q \geq 3$ and $p \geq q$. As argued in [22], it can always be recast as a supersymmetric theory realized in $(2, 0)$ AdS superspace, with $(p + q - 2)$ supersymmetries being hidden. In the case $p + q \geq 3$ and $p \geq q > 0$, every supersymmetric field theory in $\text{AdS}^{(3|p,q)}$ can be reformulated as a theory in $(1, 1)$ AdS superspace. Such reformulations were developed in [22] for general (p, q) supersymmetric nonlinear σ models in AdS_3 , with $p + q \leq 4$. It is obvious that the list of possible superspace embeddings can be continued. It is also clear that every extended supersymmetric theory in AdS_3 , with $p + q > 1$, may be realized in $\mathcal{N} = 1$ AdS superspace.

This work is devoted to implementing the $(1, 1) \rightarrow (1, 0)$ AdS reduction of the $(1, 1)$ supersymmetric higher-spin models in AdS_3 (both massless and massive), which were constructed in [23] as an extension of the earlier results in $\mathcal{N} = 2$ Minkowski superspace [24].⁴ There are two main motivations for this project. First, the off-shell structure of the half-integer superspin multiplets with $(2, 0)$ AdS supersymmetry [29] drastically differs from that of their $(1, 1)$ counterparts [23].⁵ Direct comparison of these theories is difficult in a manifestly supersymmetric setting, since they are formulated in different superspaces, $\text{AdS}^{(3|2,0)}$ and $\text{AdS}^{(3|1,1)}$, respectively. However, both families of higher-spin theories can be reformulated in the same $(1, 0)$ AdS superspace, and then the precise difference between the $(2, 0)$ and $(1, 1)$ higher-spin supermultiplets can be pinned down. The $(2, 0) \rightarrow (1, 0)$ reduction of the $(2, 0)$ supersymmetric higher-spin theories in AdS_3 [29] has already been carried out in [30]. We will show that every massless higher-spin theory with $(1, 1)$ or $(2, 0)$ AdS supersymmetry decomposes into a sum of two off-shell $(1, 0)$ supermultiplets that belong to the four series of inequivalent higher-spin gauge models constructed in [30].

The second motivation for this work is quantum mechanical. All known off-shell higher-spin $\mathcal{N} = 2$ supermultiplets in AdS_3 , with either $(2, 0)$ or $(1, 1)$ AdS supersymmetry [23,29], are reducible gauge theories (in the terminology of the Batalin-Vilkovisky quantization [31]), similar to the massless higher-spin supermultiplets in AdS_4 [27]. The Lagrangian quantization of such theories

²In order to realize field theories with the (p, q) AdS supersymmetry in $\text{AdS}^{(3|p,q)}$, for $p + q \leq 4$ one can employ the off-shell supergravity methods developed in [17–19].

³It should be pointed out that the superconformal flatness of AdS superspaces in diverse dimensions, including the $d = 3$ case, was studied in Ref. [20] where the coset spaces (1.2) with $p + q = 2$ were briefly discussed.

⁴The massless higher-spin theories proposed in [23,24] have natural counterparts in four dimensions [25–27]; see also [28] for a review.

⁵It is pertinent to mention here that the $(2, 0)$ and $(1, 1)$ AdS supersymmetries support different supercurrent multiplets [18].

proves to be a nontrivial procedure. In four dimensions, the quantization of the theories proposed in [27] was achieved in [32]. On the other hand, all off-shell higher-spin $\mathcal{N} = 1$ supermultiplets in AdS_3 studied in this paper and in [30], are irreducible gauge theories. They can be quantized using the Faddeev-Popov procedure [33], as in the nonsupersymmetric case (see, e.g., [34]). This opens the possibility to develop heat kernel techniques for higher-spin theories in $\text{AdS}^{3|2}$, by extending the four-dimensional results [28,35,36].

This paper is organized as follows. Section II provides a succinct review of $\mathcal{N} = 1$ supersymmetric field theory in AdS_3 . In Sec. III, we review the structure of the four series of off-shell massless higher-spin $\mathcal{N} = 1$ supersymmetric models in AdS_3 constructed in [30,37]. A formalism to reduce every field theory with (1, 1) AdS supersymmetry to $\mathcal{N} = 1$ AdS superspace is developed in Sec. IV. This formalism is then applied throughout Secs. V–VIII to carry out the $(1, 1) \rightarrow (1, 0)$ AdS superspace reduction of massless higher-spin models with (1, 1) AdS supersymmetry. These are detailed in Secs. V and VI for the half-integer $(s + \frac{1}{2})$ superspin case, and in Secs. VII and VIII for the integer (s) superspin case. Our results are summarized and discussed in Sec. IX, where we also analyze the difference between the (2, 0) and (1, 1) higher-spin (massless and massive) supermultiplets. Our spinor notation and conventions are collected in the Appendix A. In Appendix B, we study the component structure of the off-shell massless higher-spin $\mathcal{N} = 1$ supersymmetric models in AdS_3 reviewed in Sec. III.

II. $\mathcal{N} = 1$ SUPERSYMMETRIC FIELD THEORY IN AdS_3

In this section we provide salient facts about the geometry of $\mathcal{N} = 1$ AdS superspace, $\text{AdS}^{3|2}$, and its isometries following [16]. We also recall certain duality transformations in $\text{AdS}^{3|2}$, following [30].

Let $z^M = (x^m, \theta^\mu)$ be local coordinates parametrizing $\text{AdS}^{3|2}$. The geometry of $\text{AdS}^{3|2}$ is encoded in the set of covariant derivatives of the form

$$\nabla_A = (\nabla_a, \nabla_\alpha) = E_A^M \partial_M + \frac{1}{2} \omega_A^{bc} M_{bc}, \quad (2.1)$$

where E_A^M is the inverse vielbein and ω_A^{bc} the Lorentz connection. The explicit relation between Lorentz generators with two vector indices (M_{ab}), with one vector index (M_a), and with two spinor indices ($M_{\alpha\beta}$) is described in Appendix A. The covariant derivatives obey the following algebra [21]:

$$\{\nabla_\alpha, \nabla_\beta\} = 2i\nabla_{\alpha\beta} - 4i|\mu|M_{\alpha\beta}, \quad (2.2a)$$

$$[\nabla_a, \nabla_b] = |\mu|(\gamma_a)_\beta{}^\gamma \nabla_\gamma, \quad [\nabla_a, \nabla_b] = -4|\mu|^2 M_{ab}. \quad (2.2b)$$

In spinor notation, Eqs. (2.2b) take the form

$$\begin{aligned} [\nabla_{\alpha\beta}, \nabla_\gamma] &= -2|\mu|\varepsilon_{\gamma(\alpha}\nabla_{\beta)}, \\ [\nabla_{\alpha\beta}, \nabla_{\gamma\delta}] &= 4|\mu|^2(\varepsilon_{\gamma(\alpha}M_{\beta)\delta} + \varepsilon_{\delta(\alpha}M_{\beta)\gamma}). \end{aligned} \quad (2.2c)$$

Here $|\mu| > 0$ is a constant parameter, which determines the curvature of AdS_3 .

The parameter $|\mu|$ in (2.2) was denoted $\mathcal{S}$ in [16]. However, in this paper we prefer to make use of the notation $|\mu|$ which is more appropriate in the context of the $(1, 1) \rightarrow (1, 0)$ superspace reduction, which will be considered later. It should be remarked that the geometry of $\mathcal{N} = 1$ AdS superspace can also be described by the graded commutation relations which are obtained from (2.2) by replacing $|\mu| \rightarrow -|\mu|$. The two choices, $\mathcal{S} = |\mu|$ and $\mathcal{S} = -|\mu|$, correspond to the so-called (1, 0) and (0, 1) AdS superspaces [16], which are different realizations of $\mathcal{N} = 1$ AdS superspace. The (1, 0) and (0, 1) AdS superspaces are naturally embedded in (1, 1) AdS superspace⁶ and are related to each other by a parity transformation.

One may derive several useful identities:

$$\nabla_\alpha \nabla_\beta = i\nabla_{\alpha\beta} - 2i|\mu|M_{\alpha\beta} + \frac{1}{2}\varepsilon_{\alpha\beta}\nabla^2, \quad (2.3a)$$

$$\nabla^\beta \nabla_\alpha \nabla_\beta = 4i|\mu|\nabla_\alpha, \quad \{\nabla^2, \nabla_\alpha\} = 4i|\mu|\nabla_\alpha, \quad (2.3b)$$

$$\nabla^2 \nabla_\alpha = 2i|\mu|\nabla_\alpha + 2i\nabla_{\alpha\beta}\nabla^\beta - 4i|\mu|\nabla^\beta M_{\alpha\beta}, \quad (2.3c)$$

$$[\nabla_\alpha \nabla_\beta, \nabla^2] = 0 \Rightarrow [\nabla_{\alpha\beta}, \nabla^2] = 0, \quad (2.3d)$$

$$[\nabla_\alpha, \square] = 2|\mu|\nabla_{\alpha\beta}\nabla^\beta + 3|\mu|^2\nabla_\alpha, \quad (2.3e)$$

where we have denoted $\nabla^2 = \nabla^\alpha \nabla_\alpha$ and $\square = \nabla^a \nabla_a = -\frac{1}{2}\nabla^{\alpha\beta}\nabla_{\alpha\beta}$. In particular, it follows from the algebra of the covariant derivatives that

$$-\frac{1}{4}\nabla^2 \nabla^2 = \square - 2i|\mu|\nabla^2 + 2|\mu|\nabla^{\alpha\beta}M_{\alpha\beta} - 2|\mu|^2 M^{\alpha\beta}M_{\alpha\beta}. \quad (2.4)$$

This differential operator can be expressed in terms of the quadratic Casimir operator of the $\mathcal{N} = 1$ AdS supergroup [37]

$$\mathbb{Q} = -\frac{1}{4}\nabla^2 \nabla^2 + i|\mu|\nabla^2, \quad [\mathbb{Q}, \nabla_A] = 0. \quad (2.5)$$

Given an arbitrary superfield F and its complex conjugate $\bar{F}$, the following relation holds:

⁶As follows from (4.19), in (1, 1) AdS superspace two subsets of covariant derivatives, $(\nabla_a, \nabla_\alpha^1)$ and $(\nabla_a, \nabla_\alpha^2)$, form closed algebras of the type (2.2), with the curvature parameter being $|\mu|$ and $-|\mu|$, respectively.

$$\overline{\nabla_\alpha F} = -(-1)^{\epsilon(F)} \nabla_\alpha \bar{F}, \quad (2.6)$$

where $\epsilon(F)$ denotes the Grassmann parity of F .

According to the general formalism of [28], the isometry transformations of $\text{AdS}^{3|2}$ are generated by the Killing supervector fields,

$$\xi = \xi^B \nabla_B = \xi^b \nabla_b + \xi^\beta \nabla_\beta, \quad (2.7)$$

which by definition solve the master equation [16]

$$\left[\xi + \frac{1}{2} \xi^{bc} M_{bc}, \nabla_A \right] = 0, \quad (2.8)$$

for some Lorentz parameter $\xi^{bc} = -\xi^{cb}$. It can be shown that the Killing equation (2.8) is equivalent to the set of relations [16] (see also [37])

$$\nabla_{(\alpha} \xi_{\beta\gamma)} = 0, \quad \nabla_\beta \xi^{\beta\alpha} = -6i \xi^\alpha, \quad (2.9a)$$

$$\nabla_{(\alpha} \zeta_{\beta\gamma)} = 0, \quad \nabla_\beta \zeta^{\beta\alpha} = -12i |\mu| \xi^\alpha, \quad (2.9b)$$

$$\nabla_\alpha \xi_\beta = \frac{1}{2} \zeta_{\alpha\beta} + |\mu| \xi_{\alpha\beta}, \quad (2.9c)$$

which imply

$$\nabla_a \xi_b + \nabla_b \xi_a = 0, \quad (2.10a)$$

$$\nabla_{\alpha\beta} \xi^\beta + 3|\mu| \xi_\alpha = 0, \quad (2.10b)$$

$$(i\nabla^2 + 12|\mu|) \xi_\alpha = 0. \quad (2.10c)$$

Thus, ξ^α is a Killing vector. Given a tensor superfield $U(x, \theta)$ (with suppressed indices) on $\text{AdS}^{3|2}$, its infinitesimal isometry transformation law is given by

$$\delta_\xi U = \left(\xi^\alpha \nabla_\alpha + \xi^\alpha \nabla_\alpha + \frac{1}{2} \xi^{ab} M_{ab} \right) U. \quad (2.11)$$

To study the dynamics of $\mathcal{N} = 1$ supersymmetric field theories in AdS_3 , a manifestly supersymmetric action principle is required. Such an action is associated with a real scalar Lagrangian L and has the form

$$S = \int d^3x d^2\theta EL, \quad E^{-1} = \text{Ber}(E_A^M). \quad (2.12)$$

In what follows, we make use of the notation $d^{3|2}z := d^3x d^2\theta$.

The component form of the action (2.12) is

$$S = \frac{1}{4} \int d^3x e (i\nabla^2 + 8|\mu|) L|_{\theta=0}, \quad e^{-1} = \det(e_a^m). \quad (2.13)$$

Here the component inverse vielbein is defined as $e_a^m(x) = E_a^m|_{\theta=0}$. Making use of the $\mathcal{N} = 1$ AdS transformation law $\delta_\xi L = \xi L$ and the identities (2.9) and (2.10), it can be shown that the action (2.12) is invariant under the isometry group of $\text{AdS}^{3|2}$.

To conclude this section, we review the duality transformation described in [30]. Consider a field theory in $\text{AdS}^{3|2}$ that is formulated in terms of a real tensor superfield $V_{\alpha(n)}$ with the action

$$S^{\parallel}[V_{\alpha(n)}] = \int d^{3|2}z E \mathcal{L}(i^{n+1} \nabla_\beta V_{\alpha(n)}). \quad (2.14)$$

We call $\nabla_\beta V_{\alpha(n)}$ a longitudinal superfield, by analogy with a longitudinal vector field. The theory (2.14) has a dual formulation that is obtained by introducing a first-order action

$$S_{\text{first-order}} = \int d^{3|2}z E \{ \mathcal{L}(\Sigma_{\beta;\alpha(n)}) + i^{n+1} \mathcal{W}_{\beta;\alpha(n)} \Sigma_{\beta;\alpha(n)} \}. \quad (2.15)$$

Here $\Sigma_{\beta;\alpha(n)}$ is unconstrained and the Lagrange multiplier is

$$\mathcal{W}_{\beta;\alpha(n)} = i^{n+1} (\nabla^\gamma \nabla_\beta - 4i |\mu| \delta_\beta^\gamma) \Psi_{\gamma;\alpha(n)}, \quad \nabla^\beta \mathcal{W}_{\beta;\alpha(n)} = 0, \quad (2.16)$$

for some unconstrained prepotential $\Psi_{\gamma;\alpha(n)}$. Varying (2.15) with respect to $\Psi_{\gamma;\alpha(n)}$ gives

$$\begin{aligned} \nabla^\beta \nabla_\gamma \Sigma_{\beta;\alpha(n)} - 4i |\mu| \Sigma_{\gamma;\alpha(n)} &= 0 \\ \Rightarrow \Sigma_{\beta;\alpha(n)} &= i^{n+1} \nabla_\beta V_{\alpha(n)}, \end{aligned} \quad (2.17)$$

and thus $S_{\text{first-order}}$ reduces to the original action (2.14). On the other hand, integrating out $\Sigma_{\beta;\alpha(n)}$ from $S_{\text{first-order}}$ leads to a dual action of the form

$$S^\perp[\Psi_{\gamma;\alpha(n)}] = \int d^{3|2}z E \mathcal{L}_{\text{dual}}(\mathcal{W}_{\beta;\alpha(n)}), \quad (2.18)$$

which is invariant under gauge transformations

$$\delta \Psi_{\gamma;\alpha(n)} = i^{n+1} \nabla_\gamma \eta_{\alpha(n)}. \quad (2.19)$$

It is natural to call the gauge-invariant field strength $\mathcal{W}_{\beta;\alpha(n)}$ a transverse superfield, due to constraint (2.16). The dual formulations (2.14) and (2.18) are referred to as longitudinal and transverse, respectively. In the $n = 1$ case, the duality transformation corresponds to the standard duality between the $\mathcal{N} = 1$ scalar and vector multiplets in three dimensions, see [38].

The above duality transformation naturally extends to Minkowski superspace $\mathbb{M}^{3|2}$ in the limit $|\mu| \rightarrow 0$. Therefore,

given two dually equivalent theories in $\text{AdS}^{3|2}$, their flat superspace counterparts are also dually equivalent. The opposite is not always true. For instance, the flat superspace counterparts of the massless half-integer superspin models (3.6) and (3.10) are dual. However, these models are not dual in AdS_3 . The requirement of gauge invariance uniquely fixes the actions (3.6) and (3.10) in $\text{AdS}^{3|2}$, including the presence of certain $|\mu|$ -dependent terms that are incompatible with duality invariance.

III. MASSLESS HIGHER-SPIN $\mathcal{N}=1$ SUPERMULTIPLETS

According to [30], for each superspin value $\hat{s} \geq 1$, where $\hat{s}$ is either integer ($\hat{s} = s$) or half-integer ($\hat{s} = s + \frac{1}{2}$), there exist two off-shell formulations for a massless $\mathcal{N}=1$ superspin- $\hat{s}$ multiplet in AdS_3 . In the flat superspace limit, these models prove to be related to each other via a superfield Legendre transformation. However, when uplifted to $\mathcal{N}=1$ AdS superspace, it becomes apparent that these duality relations no longer hold. In this section, we review the explicit formulation of these respective theories, which were derived in [30,37].

As is known, any massless multiplet of superspin $\hat{s} > 1$ has no propagating degrees of freedom, and the notion of superspin is purely kinematical. It is used by analogy with massive supermultiplets. The concept of superspin is well-defined in the massive case, and we follow the definition used in [39] in the super-Poincaré case. Specifically, if n is a positive integer, an on- massive multiplet of superspin $n/2$ is described by a real symmetric rank- n spinor superfield, $\mathcal{H}_{\alpha_1 \dots \alpha_n} = \tilde{\mathcal{H}}_{\alpha_1 \dots \alpha_n} = \mathcal{H}_{(\alpha_1 \dots \alpha_n)}$, which obeys the differential conditions [40]

$$D^\beta \mathcal{H}_{\beta \alpha_1 \dots \alpha_{n-1}} = 0, \quad (3.1a)$$

$$-\frac{i}{2} D^2 \mathcal{H}_{\alpha_1 \dots \alpha_n} = m \sigma \mathcal{H}_{\alpha_1 \dots \alpha_n}, \quad \sigma = \pm 1. \quad (3.1b)$$

Here, m is a real constant of unit mass dimension, D_α is the spinor covariant derivative of $\mathcal{N}=1$ Minkowski superspace, and $D^2 = D^\alpha D_\alpha$. The superfield $\mathcal{H}_{\alpha(n)}$ contains two ordinary on- massive fields, which are

$$\begin{aligned} \phi_{\alpha_1 \dots \alpha_n} &:= \mathcal{H}_{\alpha_1 \dots \alpha_n}|_{\theta=0}, \\ \phi_{\alpha_1 \dots \alpha_{n+1}} &:= i^{n+1} D_{(\alpha_1} \mathcal{H}_{\alpha_2 \dots \alpha_{n+1})}|_{\theta=0}. \end{aligned} \quad (3.2)$$

Their helicity values are $\frac{n}{2}\sigma$ and $\frac{n+1}{2}\sigma$, respectively, see [39] for the technical details.

An off-shell massless superspin- $n/2$ gauge theory in AdS_3 can be realized in terms of two superfields, of which one is universally a superconformal gauge prepotential $H_{\alpha(n)}$ and the other is a compensating multiplet. Here $H_{\alpha(n)} := H_{\alpha_1 \dots \alpha_n} = H_{(\alpha_1 \dots \alpha_n)}$ is a real symmetric rank- n spinor, which is defined modulo gauge transformations of the form⁷

$$\delta_\zeta H_{\alpha(n)} = i^n (-1)^{\lfloor n/2 \rfloor} \nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_n)}, \quad (3.3)$$

where the gauge parameter $\zeta_{\alpha(n-1)}$ is a real unconstrained superfield, and $\lfloor x \rfloor$ stands for the floor function denoting the integer part of a real number $x \geq 0$.

A. Massless half-integer superspin theories

There exist two off-shell formulations for the massless superspin- $(s + \frac{1}{2})$ multiplet, which are referred to as longitudinal and transverse.

1. Longitudinal formulation

The longitudinal formulation is realized in terms of the real unconstrained dynamical variables

$$\mathcal{V}_{(s+\frac{1}{2})}^{\parallel} = \{H_{\alpha(2s+1)}, L_{\alpha(2s-2)}\}, \quad (3.4)$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s+1)} = i \nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s+1})}, \quad (3.5a)$$

$$\delta L_{\alpha(2s-2)} = -\frac{s}{2(2s+1)} \nabla^{\beta\gamma} \zeta_{\beta\gamma\alpha(2s-2)}, \quad (3.5b)$$

where the real gauge parameter $\zeta_{\alpha(2s)}$ is unconstrained. The unique gauge-invariant action formulated in terms of the superfields (3.4) takes the following form:

$$\begin{aligned} S_{(s+\frac{1}{2})}^{\parallel} [H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] &= \left(-\frac{1}{2}\right)^s \int d^3|2zE \left\{ -\frac{i}{2} H^{\alpha(2s+1)} \mathbb{Q} H_{\alpha(2s+1)} - \frac{i}{8} \nabla_\beta H^{\beta\alpha(2s)} \nabla^2 \nabla^\gamma H_{\gamma\alpha(2s)} \right. \\ &\quad + \frac{i s}{4} \nabla_{\beta\gamma} H^{\beta\gamma\alpha(2s-1)} \nabla^{\rho\delta} H_{\rho\delta\alpha(2s-1)} + (2s-1) L^{\alpha(2s-2)} \nabla^{\beta\gamma} \nabla^\delta H_{\beta\gamma\delta\alpha(2s-2)} \\ &\quad + 2(2s-1) \left(L^{\alpha(2s-2)} (i \nabla^2 - 4|\mu|) L_{\alpha(2s-2)} - \frac{i}{s} (s-1) \nabla_\beta L^{\beta\alpha(2s-3)} \nabla^\gamma L_{\gamma\alpha(2s-3)} \right) \\ &\quad \left. + |\mu| \left(s \nabla_\beta H^{\beta\alpha(2s)} \nabla^\gamma H_{\gamma\alpha(2s)} + \frac{1}{2} (2s+1) H^{\alpha(2s+1)} (\nabla^2 - 4i|\mu|) H_{\alpha(2s+1)} \right) \right\}, \end{aligned} \quad (3.6)$$

⁷In the flat superspace limit this gauge transformation reduces to that introduced in [39,41].

where $\mathbb{Q}$ is the quadratic Casimir operator (2.5). The action (3.6) was derived in [37]. In the flat superspace limit, $|\mu| \rightarrow 0$, the action (3.6) coincides with the model derived in [39].

2. Transverse formulation

The transverse formulation is constructed in terms of the real superfields

$$\mathcal{V}_{(s+\frac{1}{2})}^\perp = \{H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}\}, \quad (3.7)$$

where $\Upsilon_{\beta;\alpha(2s-2)}$ is a reducible superfield pertaining to the representation $\mathbf{2} \otimes (\mathbf{2s-1})$ of $\text{SL}(2, \mathbb{R})$. The superfield $\Upsilon_{\beta;\alpha(2s-2)}$ can be decomposed into irreducible components by the following rule:

$$\begin{aligned} S_{(s+\frac{1}{2})}^\perp [H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}] &= \left(-\frac{1}{2}\right)^s \int d^3|2zE \left\{ -\frac{i}{2} H^{\alpha(2s+1)} \mathbb{Q} H_{\alpha(2s+1)} - \frac{i}{8} \nabla_\beta H^{\beta\alpha(2s)} \nabla^2 \nabla^\gamma H_{\gamma\alpha(2s)} + \frac{i}{8} \nabla_{\beta\gamma} H^{\beta\gamma\alpha(2s-1)} \nabla^{\rho\delta} H_{\rho\delta\alpha(2s-1)} \right. \\ &\quad - \frac{i}{4} (2s-1) \Omega_{\beta;\alpha(2s-2)} \nabla^{\gamma\delta} H_{\gamma\delta\beta\alpha(2s-2)} + i s (2s-1) |\mu| \Upsilon_{\beta;\alpha(2s-2)} \Omega_{\beta;\alpha(2s-2)} - \frac{i}{8} (2s-1) (\Omega_{\beta;\alpha(2s-2)} \Omega_{\beta;\alpha(2s-2)} \\ &\quad \left. - 2(s-1) \Omega_{\beta;\alpha(2s-3)} \Omega_{\gamma;\alpha(2s-3)} + |\mu| \left(H^{\alpha(2s+1)} (\nabla^2 - 4i|\mu|) H_{\alpha(2s+1)} + \frac{1}{2} \nabla_\beta H^{\beta\alpha(2s)} \nabla^\gamma H_{\gamma\alpha(2s)} \right) \right\}, \end{aligned} \quad (3.10)$$

where $\Omega_{\beta;\alpha(2s-2)}$ corresponds to the real $\mathcal{N} = 1$ field strength

$$\begin{aligned} \Omega_{\beta;\alpha(2s-2)} &= -i(\nabla^\gamma \nabla_\beta - 4i|\mu| \delta_\beta^\gamma) \Upsilon_{\gamma;\alpha(2s-2)}, \\ \nabla^\beta \Omega_{\beta;\alpha(2s-2)} &= 0. \end{aligned} \quad (3.11)$$

The above theory was introduced in [30].

B. Massless integer superspin theories

Analogous to the half-integer case, there exist two off-shell formulations for the massless superspin- s multiplet, which are referred to as longitudinal and transverse.

1. Longitudinal formulation

The longitudinal formulation is realized in terms of the real unconstrained variables

$$\begin{aligned} S_{(s)}^\parallel [H_{\alpha(2s)}, V_{\alpha(2s-2)}] &= \left(-\frac{1}{2}\right)^s \int d^3|2zE \left\{ \frac{1}{2} H^{\alpha(2s)} (i\nabla^2 + 4|\mu|) H_{\alpha(2s)} \right. \\ &\quad - \frac{i}{2} \nabla_\beta H^{\beta\alpha(2s-1)} \nabla^\gamma H_{\gamma\alpha(2s-1)} - (2s-1) V^{\alpha(2s-2)} \nabla^{\beta\gamma} H_{\beta\gamma\alpha(2s-2)} \\ &\quad \left. + (2s-1) \left(\frac{1}{2} V^{\alpha(2s-2)} (i\nabla^2 + 8s|\mu|) V_{\alpha(2s-2)} + i(s-1) \nabla_\beta V^{\beta\alpha(2s-3)} \nabla^\gamma V_{\gamma\alpha(2s-3)} \right) \right\}. \end{aligned} \quad (3.14)$$

$$\Upsilon_{\beta;\alpha(2s-2)} = \Upsilon_{(\beta;\alpha_1 \dots \alpha_{2s-2})} + \frac{1}{2s-1} \sum_{k=1}^{2s-2} \epsilon_{\beta\alpha_k} \Upsilon_{\gamma;\alpha_1 \dots \hat{\alpha}_k \dots \alpha_{2s-2}}. \quad (3.8)$$

The dynamical variables (3.7) possess the following gauge freedom:

$$\delta H_{\alpha(2s+1)} = i\nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s+1})}, \quad (3.9a)$$

$$\delta \Upsilon_{\beta;\alpha(2s-2)} = \frac{i}{2s+1} (\nabla^\gamma \zeta_{\gamma\beta\alpha(2s-2)} + (2s+1) \nabla_\beta \eta_{\alpha(2s-2)}), \quad (3.9b)$$

where the gauge parameters $\zeta_{\alpha(2s)}$ and $\eta_{\alpha(2s-2)}$ are real unconstrained. The unique quadratic action which is invariant under gauge transformations (3.9) takes the form

$$\mathcal{V}_{(s)}^\parallel = \{H_{\alpha(2s)}, V_{\alpha(2s-2)}\}, \quad (3.12)$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s})}, \quad (3.13a)$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s} \nabla^\beta \zeta_{\beta\alpha(2s-2)}, \quad (3.13b)$$

where the gauge parameter $\zeta_{\alpha(2s-1)}$ is real unconstrained. The unique action that is invariant under gauge transformations (3.13) assumes the form

Up to normalization, action (3.14) coincides with the off-shell $\mathcal{N} = 1$ supersymmetric action for the massless super-spin- s multiplet derived in [37]. Taking the flat-superspace limit, action (3.14) reduces to the model derived in [39].

2. Transverse formulation

The transverse formulation is constructed in terms of the real unconstrained superfields

$$\mathcal{V}_{(s)}^\perp = \{H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}\}, \quad (3.15)$$

$$\begin{aligned} S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}] = & \left(-\frac{1}{2}\right)^s \int d^3|z| E \left\{ \frac{1}{2} H^{\alpha(2s)} (i\nabla^2 + 8s|\mu|) H_{\alpha(2s)} - is \nabla_\beta H^{\beta\alpha(2s-1)} \nabla^\gamma H_{\gamma\alpha(2s-1)} \right. \\ & - (2s-1) \mathcal{W}^{\beta;\alpha(2s-2)} \nabla^\gamma H_{\gamma\beta\alpha(2s-2)} - \frac{i}{2} (2s-1) \left(\mathcal{W}^{\beta;\alpha(2s-2)} \mathcal{W}_{\beta;\alpha(2s-2)} + \frac{s-1}{s} \mathcal{W}_{\beta;\gamma\alpha(2s-3)}^{\beta\alpha(2s-3)} \mathcal{W}^{\gamma\alpha(2s-3)}_{\gamma\alpha(2s-3)} \right) \\ & \left. - 2i(2s-1)|\mu| \Psi^{\beta;\alpha(2s-2)} \mathcal{W}_{\beta;\alpha(2s-2)} \right\}, \end{aligned} \quad (3.17)$$

where $\mathcal{W}_{\beta;\alpha(2s-2)}$ denotes the real $\mathcal{N} = 1$ field strength

$$\begin{aligned} \mathcal{W}_{\beta;\alpha(2s-2)} &= -i(\nabla^\gamma \nabla_\beta - 4i|\mu| \delta_\beta^\gamma) \Psi_{\gamma;\alpha(2s-2)}, \\ \nabla^\beta \mathcal{W}_{\beta;\alpha(2s-2)} &= 0. \end{aligned} \quad (3.18)$$

For $s > 1$, the ζ -gauge freedom (3.16) can be used to impose the gauge condition

$$\Psi_{(\alpha_1;\alpha_2\cdots\alpha_{2s-1})} = 0 \Leftrightarrow \Psi_{\beta;\alpha(2s-2)} = \sum_{k=1}^{2s-2} \varepsilon_{\beta\alpha_k} \varphi_{\alpha_1\cdots\hat{\alpha}_k\cdots\alpha_{2s-2}}, \quad (3.19)$$

for some superfield $\varphi_{\alpha(2s-3)}$. The residual gauge freedom is characterized by

$$\zeta_{\alpha(2s-1)} = i\nabla_{(\alpha_1} \eta_{\alpha_2\cdots\alpha_{2s-1})}, \quad (3.20)$$

which have the following gauge freedom:

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1} \zeta_{\alpha_2\cdots\alpha_{2s})}, \quad (3.16a)$$

$$\delta \Psi_{\beta;\alpha(2s-2)} = -\zeta_{\beta\alpha(2s-2)} + i\nabla_\beta \eta_{\alpha(2s-2)}, \quad (3.16b)$$

where the gauge parameters $\zeta_{\alpha(2s-1)}$ and $\eta_{\alpha(2s-2)}$ are real unconstrained. The gauge-invariant action is given by

which means that $\eta_{\alpha(2s-2)}$ is the only independent gauge parameter. As a consequence, the model can be reformulated in terms of the following gauge superfields:

$$\{H_{\alpha(2s)}, \varphi_{\alpha(2s-3)}\}, \quad (3.21)$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s)} = -\nabla_{(\alpha_1} \eta_{\alpha_2\cdots\alpha_{2s})}, \quad (3.22)$$

$$\delta \varphi_{\alpha(2s-3)} = i\nabla^\beta \eta_{\beta\alpha(2s-3)}. \quad (3.23)$$

The corresponding gauge-invariant action assumes the following form:

$$\begin{aligned} S_{(s)}^\perp[H_{\alpha(2s)}, \varphi_{\alpha(2s-3)}] = & \left(-\frac{1}{2}\right)^s \int d^3|z| E \left\{ \frac{1}{2} H^{\alpha(2s)} (i\nabla^2 + 8s|\mu|) H_{\alpha(2s)} \right. \\ & - is \nabla_\beta H^{\beta\alpha(2s-1)} \nabla^\gamma H_{\gamma\alpha(2s-1)} - 2(s-1) \varphi^{\alpha(2s-3)} \nabla^\beta \nabla^\gamma \delta H_{\beta\gamma\delta\alpha(2s-3)} \\ & + \frac{1}{s} (s-1) \left(8i(2s-1)|\mu|^2 \varphi^{\alpha(2s-3)} \varphi_{\alpha(2s-3)} + 2(2s-3)|\mu| \varphi^{\alpha(2s-3)} \nabla^2 \varphi_{\alpha(2s-3)} \right. \\ & - 2i\varphi^{\alpha(2s-3)} \mathbb{Q} \varphi_{\alpha(2s-3)} + \frac{i}{2(2s-1)} (2s-3) \nabla_\beta \varphi^{\beta\alpha(2s-4)} \nabla^2 \nabla^\gamma \varphi_{\gamma\alpha(2s-4)} \\ & + \frac{4}{2s-1} (2s-3)(3s-2)|\mu| \nabla_\beta \varphi^{\beta\alpha(2s-4)} \nabla^\gamma \varphi_{\gamma\alpha(2s-4)} \\ & \left. \left. - \frac{i}{2s-1} (2s-3)(s-2) \nabla_{\beta\gamma} \varphi^{\beta\gamma\alpha(2s-5)} \nabla^{\rho\sigma} \varphi_{\rho\sigma\alpha(2s-5)} \right) \right\}. \end{aligned} \quad (3.24)$$

The above theory was proposed in [30]. In the flat-superspace limit, the action (3.24) coincides with the $\mathcal{N} = 1$ model (B.25) presented in [23].

IV. (1,1) $\rightarrow$ (1,0) AdS SUPERSPACE REDUCTION

In this section, we begin by reviewing the necessary aspects of (1, 1) AdS superspace which are then utilized to develop a consistent reduction procedure for field theories with (1,1) AdS supersymmetry to $\mathcal{N} = 1$ AdS superspace.

A. (1, 1) AdS superspace: Complex basis

We summarize the key results concerning (1, 1) AdS superspace [18,23] and superfield representations of the associated isometry group $\text{OSp}(1|2; \mathbb{R}) \times \text{OSp}(1|2; \mathbb{R})$, following the presentation in [23]. The geometry of (1, 1) AdS superspace, $\text{AdS}^{(3|1,1)}$, can be described using either a real or a complex basis for the spinor covariant derivatives. In this subsection we consider the formulation in the complex basis.

The covariant derivatives of $\text{AdS}^{(3|1,1)}$ have the form

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^\alpha) = E_A + \Omega_A, \quad (4.1)$$

where $z^{\mathcal{M}} = (x^m, \theta^\mu, \bar{\theta}_\mu)$ are local complex superspace coordinates, while E_A and Ω_A denote the inverse supervielbein and Lorentz connection,

$$E_A = E_A^{\mathcal{M}} \frac{\partial}{\partial z^{\mathcal{M}}}, \quad \Omega_A = \frac{1}{2} \Omega_A^{bc} M_{bc} = \frac{1}{2} \Omega_A^{\beta\gamma} M_{\beta\gamma}. \quad (4.2)$$

The covariant derivatives satisfy the following algebra:

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = -2i\mathcal{D}_{\alpha\beta}, \quad (4.3a)$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{\mu}M_{\alpha\beta}, \quad \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 4\mu M_{\alpha\beta}, \quad (4.3b)$$

$$[\mathcal{D}_{\alpha\beta}, \mathcal{D}_\gamma] = -2i\bar{\mu}\varepsilon_{\gamma(\alpha}\bar{\mathcal{D}}_{\beta)}, \quad [\mathcal{D}_{\alpha\beta}, \bar{\mathcal{D}}_\gamma] = 2i\mu\varepsilon_{\gamma(\alpha}\mathcal{D}_{\beta)}, \quad (4.3c)$$

$$[\mathcal{D}_{\alpha\beta}, \mathcal{D}_{\gamma\delta}] = 4\bar{\mu}\mu(\varepsilon_{\gamma(\alpha}M_{\beta)\delta} + \varepsilon_{\delta(\alpha}M_{\beta)\gamma}), \quad (4.3d)$$

with $\mu \neq 0$ being a complex parameter determining the constant curvature of $\text{AdS}^{(3|1,1)}$. The phase of $\mu = |\mu|e^{i\varphi}$ can be given any fixed value by a redefinition $\mathcal{D}_\alpha \rightarrow e^{i\varphi}\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_\alpha \rightarrow e^{-i\varphi}\bar{\mathcal{D}}_\alpha$, with ρ constant.

Let $G_{\alpha(n)}$ be a symmetric rank- n spinor superfield. It is said to be longitudinal linear if it obeys the following first-order constraint

$$\bar{\mathcal{D}}_{(\alpha_1} G_{\alpha_2 \dots \alpha_{n+1})} = 0, \quad (4.4a)$$

which implies

$$(\bar{\mathcal{D}}^2 + 2n\mu)G_{\alpha(n)} = 0. \quad (4.4b)$$

In the scalar case, $n = 0$, the constraint (4.4a) becomes the condition of covariant chirality, $\bar{\mathcal{D}}_\alpha G = 0$.

A symmetric rank- n spinor superfield $\Gamma_{\alpha(n)}$ is called transverse linear if it obeys the first-order constraint

$$\bar{\mathcal{D}}^\beta \Gamma_{\beta\alpha_1 \dots \alpha_{n-1}} = 0, \quad n \neq 0, \quad (4.5a)$$

which implies

$$(\bar{\mathcal{D}}^2 - 2(n+2)\mu)\Gamma_{\alpha(n)} = 0. \quad (4.5b)$$

In the $n = 0$ case, the constraint (4.5a) is not defined. However, its corollary (4.5b) is perfectly consistent,

$$(\bar{\mathcal{D}}^2 - 4\mu)\Gamma = 0, \quad (4.6)$$

and defines a covariantly linear scalar superfield Γ .

The constraints (4.4a) and (4.5a) may be solved in terms of prepotentials $\Psi_{\alpha(n-1)}$ and $\Phi_{\alpha(n+1)}$ as follows:

$$G_{\alpha(n)} = \bar{\mathcal{D}}_{(\alpha_1} \Psi_{\alpha_2 \dots \alpha_n)}, \quad (4.7a)$$

$$\Gamma_{\alpha(n)} = \bar{\mathcal{D}}^\beta \Phi_{(\beta\alpha_1 \dots \alpha_n)}. \quad (4.7b)$$

Provided the constraints (4.4a) and (4.5a) are the only conditions imposed on $G_{\alpha(n)}$ and $\Gamma_{\alpha(n)}$, respectively, the prepotentials $\Psi_{\alpha(n-1)}$ and $\Phi_{\alpha(n+1)}$ can be chosen to be unconstrained complex. They are defined modulo gauge transformations of the form

$$\delta_\zeta \Psi_{\alpha(n-1)} = \bar{\mathcal{D}}_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{n-1})}, \quad (4.8a)$$

$$\delta_\xi \Phi_{\alpha(n+1)} = \bar{\mathcal{D}}^\gamma \xi_{(\gamma\alpha_1 \dots \alpha_{n+1})}, \quad (4.8b)$$

with the gauge parameters $\zeta_{\alpha(n-2)}$ and $\xi_{\alpha(n+2)}$ being complex unconstrained.

One can define projectors $P_n^\perp$ and $P_n^\parallel$ on the spaces of transverse linear and longitudinal linear superfields, respectively. The projectors are

$$P_n^\perp = \frac{1}{4(n+1)\mu}(\bar{\mathcal{D}}^2 + 2n\mu), \quad (4.9a)$$

$$P_n^\parallel = -\frac{1}{4(n+1)\mu}(\bar{\mathcal{D}}^2 - 2(n+2)\mu), \quad (4.9b)$$

with the properties

$$(P_n^\perp)^2 = P_n^\perp, \quad (P_n^\parallel)^2 = P_n^\parallel, \quad P_n^\perp P_n^\parallel = P_n^\parallel P_n^\perp = 0. \quad (4.10)$$

These projectors are three-dimensional (3D) cousins of those introduced by Ivanov and Sorin [42] in the case of $\mathcal{N} = 1$ AdS supersymmetry in four dimensions.

Let $V_{\alpha(n)}$ ($n \neq 0$) be an arbitrary complex tensor superfield. It can be represented as a sum of transverse linear and longitudinal linear multiplets [23]

$$V_{\alpha(n)} = -\frac{1}{2\mu(n+2)} \bar{\mathcal{D}}^\gamma \bar{\mathcal{D}}_{(\gamma} V_{\alpha_1 \dots \alpha_n)} - \frac{1}{2\mu(n+1)} \bar{\mathcal{D}}_{(\alpha_1} \bar{\mathcal{D}}^{|\gamma|} V_{\alpha_2 \dots \alpha_n) \gamma}, \quad (4.11)$$

where indices placed between vertical bars (for example, $|\gamma|$) are not subject to symmetrisation. If we choose $V_{\alpha(n)}$ to be either longitudinal linear ($G_{\alpha(n)}$) or transverse linear ($\Gamma_{\alpha(n)}$), then the above identity produces the relations (4.7a) and (4.7b) for some prepotentials $\Psi_{\alpha(n-1)}$ and $\Phi_{\alpha(n+1)}$, respectively.

The isometry transformations of $\text{AdS}^{(3|1,1)}$ are generated by real supervector fields $\lambda^A E_A$ which solve the Killing equation

$$\left[\Lambda + \frac{1}{2} l^{ab} M_{ab}, \mathcal{D}_C \right] = 0, \quad (4.12)$$

where

$$\Lambda = \lambda^A \mathcal{D}_A = \lambda^a \mathcal{D}_a + \bar{\lambda}_\alpha \bar{\mathcal{D}}^\alpha, \quad \bar{\lambda}^a = \lambda^a, \quad (4.13)$$

and l^{ab} is some local Lorentz parameter. As shown in [18], this equation implies that the parameters λ^a and l^{ab} can be uniquely determined in terms of the vector λ^a ,

$$\lambda_\alpha = \frac{i}{6} \bar{\mathcal{D}}^\beta \lambda_{\alpha\beta}, \quad l_{\alpha\beta} = 2\mathcal{D}_{(\alpha} \lambda_{\beta)}, \quad (4.14)$$

and the vector parameter obeys the equation

$$\mathcal{D}_{(\alpha} \lambda_{\beta\gamma)} = 0 \Leftrightarrow \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta\gamma)} = 0. \quad (4.15)$$

In comparison with the 3D $\mathcal{N} = 2$ Minkowski superspace, the specific feature of $\text{AdS}^{(3|1,1)}$ is that any two of the three parameters $\{\lambda_{\alpha\beta}, \lambda_\alpha, l_{\alpha\beta}\}$ can be expressed in terms of the third parameter. In particular,

$$\lambda_{\alpha\beta} = \frac{i}{\mu} \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta)}, \quad \lambda_\alpha = \frac{1}{12\mu} \mathcal{D}^\beta l_{\alpha\beta}. \quad (4.16)$$

From (4.14) and (4.16) we deduce

The solution to these equations is given in [18].

B. (1,1) AdS superspace: Real basis

It proves beneficial to realize the (1,1) AdS covariant derivatives in a real basis when performing the reduction procedure to $\mathcal{N} = 1$ AdS superspace. In accordance with [16], the algebra of covariant derivative (4.3) can be converted to the real basis by (i) making a convenient choice $\mu = -i|\mu|$; and (ii) replacing the complex operators $\mathcal{D}_\alpha, \bar{\mathcal{D}}_\alpha$ with $\nabla_\alpha^I = (\nabla_\alpha^1, \nabla_\alpha^2)$ defined by

$$\mathcal{D}_\alpha = \frac{1}{\sqrt{2}} (\nabla_\alpha^1 - i\nabla_\alpha^2), \quad \bar{\mathcal{D}}_\alpha = -\frac{1}{\sqrt{2}} (\nabla_\alpha^1 + i\nabla_\alpha^2). \quad (4.18)$$

Furthermore, we introduce the real coordinates $z^{\mathcal{M}} = (x^m, \theta_I^\mu)$ which are used to parametrize (1,1) AdS superspace. Choosing to define $\nabla_a = \mathcal{D}_a$, it can be shown that the algebra of (1,1) AdS covariant derivatives assumes the following form in the real basis (4.18):

$$\{\nabla_a^1, \nabla_b^2\} = 0, \quad (4.19a)$$

$$\begin{aligned} \{\nabla_a^1, \nabla_b^1\} &= 2i\nabla_{ab} - 4i|\mu|M_{ab}, \\ \{\nabla_a^2, \nabla_b^2\} &= 2i\nabla_{ab} + 4i|\mu|M_{ab}, \end{aligned} \quad (4.19b)$$

$$[\nabla_a, \nabla_b^1] = |\mu|(\gamma_a)_\alpha{}^\beta \nabla_\beta^1, \quad [\nabla_a, \nabla_b^2] = -|\mu|(\gamma_a)_\alpha{}^\beta \nabla_\beta^2, \quad (4.19c)$$

$$[\nabla_a, \nabla_b] = -4|\mu|^2 M_{ab}. \quad (4.19d)$$

It is apparent from (4.19) that the operators ∇_a and ∇_a^I possess the following properties:

1. These operators form a closed algebra given by

$$\{\nabla_a^1, \nabla_b^1\} = 2i\nabla_{ab} - 4i|\mu|M_{ab}, \quad (4.20a)$$

$$[\nabla_a, \nabla_b^1] = |\mu|(\gamma_a)_\beta{}^\gamma \nabla_\gamma^1, \quad [\nabla_a, \nabla_b] = -4|\mu|^2 M_{ab}. \quad (4.20b)$$

2. Relations (4.20) coincide with the algebra of covariant derivatives of $\text{AdS}^{3|2}$, Eq. (2.2).

These properties imply that $\text{AdS}^{3|2}$ is naturally realized as a surface embedded in (1,1) AdS superspace. One can make an appropriate choice in the real Grassmann variables $\theta_I^\mu = (\theta_1^\mu, \theta_2^\mu)$ such that $\text{AdS}^{3|2}$ can be identified as the surface defined by $\theta_2^\mu = 0$ in $\text{AdS}^{(3|1,1)}$. These properties enable the

consistent reduction of any field theory with (1,1) AdS supersymmetry to $\mathcal{N} = 1$ AdS superspace.

We now wish to recast the fundamental properties of the Killing supervector fields of (1,1) AdS superspace (4.13) in the real representation (4.18). The isometries of (1,1) AdS superspace are generated by the (1,1) AdS Killing supervector fields,

$$\Lambda := \lambda^A \nabla_A = \lambda^a \nabla_a + \lambda_I^a \nabla_a^I, \quad I = \underline{1}, \underline{2}, \quad (4.21)$$

which are defined to satisfy the Killing equation

$$\left[\Lambda + \frac{1}{2} l^{ab} M_{ab}, \nabla_A \right] = 0, \quad (4.22)$$

for some real Lorentz parameter $l^{ab} = -l^{ba}$. It can be shown that Eq. (4.22) is equivalent to the set of equations

$$\nabla_a^1 \lambda_\beta^1 = \frac{1}{2} l_{\alpha\beta} + |\mu| \lambda_{\alpha\beta}, \quad \nabla_a^1 \lambda_\beta^2 = 0, \quad (4.23a)$$

$$\nabla_a^2 \lambda_\beta^2 = \frac{1}{2} l_{\alpha\beta} - |\mu| \lambda_{\alpha\beta}, \quad \nabla_a^2 \lambda_\beta^1 = 0, \quad (4.23b)$$

$$\nabla_a^1 l_{\beta\gamma} = 8i|\mu| \varepsilon_{\alpha(\beta} \lambda_{\gamma)}^1, \quad \nabla_a^2 l_{\beta\gamma} = -8i|\mu| \varepsilon_{\alpha(\beta} \lambda_{\gamma)}^2, \quad (4.23c)$$

$$\nabla_a^I \lambda_a = 2i(\gamma_a)_{\alpha\beta} \lambda^{I\beta}, \quad (4.23d)$$

and

$$\nabla_a \lambda_b = l_{ab}, \quad (4.24a)$$

$$\nabla_a \lambda_\alpha^1 = |\mu| (\gamma_a)_\alpha{}^\beta \lambda_\beta^1, \quad \nabla_a \lambda_\alpha^2 = -|\mu| (\gamma_a)_\alpha{}^\beta \lambda_\beta^2, \quad (4.24b)$$

$$\nabla_a l_{bc} = 8|\mu|^2 \eta_{a[b} \lambda_{c]}, \quad (4.24c)$$

Equations (4.23) and (4.24) can be recast in the equivalent form

$$\nabla_{(\alpha}^I \lambda_{\beta\gamma)} = 0, \quad \nabla_{(\alpha}^I l_{\beta\gamma)} = 0, \quad (4.25a)$$

$$\nabla_{(\alpha}^1 \lambda_{\beta)}^1 = \frac{1}{2} l_{\alpha\beta} + |\mu| \lambda_{\alpha\beta}, \quad \nabla_{(\alpha}^2 \lambda_{\beta)}^2 = \frac{1}{2} l_{\alpha\beta} - |\mu| \lambda_{\alpha\beta}, \quad (4.25b)$$

$$\nabla_{(\alpha}^1 \lambda_{\beta)}^2 = \nabla_{(\alpha}^2 \lambda_{\beta)}^1 = 0, \quad \nabla^{\alpha(I} l_{\alpha}^{J)} = 0, \quad (4.25c)$$

$$\lambda^{1\alpha} = \frac{i}{6} \nabla_\beta^1 \lambda^{\alpha\beta} = \frac{i}{12|\mu|} \nabla_\beta^1 l^{\alpha\beta}, \quad (4.25d)$$

$$\lambda^{2\alpha} = \frac{i}{6} \nabla_\beta^2 \lambda^{\alpha\beta} = -\frac{i}{12|\mu|} \nabla_\beta^2 l^{\alpha\beta}. \quad (4.25e)$$

It follows from (4.24a) that the parameter λ_a is a Killing vector field

$$\nabla_a \lambda_b + \nabla_b \lambda_a = 0, \quad (4.26)$$

and relations (4.24b) are Killing spinor equations.

C. Reduction from (1,1) to $\mathcal{N} = 1$ AdS superspace

Given a tensor superfield $U(x, \theta_I)$ on (1,1) AdS superspace, where indices have been suppressed, we define its bar projection to $\mathcal{N} = 1$ AdS superspace by the rule

$$U| := U(x, \theta_I)|_{\theta_{\underline{2}}=0} \quad (4.27)$$

in a *special coordinate system*, which will be described below. Given the (1,1) covariant derivative in the real representation (4.18)

$$\nabla_A = (\nabla_a, \nabla_a^I) = E_A^{\mathcal{M}} \frac{\partial}{\partial z^{\mathcal{M}}} + \frac{1}{2} \Omega_A^{bc} M_{bc}, \quad (4.28)$$

we define its $\mathcal{N} = 1$ projection by the rule

$$\nabla_A| = E_A^{\mathcal{M}} \frac{\partial}{\partial z^{\mathcal{M}}} + \frac{1}{2} \Omega_A^{bc} M_{bc}. \quad (4.29)$$

We use the freedom to perform general coordinate and local Lorentz transformations to impose the gauge

$$\nabla_a| = \nabla_a, \quad \nabla_a^1| = \nabla_a, \quad (4.30)$$

where $\nabla_A = (\nabla_a, \nabla_a^I)$ is the set of covariant derivatives for $\text{AdS}^{3|2}$; see Eq. (2.1). In the chosen coordinate system, the operator ∇_a^1 does not involve any partial derivatives with respect to $\theta_{\underline{2}}$. Thus for any positive integer k , it follows that $(\nabla_{\alpha_1}^1 \cdots \nabla_{\alpha_k}^1 U)| = \nabla_{\alpha_1} \cdots \nabla_{\alpha_k} U|$.

We now consider the $\mathcal{N} = 1$ projection of the (1,1) AdS Killing supervector (4.21)

$$\Lambda| = \xi^a \nabla_a + \xi^\alpha \nabla_\alpha + \epsilon^\alpha \nabla_\alpha^2, \quad (4.31)$$

where we have introduced the $\mathcal{N} = 1$ superfields

$$\xi^a := \lambda^a|, \quad \xi^\alpha := \lambda_\alpha^1|, \quad \epsilon^\alpha := \lambda_\alpha^2|. \quad (4.32)$$

Additionally, we define the $\mathcal{N} = 1$ projection of the Lorentz parameter l^{ab} to be

$$\zeta^{ab} := l^{ab}|. \quad (4.33)$$

It is important to note that the superfields $(\xi^a, \xi^\alpha, \zeta^{ab})$ parametrize the infinitesimal isometries of $\text{AdS}^{3|2}$. Such transformations are generated by the Killing supervector fields, $\xi = \xi^a \nabla_a + \xi^\alpha \nabla_\alpha$, satisfying the $\mathcal{N} = 1$ Killing

equation, Eq. (2.8). Indeed, the relations (2.9) and (2.10) automatically follow from the (1,1) AdS Killing equations (4.25), upon projection. The parameter ϵ_α , which generates the second supersymmetry transformation, has the property

$$\nabla_a \epsilon_\alpha = -|\mu|(\gamma_a)_\alpha{}^\beta \epsilon_\beta. \quad (4.34)$$

Given the transformation law of a tensor superfield $U(x, \theta_I)$ on (1,1) AdS superspace

$$\delta_\Lambda U = \left(\Lambda + \frac{1}{2} I^{ab} M_{ab} \right) U, \quad (4.35)$$

we find its projection to $\mathcal{N} = 1$ AdS superspace to be

$$\delta_\Lambda U| = \delta_\xi U| + \delta_\epsilon U|, \quad (4.36)$$

where

$$\delta_\xi U| = \left(\xi^a \nabla_a + \xi^\alpha \nabla_\alpha + \frac{1}{2} \zeta^{ab} M_{ab} \right) U|, \quad (4.37a)$$

$$\delta_\epsilon U| = \epsilon^\alpha (\nabla_\alpha^2 U)|. \quad (4.37b)$$

The first transformation (4.37a) coincides with the infinitesimal transformation generated by a Killing supervector in $\text{AdS}^{3|2}$, Eq. (2.11). Thus, $U|$ can be identified as a tensor superfield on $\mathcal{N} = 1$ AdS superspace. The other transformation (4.37b) corresponds to the second supersymmetry transformation, which is generated by ϵ^α .

D. The (1,1) AdS supersymmetric actions in $\text{AdS}^{3|2}$

Every supersymmetric field theory in (1,1) AdS superspace can be reduced to $\mathcal{N} = 1$ AdS superspace. In the following subsection, we explore the necessary mathematical framework which will be employed to develop such a reduction procedure. As presented in [17–19,22], manifestly supersymmetric actions in (1,1) AdS superspace can be constructed by

1. Integrating a real scalar Lagrangian $\mathcal{L}$ over the full (1,1) AdS superspace,

$$\begin{aligned} & \int d^3x d^2\theta d^2\bar{\theta} E \mathcal{L} \\ &= \frac{1}{16} \int d^3x e(\mathcal{D}^2 - 16\bar{\mu})(\bar{\mathcal{D}}^2 - 4\mu) \mathcal{L}|_{\theta=0} \\ &= \int d^3x e \left(\frac{1}{16} \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\alpha - \frac{\mu}{4} \mathcal{D}^2 \right. \\ & \quad \left. - \frac{\bar{\mu}}{4} \bar{\mathcal{D}}^2 + 4\mu\bar{\mu} \right) \mathcal{L}|_{\theta=0}, \end{aligned} \quad (4.38)$$

with $E^{-1} = \text{Ber}(E_{\mathcal{A}}^{\mathcal{M}})$.

2. Integrating a covariantly chiral Lagrangian $\mathcal{L}_c$ over the chiral subspace,

$$\begin{aligned} & \int d^3x d^2\theta \mathcal{E} \mathcal{L}_c = -\frac{1}{4} \int d^3x e (\mathcal{D}^2 - 16\bar{\mu}) \mathcal{L}_c|_{\theta=0}, \\ & \bar{\mathcal{D}}_\alpha \mathcal{L}_c = 0, \end{aligned} \quad (4.39)$$

where $\mathcal{E}$ is the chiral density. The two supersymmetric invariants are related by the rule

$$\begin{aligned} & \int d^3x d^2\theta d^2\bar{\theta} E \mathcal{L} = \int d^3x d^2\theta \mathcal{E} \mathcal{L}_c, \\ & \mathcal{L}_c := -\frac{1}{4} (\bar{\mathcal{D}}^2 - 4\mu) \mathcal{L}. \end{aligned} \quad (4.40)$$

In (1,1) AdS superspace, every chiral action can always be recast as an integral over the full superspace

$$\int d^3x d^2\theta \mathcal{E} \mathcal{L}_c = \frac{1}{\mu} \int d^3x d^2\theta d^2\bar{\theta} E \mathcal{L}_c. \quad (4.41)$$

We will use the notation $d^{3|4}z := d^3x d^2\theta d^2\bar{\theta}$ for the full superspace measure.

Instead of reducing the above supersymmetric actions to components, we wish to obtain a prescription that allows for their reduction to $\mathcal{N} = 1$ AdS superspace. The supersymmetric action in $\text{AdS}^{3|2}$ is described by a real scalar Lagrangian L

$$S = \int d^{3|2}z E L = \frac{1}{4} \int d^3x e (i\nabla^2 + 8|\mu|) L|_{\theta=0}. \quad (4.42)$$

The action (4.38) reduces to $\text{AdS}^{3|2}$ as follows:

$$\mathbb{S} = \int d^{3|4}z E \mathcal{L} = -\frac{i}{4} \int d^{3|2}z E \{ (\nabla^2)^2 + 8i|\mu| \} \mathcal{L}|. \quad (4.43)$$

By making use of the Killing equations (4.25) and (4.34), it can be shown that the action (4.43) is invariant under (1,1) AdS isometry transformations given by Eq. (4.36).

In the remainder of this paper, we will carry out the $(1,1) \rightarrow (1,0)$ AdS superspace reduction of the massless higher-spin supermultiplets and show that the reduced actions coincide with those presented in Sec. III.

V. MASSLESS HALF-INTEGER SUPERSPIN: TRANSVERSE FORMULATION

In (1,1) AdS superspace, there exist two off-shell formulations for the massless multiplet of half-integer superspin- $(s + \frac{1}{2})$, with $s \geq 2$ [23]. These two theories, which are called transverse and longitudinal, prove to be dual to each other. In the following section, we develop the

$(1, 1) \rightarrow (1, 0)$ reduction procedure for the transverse formulation.

A. Transverse formulation

According to [23], the transverse formulation for the massless superspin- $(s + \frac{1}{2})$ multiplet is described by the dynamical variables

$$\mathcal{V}_{(s+\frac{1}{2})}^\perp = \{\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}\}, \quad (5.1)$$

where $\mathfrak{H}_{\alpha(2s)} = \mathfrak{H}_{(\alpha_1 \dots \alpha_{2s})}$ is an unconstrained real superfield, and the complex superfield $\Gamma_{\alpha(2s-2)} = \Gamma_{(\alpha_1 \dots \alpha_{2s-2})}$ is transverse linear (4.5a). The superfields $\mathfrak{H}_{\alpha(2s)}$ and $\Gamma_{\alpha(2s-2)}$ are defined modulo gauge transformations of the form

$$\delta_\lambda \mathfrak{H}_{\alpha(2s)} = \bar{\mathcal{D}}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_{2s})} - \mathcal{D}_{(\alpha_1} \bar{\lambda}_{\alpha_2 \dots \alpha_{2s})} \equiv g_{\alpha(2s)} + \bar{g}_{\alpha(2s)}, \quad (5.2a)$$

$$\begin{aligned} \delta_\lambda \Gamma_{\alpha(2s-2)} &= -\frac{1}{4} \bar{\mathcal{D}}^\beta (\mathcal{D}^2 + 2(2s-1)\bar{\mu}) \bar{\lambda}_{\beta\alpha(2s-2)} \\ &\equiv \frac{s}{2s+1} \bar{\mathcal{D}}^\beta \mathcal{D}'_{\beta\gamma\alpha(2s-2)}, \end{aligned} \quad (5.2b)$$

where the gauge parameter $\lambda_{\alpha(2s-1)}$ is complex unconstrained. By construction, the complex gauge parameter $g_{\alpha(2s)} := \bar{\mathcal{D}}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_{2s})}$ is longitudinal linear (4.4a). Note that gauge transformation (5.2a) indicates that the superfield $\mathfrak{H}_{\alpha(2s)}$ corresponds to the $\mathcal{N} = 1$ superconformal gauge prepotential [24]. Up to normalization, there exists a unique quadratic action which is invariant under the gauge transformations (5.2). This action takes the form

$$\begin{aligned} \mathbb{S}_{(s+\frac{1}{2})}^\perp[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}] &= \left(-\frac{1}{2}\right)^s \int d^{3|4}z E \left\{ 2s(2s-1) \bar{\mu} \mu \mathfrak{H}^{\alpha(2s)} \mathfrak{H}_{\alpha(2s)} + \frac{1}{8} \mathfrak{H}^{\alpha(2s)} \mathcal{D}^\beta (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\beta \mathfrak{H}_{\alpha(2s)} \right. \\ &\quad + \mathfrak{H}^{\alpha(2s)} (\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}_{\alpha_2} \Gamma_{\alpha_3 \dots \alpha_{2s}} - \bar{\mathcal{D}}_{\alpha_1} \mathcal{D}_{\alpha_2} \bar{\Gamma}_{\alpha_3 \dots \alpha_{2s}}) + \frac{2s-1}{s} \bar{\Gamma}^{\alpha(2s-2)} \Gamma_{\alpha(2s-2)} \\ &\quad \left. + \frac{2s+1}{2s} (\Gamma^{\alpha(2s-2)} \Gamma_{\alpha(2s-2)} + \bar{\Gamma}^{\alpha(2s-2)} \bar{\Gamma}_{\alpha(2s-2)}) \right\}. \end{aligned} \quad (5.3)$$

The above construction does not take into consideration the case where $s = 1$, since the transverse linear constraint (4.5a) is ill-defined for $n = 0$. However, corollary (4.5b) is consistent for $n = 0$ and defines a covariantly transverse linear scalar superfield (4.5a). So Γ and $\bar{\Gamma}$ can be interpreted as compensators, which have the corresponding gauge transformations

$$\delta_\lambda \mathfrak{H}_{\alpha\beta} = \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta)} - \mathcal{D}_{(\alpha} \bar{\lambda}_{\beta)}, \quad (5.4a)$$

$$\delta_\lambda \Gamma = -\frac{1}{4} \bar{\mathcal{D}}^\beta (\mathcal{D}^2 + 2\bar{\mu}) \bar{\lambda}_\beta, \quad (5.4b)$$

as a result of (5.2). It is simple to show that constraint (4.5b) is consistent with the variation (5.4b). Choosing $s = 1$ in (5.3) yields a gauge-invariant action that can be identified as the linearized action for nonminimal (1,1) AdS supergravity [18].

B. Reduction of gauge prepotentials to AdS^{3|2}

We wish to reduce the gauge prepotentials (5.1) to $\mathcal{N} = 1$ AdS superspace. We start by reducing the superconformal gauge multiplet $\mathfrak{H}_{\alpha(2s)}$. Converting the longitudinal linear constraint of $g_{\alpha(2s)}$ (4.4a) to the real representation (4.18) yields

$$\nabla^2_{(\alpha_1} g_{\alpha_2 \dots \alpha_{2s+1})} = i \nabla^1_{(\alpha_1} g_{\alpha_2 \dots \alpha_{2s+1})}. \quad (5.5)$$

Performing a Taylor expansion of $g_{\alpha(2s)}(\theta^I)$ about θ^2 , and using (5.5), we find the independent θ^2 components of $g_{\alpha(2s)}$ to be

$$g_{\alpha(2s)}|, \quad \nabla^{2\beta} g_{\beta\alpha(2s-1)}|. \quad (5.6)$$

The gauge transformation (5.2a) allows us to impose the gauge conditions

$$\mathfrak{H}_{\alpha(2s)}| = 0, \quad \nabla^{2\beta} \mathfrak{H}_{\beta\alpha(2s-1)}| = 0. \quad (5.7)$$

In this Wess-Zumino (WZ) gauge, we stay with the unconstrained real $\mathcal{N} = 1$ superfields

$$H_{\alpha(2s+1)} := i \nabla^2_{(\alpha_1} \mathfrak{H}_{\alpha_2 \dots \alpha_{2s+1})}|, \quad (5.8a)$$

$$H_{\alpha(2s)} := \frac{i}{4} (\nabla^2)^2 \mathfrak{H}_{\alpha(2s)}|. \quad (5.8b)$$

The residual gauge freedom which preserves gauge conditions (5.7) are described by the unconstrained real $\mathcal{N} = 1$ superfields

$$g_{\alpha(2s)}| = -\frac{i}{2}\zeta_{\alpha(2s)}, \quad \bar{\zeta}_{\alpha(2s)} = \zeta_{\alpha(2s)}, \quad (5.9a)$$

$$\nabla^{2\beta} g_{\beta\alpha(2s-1)}| = \frac{2s+1}{2s}\zeta_{\alpha(2s-1)}, \quad \bar{\zeta}_{\alpha(2s-1)} = \zeta_{\alpha(2s-1)}. \quad (5.9b)$$

From (5.9), we can readily determine the gauge transformations of (5.8),

$$\delta H_{\alpha(2s+1)} = i\nabla_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s+1})}, \quad (5.10a)$$

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s})}. \quad (5.10b)$$

Next, we wish to reduce $\Gamma_{\alpha(2s-2)}$ to $\mathcal{N} = 1$ AdS superspace. The superfield $\Gamma_{\alpha(2s-2)}$ obeys the transverse linear constraint (4.5a), which takes the following form in the real representation (4.18):

$$\nabla^{2\beta}\Gamma_{\beta\alpha(2s-3)} = i\nabla^{1\beta}\Gamma_{\beta\alpha(2s-3)}. \quad (5.11)$$

It follows that $\Gamma_{\alpha(2s-2)}(\theta^I)$ has two independent θ^2 components

$$\Gamma_{\alpha(2s-2)}|, \quad \nabla^2_{(\alpha_1}\Gamma_{\alpha_2\cdots\alpha_{2s-1})}|. \quad (5.12)$$

Utilizing the gauge transformation of $\Gamma_{\alpha(2s-2)}$ (5.2b) and the real representation (4.18), we find

$$\delta\Gamma_{\alpha(2s-2)} = \frac{is}{2s+1}(\nabla^{1\beta}\nabla^{2\gamma} - \nabla^{\beta\gamma})\bar{g}_{\beta\gamma\alpha(2s-2)}, \quad (5.13a)$$

$$\begin{aligned} \nabla^2_{(\alpha_1}\delta\Gamma_{\alpha_2\cdots\alpha_{2s-1})} &= \frac{s}{2s+1}(\nabla^{1\beta}\nabla_{(\alpha_1}{}^{\gamma}\bar{g}_{\alpha_2\cdots\alpha_{2s-1})\beta\gamma} - (4s+1)|\mu|\nabla^{1\beta}\bar{g}_{\beta\alpha(2s-1)} + \nabla_{\beta}{}^{\gamma}\nabla^{1\beta}\bar{g}_{\gamma\alpha(2s-1)} - i\nabla_{\beta}{}^{\gamma}\nabla^{2\beta}\bar{g}_{\gamma\alpha(2s-1)} \\ &\quad - \frac{1}{2}(\nabla^1)^2\nabla^{2\beta}\bar{g}_{\beta\alpha(2s-1)} - (2s+1)i|\mu|\nabla^{2\beta}\bar{g}_{\beta\alpha(2s-1)} - i\nabla^{\beta\gamma}\nabla^2_{(\alpha_1}\bar{g}_{\alpha_2\cdots\alpha_{2s-1})\beta\gamma}). \end{aligned} \quad (5.13b)$$

From (5.13), we can immediately read off the gauge transformations of the complex $\mathcal{N} = 1$ superfields (5.12),

$$\delta\Gamma_{\alpha(2s-2)}| = -\frac{i}{2}\nabla^{\beta}\zeta_{\beta\alpha(2s-2)} + \frac{s}{2(2s+1)}\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-2)}, \quad (5.14a)$$

$$\begin{aligned} \nabla^2_{(\alpha_1}\delta\Gamma_{\alpha_2\cdots\alpha_{2s-1})}| &= \frac{s}{2(2s+1)}\left(i\nabla^{\beta}\nabla_{(\alpha_1}{}^{\gamma}\zeta_{\alpha_2\cdots\alpha_{2s-1})\beta\gamma} + i\nabla_{\beta}{}^{\gamma}\nabla^{\beta}\zeta_{\gamma\alpha(2s-1)} - (4s+1)i|\mu|\nabla^{\beta}\zeta_{\beta\alpha(2s-1)} \right. \\ &\quad \left. + \frac{2s+1}{s}\left((2s+1)i|\mu|\zeta_{\alpha(2s-1)} + i\nabla_{(\alpha_1}{}^{\beta}\zeta_{\alpha_2\cdots\alpha_{2s-1})\beta} + \frac{1}{2}\nabla^2\zeta_{\alpha(2s-1)}\right)\right). \end{aligned} \quad (5.14b)$$

Let us express the $\mathcal{N} = 1$ superfields (5.12) in terms of their real and imaginary parts,

$$\Gamma_{\alpha(2s-2)}| = -L_{\alpha(2s-2)} - isV_{\alpha(2s-2)}, \quad (5.15a)$$

$$\nabla^2_{(\alpha_1}\Gamma_{\alpha_2\cdots\alpha_{2s-1})}| = \frac{1}{2}(\Phi_{\alpha(2s-1)} + i\Omega_{\alpha(2s-1)}). \quad (5.15b)$$

It then follows from gauge transformations (5.10) and (5.14) that we are, in fact, dealing with two different gauge theories. The first model is formulated in terms of the real unconstrained gauge superfields

$$\mathcal{V}_{(s+\frac{1}{2})}^{\parallel} = \{H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-1)}\}, \quad (5.16)$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s+1)} = i\nabla_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s+1})}, \quad (5.17a)$$

$$\delta L_{\alpha(2s-2)} = -\frac{s}{2(2s+1)}\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-2)}, \quad (5.17b)$$

$$\begin{aligned} \delta\Phi_{\alpha(2s-1)} &= \frac{is}{2s+1}(\nabla^{\beta}\nabla_{(\alpha_1}{}^{\gamma}\zeta_{\alpha_2\cdots\alpha_{2s-1})\beta\gamma} \\ &\quad - (4s+1)|\mu|\nabla^{\beta}\zeta_{\beta\alpha(2s-1)} \\ &\quad - \nabla^{\beta\gamma}\nabla_{\beta}\zeta_{\gamma\alpha(2s-1)}), \end{aligned} \quad (5.17c)$$

where the gauge parameter $\zeta_{\alpha(2s)}$ is real unconstrained. The other gauge theory is constructed in terms of the superfields

$$\mathcal{V}_{(s)}^{\parallel} = \{H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}\}, \quad (5.18)$$

which possesses the gauge freedom

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s})}, \quad (5.19a)$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s} \nabla^\beta \zeta_{\beta\alpha(2s-2)}, \quad (5.19b)$$

$$\begin{aligned} \delta \Omega_{\alpha(2s-1)} &= (2s+1) |\mu| \zeta_{\alpha(2s-1)} + \nabla_{(\alpha_1}{}^{\beta} \zeta_{\alpha_2 \dots \alpha_{2s-1})\beta} \\ &\quad - \frac{i}{2} \nabla^2 \zeta_{\alpha(2s-1)}, \end{aligned} \quad (5.19c)$$

where the parameter $\zeta_{\alpha(2s-1)}$ is real unconstrained. Applying the superspace reduction procedure to the action (5.3) yields two decoupled $\mathcal{N} = 1$ supersymmetric theories, each formulated in terms of the gauge fields (5.16) and (5.18), respectively,

$$\begin{aligned} &\mathbb{S}_{(s+\frac{1}{2})}^\perp [\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}] \\ &= S_{(s+\frac{1}{2})}^\parallel [H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-1)}] \\ &\quad + S_{(s)}^\parallel [H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}]. \end{aligned} \quad (5.20)$$

Explicit expressions for the decoupled supersymmetric actions are given in the following subsection.

C. Massless higher-spin $\mathcal{N} = 1$ supermultiplets

The first of the decoupled $\mathcal{N} = 1$ supersymmetric actions, which is realized by the dynamical variables (5.16), takes the following form:

$$\begin{aligned} S_{(s+\frac{1}{2})}^\parallel [H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-1)}] &= \left(-\frac{1}{2}\right)^s \left(-\frac{i}{8}\right) \int d^3|z| E \left\{ 2H^{\alpha(2s+1)} \mathbb{Q} H_{\alpha(2s+1)} \right. \\ &\quad - 8(s-2)(2s+1) |\mu|^2 H^{\alpha(2s+1)} H_{\alpha(2s+1)} + (2s+1) i |\mu| H^{\alpha(2s+1)} \nabla^2 H_{\alpha(2s+1)} \\ &\quad - 4(2s+1) |\mu| H^{\beta\alpha(2s)} \nabla_\beta{}^\gamma H_{\gamma\alpha(2s)} - i H^{\beta\alpha(2s)} \nabla^2 \nabla_\beta{}^\gamma H_{\gamma\alpha(2s)} - 4H^{\beta\gamma\alpha(2s-1)} \nabla_{\beta\gamma} \Phi_{\alpha(2s-1)} \\ &\quad - 8i H^{\beta\gamma\delta\alpha(2s-2)} \nabla_{\beta\gamma} \nabla_\delta L_{\alpha(2s-2)} - 16i L^{\alpha(2s-2)} \nabla^\beta \Phi_{\beta\alpha(2s-2)} + \frac{2}{s} \Phi^{\alpha(2s-1)} \Phi_{\alpha(2s-1)} \\ &\quad + \frac{8i}{s(2s-1)} (2(s-1)(4s-1) L^{\beta\alpha(2s-3)} \nabla_\beta{}^\gamma L_{\gamma\alpha(2s-3)} \\ &\quad \left. - 4s(4s^2 - 3s + 1) |\mu| L^{\alpha(2s-2)} L_{\alpha(2s-2)} - (3s-1) i L^{\alpha(2s-2)} \nabla^2 L_{\alpha(2s-2)} \right\}. \end{aligned} \quad (5.21)$$

Upon inspection, it is apparent that the superfield $\Phi_{\alpha(2s-1)}$ is auxiliary. So by making use of its equation of motion,

$$\Phi_{\alpha(2s-1)} = -s \nabla^{\beta\gamma} H_{\beta\gamma\alpha(2s-1)} - 4is \nabla_{(\alpha_1} L_{\alpha_2 \dots \alpha_{2s-1})}, \quad (5.22)$$

we can eliminate the auxiliary field $\Phi_{\alpha(2s-1)}$ from the action (5.21). It can be shown that the resulting action coincides with the off-shell $\mathcal{N} = 1$ supersymmetric action for massless half-integer superspin in AdS_3 (3.6).

The other $\mathcal{N} = 1$ theory is formulated in terms of the gauge superfields (5.18),

$$\begin{aligned} S_{(s)}^\parallel [H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}] &= \left(-\frac{1}{2}\right)^s \int d^3|z| E \left\{ 2|\mu| H^{\alpha(2s)} H_{\alpha(2s)} \right. \\ &\quad + \frac{i}{2} H^{\alpha(2s)} \nabla^2 H_{\alpha(2s)} + 2s H^{\beta\gamma\alpha(2s-2)} \nabla_{\beta\gamma} V_{\alpha(2s-2)} - H^{\beta\alpha(2s-1)} \nabla_\beta \Omega_{\alpha(2s-1)} \\ &\quad + \frac{i}{2} \Omega^{\alpha(2s-1)} \Omega_{\alpha(2s-1)} + \frac{1}{2s-1} (4s(s-1)^2 V^{\beta\alpha(2s-3)} \nabla_\beta{}^\gamma V_{\gamma\alpha(2s-3)} \\ &\quad + 4s(2s^3 - 2s + 1) |\mu| V^{\alpha(2s-2)} V_{\alpha(2s-2)} + is(2s^2 - 2s + 1) V^{\alpha(2s-2)} \nabla^2 V_{\alpha(2s-2)} \\ &\quad \left. + (2s-1) V^{\alpha(2s-2)} \nabla^\beta \Omega_{\beta\alpha(2s-2)} \right\}. \end{aligned} \quad (5.23)$$

The superfield $\Omega_{\alpha(2s-1)}$ is auxiliary, so upon elimination via its equation of motion

$$\Omega_{\alpha(2s-1)} = -i \nabla^\beta H_{\beta\alpha(2s-1)} - i \nabla_{(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-1})}, \quad (5.24)$$

we find that the resulting action coincides with the off-shell $\mathcal{N} = 1$ action massless superspin- s multiplet in AdS_3 (3.14).

D. Second supersymmetry transformations

As discussed in Sec. IV D, by construction, the $\mathcal{N} = 1$ reduced actions (5.21) and (5.23) are invariant under the second supersymmetry transformations (4.37b). For convenience, we recall the form of these transformations

$$\delta_\epsilon U| = \epsilon^\alpha (\nabla_\alpha^2 U)|, \quad (5.25)$$

where $U(x, \theta^I)$ is a $\mathcal{N} = (1, 1)$ superfield, with indices being suppressed. The second supersymmetry transformations act on the $\mathcal{N} = 1$ fields (5.16) and (5.18) in the following fashion:

$$\delta_\epsilon \mathfrak{H}_{\alpha(2s)}| = -i\epsilon^\beta H_{\beta\alpha(2s)}, \quad (5.26a)$$

$$\delta_\epsilon \nabla^{2\beta} \mathfrak{H}_{\beta\alpha(2s-1)}| = 2i\epsilon^\beta H_{\beta\alpha(2s-1)}, \quad (5.26b)$$

$$\delta_\epsilon H_{\alpha(2s+1)} = -2\epsilon_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})}, \quad (5.26c)$$

$$\delta_\epsilon H_{\alpha(2s)} = -\frac{i}{2}((2s+1)|\mu| \epsilon^\beta H_{\beta\alpha(2s)} - \epsilon^\beta \nabla_\beta^\gamma H_{\gamma\alpha(2s)}), \quad (5.26d)$$

$$\delta_\epsilon L_{\alpha(2s-2)} = -\frac{i}{2} \epsilon^\beta \Omega_{\beta\alpha(2s-2)} + \frac{2s}{2s-1} (s-1) \epsilon_{(\alpha_1} \nabla^\beta V_{\alpha_2 \dots \alpha_{2s-2})\beta}, \quad (5.26e)$$

$$\delta_\epsilon V_{\alpha(2s-2)} = \frac{i}{2s} \epsilon^\beta \Phi_{\beta\alpha(2s-2)} - \frac{2}{s(2s-1)} (s-1) \epsilon_{(\alpha_1} \nabla^\beta L_{\alpha_2 \dots \alpha_{2s-2})\beta}, \quad (5.26f)$$

$$\begin{aligned} \delta_\epsilon \Phi_{\alpha(2s-1)} &= \frac{1}{2s-1} (2s(2s-1) \epsilon^\beta \nabla_{\beta(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-1})} + i s \epsilon_{(\alpha_1} \nabla^2 V_{\alpha_2 \dots \alpha_{2s-1})} \\ &\quad + 4s(4s^2 - 3s + 1) |\mu| \epsilon_{(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-1})} + (2s-1) \epsilon_{(\alpha_1} \nabla^\beta \Omega_{\alpha_2 \dots \alpha_{2s-1})\beta} - 4s(s-1) \epsilon_{(\alpha_1} \nabla_{\alpha_2}^\beta V_{\alpha_3 \dots \alpha_{2s-1})\beta}), \end{aligned} \quad (5.26g)$$

$$\begin{aligned} \delta_\epsilon \Omega_{\alpha(2s-1)} &= -\frac{1}{2s-1} (2(2s-1) \epsilon^\beta \nabla_{\beta(\alpha_1} L_{\alpha_2 \dots \alpha_{2s-1})} + i \epsilon_{(\alpha_1} \nabla^2 L_{\alpha_2 \dots \alpha_{2s-1})} \\ &\quad + 4(4s^2 - 3s + 1) |\mu| \epsilon_{(\alpha_1} L_{\alpha_2 \dots \alpha_{2s-1})} + (2s-1) \epsilon_{(\alpha_1} \nabla^\beta \Phi_{\alpha_2 \dots \alpha_{2s-1})\beta} - 4(s-1) \epsilon_{(\alpha_1} \nabla_{\alpha_2}^\beta L_{\alpha_3 \dots \alpha_{2s-1})\beta}). \end{aligned} \quad (5.26h)$$

It is apparent from (5.26a) and (5.26b) that the second supersymmetry transformation (5.25) breaks the WZ gauge conditions (5.7), which we recall for convenience

$$\mathfrak{H}_{\alpha(2s)}| = 0, \quad \nabla^{2\beta} \mathfrak{H}_{\beta\alpha(2s-1)}| = 0. \quad (5.27)$$

In order to resolve this, it is necessary to supplement the variation (5.25) with the ϵ -dependent gauge transformations:

$$\delta_{g(\epsilon)} \mathfrak{H}_{\alpha(2s)} = g_{\alpha(2s)}(\epsilon) + \bar{g}_{\alpha(2s)}(\epsilon), \quad (5.28a)$$

$$\delta_{g(\epsilon)} \Gamma_{\alpha(2s-2)} = \frac{s}{2s+1} \bar{\mathcal{D}}^\beta \mathcal{D}^\gamma \bar{g}_{\beta\gamma\alpha(2s-2)}(\epsilon). \quad (5.28b)$$

The modified second supersymmetry transformations now take the following form:

$$\hat{\delta}_\epsilon \mathfrak{H}_{\alpha(2s)} = \delta_\epsilon \mathfrak{H}_{\alpha(2s)} + \delta_{g(\epsilon)} \mathfrak{H}_{\alpha(2s)}, \quad (5.29a)$$

$$\hat{\delta}_\epsilon \Gamma_{\alpha(2s-2)} = \delta_\epsilon \Gamma_{\alpha(2s-2)} + \delta_{g(\epsilon)} \Gamma_{\alpha(2s-2)}. \quad (5.29b)$$

The sole purpose of introducing the ϵ -dependent gauge transformations (5.28) is to restore the original Wess-Zumino gauge (5.27). Fixing the form of the $\mathcal{N} = 1$ components of $g_{\alpha(2s)}(\epsilon)$ as

$$g_{\alpha(2s)}(\epsilon)| = \frac{i}{2} \epsilon^\beta H_{\beta\alpha(2s)}, \quad (5.30a)$$

$$\nabla^{2\beta} g_{\beta\alpha(2s-1)}(\epsilon)| = -i\epsilon^\beta H_{\beta\alpha(2s-1)}, \quad (5.30b)$$

ensures that (5.29) takes us back to the original gauge (5.27). The transformations (5.29) act on the $\mathcal{N} = 1$ superfields by the rule

$$\hat{\delta}_\epsilon H_{\alpha(2s+1)} = -2\epsilon_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})}, \quad (5.31a)$$

$$\begin{aligned} \hat{\delta}_\epsilon H_{\alpha(2s)} &= -\frac{i}{4(2s+1)} (i\epsilon^\beta \nabla^2 H_{\beta\alpha(2s)} + 4s\epsilon_{(\alpha_1} \nabla^{\beta\gamma} H_{\alpha_2 \dots \alpha_{2s})\beta\gamma} \\ &\quad + 2(2s+1)(4s+1) |\mu| \epsilon^\beta H_{\beta\alpha(2s)} - 2\epsilon^\beta \nabla_\beta^\gamma H_{\gamma\alpha(2s)}), \end{aligned} \quad (5.31b)$$

$$\hat{\delta}_\epsilon L_{\alpha(2s-2)} = -\frac{i}{2} \epsilon^\beta \Omega_{\beta\alpha(2s-2)} + \frac{2s}{2s-1} (s-1) \epsilon_{(\alpha_1} \nabla^\beta V_{\alpha_2 \dots \alpha_{2s-2})\beta} + \frac{s}{2s+1} \epsilon^\beta \nabla^\gamma H_{\beta\gamma\alpha(2s-2)}, \quad (5.31c)$$

$$\hat{\delta}_\epsilon V_{\alpha(2s-2)} = \frac{i}{2s} \epsilon^\beta \Phi_{\beta\alpha(2s-2)} - \frac{2}{s(2s-1)} (s-1) \epsilon_{(\alpha_1} \nabla^\beta L_{\alpha_2 \dots \alpha_{2s-2})\beta} + \frac{i}{2(2s+1)} \epsilon^\beta \nabla^\gamma \delta H_{\beta\gamma\alpha(2s-2)}, \quad (5.31d)$$

$$\begin{aligned} \hat{\delta}_\epsilon \Phi_{\alpha(2s-1)} &= \frac{1}{2s-1} (2s(2s-1) \epsilon^\beta \nabla_{\beta(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-1})} + i s \epsilon_{(\alpha_1} \nabla^2 V_{\alpha_2 \dots \alpha_{2s-1})} + 4s(4s^2 - 3s + 1) |\mu| \epsilon_{(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-1})} \\ &\quad + (2s-1) \epsilon_{(\alpha_1} \nabla^\beta \Omega_{\alpha_2 \dots \alpha_{2s-1})\beta} - 4s(s-1) \epsilon_{(\alpha_1} \nabla_{\alpha_2}^\beta V_{\alpha_3 \dots \alpha_{2s-1})\beta}) \\ &\quad - \frac{s}{2s+1} \epsilon^\beta (4(s+1) |\mu| H_{\beta\alpha(2s-1)} + 2 \nabla_{(\alpha_1}{}^\gamma H_{\alpha_2 \dots \alpha_{2s-1})\beta\gamma} - i \nabla^2 H_{\beta\alpha(2s-1)}), \end{aligned} \quad (5.31e)$$

$$\begin{aligned} \hat{\delta}_\epsilon \Omega_{\alpha(2s-1)} &= -\frac{1}{2s-1} (2(2s-1) \epsilon^\beta \nabla_{\beta(\alpha_1} L_{\alpha_2 \dots \alpha_{2s-1})} + i \epsilon_{(\alpha_1} \nabla^2 L_{\alpha_2 \dots \alpha_{2s-1})} \\ &\quad + 4(4s^2 - 3s + 1) |\mu| \epsilon_{(\alpha_1} L_{\alpha_2 \dots \alpha_{2s-1})} + (2s-1) \epsilon_{(\alpha_1} \nabla^\beta \Phi_{\alpha_2 \dots \alpha_{2s-1})\beta} \\ &\quad - 4(s-1) \epsilon_{(\alpha_1} \nabla_{\alpha_2}^\beta L_{\alpha_3 \dots \alpha_{2s-1})\beta}) + \frac{s}{2s+1} \epsilon^\beta (2(2s-1) |\mu| \nabla^\gamma H_{\beta\gamma\alpha(2s-1)} \\ &\quad - \nabla_{(\alpha_1}{}^\gamma \nabla^\delta H_{\alpha_2 \dots \alpha_{2s-1})\beta\gamma\delta} + \nabla^\gamma \delta \nabla_\gamma H_{\beta\delta\alpha(2s-1)}). \end{aligned} \quad (5.31f)$$

VI. MASSLESS HALF-INTEGERSUPERSPIN: LONGITUDINAL FORMULATION

In this section, we develop the superspace reduction procedure for the longitudinal formulation, following the prescription advocated in Sec. V.

A. Longitudinal formulation

For $s \geq 2$, the longitudinal formulation for the massless superspin- $(s + \frac{1}{2})$ multiplet is described in terms of the variables

$$\mathcal{V}_{(s+\frac{1}{2})}^\parallel = \{\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}\}. \quad (6.1)$$

Here, the real superfield $\mathfrak{H}_{\alpha(2s)}$ is identical to that of (5.1), and the complex superfield $G_{\alpha(2s-2)}$ is longitudinal linear (4.4a). Modulo an overall normalization factor, the longitudinal formulation is uniquely described by the action

$$\begin{aligned} \mathbb{S}_{(s+\frac{1}{2})}^\parallel[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}] &= \left(-\frac{1}{2} \right)^s \int d^3 4 z E \left\{ \frac{1}{8} \mathfrak{H}^{\alpha(2s)} \mathcal{D}^\beta (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\beta \mathfrak{H}_{\alpha(2s)} + 2s(s-1) \mu \bar{\mu} \mathfrak{H}^{\alpha(2s)} \mathfrak{H}_{\alpha(2s)} \right. \\ &\quad - \frac{1}{16} ([\mathcal{D}_\beta, \bar{\mathcal{D}}_\gamma] \mathfrak{H}^{\beta\gamma\alpha(2s-2)}) [\mathcal{D}^\delta, \bar{\mathcal{D}}^\rho] \mathfrak{H}_{\delta\rho\alpha(2s-2)} + \frac{s}{2} (\mathcal{D}_{\beta\gamma} \mathfrak{H}^{\beta\gamma\alpha(2s-2)}) \mathcal{D}^{\delta\rho} \mathfrak{H}_{\delta\rho\alpha(2s-2)} \\ &\quad + \frac{2s-1}{2s} \left(i \mathcal{D}_{\beta\gamma} \mathfrak{H}^{\beta\gamma\alpha(2s-2)} (G_{\alpha(2s-2)} - \bar{G}_{\alpha(2s-2)}) + \frac{1}{s} \bar{G}^{\alpha(2s-2)} G_{\alpha(2s-2)} \right) \\ &\quad \left. - \frac{2s+1}{4s^2} (G^{\alpha(2s-2)} G_{\alpha(2s-2)} + \bar{G}^{\alpha(2s-2)} \bar{G}_{\alpha(2s-2)}) \right\}, \end{aligned} \quad (6.2)$$

which is invariant under the gauge transformations

$$\delta_\lambda \mathfrak{H}_{\alpha(2s)} = \bar{\mathcal{D}}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_{2s})} - \mathcal{D}_{(\alpha_1} \bar{\lambda}_{\alpha_2 \dots \alpha_{2s})} \equiv g_{\alpha(2s)} + \bar{g}_{\alpha(2s)}, \quad (6.3a)$$

$$\begin{aligned} \delta_\lambda G_{\alpha(2s-2)} &= -\frac{1}{4} (\bar{\mathcal{D}}^2 - 4s\mu) \mathcal{D}^\beta \lambda_{\alpha(2s-2)\beta} + i(s-1) \bar{\mathcal{D}}_{(\alpha_1} \mathcal{D}^{|\beta\gamma|} \lambda_{\alpha_2 \dots \alpha_{2s-2})\beta\gamma} \\ &\equiv \left(\frac{s}{2s+1} \mathcal{D}^\beta \bar{\mathcal{D}}^\gamma + i s \mathcal{D}^{\beta\gamma} \right) g_{\beta\gamma\alpha(2s-2)}, \end{aligned} \quad (6.3b)$$

where $\lambda_{\alpha(2s-1)}$ is complex unconstrained and $g_{\alpha(2s)}$ is longitudinal linear, as in (5.2a).

In the case where $s = 1$, the compensator G is covariantly chiral. If we introduce the field redefinition $G = 3\sigma$ in action (6.2), along with choosing $s = 1$, then the model can be shown to coincide with the linearized action for minimal (1,1) AdS supergravity [18]. In the case $s = 1$, the gauge transformations (6.3) yield

$$\delta_\lambda \mathfrak{H}_{\alpha\beta} = \bar{\mathcal{D}}_{(\alpha} \lambda_{\beta)} - \mathcal{D}_{(\alpha} \bar{\lambda}_{\beta)}, \quad (6.4)$$

$$\delta_\lambda G = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\bar{\mu})\mathcal{D}^\beta \lambda_\beta. \quad (6.5)$$

It is an easy exercise to show that $\delta_\lambda G$ is covariantly chiral.

B. Reduction of gauge prepotentials to $\text{AdS}^{3|2}$

The reduction of the superconformal gauge prepotential $\mathfrak{H}_{\alpha(2s)}$ was addressed in Sec. V. We need only reduce the compensator $G_{\alpha(2s-2)}$ to $\mathcal{N} = 1$ AdS superspace. We start by converting the longitudinal constraint of $G_{\alpha(2s-2)}$ (4.4a) to the real basis (4.18)

$$\nabla^2_{(\alpha_1} G_{\alpha_2 \dots \alpha_{2s-1})} = i\nabla^1_{(\alpha_1} G_{\alpha_2 \dots \alpha_{2s-1})}. \quad (6.6)$$

Performing a Taylor expansion of $G_{\alpha(2s-2)}(\theta^I)$ about θ^2 , and using (6.6), we find the independent θ^2 components of $G_{\alpha(2s-2)}$ to be

$$G_{\alpha(2s-2)}|, \quad \nabla^{2\beta} G_{\beta\alpha(2s-3)}|. \quad (6.7)$$

Utilizing the gauge transformations (6.3b) and the real representation (4.18), we find

$$\delta G_{\alpha(2s-2)} = \frac{is}{2s+1}(2s\nabla^{\beta\gamma} - \nabla^{1\beta}\nabla^{2\gamma})g_{\beta\gamma\alpha(2s-2)}, \quad (6.8a)$$

$$\nabla^{2\beta}\delta G_{\beta\alpha(2s-3)} = \frac{s}{2s+1}(2is\nabla^{\beta\gamma}\nabla^{2\delta} - \nabla^{\beta\gamma}\nabla^{1\delta})g_{\beta\gamma\delta\alpha(2s-3)}. \quad (6.8b)$$

We can use the residual gauge freedom (5.9) to compute the gauge transformations of the complex $\mathcal{N} = 1$ superfields (6.7)

$$\delta G_{\alpha(2s-2)}| = \frac{s^2}{2s+1}\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-2)} - \frac{i}{2}\nabla^\beta\zeta_{\beta\alpha(2s-2)}, \quad (6.9a)$$

$$\begin{aligned} \nabla^{2\beta}\delta G_{\beta\alpha(2s-3)}| \\ = is\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-3)} + \frac{is}{2(2s+1)}\nabla^{\beta\gamma}\nabla^\delta\zeta_{\beta\gamma\delta\alpha(2s-3)}. \end{aligned} \quad (6.9b)$$

It is useful to separate the complex superfields (6.7) into real and imaginary components

$$G_{\alpha(2s-2)}| = -2sL_{\alpha(2s-2)} - isV_{\alpha(2s-2)}, \quad (6.10a)$$

$$\nabla^{2\beta}G_{\beta\alpha(2s-3)}| = \frac{1}{2}(\Phi_{\alpha(2s-3)} + i\Omega_{\alpha(2s-3)}). \quad (6.10b)$$

It then becomes apparent from gauge transformations (5.10) and (6.9) that we are, in fact, dealing with two different gauge theories. The first gauge theory is formulated in terms of the dynamical variables

$$\mathcal{V}_{(s+\frac{1}{2})}^\parallel = \{H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-3)}\}, \quad (6.11)$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s+1)} = i\nabla_{(\alpha_1}\zeta_{\alpha_2 \dots \alpha_{2s+1})}, \quad (6.12a)$$

$$\delta L_{\alpha(2s-2)} = -\frac{s}{2(2s+1)}\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-2)}, \quad (6.12b)$$

$$\delta\Phi_{\alpha(2s-3)} = \frac{is}{2s+1}\nabla^{\beta\gamma}\nabla^\delta\zeta_{\beta\gamma\delta\alpha(2s-3)}. \quad (6.12c)$$

The other gauge model is described in terms of the superfields

$$\mathcal{V}_{(s)}^\parallel = \{H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-3)}\}, \quad (6.13)$$

which possess the following gauge freedom:

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1}\zeta_{\alpha_2 \dots \alpha_{2s})}, \quad (6.14a)$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s}\nabla^\beta\zeta_{\beta\alpha(2s-2)}, \quad (6.14b)$$

$$\delta\Omega_{\alpha(2s-3)} = 2s\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-3)}. \quad (6.14c)$$

After carrying out the reduction to $\mathcal{N} = 1$ AdS superspace, the action (6.2) decouples into two $\mathcal{N} = 1$ theories, which are formulated in terms of superfields (6.11) and (6.13), respectively,

$$\begin{aligned} \mathbb{S}_{(s+\frac{1}{2})}^\parallel[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}] \\ = \mathbb{S}_{(s+\frac{1}{2})}^\parallel[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-3)}] \\ + \mathbb{S}_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-3)}]. \end{aligned} \quad (6.15)$$

In the following subsection, we provide the explicit forms of the decoupled $\mathcal{N} = 1$ actions.

C. Massless higher-spin $\mathcal{N} = 1$ supermultiplets

The first $\mathcal{N} = 1$ gauge theory, described in terms of the dynamical variables (6.11), takes the form

$$\begin{aligned}
S_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-3)}] = & \left(-\frac{1}{2}\right)^s \left(-\frac{i}{8}\right) \int d^3|z| E \left\{ 2H^{\alpha(2s+1)} \mathbb{Q} H_{\alpha(2s+1)} + (2s+1)i|\mu| H^{\alpha(2s+1)} \nabla^2 H_{\alpha(2s+1)} \right. \\
& - 4(2s+1)|\mu| H^{\beta\alpha(2s)} \nabla_{\beta}{}^{\gamma} H_{\gamma\alpha(2s)} - iH^{\beta\alpha(2s)} \nabla^2 \nabla_{\beta}{}^{\gamma} H_{\gamma\alpha(2s)} \\
& - 8(s-2)(2s+1)|\mu|^2 H^{\alpha(2s+1)} H_{\alpha(2s+1)} - 2s \nabla_{\beta\gamma} H^{\beta\gamma\alpha(2s-1)} \nabla^{\delta\rho} H_{\delta\rho\alpha(2s-1)} \\
& - \frac{4}{2s-1} \left(16i|\mu|(2s^3-2s+1)L^{\alpha(2s-2)} L_{\alpha(2s-2)} + 4(2s^2-1)L^{\alpha(2s-2)} \nabla^2 L_{\alpha(2s-2)} \right. \\
& + 16i(s-1)^2 L^{\beta\alpha(2s-3)} \nabla_{\beta}{}^{\gamma} L_{\gamma\alpha(2s-3)} + 2i(2s-1)^2 \nabla_{\beta\gamma} H^{\beta\gamma\delta\alpha(2s-2)} \nabla_{\delta} L_{\alpha(2s-2)} \\
& \left. \left. + \frac{4i}{s}(s-1)L^{\beta\alpha(2s-3)} \nabla_{\beta} \Phi_{\alpha(2s-3)} + \frac{1}{s}(s-1)\Phi^{\alpha(2s-3)} \Phi_{\alpha(2s-3)} \right) \right\}. \quad (6.16)
\end{aligned}$$

The action (6.16) is invariant under the gauge transformations (6.12). We can eliminate the auxiliary field $\Phi_{\alpha(2s-3)}$ from (6.16) by using the equation of motion

$$\Phi_{\alpha(2s-3)} = -2i\nabla^{\beta} L_{\beta\alpha(2s-3)}. \quad (6.17)$$

The resulting action, up to an overall factor, then coincides with model (3.6).

The other $\mathcal{N} = 1$ model that is constructed in terms of the dynamical variables (6.13) assumes the form

$$\begin{aligned}
S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-3)}] = & -\left(-\frac{1}{2}\right)^s \int d^3|z| E \left\{ (s-1)|\mu| H^{\alpha(2s)} H_{\alpha(2s)} \right. \\
& - \frac{i}{4} H^{\alpha(2s)} \nabla^2 H_{\alpha(2s)} + \frac{1}{2} H^{\beta\alpha(2s-1)} \nabla_{\beta}{}^{\gamma} H_{\gamma\alpha(2s-1)} - (2s-1) H^{\alpha(2s)} \nabla_{(\alpha_1\alpha_2} V_{\alpha_3\cdots\alpha_{2s})} \\
& - \frac{1}{4(2s-1)} \left(2is V^{\alpha(2s-2)} \nabla^2 V_{\alpha(2s-2)} - 4(s-1)(4s-1) V^{\beta\alpha(2s-3)} \nabla_{\beta}{}^{\gamma} V_{\gamma\alpha(2s-3)} \right. \\
& + 8s(4s^2-3s+1)|\mu| V^{\alpha(2s-2)} V_{\alpha(2s-2)} + 8(s-1) V^{\beta\alpha(2s-3)} \nabla_{\beta} \Omega_{\alpha(2s-3)} \\
& \left. \left. - \frac{i}{s^2}(s-1)\Omega^{\alpha(2s-3)} \Omega_{\alpha(2s-3)} \right) \right\}, \quad (6.18)
\end{aligned}$$

which is invariant under the gauge transformations (6.14). It is evident that the superfield $\Omega_{\alpha(2s-3)}$ is auxiliary, so upon elimination via its equation of motion

$$\Omega_{\alpha(2s-3)} = -4is^2 \nabla^{\beta} V_{\beta\alpha(2s-3)}, \quad (6.19)$$

one obtains the action (3.14).

D. Second supersymmetry transformations

Let us note that the reduced actions (6.15)

$$S_{(s+\frac{1}{2})}^{\parallel}[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}] = S_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-3)}] + S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-3)}] \quad (6.20)$$

are invariant under second supersymmetry. More explicitly, the second supersymmetry transformation acts on the dynamical superfields (6.11) and (6.13) by the following rule:

$$\delta_{\epsilon} \mathfrak{H}_{\alpha(2s)} = -i\epsilon^{\beta} H_{\beta\alpha(2s)}, \quad (6.21a)$$

$$\delta_{\epsilon} \nabla^2 \mathfrak{H}_{\beta\alpha(2s-1)} = 2i\epsilon^{\beta} H_{\beta\alpha(2s-1)}, \quad (6.21b)$$

$$\delta_{\epsilon} H_{\alpha(2s+1)} = -2\epsilon_{(\alpha_1} H_{\alpha_2\cdots\alpha_{2s+1})}, \quad (6.21c)$$

$$\delta_\epsilon H_{\alpha(2s)} = -\frac{i}{2}((2s+1)|\mu|\epsilon^\beta H_{\beta\alpha(2s)} - \epsilon^\beta \nabla_\beta{}^\gamma H_{\gamma\alpha(2s)}), \quad (6.21d)$$

$$\delta_\epsilon L_{\alpha(2s-2)} = -\frac{1}{2}\epsilon^\beta \nabla_{(\beta} V_{\alpha(2s-2))} + \frac{i(s-1)}{2s(2s-1)}\epsilon_{(\alpha_1} \Omega_{\alpha_2 \dots \alpha_{2s-2})}, \quad (6.21e)$$

$$\delta_\epsilon V_{\alpha(2s-2)} = 2\epsilon^\beta \nabla_{(\beta} L_{\alpha(2s-2))} - \frac{i(s-1)}{s(2s-1)}\epsilon_{(\alpha_1} \Phi_{\alpha_2 \dots \alpha_{2s-2})}, \quad (6.21f)$$

$$\begin{aligned} \delta_\epsilon \Phi_{\alpha(2s-3)} &= \frac{1}{2s-1}\epsilon^\beta(2s(2s-3)\nabla_{(\alpha_1}{}^\gamma V_{\alpha_2 \dots \alpha_{2s-3})\beta\gamma} - i s \nabla^2 V_{\beta\alpha(2s-3)} \\ &\quad + 4s^2 \nabla_{\beta}{}^\gamma V_{\gamma\alpha(2s-3)} - 4s(4s^2 - 5s + 2)|\mu|V_{\beta\alpha(2s-3)} - 2(s-1)\nabla_{(\beta} \Omega_{\alpha(2s-3))}), \end{aligned} \quad (6.21g)$$

$$\begin{aligned} \delta_\epsilon \Omega_{\alpha(2s-3)} &= \frac{1}{2s-1}\epsilon^\beta(2is\nabla^2 L_{\beta\alpha(2s-3)} - 4s(2s-3)\nabla_{(\alpha_1}{}^\gamma L_{\alpha_2 \dots \alpha_{2s-3})\beta\gamma} \\ &\quad - 8s^2 \nabla_{\beta}{}^\gamma L_{\gamma\alpha(2s-3)} + 8s(4s^2 - 5s + 2)|\mu|L_{\beta\alpha(2s-3)} + 2(s-1)\nabla_{(\beta} \Phi_{\alpha(2s-3))}). \end{aligned} \quad (6.21h)$$

It is evident from the variations (6.21a) and (6.21b) that second supersymmetry is not compatible with the imposed WZ gauge conditions (5.27). Recall that the longitudinal and transverse formulations are both constructed in terms of the real superfield $\mathfrak{H}_{\alpha(2s)}$, which is defined modulo gauge transformations (5.2a). We used this gauge freedom to impose (5.27) in both theories.

Using a similar approach as in Sec. V D, the WZ gauge conditions (5.27) in the longitudinal formulation can be restored provided we accompany the second supersymmetry transformations with the following ϵ -dependent gauge transformations:

$$\delta_{g(\epsilon)} \mathfrak{H}_{\alpha(2s)} = g_{\alpha(2s)}(\epsilon) + \bar{g}_{\alpha(2s)}(\epsilon), \quad (6.22a)$$

$$\delta_{g(\epsilon)} G_{\alpha(2s-2)} = \left(\frac{s}{2s+1} \mathcal{D}^\beta \bar{\mathcal{D}}^\gamma + i s \mathcal{D}^{\beta\gamma} \right) g_{\beta\gamma\alpha(2s-2)}(\epsilon), \quad (6.22b)$$

where

$$g_{\alpha(2s)}(\epsilon) = \frac{i}{2}\epsilon^\beta H_{\beta\alpha(2s)}, \quad (6.23a)$$

$$\nabla^{2\beta} g_{\beta\alpha(2s-1)}(\epsilon) = -i\epsilon^\beta H_{\beta\alpha(2s-1)}. \quad (6.23b)$$

The modified second supersymmetry transformations now read

$$\hat{\delta}_\epsilon \mathfrak{H}_{\alpha(2s)} = \delta_\epsilon \mathfrak{H}_{\alpha(2s)} + \delta_{g(\epsilon)} \mathfrak{H}_{\alpha(2s)}, \quad (6.24a)$$

$$\hat{\delta}_\epsilon G_{\alpha(2s-2)} = \delta_\epsilon G_{\alpha(2s-2)} + \delta_{g(\epsilon)} G_{\alpha(2s-2)}. \quad (6.24b)$$

The transformations (6.24) act on the $\mathcal{N} = 1$ superfields (6.11) and (6.13) as follows:

$$\hat{\delta}_\epsilon H_{\alpha(2s+1)} = -2\epsilon_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})}, \quad (6.25a)$$

$$\hat{\delta}_\epsilon H_{\alpha(2s)} = -\frac{i}{4(2s+1)}(i\epsilon^\beta \nabla^2 H_{\beta\alpha(2s)} + 4s\epsilon_{(\alpha_1} \nabla^{\beta\gamma} H_{\alpha_2 \dots \alpha_{2s})\beta\gamma} + 2(2s+1)(4s+1)|\mu|\epsilon^\beta H_{\beta\alpha(2s)} - 2\epsilon^\beta \nabla_\beta{}^\gamma H_{\gamma\alpha(2s)}), \quad (6.25b)$$

$$\hat{\delta}_\epsilon L_{\alpha(2s-2)} = -\frac{1}{2}\epsilon^\beta \nabla_{(\beta} V_{\alpha(2s-2))} + \frac{i(s-1)}{2s(2s-1)}\epsilon_{(\alpha_1} \Omega_{\alpha_2 \dots \alpha_{2s-2})} - \frac{1}{2(2s+1)}\epsilon^\beta \nabla^\gamma H_{\beta\gamma\alpha(2s-2)}, \quad (6.25c)$$

$$\hat{\delta}_\epsilon V_{\alpha(2s-2)} = 2\epsilon^\beta \nabla_{(\beta} L_{\alpha(2s-2))} - \frac{i(s-1)}{s(2s-1)}\epsilon_{(\alpha_1} \Phi_{\alpha_2 \dots \alpha_{2s-2})} - \frac{is}{2s+1}\epsilon^\beta \nabla^\gamma \delta H_{\beta\gamma\alpha(2s-2)}, \quad (6.25d)$$

$$\begin{aligned}\hat{\delta}_\epsilon \Phi_{\alpha(2s-3)} &= \frac{1}{2s-1} \epsilon^\beta (2s(2s-3) \nabla_{(\alpha_1}{}^\gamma V_{\alpha_2 \dots \alpha_{2s-3})\beta\gamma} - i s \nabla^2 V_{\beta\alpha(2s-3)} + 4s^2 \nabla_\beta{}^\gamma V_{\gamma\alpha(2s-3)} - 4s(4s^2 - 5s + 2) |\mu| V_{\beta\alpha(2s-3)} \\ &\quad - 2(s-1) \nabla_{(\beta} \Omega_{\alpha(2s-3))}) + \frac{4s^2}{2s+1} \epsilon^\beta \nabla^\gamma \delta H_{\beta\gamma\delta\alpha(2s-3)},\end{aligned}\quad (6.25e)$$

$$\begin{aligned}\hat{\delta}_\epsilon \Omega_{\alpha(2s-3)} &= \frac{1}{2s-1} \epsilon^\beta (2is \nabla^2 L_{\beta\alpha(2s-3)} - 4s(2s-3) \nabla_{(\alpha_1}{}^\gamma L_{\alpha_2 \dots \alpha_{2s-3})\beta\gamma} - 8s^2 \nabla_\beta{}^\gamma L_{\gamma\alpha(2s-3)} + 8s(4s^2 - 5s + 2) |\mu| L_{\beta\alpha(2s-3)} \\ &\quad + 2(s-1) \nabla_{(\beta} \Phi_{\alpha(2s-3))}) + \frac{s}{2s+1} \epsilon^\beta \nabla^\gamma \delta \nabla^\lambda H_{\beta\gamma\delta\lambda\alpha(2s-3)}.\end{aligned}\quad (6.25f)$$

VII. MASSLESS INTEGER SUPERSPIN: LONGITUDINAL FORMULATION

In [23], two dually equivalent off-shell formulations, called transverse and longitudinal, for the massless multiplets of integer superspin were developed in (1,1) AdS superspace. In the following section, we reduce the longitudinal model to $\mathcal{N} = 1$ AdS superspace.

A. Longitudinal formulation

Given an integer $s \geq 1$, the longitudinal formulation for the massless superspin- s multiplet is realized in terms of the following dynamical variables:

$$\mathcal{V}_{(s)}^\parallel = \{U_{\alpha(2s-2)}, G_{\alpha(2s)}, \bar{G}_{\alpha(2s)}\}. \quad (7.1)$$

Here $U_{\alpha(2s-2)}$ is an unconstrained real superfield, and the complex superfield $G_{\alpha(2s)}$ is longitudinal linear. The

dynamical superfields $U_{\alpha(2s-2)}$ and $G_{\alpha(2s)}$ are defined modulo gauge transformations of the form

$$\begin{aligned}\delta_L U_{\alpha(2s-2)} &= \mathcal{D}^\beta L_{\beta\alpha(2s-2)} - \bar{\mathcal{D}}^\beta \bar{L}_{\beta\alpha(2s-2)} \\ &\equiv \bar{\gamma}_{\alpha(2s-2)} + \gamma_{\alpha(2s-2)},\end{aligned}\quad (7.2a)$$

$$\begin{aligned}\delta_L G_{\alpha(2s)} &= -\frac{1}{2} \bar{\mathcal{D}}_{(\alpha_1} (\mathcal{D}^2 - 2(2s+1)\bar{\mu}) L_{\alpha_2 \dots \alpha_{2s})} \\ &\equiv \bar{\mathcal{D}}_{(\alpha_1} \mathcal{D}_{\alpha_2} \bar{\gamma}_{\alpha_3 \dots \alpha_{2s}}).\end{aligned}\quad (7.2b)$$

Here the gauge parameter $L_{\alpha(2s-1)}$ is an unconstrained complex superfield, and $\gamma^{\alpha(2s-2)} := \bar{\mathcal{D}}_\beta \bar{L}^{\beta\alpha(2s-2)}$ is transverse linear.

Modulo an overall normalization factor, the longitudinal formulation for the massless superspin- s multiplet is described by the action

$$\begin{aligned}\mathbb{S}_{(s)}^\parallel[U_{\alpha(2s-2)}, G_{\alpha(2s)}, \bar{G}_{\alpha(2s)}] &= \left(-\frac{1}{2}\right)^s \int d^3|4 z E \left\{ \frac{1}{8} U^{\alpha(2s-2)} \mathcal{D}^\gamma (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\gamma U_{\alpha(2s-2)} \right. \\ &\quad + \frac{s}{2s+1} U^{\alpha(2s-2)} (\mathcal{D}^\beta \bar{\mathcal{D}}^\gamma G_{\beta\gamma\alpha(2s-2)} - \bar{\mathcal{D}}^\beta \mathcal{D}^\gamma \bar{G}_{\beta\gamma\alpha(2s-2)}) + \frac{s}{2s-1} \bar{G}^{\alpha(2s)} G_{\alpha(2s)} \\ &\quad \left. + \frac{s}{2(2s+1)} (G^{\alpha(2s)} G_{\alpha(2s)} + \bar{G}^{\alpha(2s)} \bar{G}_{\alpha(2s)}) + 2s(s+1) \mu \bar{\mu} U^{\alpha(2s-2)} U_{\alpha(2s-2)} \right\},\end{aligned}\quad (7.3)$$

which is invariant under the gauge transformations (7.2).

B. Reduction of gauge prepotentials to $\text{AdS}^{3|2}$

We wish to reduce the gauge superfields (7.1) to $\mathcal{N} = 1$ AdS superspace. We start by reducing the superfield $U_{\alpha(2s-2)}$. Converting the transverse linear constraint of $\gamma_{\alpha(2s-2)}$ (4.5a) to the real basis (4.18) gives

$$\nabla^{2\beta} \gamma_{\beta\alpha(2s-3)} = i \nabla^{1\beta} \gamma_{\beta\alpha(2s-3)}. \quad (7.4)$$

Taking a Taylor expansion of $\gamma_{\alpha(2s-2)}(\theta^I)$ about θ^2 , and then using (7.4), we find the independent θ^2 components of $\gamma_{\alpha(2s-2)}$ to be

$$\gamma_{\alpha(2s-2)}|, \quad \nabla^2_{(\alpha_1} \gamma_{\alpha_2 \dots \alpha_{2s-1})}|. \quad (7.5)$$

The gauge transformation (7.2a) allows us to choose the gauge conditions

$$U_{\alpha(2s-2)}| = 0. \quad (7.6)$$

It must be noted that the gauge condition (7.6) is less restrictive than those proposed in the analogous reduction procedure performed in Minkowski superspace, as given in Appendix B of [23]. This was done in order to ensure that one of the decoupled $\mathcal{N} = 1$ actions coincides with (3.17). It can be shown that if one chooses to impose the same gauge conditions as detailed in [23], i.e.,

$$U_{\alpha(2s-2)}| = 0, \quad \nabla^2_{(\alpha_1} U_{\alpha_2 \dots \alpha_{2s-1})}| = 0, \quad (7.7)$$

then one will obtain a $\mathcal{N} = 1$ action, which would instead coincide with (3.24).

The residual gauge symmetry that preserves the gauge condition (7.6) is described by the real unconstrained $\mathcal{N} = 1$ superfield

$$\gamma_{\alpha(2s-2)}| = -\frac{i}{2}\zeta_{\alpha(2s-2)}, \quad \bar{\zeta}_{\alpha(2s-2)} = \zeta_{\alpha(2s-2)}. \quad (7.8)$$

We now wish to reduce the superfield $\bar{G}_{\alpha(2s)}$ to $\mathcal{N} = 1$ AdS superspace. Converting the longitudinal linear constraint of $\bar{G}_{\alpha(2s)}$ (4.4a) to the real basis yields

$$\nabla^2_{(\alpha_1} \bar{G}_{\alpha_2 \dots \alpha_{2s+1})} = -i\nabla^1_{(\alpha_1} \bar{G}_{\alpha_1 \dots \alpha_{2s+1})}. \quad (7.9)$$

Upon reduction to $\mathcal{N} = 1$ AdS superspace, it follows from constraint (7.9) that $\bar{G}_{\alpha(2s)}$ has two independent θ^2 components:

$$\bar{G}_{\alpha(2s)}|, \quad \nabla^{2\beta} \bar{G}_{\beta\alpha(2s-1)}|. \quad (7.10)$$

Making use of the gauge transformations (7.2b) and the real representation (4.18), we find

$$\delta \bar{G}_{\alpha(2s)} = i(\nabla^1_{(\alpha_1} \nabla^2_{\alpha_2} + \nabla_{(\alpha_1 \alpha_2}) \gamma_{\alpha_3 \dots \alpha_{2s}}), \quad (7.11a)$$

$$\begin{aligned} \nabla^{2\beta} \delta \bar{G}_{\beta\alpha(2s-1)} &= \frac{i}{2s} (2s \nabla_{(\alpha_1}{}^\beta \nabla^2_{\alpha_2} \gamma_{\alpha_3 \dots \alpha_{2s-1}\beta}) + (4s^2 - 1)|\mu| \nabla^2_{(\alpha_1} \gamma_{\alpha_2 \dots \alpha_{2s-1})} \\ &\quad + 2i(s+1) \nabla_{\beta(\alpha_1} \nabla^{1\beta} \gamma_{\alpha_2 \dots \alpha_{2s-1})} + 6is(2s+1)|\mu| \nabla^1_{(\alpha_1} \gamma_{\alpha_2 \dots \alpha_{2s-1})} \\ &\quad + \frac{i}{2} (2s+1) (\nabla^1)^2 \nabla^2_{(\alpha_1} \gamma_{\alpha_2 \dots \alpha_{2s-1})} - 2i(s-1) \nabla_{(\alpha_1}{}^\beta \nabla^1_{\alpha_2} \gamma_{\alpha_3 \dots \alpha_{2s-1})\beta}). \end{aligned} \quad (7.11b)$$

Using the residual gauge freedom (7.8), we can directly compute the gauge transformations of the $\mathcal{N} = 1$ superfields (7.10) from (7.11)

$$\delta \bar{G}_{\alpha(2s)}| = \frac{1}{2} (\nabla_{(\alpha_1 \alpha_2} \zeta_{\alpha_3 \dots \alpha_{2s}}) + 2i \nabla_{(\alpha_1} (\tau_{\alpha_2 \dots \alpha_{2s}}) + i \tilde{\tau}_{\alpha_2 \dots \alpha_{2s}}))), \quad (7.12a)$$

$$\begin{aligned} \nabla^{2\beta} \delta \bar{G}_{\beta\alpha(2s-1)}| &= \frac{i}{2s} \left(2s(2s+1)|\mu| \nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s-1})} + (s+1) \nabla_{\beta(\alpha_1} \nabla^\beta \zeta_{\alpha_2 \dots \alpha_{2s-1})} \right. \\ &\quad + (4s^2 - 1)|\mu| (\tau_{\alpha(2s-1)} + i \tilde{\tau}_{\alpha(2s-1)}) + \frac{i}{2} (2s+1) \nabla^2 (\tau_{\alpha(2s-1)} + i \tilde{\tau}_{\alpha(2s-1)}) \\ &\quad \left. + (2s-1) \nabla_{(\alpha_1}{}^\beta (\tau_{\alpha_2 \dots \alpha_{2s-1})\beta} + i \tilde{\tau}_{\alpha_2 \dots \alpha_{2s-1})\beta}) - (s-1) \nabla_{(\alpha_1}{}^\beta \nabla_{\alpha_2} \zeta_{\alpha_3 \dots \alpha_{2s-1})\beta} \right), \end{aligned} \quad (7.12b)$$

where we have defined

$$\nabla^2_{(\alpha_1} \gamma_{\alpha_2 \dots \alpha_{2s-1})}| = \tau_{\alpha(2s-1)} + i \tilde{\tau}_{\alpha(2s-1)}. \quad (7.13)$$

It is useful to decompose the complex $\mathcal{N} = 1$ superfields (7.10) into real and imaginary parts

$$\bar{G}_{\alpha(2s)}| = -\frac{1}{2} (H_{\alpha(2s)} - 2i \tilde{H}_{\alpha(2s)}), \quad (7.14a)$$

$$\nabla^{2\beta} \bar{G}_{\beta\alpha(2s-1)}| = \Phi_{\alpha(2s-1)} - 2i \Omega_{\alpha(2s-1)}. \quad (7.14b)$$

The gauge transformations of these four real $\mathcal{N} = 1$ superfields immediately follow from gauge transformations (7.12)

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1} \rho_{\alpha_2 \dots \alpha_{2s}}), \quad (7.15a)$$

$$\delta \tilde{H}_{\alpha(2s)} = \nabla_{(\alpha_1} \tau_{\alpha_2 \dots \alpha_{2s}}), \quad (7.15b)$$

$$\delta\Phi_{\alpha(2s-1)} = -\frac{1}{8s}(2(2s-1)\nabla_{(\alpha_1}{}^\beta\rho_{\alpha_2\cdots\alpha_{2s-1})\beta} + 2(4s^2-1)|\mu|\rho_{\alpha(2s-1)} + (2s+1)i\nabla^2\rho_{\alpha(2s-1)} - 8i(4s^2-1)|\mu|\nabla_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s-1})}), \quad (7.15c)$$

$$\delta\Omega_{\alpha(2s-1)} = -\frac{1}{8s}(2(2s-1)\nabla_{(\alpha_1}{}^\beta\tau_{\alpha_2\cdots\alpha_{2s-1})\beta} + 2(4s^2-1)|\mu|\tau_{\alpha(2s-1)} + i(2s+1)\nabla^2\tau_{\alpha(2s-1)}), \quad (7.15d)$$

where we have introduced the gauge parameter redefinition

$$\rho_{\alpha(2s-1)} := 2\tilde{\tau}_{\alpha(2s-1)} + i\nabla_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s-1})}. \quad (7.16)$$

We are now left with the remaining unconstrained real $\mathcal{N} = 1$ superfields

$$\mathcal{U}_{\beta;\alpha(2s-2)} := i\nabla_\beta^2 U_{\alpha(2s-2)}, \quad (7.17a)$$

$$V_{\alpha(2s-2)} := -\frac{i}{8s}(\nabla^2)^2 U_{\alpha(2s-2)}. \quad (7.17b)$$

Utilizing the gauge transformations (7.2a), in conjunction with the residual gauge symmetry (7.8) and definition (7.13), we find that the superfields (7.17) possess the following gauge freedom:

$$\delta\mathcal{U}_{\beta;\alpha(2s-2)} = -\rho_{\beta\alpha(2s-2)} + i\nabla_\beta\zeta_{\alpha(2s-2)}, \quad (7.18a)$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s}\nabla^\beta\tau_{\beta\alpha(2s-2)}. \quad (7.18b)$$

It then becomes apparent from gauge transformations (7.15) and (7.18) that we are dealing with two different gauge theories. The first of these gauge theories is formulated in terms of the dynamical variables

$$\mathcal{V}_{(s)}^\perp = \{H_{\alpha(2s)}, \mathcal{U}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s-1)}\}, \quad (7.19)$$

which are defined modulo gauge transformations

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1}\rho_{\alpha_2\cdots\alpha_{2s})}, \quad (7.20a)$$

$$\delta\mathcal{U}_{\beta;\alpha(2s-2)} = -\rho_{\beta\alpha(2s-2)} + i\nabla_\beta\zeta_{\alpha(2s-2)}, \quad (7.20b)$$

$$\begin{aligned} \delta\Phi_{\alpha(2s-1)} = & -\frac{1}{8s}(2(2s-1)\nabla_{(\alpha_1}{}^\beta\rho_{\alpha_2\cdots\alpha_{2s-1})\beta} \\ & + 2(4s^2-1)|\mu|\rho_{\alpha(2s-1)} + (2s+1)i\nabla^2\rho_{\alpha(2s-1)} \\ & - 8i(4s^2-1)|\mu|\nabla_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s-1})}). \end{aligned} \quad (7.20c)$$

The other gauge model is described by the superfields

$$\mathcal{V}_{(s)}^\parallel = \{\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}\}, \quad (7.21)$$

which possesses the following gauge freedom:

$$\delta\tilde{H}_{\alpha(2s)} = \nabla_{(\alpha_1}\tau_{\alpha_2\cdots\alpha_{2s})}, \quad (7.22a)$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s}\nabla^\beta\tau_{\beta\alpha(2s-2)}, \quad (7.22b)$$

$$\begin{aligned} \delta\Omega_{\alpha(2s-1)} = & -\frac{1}{8s}(2(2s-1)\nabla_{(\alpha_1}{}^\beta\tau_{\alpha_2\cdots\alpha_{2s-1})\beta} \\ & + 2(4s^2-1)|\mu|\tau_{\alpha(2s-1)} \\ & + i(2s+1)\nabla^2\tau_{\alpha(2s-1)}). \end{aligned} \quad (7.22c)$$

Applying the reduction prescription to the action (7.3), we obtain two decoupled $\mathcal{N} = 1$ supersymmetric actions, which are formulated in terms of superfields (7.19) and (7.21), respectively,

$$\begin{aligned} \mathbb{S}_{(s)}^\parallel[U_{\alpha(2s-2)}, G_{\alpha(2s)}, \tilde{G}_{\alpha(2s)}] \\ = S_{(s)}^\perp[H_{\alpha(2s)}, \mathcal{U}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s-1)}] \\ + S_{(s)}^\parallel[\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}]. \end{aligned} \quad (7.23)$$

The explicit form of the $\mathcal{N} = 1$ decoupled actions are provided in the following subsection.

Let us point out that there also appeared a new off-shell formulation for the massless integer superspin multiplet in (1,1) AdS superspace [23]. This formulation proves to be a generalized version of the longitudinal action (7.3), for the gauge-invariant action involves not only $U_{\alpha(2s-2)}$, $\Psi_{\alpha(2s-1)}$, and $\bar{\Psi}_{\alpha(2s-1)}$, but also new compensating superfields. Furthermore, the prepotential $\Psi_{\alpha(2s-1)}$ (associated with the longitudinal linear field strength $G_{\alpha(2s)}$), enjoys a larger gauge symmetry, which is that of the superconformal complex superspin- s multiplet [23]. Upon reduction to $\mathcal{N} = 1$ AdS superspace, we found that this new formulation decoupled into two $\mathcal{N} = 1$ supersymmetric higher-spin models that coincide with the right-hand side of Eq. (7.23).

C. Massless higher-spin $\mathcal{N} = 1$ supermultiplets

The first $\mathcal{N} = 1$ supersymmetric action, which is described by the dynamical variables (7.19), takes the form

$$\begin{aligned}
S_{(s)}^\perp[H_{\alpha(2s)}, \mathcal{U}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s-1)}] = & \left(-\frac{1}{2}\right)^s \frac{1}{4} \int d^3z E \left\{ -i\mathcal{U}^{\beta;\alpha(2s-2)} \square \mathcal{U}_{\beta;\alpha(2s-2)} - \frac{1}{2} \mathcal{U}^{\beta;\alpha(2s-2)} \nabla^2 \nabla_\beta^\gamma \mathcal{U}_{\gamma;\alpha(2s-2)} \right. \\
& + \frac{1}{2} (2s-3) |\mu| \mathcal{U}^{\beta;\alpha(2s-2)} \nabla^2 \mathcal{U}_{\beta;\alpha(2s-2)} + 2(s-1) |\mu| \mathcal{U}_{\beta;\alpha(2s-3)}^{\beta;\alpha(2s-3)} \nabla^2 \mathcal{U}_{\gamma;\alpha(2s-3)}^\gamma \\
& + i(4s^2-1) |\mu|^2 \mathcal{U}^{\beta;\alpha(2s-2)} \mathcal{U}_{\beta;\alpha(2s-2)} - 8i(s-1) |\mu|^2 \mathcal{U}_{\beta;\alpha(2s-3)}^{\beta;\alpha(2s-3)} \mathcal{U}_{\gamma;\alpha(2s-3)}^\gamma \\
& - \frac{2s}{2s+1} \mathcal{U}^{\beta;\alpha(2s-2)} (-2\nabla^\gamma \delta \nabla_\gamma H_{\delta\beta\alpha(2s-2)} + \nabla^\gamma \delta \nabla_\beta H_{\gamma\delta\alpha(2s-2)}) \\
& + \nabla^2 \Phi_{\beta\alpha(2s-2)} + 2i\nabla_\beta^\gamma \Phi_{\alpha(2s-2)\gamma} - 4s |\mu| \nabla^\gamma H_{\gamma\beta\alpha(2s-2)} + 2i(2s+1) |\mu| \Phi_{\beta\alpha(2s-2)} \\
& - \frac{s}{(2s+1)^2(2s-1)} (-i(3s+1) H^{\alpha(2s)} \nabla^2 H_{\alpha(2s)}) \\
& + 2s(4s+1) H^{\alpha(2s-1)\beta} \nabla_\beta^\gamma H_{\alpha(2s-1)\gamma} + 16s^2 H^{\alpha(2s)} \nabla_{(\alpha_1} \Phi_{\alpha_2 \dots \alpha_{2s})} \\
& \left. - 8is \Phi^{\alpha(2s-1)} \Phi_{\alpha(2s-1)} - 4s(4s^2+3s+1) |\mu| H^{\alpha(2s)} H_{\alpha(2s)} \right\}. \tag{7.24}
\end{aligned}$$

It is apparent that the superfield $\Phi_{\alpha(2s-1)}$ is auxiliary, thus, upon elimination via its equation of motion,

$$\Phi_{\alpha(2s-1)} = -\frac{i}{8s} (2i(2s+1)(4s^2-1) |\mu| \mathcal{U}_{\alpha(2s-1)} + (4s^2-1) \nabla^2 \mathcal{U}_{\alpha(2s-1)} - 2i(4s^2-1) \nabla_{\beta(\alpha_1} \mathcal{U}_{\alpha_2 \dots \alpha_{2s-1})}^{\beta;\alpha(2s-1)} + 8s^2 \nabla^\beta H_{\beta\alpha(2s-1)}), \tag{7.25}$$

we arrive at the resulting action

$$\begin{aligned}
S_{(s)}^\perp[H_{\alpha(2s)}, \mathcal{U}_{\beta;\alpha(2s-2)}] = & \left(-\frac{1}{2}\right)^s \frac{1}{4} \int d^3z E \left\{ -\frac{is(s-1)}{2s-1} H^{\alpha(2s)} \nabla^2 H_{\alpha(2s)} \right. \\
& - \frac{2s^2}{2s-1} H^{\alpha(2s-1)\beta} \nabla_\beta^\gamma H_{\alpha(2s-1)\gamma} - \frac{4s^2(s-1)}{2s-1} |\mu| H^{\alpha(2s)} H_{\alpha(2s)} \\
& - 2s \mathcal{U}^{\delta\alpha(2s-2)} \nabla^{\beta\gamma} (\nabla_{(\delta} H_{\alpha_1 \dots \alpha_{2s-2})\beta\gamma} - 2\nabla_\beta H_{\gamma\delta\alpha(2s-2)}) \\
& + \frac{4s(s-1)}{2s-1} \mathcal{U}_{\lambda;\alpha(2s-3)}^{\lambda\alpha(2s-3)} \nabla^{\beta\gamma} \nabla^\delta H_{\beta\gamma\delta\alpha(2s-3)} - 8s^2 |\mu| \mathcal{U}^{\beta;\alpha(2s-2)} \nabla^\gamma H_{\gamma\beta\alpha(2s-2)} \\
& + \mathcal{U}^{\alpha(2s-1)} \left(-2is \square \mathcal{U}_{\alpha(2s-1)} - s \nabla^2 \nabla_{(\alpha_1} \mathcal{U}_{\alpha_2 \dots \alpha_{2s-1})\beta} \right. \\
& - i(s-1) \nabla_{(\alpha_1 \alpha_2} \nabla^{\beta\gamma} \mathcal{U}_{\alpha_3 \dots \alpha_{2s-1})\beta\gamma} + 2i \frac{(s-1)(2s-3)}{2s-1} \nabla_{(\alpha_1 \alpha_2} \nabla_{\alpha_3}^\beta \mathcal{U}_{\alpha_4 \dots \alpha_{2s-1})\beta\gamma}^\gamma \\
& + \frac{(s-1)(2s+1)}{2s-1} \nabla^2 \nabla_{(\alpha_1 \alpha_2} \mathcal{U}_{\alpha_3 \dots \alpha_{2s-1})\beta}^{\beta;\alpha(2s-1)} - s(2s+1) |\mu| \nabla^2 \mathcal{U}_{\alpha(2s-1)} \\
& - 4is(2s-1) |\mu| \nabla_{(\alpha_1}^\beta \mathcal{U}_{\alpha_2 \dots \alpha_{2s-1})\beta} + 2i(s-1)(2s+1) |\mu| \nabla_{(\alpha_1 \alpha_2} \mathcal{U}_{\alpha_3 \dots \alpha_{2s-1})\beta}^{\beta;\alpha(2s-1)} \\
& \left. - 2is(2s-1)(2s+1) |\mu|^2 \mathcal{U}_{\alpha(2s-1)} \right) \\
& + \mathcal{U}_{\lambda;\alpha(2s-3)}^{\lambda\alpha(2s-3)} \left(2i \frac{(s-1)(6s-5)}{(2s-1)^2} \square \mathcal{U}_{\beta\alpha(2s-3)}^{\beta;\alpha(2s-3)} \right. \\
& - \frac{(s-1)(2s-3)}{(2s-1)^2} (2i(s-2) \nabla^{\beta\gamma} \nabla_{(\alpha_1 \alpha_2} \mathcal{U}_{\alpha_3 \dots \alpha_{2s-3})\beta\gamma}^\delta \\
& + \nabla^2 \nabla_{(\alpha_1}^\beta \mathcal{U}_{\alpha_2 \dots \alpha_{2s-3})\beta\gamma}^\gamma) + \frac{(s-1)(2s+1)}{(2s-1)} |\mu| \nabla^2 \mathcal{U}_{\beta\alpha(2s-3)}^{\beta;\alpha(2s-3)} \\
& \left. - 2i \frac{s-1}{2s-1} (4s^2+16s-17) |\mu|^2 \mathcal{U}_{\beta\alpha(2s-3)}^{\beta;\alpha(2s-3)} \right) \Big\}. \tag{7.26}
\end{aligned}$$

Here we have made use of the notation $\mathcal{U}_{\alpha(2s-1)} = \mathcal{U}_{(\alpha_1; \alpha_2 \dots \alpha_{2s-1})}$. It is a tedious but straightforward exercise to verify that the above $\mathcal{N} = 1$ action coincides with the transverse formulation for massless superspin- s multiplet given by (3.17). To prove this, one first needs to introduce the $\mathcal{N} = 1$ field strength $\mathcal{W}_{\beta; \alpha(2s-2)}$ associated with the prepotential $\mathcal{U}_{\beta; \alpha(2s-2)}$,

$$\mathcal{W}_{\beta; \alpha(2s-2)} = -i(\nabla^\gamma \nabla_\beta - 4i|\mu|\delta_{\beta}^{\gamma})\mathcal{U}_{\gamma; \alpha(2s-2)}, \quad \nabla^\beta \mathcal{W}_{\beta; \alpha(2s-2)} = 0. \quad (7.27)$$

The action (7.26) can then be written more compactly as

$$\begin{aligned} S_{(s)}^\perp[\mathcal{H}_{\alpha(2s)}, \mathcal{U}_{\beta; \alpha(2s-2)}] = & \left(-\frac{1}{2}\right)^s \left(\frac{2s}{2s-1}\right) \int d^3|z|E \left\{ \frac{1}{2} H^{\alpha(2s)} (i\nabla^2 + 8s|\mu|) H_{\alpha(2s)} \right. \\ & - i s \nabla_\beta H^{\beta\alpha(2s-1)} \nabla^\gamma H_{\gamma\alpha(2s-1)} - (2s-1) \mathcal{W}^{\beta; \alpha(2s-2)} \nabla^\gamma H_{\gamma\beta\alpha(2s-2)} \\ & - \frac{i}{2} (2s-1) \left(\mathcal{W}^{\beta; \alpha(2s-2)} \mathcal{W}_{\beta; \alpha(2s-2)} + \frac{s-1}{s} \mathcal{W}_{\beta; \gamma\alpha(2s-3)}^{\beta\alpha(2s-3)} \mathcal{W}_{\gamma\alpha(2s-3)} \right) \\ & \left. - 2i(2s-1)|\mu| \mathcal{U}^{\beta; \alpha(2s-2)} \mathcal{W}_{\beta; \alpha(2s-2)} \right\}, \end{aligned} \quad (7.28)$$

which is exactly (3.17) modulo an overall normalization factor.

The other decoupled $\mathcal{N} = 1$ action that is formulated in terms of the dynamical variables (7.21) assumes the form

$$\begin{aligned} S_{(s)}^\parallel[\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}] = & \left(-\frac{1}{2}\right)^s \int d^3|z|E \left\{ 2s^2 V^{\alpha(2s-2)} (i\nabla^2 + 4|\mu|) V_{\alpha(2s-2)} \right. \\ & - \frac{4s^2}{2s+1} V^{\alpha(2s-2)} (\nabla^{\beta\gamma} \tilde{H}_{\beta\gamma\alpha(2s-2)} + 2\nabla^\beta \Omega_{\beta\alpha(2s-2)}) \\ & + \frac{s}{(2s+1)^2(2s-1)} ((2s+1)^2 \tilde{H}^{\alpha(2s)} (i\nabla^2 + 4|\mu|) \tilde{H}_{\alpha(2s)} \\ & - 4is(s+1) \nabla_\beta \tilde{H}^{\beta\alpha(2s-1)} \nabla^\gamma \tilde{H}_{\gamma\alpha(2s-1)} - 8s \tilde{H}^{\alpha(2s)} \nabla_{(\alpha_1} \Omega_{\alpha_2 \dots \alpha_{2s})} \\ & \left. + 16is^2 \Omega^{\alpha(2s-1)} \Omega_{\alpha(2s-1)}) \right\}. \end{aligned} \quad (7.29)$$

It is clear from the action (7.29) that the superfield $\Omega_{\alpha(2s-1)}$ is auxiliary. Integrating it out by using its equation of motion,

$$\Omega_{\alpha(2s-1)} = \frac{i}{4s} ((4s^2 - 1) \nabla_{(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-1})} - \nabla^\beta \tilde{H}_{\beta\alpha(2s-1)}), \quad (7.30)$$

we find that the resulting action, modulo a normalization factor, coincides with (3.14).

D. Second supersymmetry transformations

We begin by computing the second supersymmetry transformations (5.25) of the superfields (7.19) and (7.21),

$$\delta_\epsilon U_{\alpha(2s-2)} = -i\epsilon^\beta \mathcal{U}_{\beta; \alpha(2s-2)}, \quad (7.31a)$$

$$\delta_\epsilon \mathcal{U}_{\beta; \alpha(2s-2)} = 4s\epsilon_\beta V_{\alpha(2s-2)}, \quad (7.31b)$$

$$\delta_\epsilon V_{\alpha(2s-2)} = \frac{i}{4s} ((2s-1)|\mu|\epsilon^\beta \mathcal{U}_{\beta; \alpha(2s-2)} - \epsilon^\beta \nabla_\beta \mathcal{U}_{\gamma; \alpha(2s-2)} + 4(s-1)|\mu|\epsilon_{(\alpha_1} \mathcal{U}_{\alpha_2 \dots \alpha_{2s-2})}^{\beta; \alpha(2s-2)}), \quad (7.31c)$$

$$\delta_\epsilon H_{\alpha(2s)} = -2\epsilon^\beta \nabla_{(\beta} \tilde{H}_{\alpha(2s))} - \frac{8is}{2s+1} \epsilon_{(\alpha_1} \Omega_{\alpha_2 \dots \alpha_{2s})}, \quad (7.31d)$$

$$\delta_\epsilon \tilde{H}_{\alpha(2s)} = \frac{1}{2} \epsilon^\beta \nabla_{(\beta} H_{\alpha(2s))} + \frac{2is}{2s+1} \epsilon_{(\alpha_1} \Phi_{\alpha_2 \dots \alpha_{2s})}, \quad (7.31e)$$

$$\begin{aligned}\delta_\epsilon \Phi_{\alpha(2s-1)} &= \frac{1}{2(2s+1)} \epsilon^\beta (2(4s^2 + 3s + 1) |\mu| \tilde{H}_{\beta\alpha(2s-1)} + i \nabla^2 \tilde{H}_{\beta\alpha(2s-1)} - 2(4s + 3) \nabla_\beta{}^\gamma \tilde{H}_{\gamma\alpha(2s-1)} \\ &\quad - 2(2s - 1) \nabla_{(\alpha_1}{}^\gamma \tilde{H}_{\alpha_2 \dots \alpha_{2s-1})\beta\gamma} + 8s \nabla_{(\beta} \Omega_{\alpha(2s-1))}),\end{aligned}\quad (7.31f)$$

$$\begin{aligned}\delta_\epsilon \Omega_{\alpha(2s-1)} &= -\frac{1}{8(2s+1)} \epsilon^\beta (4(4s^2 + 3s + 1) |\mu| H_{\beta\alpha(2s-1)} + i \nabla^2 H_{\beta\alpha(2s-1)} - 2(4s + 3) \nabla_\beta{}^\gamma H_{\gamma\alpha(2s-1)} \\ &\quad - 2(2s - 1) \nabla_{(\alpha_1}{}^\gamma H_{\alpha_2 \dots \alpha_{2s-1})\beta\gamma} + 8s \nabla_{(\beta} \Phi_{\alpha(2s-1))}).\end{aligned}\quad (7.31g)$$

It is clear from the variation (7.31a) that the second supersymmetry transformation (5.25) breaks the WZ gauge (7.6). To restore this gauge condition, it is necessary to supplement the second supersymmetry transformation with the ϵ -dependent gauge transformations:

$$\delta_{\gamma(\epsilon)} U_{\alpha(2s-2)} = \gamma_{\alpha(2s-2)}(\epsilon) + \bar{\gamma}_{\alpha(2s-2)}(\epsilon), \quad (7.32a)$$

$$\begin{aligned}\delta_{\gamma(\epsilon)} G_{\alpha(2s)} &= \bar{D}_{(\alpha_1} \mathcal{D}_{\alpha_2} \bar{\gamma}_{\alpha_3 \dots \alpha_{2s}})(\epsilon) \\ &= i(\nabla^1_{(\alpha_1} \nabla^2_{\alpha_2} + \nabla_{(\alpha_1 \alpha_2}) \gamma_{\alpha_3 \dots \alpha_{2s}})(\epsilon),\end{aligned}\quad (7.32b)$$

with

$$\gamma_{\alpha(2s-2)}(\epsilon) = \frac{i}{2} \epsilon^\beta \mathcal{U}_{\beta;\alpha(2s-2)}. \quad (7.33)$$

The modified second supersymmetry transformations have the form

$$\hat{\delta}_\epsilon U_{\alpha(2s-2)} = \delta_\epsilon U_{\alpha(2s-2)} + \delta_{\gamma(\epsilon)} U_{\alpha(2s-2)}, \quad (7.34a)$$

$$\hat{\delta}_\epsilon G_{\alpha(2s)} = \delta_\epsilon G_{\alpha(2s)} + \delta_{\gamma(\epsilon)} G_{\alpha(2s)}. \quad (7.34b)$$

They act on the $\mathcal{N} = 1$ gauge superfields (7.19) and (7.21) by the rule

$$\hat{\delta}_\epsilon \mathcal{U}_{\beta;\alpha(2s-2)} = 4s \epsilon_\beta V_{\alpha(2s-2)}, \quad (7.35a)$$

$$\hat{\delta}_\epsilon V_{\alpha(2s-2)} = -\frac{i}{8s} (2(2s+1) |\mu| \epsilon^\beta \mathcal{U}_{\beta;\alpha(2s-2)} + 2\epsilon^\beta \nabla_\beta{}^\gamma \mathcal{U}_{\gamma;\alpha(2s-2)} - 8(s-1) |\mu| \epsilon_{(\alpha_1} \mathcal{U}^{\beta;}_{\alpha_2 \dots \alpha_{2s-2})\beta} + i\epsilon^\beta \nabla^2 \mathcal{U}_{\beta;\alpha(2s-2)}), \quad (7.35b)$$

$$\begin{aligned}\hat{\delta}_\epsilon H_{\alpha(2s)} &= -2\epsilon^\beta \nabla_{(\beta} \tilde{H}_{\alpha(2s))} - \frac{8is}{2s+1} \epsilon_{(\alpha_1} \Omega_{\alpha_2 \dots \alpha_{2s})}, \\ \hat{\delta}_\epsilon \tilde{H}_{\alpha(2s)} &= \frac{1}{2} \epsilon^\beta \nabla_{(\beta} H_{\alpha(2s))} + \frac{2is}{2s+1} \epsilon_{(\alpha_1} \Phi_{\alpha_2 \dots \alpha_{2s})} + i |\mu| \epsilon_{(\alpha_1} \mathcal{U}_{\alpha_2 \dots \alpha_{2s})} - \frac{i}{2} \epsilon_\beta \nabla_{(\alpha_1 \alpha_2} \mathcal{U}^{\beta;}_{\alpha_3 \dots \alpha_{2s})},\end{aligned}\quad (7.35c)$$

$$\begin{aligned}\hat{\delta}_\epsilon \Phi_{\alpha(2s-1)} &= \frac{1}{2(2s+1)} \epsilon^\beta (2(4s^2 + 3s + 1) |\mu| \tilde{H}_{\beta\alpha(2s-1)} + i \nabla^2 \tilde{H}_{\beta\alpha(2s-1)} - 2(4s + 3) \nabla_\beta{}^\gamma \tilde{H}_{\gamma\alpha(2s-1)} \\ &\quad - 2(2s - 1) \nabla_{(\alpha_1}{}^\gamma \tilde{H}_{\alpha_2 \dots \alpha_{2s-1})\beta\gamma} + 8s \nabla_{(\beta} \Omega_{\alpha(2s-1))}),\end{aligned}\quad (7.35d)$$

$$\begin{aligned}\hat{\delta}_\epsilon \Omega_{\alpha(2s-1)} &= -\frac{1}{8(2s+1)} \epsilon^\beta (4(4s^2 + 3s + 1) |\mu| H_{\beta\alpha(2s-1)} + i \nabla^2 H_{\beta\alpha(2s-1)} - 2(4s + 3) \nabla_\beta{}^\gamma H_{\gamma\alpha(2s-1)} \\ &\quad - 2(2s - 1) \nabla_{(\alpha_1}{}^\gamma H_{\alpha_2 \dots \alpha_{2s-1})\beta\gamma} + 8s \nabla_{(\beta} \Phi_{\alpha(2s-1))}) + \frac{1}{4s} ((2s+1) |\mu| \epsilon^\beta \nabla_\beta \mathcal{U}_{\alpha(2s-1)} \\ &\quad + (s+1) \epsilon_\beta \nabla_{\gamma(\alpha_1} \nabla^\gamma \mathcal{U}^{\beta;}_{\alpha_2 \dots \alpha_{2s-1})} + 2(s-1) |\mu| \epsilon_{(\alpha_1} \nabla_{\alpha_2} \mathcal{U}^{\beta;}_{\alpha_3 \dots \alpha_{2s-1})\beta} - (s-1) \epsilon_\beta \nabla_{(\alpha_1}{}^\gamma \nabla_{\alpha_2} \mathcal{U}^{\beta;}_{\alpha_3 \dots \alpha_{2s-1})\gamma} \\ &\quad + (6s^2 + 3s + 2) |\mu| \epsilon_\beta \nabla_{(\alpha_1} \mathcal{U}^{\beta;}_{\alpha_2 \dots \alpha_{2s-1})}).\end{aligned}\quad (7.35e)$$

VIII. MASSLESS INTEGER SUPERSPIN: TRANSVERSE FORMULATION

We now apply the same reduction procedure to the off-shell transverse formulation for the massless multiplets of integer superspin.

A. Transverse formulation

For any integer $s \geq 1$, the transverse formulation for the massless superspin- s multiplet is realized in terms of the following dynamical variables:

$$\begin{aligned} \mathbb{S}_{(s)}^\perp[U_{\alpha(2s-2)}, \Gamma_{\alpha(2s)}, \bar{\Gamma}_{\alpha(2s)}] = & \left(-\frac{1}{2} \right)^s \int d^3z E \left\{ 2s(s+1) \mu \bar{\mu} U^{\alpha(2s-2)} U_{\alpha(2s-2)} + \frac{1}{8} U^{\alpha(2s-2)} \mathcal{D}^\gamma (\bar{\mathcal{D}}^2 - 6\mu) \mathcal{D}_\gamma U_{\alpha(2s-2)} \right. \\ & - i U^{\alpha_1 \dots \alpha_{2s-2}} \mathcal{D}^{\alpha_{2s-1} \alpha_{2s}} (\Gamma_{\alpha(2s)} - \bar{\Gamma}_{\alpha(2s)}) - \frac{2s-1}{16(2s+1)} (8s \mathcal{D}^{\alpha_1 \alpha_2} U^{\alpha_3 \dots \alpha_{2s}} \mathcal{D}_{(\alpha_1 \alpha_2} U_{\alpha_3 \dots \alpha_{2s}}) \\ & + [\mathcal{D}^{\alpha_1}, \bar{\mathcal{D}}^{\alpha_2}] U^{\alpha_3 \dots \alpha_{2s}} [\mathcal{D}_{(\alpha_1}, \bar{\mathcal{D}}_{\alpha_2)} U_{\alpha_3 \dots \alpha_{2s}}) - \frac{2}{2s-1} \bar{\Gamma}^{\alpha(2s)} \Gamma_{\alpha(2s)} \\ & \left. + \frac{1}{2s+1} (\Gamma^{\alpha(2s)} \Gamma_{\alpha(2s)} + \bar{\Gamma}^{\alpha(2s)} \bar{\Gamma}_{\alpha(2s)}) \right\}, \end{aligned} \quad (8.2)$$

which is invariant under the gauge transformations

$$\begin{aligned} \delta_L U_{\alpha(2s-2)} &= \mathcal{D}^\beta L_{\beta\alpha(2s-2)} - \bar{\mathcal{D}}^\beta \bar{L}_{\beta\alpha(2s-2)} \\ &\equiv \bar{\gamma}_{\alpha(2s-2)} + \gamma_{\alpha(2s-2)}, \end{aligned} \quad (8.3a)$$

$$\begin{aligned} \delta_L \Gamma_{\alpha(2s)} &= -\frac{1}{4} (\bar{\mathcal{D}}^2 + 4s\mu) \mathcal{D}_{(\alpha_1} \bar{L}_{\alpha_2 \dots \alpha_{2s})} \\ &\quad + \frac{i}{2} (2s+1) \bar{\mathcal{D}}^\gamma \mathcal{D}_{(\gamma \alpha_1} \bar{L}_{\alpha_2 \dots \alpha_{2s})} \\ &\equiv \frac{1}{2} \mathcal{D}_{(\alpha_1} \bar{\mathcal{D}}_{\alpha_2} \gamma_{\alpha_3 \dots \alpha_{2s})} - \frac{i}{2} (2s-1) \mathcal{D}_{(\alpha_1 \alpha_2} \gamma_{\alpha_3 \dots \alpha_{2s})}, \end{aligned} \quad (8.3b)$$

where the gauge parameter $L_{\alpha(2s-1)}$ is complex unconstrained and the superfield $\gamma_{\alpha(2s-2)}$ is transverse linear, as in (7.2a).

B. Reduction of gauge prepotentials to $\text{AdS}^{3|2}$

We begin by reducing the real superfield $U_{\alpha(2s-2)}$. Following the prescription endorsed in Sec. VII, we have the freedom to impose the gauge condition

$$U_{\alpha(2s-2)}| = 0. \quad (8.4)$$

The residual gauge symmetry that preserves this gauge is described by the real unconstrained $\mathcal{N} = 1$ superfield

$$\gamma_{\alpha(2s-2)}| = -\frac{i}{2} \zeta_{\alpha(2s-2)}, \quad \zeta_{\alpha(2s-2)} = \bar{\zeta}_{\alpha(2s-2)}. \quad (8.5)$$

$$\mathcal{V}_{(s)}^\perp = \{U_{\alpha(2s-2)}, \Gamma_{\alpha(2s)}, \bar{\Gamma}_{\alpha(2s)}\}. \quad (8.1)$$

The real superfield $U_{\alpha(2s-2)}$ is the same as in (7.1) and the complex superfield $\Gamma_{\alpha(2s)}$ is transverse linear. The transverse formulation for the massless superspin- s multiplet was derived by performing a superfield duality transformation on the longitudinal model reviewed in Sec. VII. Thus, it can be shown that the transverse action takes the form

As a result of choosing the gauge condition (8.4), we are left with the remaining independent $\mathcal{N} = 1$ superfields with respect to $U_{\alpha(2s-2)}$

$$\mathcal{U}_{\beta;\alpha(2s-2)} := i \nabla_\beta^2 U_{\alpha(2s-2)}|, \quad (8.6a)$$

$$V_{\alpha(2s-2)} := -\frac{i}{8s} (\nabla^2)^2 U_{\alpha(2s-2)}|. \quad (8.6b)$$

Using (8.3a) enables the computation of the corresponding gauge transformations,

$$\begin{aligned} \delta \mathcal{U}_{\beta;\alpha(2s-2)} &= -2s \rho_{\beta\alpha(2s-2)} + i \nabla_\beta \zeta_{\alpha(2s-2)} \\ &\quad + i(2s-1) \nabla_{(\beta} \zeta_{\alpha(2s-2))}, \end{aligned} \quad (8.7a)$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s} \nabla^\beta \tau_{\beta\alpha(2s-2)}, \quad (8.7b)$$

where we have introduced the following definition:

$$\nabla_{(\alpha_1}^2 \gamma_{\alpha_2 \dots \alpha_{2s-1})}| = \tau_{\alpha(2s-1)} + i s \tilde{\tau}_{\alpha(2s-1)}, \quad (8.8)$$

and gauge parameter redefinition

$$\rho_{\alpha(2s-1)} = \tilde{\tau}_{\alpha(2s-1)} + i \nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s-1})}. \quad (8.9)$$

It is immediately obvious that the gauge transformation of $\mathcal{U}_{\beta;\alpha(2s-2)}$ does not coincide with its analog from the longitudinal formulation (7.18a). Addressing this, we introduce the following $\mathcal{N} = 1$ superfield redefinition:

$$\tilde{\mathcal{U}}_{\beta;\alpha(2s-2)} = \mathcal{U}_{\beta;\alpha(2s-2)} - \frac{2s-1}{2s} \mathcal{U}_{\beta\alpha(2s-2)}. \quad (8.10)$$

By using the gauge transformation (8.7a), it can be shown that $\tilde{\mathcal{U}}_{\beta;\alpha(2s-2)}$ possesses the desired gauge freedom

$$\delta\tilde{\mathcal{U}}_{\beta;\alpha(2s-2)} = -\rho_{\beta\alpha(2s-2)} + i\nabla_{\beta}\zeta_{\alpha(2s-2)}. \quad (8.11)$$

Next, we wish to go about reducing $\Gamma_{\alpha(2s)}$ to $\mathcal{N} = 1$ AdS superspace. We begin by converting the transverse linear constraint $\Gamma_{\alpha(2s)}$ (4.5a) to the real representation (4.18)

$$\nabla^{2\beta}\Gamma_{\beta\alpha(2s-1)} = i\nabla^{1\beta}\Gamma_{\beta\alpha(2s-1)}. \quad (8.12)$$

Taking a Taylor expansion of $\Gamma_{\alpha(2s)}(\theta^I)$ about θ^2 , and using constraint (8.12), we find that the independent θ^2 components of $\Gamma_{\alpha(2s)}$ are

$$\Gamma_{\alpha(2s)}|, \quad \nabla^2_{(\alpha_1}\Gamma_{\alpha_2\cdots\alpha_{2s+1})}|. \quad (8.13)$$

Using the gauge transformations (8.3b) and the real representation (4.18), we find

$$\delta\Gamma_{\alpha(2s)} = -\frac{i}{2}\nabla^1_{(\alpha_1}\nabla^2_{\alpha_2}\gamma_{\alpha_3\cdots\alpha_{2s})} - is\nabla_{(\alpha_1\alpha_2}\gamma_{\alpha_3\cdots\alpha_{2s})}, \quad (8.14a)$$

$$\begin{aligned} \nabla^2_{(\alpha_1}\delta\Gamma_{\alpha_2\cdots\alpha_{2s+1})} &= -\frac{1}{2}\nabla_{(\alpha_1\alpha_2}\nabla^1_{\alpha_3}\gamma_{\alpha_4\cdots\alpha_{2s+1})} \\ &\quad - is\nabla_{(\alpha_1\alpha_2}\nabla^2_{\alpha_3}\gamma_{\alpha_4\cdots\alpha_{2s+1})}. \end{aligned} \quad (8.14b)$$

Making use of (8.14) and the gauge freedom (7.8) allows us to directly compute the gauge transformations of the $\mathcal{N} = 1$ complex superfields (8.13),

$$\begin{aligned} \delta\Gamma_{\alpha(2s)}| &= -\frac{i}{2}\nabla_{(\alpha_1}\tau_{\alpha_2\cdots\alpha_{2s})} + \frac{s}{2}\nabla_{(\alpha_1}\tilde{\tau}_{\alpha_2\cdots\alpha_{2s})} \\ &\quad - \frac{s}{2}\nabla_{(\alpha_1\alpha_2}\zeta_{\alpha_3\cdots\alpha_{2s})}, \end{aligned} \quad (8.15a)$$

$$\begin{aligned} \nabla^2_{(\alpha_1}\delta\Gamma_{\alpha_2\cdots\alpha_{2s+1})}| &= \frac{i}{4}\nabla_{(\alpha_1\alpha_2}\nabla_{\alpha_3}\zeta_{\alpha_4\cdots\alpha_{2s+1})} - is\nabla_{(\alpha_1\alpha_2}\tau_{\alpha_3\cdots\alpha_{2s+1})} \\ &\quad + s^2\nabla_{(\alpha_1\alpha_2}\tilde{\tau}_{\alpha_3\cdots\alpha_{2s+1})}. \end{aligned} \quad (8.15b)$$

Let us split the complex $\mathcal{N} = 1$ superfields (8.13) into real and imaginary parts

$$\Gamma_{\alpha(2s)}| = \frac{1}{2}(sH_{\alpha(2s)} - i\tilde{H}_{\alpha(2s)}), \quad (8.16a)$$

$$\nabla^2_{(\alpha_1}\Gamma_{\alpha_2\cdots\alpha_{2s+1})}| = \Phi_{\alpha(2s+1)} - is\Omega_{\alpha(2s+1)}. \quad (8.16b)$$

The gauge transformations of these four real $\mathcal{N} = 1$ superfields can be read off from the gauge transformations (8.14),

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1}\rho_{\alpha_2\cdots\alpha_{2s})}, \quad (8.17a)$$

$$\delta\tilde{H}_{\alpha(2s)} = \nabla_{(\alpha_1}\tau_{\alpha_2\cdots\alpha_{2s})}, \quad (8.17b)$$

$$\begin{aligned} \delta\Phi_{\alpha(2s+1)} &= s^2\nabla_{(\alpha_1\alpha_2}\rho_{\alpha_3\cdots\alpha_{2s+1})} \\ &\quad - \frac{i}{4}(4s^2 - 1)\nabla_{(\alpha_1\alpha_2}\nabla_{\alpha_3}\zeta_{\alpha_4\cdots\alpha_{2s+1})}, \end{aligned} \quad (8.17c)$$

$$\delta\Omega_{\alpha(2s+1)} = \nabla_{(\alpha_1\alpha_2}\tau_{\alpha_3\cdots\alpha_{2s+1})}. \quad (8.17d)$$

It then follows from gauge transformations (8.7b), (8.11), and (8.17) that we are indeed dealing with two different gauge theories. The first of the models is formulated in terms of the dynamical variables

$$\mathcal{V}_{(s)}^\perp = \{H_{\alpha(2s)}, \tilde{\mathcal{U}}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s+1)}\}, \quad (8.18)$$

which are defined modulo gauge transformations

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1}\rho_{\alpha_2\cdots\alpha_{2s})}, \quad (8.19a)$$

$$\delta\tilde{\mathcal{U}}_{\beta;\alpha(2s-2)} = -\rho_{\beta\alpha(2s-2)} + i\nabla_{\beta}\zeta_{\alpha(2s-2)}, \quad (8.19b)$$

$$\begin{aligned} \delta\Phi_{\alpha(2s+1)} &= s^2\nabla_{(\alpha_1\alpha_2}\rho_{\alpha_3\cdots\alpha_{2s})} \\ &\quad - \frac{i}{4}(4s^2 - 1)\nabla_{(\alpha_1\alpha_2}\nabla_{\alpha_3}\zeta_{\alpha_4\cdots\alpha_{2s+1})}. \end{aligned} \quad (8.19c)$$

The other gauge theory is described by the superfields

$$\mathcal{V}_{(s)}^\parallel = \{\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s+1)}\}, \quad (8.20)$$

which has the corresponding gauge freedom

$$\delta\tilde{H}_{\alpha(2s)} = \nabla_{(\alpha_1}\tau_{\alpha_2\cdots\alpha_{2s})}, \quad (8.21a)$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s}\nabla^\beta\tau_{\beta\alpha(2s-2)}, \quad (8.21b)$$

$$\delta\Omega_{\alpha(2s+1)} = \nabla_{(\alpha_1\alpha_2}\tau_{\alpha_3\cdots\alpha_{2s+1})}. \quad (8.21c)$$

From the perspective of $\mathcal{N} = 1$ AdS superspace, the reduction of the action (8.2) gives rise to two decoupled supersymmetric actions, which are formulated in terms of superfields (8.18) and (8.20), respectively,

$$\begin{aligned} \mathbb{S}_{(s)}^\perp[U_{\alpha(2s-2)}, \Gamma_{\alpha(2s)}, \bar{\Gamma}_{\alpha(2s)}] \\ = S_{(s)}^\perp[H_{\alpha(2s)}, \tilde{\mathcal{U}}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s+1)}] \\ + S_{(s)}^\parallel[\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s+1)}]. \end{aligned} \quad (8.22)$$

We provide the explicit realizations of these two $\mathcal{N} = 1$ decoupled actions in the following subsection.

C. Massless higher-spin $\mathcal{N} = 1$ supermultiplets

The first of the decoupled $\mathcal{N} = 1$ supersymmetric actions which is formulated in terms of the dynamical variables (8.18) takes the form

$$\begin{aligned}
S_{(s)}^\perp[H_{\alpha(2s)}, \tilde{\mathcal{U}}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s+1)}] = & \left(-\frac{1}{2} \right)^s \frac{1}{4} \int d^3|z| E \left\{ \tilde{\mathcal{U}}^{\alpha(2s-1)} \left(-2is(4s-1) \square \tilde{\mathcal{U}}_{\alpha(2s-1)} \right. \right. \\
& - s \nabla^2 \nabla^\beta{}_{(\alpha_1} \tilde{\mathcal{U}}_{\alpha_2 \dots \alpha_{2s-1})\beta} + 4is(s-1)^2 \nabla_{(\alpha_1 \alpha_2} \nabla^{\beta\gamma} \tilde{\mathcal{U}}_{\alpha_3 \dots \alpha_{2s-1})\beta\gamma} - s(2s+1) |\mu| \nabla^2 \tilde{\mathcal{U}}_{\alpha(2s-1)} \\
& + \frac{2i(s-1)(2s-3)}{2s-1} \nabla_{(\alpha_1 \alpha_2} \nabla_{\alpha_3}{}^\beta \tilde{\mathcal{U}}^{\gamma\delta}{}_{\alpha_4 \dots \alpha_{2s-1})\beta\gamma} + \frac{(s-1)(2s+1)}{2s-1} \nabla^2 \nabla_{(\alpha_1 \alpha_2} \tilde{\mathcal{U}}^{\beta\gamma}{}_{\alpha_3 \dots \alpha_{2s-1})\beta} \\
& - 4is(2s-1) |\mu| \nabla_{(\alpha_1}{}^\beta \tilde{\mathcal{U}}_{\alpha_2 \dots \alpha_{2s-1})\beta} + 2i(s-1)(2s+1) |\mu| \nabla_{(\alpha_1 \alpha_2} \tilde{\mathcal{U}}^{\beta\gamma}{}_{\alpha_3 \dots \alpha_{2s-1})\beta} \\
& \left. \left. + 2is(2s-1)(2s+1)(4s-3) |\mu|^2 \tilde{\mathcal{U}}_{\alpha(2s-1)} \right) \right. \\
& + \tilde{\mathcal{U}}_{\beta;\alpha(2s-3)}^{\beta\alpha(2s-3)} \left(2i \frac{(s-1)(6s-5)}{(2s-1)^2} \square \tilde{\mathcal{U}}^{\gamma\delta}{}_{\gamma\alpha(2s-3)} + \frac{(s-1)(2s+1)}{(2s-1)} |\mu| \nabla^2 \tilde{\mathcal{U}}^{\gamma\delta}{}_{\gamma\alpha(2s-3)} \right. \\
& - \frac{(s-1)(2s-3)}{(2s-1)^2} (2i(s-2) \nabla^{\gamma\delta} \nabla_{(\alpha_1 \alpha_2} \tilde{\mathcal{U}}^{\lambda\kappa}{}_{\alpha_3 \dots \alpha_{2s-3})\gamma\delta\lambda} + \nabla^2 \nabla_{(\alpha_1} \tilde{\mathcal{U}}^{\delta\epsilon}{}_{\alpha_2 \dots \alpha_{2s-3})\gamma\delta}) \\
& \left. - \frac{2i(s-1)}{2s-1} (4s^2 + 16s - 17) |\mu|^2 \tilde{\mathcal{U}}^{\gamma\delta}{}_{\gamma\alpha(2s-3)} \right) \\
& - \frac{8s}{(2s+1)^2(2s-1)} \left(s(2s^3 - 2s - 1) |\mu| H^{\alpha(2s)} H_{\alpha(2s)} + s^2(s+1) H^{\beta\alpha(2s-1)} \nabla_\beta{}^\gamma H_{\gamma\alpha(2s-1)} \right. \\
& + \frac{is}{4} (2s^2 - 1) H^{\alpha(2s)} \nabla^2 H_{\alpha(2s)} + (2s+1) H^{\alpha(2s)} \nabla^\beta \Phi_{\beta\alpha(2s)} + 2i(2s+1) \Phi^{\alpha(2s+1)} \Phi_{\alpha(2s+1)} \left. \right) \\
& + \frac{8s}{2s+1} \tilde{\mathcal{U}}^{\alpha(2s-1)} (s \nabla^{\beta\gamma} \nabla_\beta H_{\gamma\alpha(2s-1)} - s \nabla^{\beta\gamma} \nabla_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s-1})\beta\gamma} + s(2s+1) |\mu| \nabla^\beta H_{\beta\alpha(2s-1)} \\
& \left. + i(2s+1) \nabla^{\beta\gamma} \Phi_{\beta\gamma\alpha(2s-1)} \right) + \frac{s(s-1)}{2s-1} \tilde{\mathcal{U}}_{\beta;\alpha(2s-3)}^{\beta\alpha(2s-3)} \nabla^{\beta\gamma} \nabla^\delta H_{\beta\gamma\delta\alpha(2s-3)} \left. \right\}, \quad (8.23)
\end{aligned}$$

which is invariant under the gauge transformations (8.19). It is apparent that the superfield $\Phi_{\alpha(2s+1)}$ is auxiliary, so upon elimination via its equation of motion,

$$\Phi_{\alpha(2s+1)} = -\frac{1}{4} ((4s^2 - 1) \nabla_{(\alpha_1 \alpha_2} \tilde{\mathcal{U}}_{\alpha_3 \dots \alpha_{2s+1})} + i \nabla_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})}), \quad (8.24)$$

one arrives at the action, which, up to an overall normalization factor, coincides with (7.26).

The second $\mathcal{N} = 1$ gauge theory formulated in terms of the dynamical variables (8.20) assumes the form

$$\begin{aligned}
S_{(s)}^\parallel[\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s+1)}] = & \left(-\frac{1}{2} \right)^s \int d^3|z| E \left\{ 4s^2(s+1) |\mu| V^{\alpha(2s-2)} V_{\alpha(2s-2)} \right. \\
& + 2s(s-1) V^{\alpha(2s-3)\beta} \nabla_\beta{}^\gamma V_{\gamma\alpha(2s-3)} - 2s V^{\alpha(2s-2)} \nabla^{\beta\gamma} \tilde{H}_{\beta\gamma\alpha(2s-2)} + is^2 V^{\alpha(2s-2)} \nabla^2 V_{\alpha(2s-2)} \\
& - \frac{1}{2(2s+1)^2(2s-1)} (4is^2(2s+1) \Omega^{\alpha(2s+1)} \Omega_{\alpha(2s+1)} + 2s(4s+1) \tilde{H}^{\alpha(2s-1)\beta} \nabla_\beta{}^\gamma \tilde{H}_{\gamma\alpha(2s-1)} \\
& \left. - is \tilde{H}^{\alpha(2s)} \nabla^2 \tilde{H}_{\alpha(2s)} - 4s(4s^2 + 3s + 1) |\mu| \tilde{H}^{\alpha(2s)} \tilde{H}_{\alpha(2s)} + 8s^2(2s+1) \tilde{H}^{\alpha(2s)} \nabla^\beta \Omega_{\beta\alpha(2s)} \right\}. \quad (8.25)
\end{aligned}$$

The superfield $\Omega_{\alpha(2s+1)}$ is auxiliary, so integrating it out using its equation of motion,

$$\Omega_{\alpha(2s+1)} = -i\nabla_{(\alpha_1} \tilde{H}_{\alpha_2 \dots \alpha_{2s+1})}, \quad (8.26)$$

we obtain action (3.14) up to an overall normalization factor.

D. Second supersymmetry transformations

We begin by computing the second supersymmetry transformations of the real $\mathcal{N} = 1$ fields (8.18) and (8.20),

$$\delta_\epsilon U_{\alpha(2s-2)} = -2is\epsilon^\beta \tilde{\mathcal{U}}_{\beta\alpha(2s-2)} + \frac{2i(s-1)}{2s-1} \epsilon_{(\alpha_1} \tilde{\mathcal{U}}^{\beta; \alpha_2 \dots \alpha_{2s-2})\beta}, \quad (8.27a)$$

$$\delta_\epsilon \tilde{\mathcal{U}}_{\beta;\alpha(2s-2)} = 2(2s-1)\epsilon_\beta V_{\alpha(2s-2)} - 4(s-1)\epsilon_{(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-2})\beta}, \quad (8.27b)$$

$$\delta_\epsilon V_{\alpha(2s-2)} = \frac{i}{2s} \left(s(2s-1)|\mu| \epsilon^\beta \tilde{\mathcal{U}}_{\beta\alpha(2s-2)} + (s-1)|\mu| \epsilon_{(\alpha_1} \tilde{\mathcal{U}}^{\beta; \alpha_2 \dots \alpha_{2s-2})\beta} - s\epsilon^\beta \nabla_\beta \gamma \tilde{\mathcal{U}}_{\gamma\alpha(2s-2)} + \frac{s-1}{2s-1} \epsilon^\beta \nabla_{\beta(\alpha_1} \tilde{\mathcal{U}}^{\gamma; \alpha_2 \dots \alpha_{2s-2})\gamma} \right), \quad (8.27c)$$

$$\delta_\epsilon H_{\alpha(2s)} = -2i\epsilon^\beta \Omega_{\beta\alpha(2s)} - \frac{2}{2s+1} \epsilon_{(\alpha_1} \nabla^\beta \tilde{H}_{\alpha_2 \dots \alpha_{2s})\beta}, \quad (8.27d)$$

$$\delta_\epsilon \tilde{H}_{\alpha(2s)} = 2i\epsilon^\beta \Phi_{\beta\alpha(2s)} + \frac{2s^2}{2s+1} \epsilon_{(\alpha_1} \nabla^\beta H_{\alpha_2 \dots \alpha_{2s})\beta}, \quad (8.27e)$$

$$\begin{aligned} \delta_\epsilon \Phi_{\alpha(2s+1)} &= \frac{1}{2} \epsilon^\beta \nabla_{\beta(\alpha_1} \tilde{H}_{\alpha_2 \dots \alpha_{2s+1})} - s\epsilon_{(\alpha_1} \nabla^\beta \Omega_{\alpha_2 \dots \alpha_{2s+1})\beta} + \frac{1}{4(2s+1)} (i\epsilon_{(\alpha_1} \nabla^2 \tilde{H}_{\alpha_2 \dots \alpha_{2s+1})} - 4s\epsilon_{(\alpha_1} \nabla_{\alpha_2}{}^\beta \tilde{H}_{\alpha_3 \dots \alpha_{2s+1})\beta} \\ &\quad + 4(4s^2 + 5s + 2)|\mu| \epsilon_{(\alpha_1} \tilde{H}_{\alpha_2 \dots \alpha_{2s+1})}), \end{aligned} \quad (8.27f)$$

$$\begin{aligned} \delta_\epsilon \Omega_{\alpha(2s+1)} &= -\frac{1}{2} \epsilon^\beta \nabla_{\beta(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})} + \frac{1}{s} \epsilon_{(\alpha_1} \nabla^\beta \Phi_{\alpha_2 \dots \alpha_{2s+1})\beta} - \frac{1}{4(2s+1)} (i\epsilon_{(\alpha_1} \nabla^2 H_{\alpha_2 \dots \alpha_{2s+1})} \\ &\quad - 4s\epsilon_{(\alpha_1} \nabla_{\alpha_2}{}^\beta H_{\alpha_3 \dots \alpha_{2s+1})\beta} + 4(4s^2 + 5s + 2)|\mu| \epsilon_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})}). \end{aligned} \quad (8.27g)$$

It is clear from the variation (8.27a) that the second supersymmetry transformation (5.25) breaks the gauge condition (8.4). We should then supplement the second supersymmetry transformation with the ϵ -dependent gauge transformations:

$$\delta_{\gamma(\epsilon)} U_{\alpha(2s-2)} = \gamma_{\alpha(2s-2)}(\epsilon) + \bar{\gamma}_{\alpha(2s-2)}(\epsilon), \quad (8.28a)$$

$$\delta_{\gamma(\epsilon)} \Gamma_{\alpha(2s)} = \frac{1}{2} \mathcal{D}_{(\alpha_1} \bar{\mathcal{D}}_{\alpha_2} \gamma_{\alpha_3 \dots \alpha_{2s})}(\epsilon) - \frac{i}{2} (2s-1) \mathcal{D}_{(\alpha_1 \alpha_2} \gamma_{\alpha_3 \dots \alpha_{2s})}(\epsilon), \quad (8.28b)$$

with

$$\gamma_{\alpha(2s-2)}(\epsilon) = is\epsilon^\beta \tilde{\mathcal{U}}_{\beta\alpha(2s-2)} - \frac{i(s-1)}{2s-1} \epsilon_{(\alpha_1} \tilde{\mathcal{U}}^{\beta; \alpha_2 \dots \alpha_{2s-2})\beta}. \quad (8.29)$$

The modified second supersymmetry transformations have the form

$$\hat{\delta}_\epsilon U_{\alpha(2s-2)} = \delta_\epsilon U_{\alpha(2s-2)} + \delta_{\gamma(\epsilon)} U_{\alpha(2s-2)}, \quad (8.30a)$$

$$\hat{\delta}_\epsilon \Gamma_{\alpha(2s)} = \delta_\epsilon \Gamma_{\alpha(2s)} + \delta_{\gamma(\epsilon)} \Gamma_{\alpha(2s)}. \quad (8.30b)$$

They act on the $\mathcal{N} = 1$ gauge superfields (8.18) and (8.20) by the rule

$$\hat{\delta}_\epsilon \tilde{\mathcal{U}}_{\beta;\alpha(2s-2)} = 2(2s-1)\epsilon_\beta V_{\alpha(2s-2)} - 4(s-1)\epsilon_{(\alpha_1} V_{\alpha_2 \dots \alpha_{2s-2})\beta}, \quad (8.31a)$$

$$\begin{aligned} \hat{\delta}_\epsilon V_{\alpha(2s-2)} = & -\frac{i}{4} \left(2(2s+1) |\mu| \epsilon^\beta \tilde{\mathcal{U}}_{\beta\alpha(2s-2)} + 2\epsilon^\beta \nabla_{\beta'} \tilde{\mathcal{U}}_{\gamma\alpha(2s-2)} + i\epsilon^\beta \nabla^2 \tilde{\mathcal{U}}_{\beta\alpha(2s-2)} - \frac{(s-1)}{s(2s-1)} (2(6s-1) |\mu| \epsilon_{(\alpha_1} \tilde{\mathcal{U}}^{\beta; \alpha_2 \dots \alpha_{2s-2})\beta} \right. \\ & \left. + 2\epsilon^\beta \nabla_{\beta(\alpha_1} \tilde{\mathcal{U}}^{\gamma; \alpha_2 \dots \alpha_{2s-2})\gamma} + i\epsilon_{(\alpha_1} \nabla^2 \tilde{\mathcal{U}}^{\beta; \alpha_2 \dots \alpha_{2s-2})\beta} \right), \end{aligned} \quad (8.31b)$$

$$\hat{\delta}_\epsilon H_{\alpha(2s)} = -2i\epsilon^\beta \Omega_{\beta\alpha(2s)} - \frac{2}{2s+1} \epsilon_{(\alpha_1} \nabla^\beta \tilde{H}_{\alpha_2 \dots \alpha_{2s})\beta}, \quad (8.31c)$$

$$\begin{aligned} \hat{\delta}_\epsilon \tilde{H}_{\alpha(2s)} = & 2i\epsilon^\beta \Phi_{\beta\alpha(2s)} + \frac{2s^2}{2s+1} \epsilon_{(\alpha_1} \nabla^\beta H_{\alpha_2 \dots \alpha_{2s})\beta} + 4is^2 |\mu| \epsilon_{(\alpha_1} \tilde{\mathcal{U}}_{\alpha_2 \dots \alpha_{2s})} \\ & + 2is^2 \epsilon^\beta \nabla_{(\alpha_1 \alpha_2} \tilde{\mathcal{U}}_{\alpha_3 \dots \alpha_{2s})\beta} - \frac{2is}{2s-1} (s-1) \epsilon_{(\alpha_1} \nabla_{\alpha_2 \alpha_3} \tilde{\mathcal{U}}^{\beta; \alpha_4 \dots \alpha_{2s})\beta}, \end{aligned} \quad (8.31d)$$

$$\begin{aligned} \hat{\delta}_\epsilon \Phi_{\alpha(2s+1)} = & \frac{1}{2} \epsilon^\beta \nabla_{\beta(\alpha_1} \tilde{H}_{\alpha_2 \dots \alpha_{2s+1})} - s\epsilon_{(\alpha_1} \nabla^\beta \Omega_{\alpha_2 \dots \alpha_{2s+1})\beta} + \frac{1}{4(2s+1)} (i\epsilon_{(\alpha_1} \nabla^2 \tilde{H}_{\alpha_2 \dots \alpha_{2s+1})} - 4s\epsilon_{(\alpha_1} \nabla_{\alpha_2}^\beta \tilde{H}_{\alpha_3 \dots \alpha_{2s+1})\beta} \\ & + 4(4s^2 + 5s + 2) |\mu| \epsilon_{(\alpha_1} \tilde{H}_{\alpha_2 \dots \alpha_{2s+1})}), \end{aligned} \quad (8.31e)$$

$$\begin{aligned} \hat{\delta}_\epsilon \Omega_{\alpha(2s+1)} = & -\frac{1}{2} \epsilon^\beta \nabla_{\beta(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})} + \frac{1}{s} \epsilon_{(\alpha_1} \nabla^\beta \Phi_{\alpha_2 \dots \alpha_{2s+1})\beta} - \frac{1}{4(2s+1)} (i\epsilon_{(\alpha_1} \nabla^2 H_{\alpha_2 \dots \alpha_{2s+1})} - 4s\epsilon_{(\alpha_1} \nabla_{\alpha_2}^\beta H_{\alpha_3 \dots \alpha_{2s+1})\beta} \\ & + 4(4s^2 + 5s + 2) |\mu| \epsilon_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+1})}) - \frac{1}{2} \epsilon^\beta \nabla_{(\alpha_1 \alpha_2} \nabla_{\alpha_3} \tilde{\mathcal{U}}_{\alpha_4 \dots \alpha_{2s+1})\beta} \\ & - |\mu| \epsilon_{(\alpha_1} \nabla_{\alpha_2} \tilde{\mathcal{U}}_{\alpha_3 \dots \alpha_{2s+1})} + \frac{1}{2s(2s-1)} (s-1) \epsilon_{(\alpha_1} \nabla_{\alpha_2 \alpha_3} \nabla_{\alpha_4} \tilde{\mathcal{U}}^{\beta; \alpha_5 \dots \alpha_{2s+1})\beta}. \end{aligned} \quad (8.31f)$$

IX. DISCUSSION

This section provides a summary of the results obtained in Secs. V–VIII, where we carried out the $(1, 1) \rightarrow (1, 0)$ AdS superspace reduction of the four massless higher-spin multiplets with $(1, 1)$ AdS supersymmetry.

It is useful to recall from Sec. III that there exist four off-shell series of massless higher-spin $\mathcal{N} = 1$ multiplets in AdS₃ [two series for each half-integer ($\hat{s} = s + \frac{1}{2}$) and integer ($\hat{s} = s$) superspin $\hat{s} \geq 1$]:

$$S_{(s+\frac{1}{2})}^{\parallel} [H_{\alpha(2s+1)}, L_{\alpha(2s-2)}], \quad S_{(s+\frac{1}{2})}^{\perp} [H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}]; \quad (9.1a)$$

$$S_{(s)}^{\parallel} [H_{\alpha(2s)}, V_{\alpha(2s-2)}], \quad S_{(s)}^{\perp} [H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}]. \quad (9.1b)$$

As demonstrated in [30], the models $S_{(s+\frac{1}{2})}^{\parallel} [H_{\alpha(2s+1)}, L_{\alpha(2s-2)}]$ and $S_{(s+\frac{1}{2})}^{\perp} [H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}]$ are dually equivalent in the flat superspace limit. However, this duality no longer holds in AdS^{3|2}. The same feature characterizes the two formulations for the massless integer superspin multiplet: $S_{(s)}^{\perp} [H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}]$ is dual to $S_{(s)}^{\parallel} [H_{\alpha(2s)}, V_{\alpha(2s-2)}]$ *only* in flat superspace. In Appendix B, we study in detail the component structure of these four massless $\mathcal{N} = 1$ supersymmetric higher-spin gauge models in AdS₃. Upon elimination of the corresponding auxiliary fields, the results are as follows:

$$S_{(s+\frac{1}{2})}^{\perp} [H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}] = S_{\text{bos}} [h_{\alpha(2s+2)}, \Phi_{\beta\gamma; \alpha(2s-2)}] + S_{\text{FF}}^{(2s+1, +)} [h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}], \quad (9.2a)$$

$$S_{(s+\frac{1}{2})}^{\parallel} [H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] = S_{\text{F}}^{(2s+2)} [h_{\alpha(2s+2)}, y_{\alpha(2s-2)}] + S_{\text{FF}}^{(2s+1, -)} [h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}], \quad (9.2b)$$

$$S_{(s)}^{\perp} [H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}] = \frac{2s-1}{s-1} S_{\text{F}}^{(2s)} [h_{\alpha(2s)}, y_{\alpha(2s-4)}] + S_{\text{FF}}^{(2s+1, -)} [h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}], \quad (9.2c)$$

$$S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int d^3x e \{ h^{\alpha(2s)} h_{\alpha(2s)} + 2s(2s-1) y^{\alpha(2s-2)} y_{\alpha(2s-2)} \} + S_{\text{FF}}^{(2s+1,+)}[h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}]. \quad (9.2d)$$

Let us consider the bosonic sector. The action $S_{\text{bos}}[h_{\alpha(2s+2)}, \Phi_{\beta\gamma;\alpha(2s-2)}]$ is dual to the Fronsdal action $S_{\text{F}}^{(2s+2)}[h_{\alpha(2s+2)}, y_{\alpha(2s-2)}]$ in the flat-space limit [see Eq. (B43)]. The actions $S_{\text{F}}^{(2s+2)}[h_{\alpha(2s+2)}, y_{\alpha(2s-2)}]$ and $S_{\text{F}}^{(2s)}[h_{\alpha(2s)}, y_{\alpha(2s-4)}]$ correspond to the Fronsdal actions in AdS_3 describing massless spin- $(s+1)$ and spin- s fields, respectively. Although the bosonic fields $h_{\alpha(2s)}$ and $y_{\alpha(2s-2)}$ in Eq. (9.2d) are pure auxiliary, the action is still consistent with the fact that the (Fang-)Fronsdal action in three dimensions has no propagating degrees of freedom. In the fermionic sector, our results agree with the fact that there are two AdS_3 extensions of the (Fang-)Fronsdal action, $S_{\text{FF}}^{(2s+1,+)}$ and $S_{\text{FF}}^{(2s+1,-)}$. They are related to each other by the replacement $|\mu| \rightarrow -|\mu|$.

Sections V and VI detail the $(1, 1) \rightarrow (1, 0)$ AdS superspace reduction of the transverse and longitudinal formulations for the massless *half-integer* superspin multiplets, respectively. In $(1, 1)$ AdS superspace, these off-shell formulations are dually equivalent. When reduced to $\mathcal{N} = 1$ AdS superspace, we found that the actual difference between the two models lies in the structure of auxiliary superfields. Let us refer to Eq. (5.20), which shows that the transverse formulation gives rise to two decoupled $\mathcal{N} = 1$ supersymmetric theories:

$$\mathbb{S}_{(s+\frac{1}{2})}^{\perp}[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}] = S_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-1)}] + S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}]. \quad (9.3)$$

In the longitudinal case, from Eq. (6.15) we have that

$$\mathbb{S}_{(s+\frac{1}{2})}^{\parallel}[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}] = S_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}, \Phi_{\alpha(2s-3)}] + S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-3)}]. \quad (9.4)$$

We further showed that the superfields $\Phi_{\alpha(2s-1)}, \Omega_{\alpha(2s-1)}, \Phi_{\alpha(2s-3)}, \Omega_{\alpha(2s-3)}$ are all auxiliary. Once they are eliminated, each formulation then leads to the same $\mathcal{N} = 1$ supersymmetric higher-spin actions:

$$\text{Transverse: } \mathbb{S}_{(s+\frac{1}{2})}^{\perp}[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}] \begin{cases} S_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] \\ S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}] \end{cases}, \quad (9.5a)$$

$$\text{Longitudinal: } \mathbb{S}_{(s+\frac{1}{2})}^{\parallel}[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}] \begin{cases} S_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] \\ S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}] \end{cases}. \quad (9.5b)$$

Sections VII and VIII concern the $(1, 1) \rightarrow (1, 0)$ AdS superspace reduction of the longitudinal and transverse formulations for the massless *integer* superspin multiplets, respectively. In $(1, 1)$ AdS superspace, they are dual to each other. As in the half-integer case, we demonstrated that upon reduction to $\mathcal{N} = 1$ AdS superspace, these formulations differ by the structure of the auxiliary superfields. As shown in Eq. (7.23), the longitudinal model is equivalent to

$$\mathbb{S}_{(s)}^{\parallel}[U_{\alpha(2s-2)}, G_{\alpha(2s)}, \bar{G}_{\alpha(2s)}] = S_{(s)}^{\perp}[H_{\alpha(2s)}, \mathcal{U}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s-1)}] + S_{(s)}^{\parallel}[\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s-1)}]. \quad (9.6)$$

On the other hand, Eq. (8.22) shows that the transverse model yields

$$\mathbb{S}_{(s)}^{\perp}[U_{\alpha(2s-2)}, \Gamma_{\alpha(2s)}, \bar{\Gamma}_{\alpha(2s)}] = S_{(s)}^{\perp}[H_{\alpha(2s)}, \tilde{\mathcal{U}}_{\beta;\alpha(2s-2)}, \Phi_{\alpha(2s+1)}] + S_{(s)}^{\parallel}[\tilde{H}_{\alpha(2s)}, V_{\alpha(2s-2)}, \Omega_{\alpha(2s+1)}]. \quad (9.7)$$

The superfields $\Phi_{\alpha(2s-1)}, \Omega_{\alpha(2s-1)}, \Phi_{\alpha(2s+1)}, \Omega_{\alpha(2s+1)}$ are all auxiliary and thus can be integrated out from their corresponding actions. We showed that the resulting $\mathcal{N} = 1$ actions are the same in both formulations:

$$\text{Transverse:} \quad \mathbb{S}_{(s)}^\perp[U_{\alpha(2s-2)}, \Gamma_{\alpha(2s)}, \bar{\Gamma}_{\alpha(2s)}] \begin{cases} \rightarrow S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}] \\ \rightarrow S_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}] \end{cases} \quad (9.8a)$$

$$\text{Longitudinal:} \quad \mathbb{S}_{(s)}^\parallel[U_{\alpha(2s-2)}, G_{\alpha(2s)}, \bar{G}_{\alpha(2s)}] \begin{cases} \rightarrow S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}] \\ \rightarrow S_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}] \end{cases} \quad (9.8b)$$

We are now in a position to compare the above $(1, 1) \rightarrow (1, 0)$ AdS reduction results with the $\mathcal{N} = 1$ supersymmetric higher-spin gauge theories obtained via $(2, 0) \rightarrow (1, 0)$ AdS reduction [30]. There exist two off-shell formulations for massless *half-integer* superspin multiplets with $(2, 0)$ AdS supersymmetry, which are not dual to each other.⁸ They are known as type II and type III series [29]. Upon reduction to $\mathcal{N} = 1$ superspace, type II series yields

$$\text{Type II:} \quad \mathbb{S}_{(s+\frac{1}{2})}^{\text{II}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{L}_{\alpha(2s-2)}] \begin{cases} \rightarrow S_{(s+\frac{1}{2})}^\parallel[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] \\ \rightarrow S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}] \end{cases} \quad (9.9)$$

On the other hand, type III series leads to

$$\text{Type III:} \quad \mathbb{S}_{(s+\frac{1}{2})}^{\text{III}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{V}_{\alpha(2s-2)}] \begin{cases} \rightarrow S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}] \\ \rightarrow S_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}] \end{cases} \quad (9.10)$$

We point out that unlike the situation for the $(1, 1)$ AdS multiplets, the $\mathcal{N} = 1$ models obtained from the $(2, 0) \rightarrow (1, 0)$ AdS reduction do not involve any auxiliary superfields. Additionally, reductions of the $(1, 1)$ AdS multiplets only produce three of the four series of off-shell $\mathcal{N} = 1$ multiplets, with the exception of the transverse half-integer model, $S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}]$. The latter can only be derived via $(2, 0) \rightarrow (1, 0)$ AdS reduction of the type III series. This formulation corresponds to the linearized supergravity model (for $s = 1$), which does not admit a nonlinear extension [18]. In accordance with (9.2a) (and the component analysis in B 3), the bosonic sector of $S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}]$ does not correspond to the standard Fronsdal action, but to its dual version. However, this duality is only present in the flat-space limit.

The dualities relating the four $\mathcal{N} = 1$ supersymmetric models in the flat limit, $|\mu| \rightarrow 0$, deserve further comment. In flat superspace, one may take any of the above $\mathcal{N} = 2$ superfield formulations and construct several dual models. This can be done by replacing its $\mathcal{N} = 1$ sector(s) by its dual versions. For concreteness, let us consider the transverse formulation for the massless half-integer superspin, Eq. (9.5a), in flat superspace. We denote the flat superspace action by $\mathbb{S}_{(s+\frac{1}{2})}^\perp[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}]_{\text{FS}}$. We can dualize one of its $\mathcal{N} = 1$ sectors, or both. This procedure gives us four different gauge theories:

- (i) $S_{(s+\frac{1}{2})}^\parallel[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}]_{\text{FS}} + S_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}]_{\text{FS}}$,
- (ii) $S_{(s+\frac{1}{2})}^\parallel[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}]_{\text{FS}} + S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}]_{\text{FS}}$,
- (iii) $S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}]_{\text{FS}} + S_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}]_{\text{FS}}$,
- (iv) $S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta; \alpha(2s-2)}]_{\text{FS}} + S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta; \alpha(2s-2)}]_{\text{FS}}$.

⁸Off-shell construction for the massless integer superspin $(2, 0)$ multiplets remains an open problem.

Case (i) describes $\mathbb{S}_{(s+\frac{1}{2})}^\perp[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}]_{\text{FS}}$. Cases (ii) and (iii) correspond to $\mathbb{S}_{(s+\frac{1}{2})}^{\text{II}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{L}_{\alpha(2s-2)}]_{\text{FS}}$ and $\mathbb{S}_{(s+\frac{1}{2})}^{\text{III}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{B}_{\alpha(2s-2)}]_{\text{FS}}$, respectively. At this stage, it is unknown if there exists a $\mathcal{N} = 2$ superfield formulation that leads to the two transverse models described by (iv). This will be an interesting open problem to investigate further.

The massless higher-spin $\mathcal{N} = 1$ models (9.1) do not have any propagating degrees of freedom. However, they can be deformed in order to generate off-shell topologically massive higher-spin supersymmetric theories. There has been recent progress made in the construction of these off-shell theories with various supersymmetries. It is based on the approaches developed in [39] for $\mathcal{N} = 1$ and in [24] for $\mathcal{N} = 2$ Poincaré supersymmetries. They were later extended to AdS₃ for the following cases: $\mathcal{N} = 1$ [30,37], $\mathcal{N} = (1, 1)$ [23], and $\mathcal{N} = (2, 0)$ [29]. The gauge-invariant actions for such massive multiplets are obtained by adding a superconformal and massless higher-spin action together, following the philosophy of topologically massive theories [43–46].

For a positive integer n , the conformal superspin- $\frac{n}{2}$ action in AdS^{3|2} [37,39,41] is

$$S_{\text{SCS}}^{(n/2)}[H_{\alpha(n)}] = -\frac{i^n}{2^{[n/2]+1}} \int d^3z z E H^{\alpha(n)} W_{\alpha(n)}(H). \quad (9.11)$$

Here $H_{\alpha(n)}$ is the superconformal gauge prepotential and $W_{\alpha(n)}(H)$ is the higher-spin $\mathcal{N} = 1$ super-Cotton tensor. The latter possesses the defining properties of being (i) gauge-invariant, $\delta_\zeta W_{\alpha(n)} = 0$; and (ii) transverse, $\nabla^\beta W_{\beta\alpha_1 \dots \alpha_{n-1}} = 0$. The action (9.11) is invariant under the gauge transformations (3.3). The explicit expression for $W_{\alpha(n)}(H)$ was given for the first time in $\mathcal{N} = 1$ Minkowski superspace in [39,41]. A few years later, it was generalized to an arbitrary conformally flat superspace in [47] by using the formalism of $\mathcal{N} = 1$ conformal superspace [48]. Recently, the closed form expression for $W_{\alpha(n)}(H)$ in AdS^{3|2} has been derived in [49].

In accordance with [30,37], there exist four off-shell formulations for topologically massive higher-spin $\mathcal{N} = 1$ multiplets in AdS₃. For a positive integer s , there are two off-shell gauge-invariant models for a topologically massive superspin- $(s + \frac{1}{2})$ multiplet in AdS₃:

$$\begin{aligned} S_{(s+\frac{1}{2})}^\parallel[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}|m] \\ = \kappa S_{\text{SCS}}^{(s+\frac{1}{2})}[H_{\alpha(2s+1)}] + m^{2s-1} S_{(s+\frac{1}{2})}^\parallel[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}], \end{aligned} \quad (9.12a)$$

$$\begin{aligned} S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}|m] \\ = \kappa S_{\text{SCS}}^{(s+\frac{1}{2})}[H_{\alpha(2s+1)}] + m^{2s-1} S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}]. \end{aligned} \quad (9.12b)$$

Here κ is a dimensionless parameter and m is a real massive parameter. For a topologically massive superspin- s multiplet in AdS₃, we have the following models:

$$\begin{aligned} S_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}|m] \\ = S_{\text{SCS}}^{(s)}[H_{\alpha(2s)}] + m^{2s-1} S_{(s)}^\parallel[H_{\alpha(2s)}, V_{\alpha(2s-2)}], \end{aligned} \quad (9.13a)$$

$$\begin{aligned} S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}|m] \\ = S_{\text{SCS}}^{(s)}[H_{\alpha(2s)}] + m^{2s-1} S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}]. \end{aligned} \quad (9.13b)$$

Another way of deriving the four massive $\mathcal{N} = 1$ models, (9.12) and (9.13), is by performing either a $(1, 1) \rightarrow (1, 0)$ or $(2, 0) \rightarrow (1, 0)$ AdS reduction of the off-shell topologically massive higher-spin supersymmetric theories presented in [23,29], respectively. In what follows, we will recall the structure of off-shell massive higher-spin supermultiplets in (1,1) AdS superspace [23] and take a closer look at their reduction.

Given a positive integer n , the superconformal higher-spin action in AdS^(3|1,1) [23,24,37,50] is given by

$$S_{\text{SCS}}[\mathfrak{H}_{\alpha(n)}] = -\frac{i^n}{2^{[n/2]+1}} \int d^3z z E \mathfrak{H}^{\alpha(n)} \mathfrak{W}_{\alpha(n)}(\mathfrak{H}), \quad (9.14)$$

where $\mathfrak{H}_{\alpha(n)}$ is the superconformal gauge multiplet and $\mathfrak{W}_{\alpha(n)}(\mathfrak{H})$ is the higher-spin $\mathcal{N} = 2$ super-Cotton tensor. The action (9.14) is invariant under the gauge transformations

$$\delta_\lambda \mathfrak{H}_{\alpha(n)} = \bar{\mathcal{D}}_{(\alpha_1} \lambda_{\alpha_2 \dots \alpha_n)} - (-1)^n \mathcal{D}_{(\alpha_1} \bar{\lambda}_{\alpha_2 \dots \alpha_n)}, \quad (9.15)$$

where $\lambda_{\alpha(n-1)}$ is unconstrained complex. To check the gauge invariance, one needs to make use of the properties that $\mathfrak{W}_{\alpha(n)}(\mathfrak{H})$ is (i) transverse linear, $\bar{\mathcal{D}}^\beta \mathfrak{W}_{\beta\alpha_1 \dots \alpha_{n-1}} = \mathcal{D}^\beta \mathfrak{W}_{\beta\alpha_1 \dots \alpha_{n-1}} = 0$; and (ii) gauge invariant, $\delta_\lambda \mathfrak{W}_{\alpha(n)} = 0$. In $\mathcal{N} = 2$ Minkowski superspace, the closed form expression for $\mathfrak{W}_{\alpha(n)}(\mathfrak{H})$ was given for the first time in [24], and a much simpler expression was derived in [50]. It was generalized to an arbitrary conformally flat superspace in [47] by using the formalism of $\mathcal{N} = 2$ conformal superspace [48].

In (1,1) AdS superspace, let us consider the two dually equivalent off-shell formulations for a massive superspin- $(s + \frac{1}{2})$ multiplet [23]:

$$\begin{aligned} S_{(s+\frac{1}{2})}^\perp[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}|m] \\ = \kappa S_{\text{SCS}}[\mathfrak{H}_{\alpha(2s)}] + m^{2s-1} S_{(s+\frac{1}{2})}^\perp[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}], \end{aligned} \quad (9.16a)$$

$$\begin{aligned} S_{(s+\frac{1}{2})}^\parallel[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}|m] \\ = \kappa S_{\text{SCS}}[\mathfrak{H}_{\alpha(2s)}] + m^{2s-1} S_{(s+\frac{1}{2})}^\parallel[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}]. \end{aligned} \quad (9.16b)$$

The superconformal action $\mathbb{S}_{\text{SCS}}[\mathfrak{H}_{\alpha(2s)}]$ is given by (9.14), for the case $n = 2s$. The transverse $\mathbb{S}_{(s+\frac{1}{2})}^{\perp}[\mathfrak{H}_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}]$ and longitudinal $\mathbb{S}_{(s+\frac{1}{2})}^{\parallel}[\mathfrak{H}_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}]$ actions are given by (5.3) and (6.2), respectively.

As pointed out in [23], it is expected that the topologically massive actions (9.16a) and (9.16b) describe the on-shell massive supermultiplets in (1,1) AdS superspace [40]. Given a positive integer $n > 0$, a massive on-shell multiplet of superspin $(n+1)/2$ is realized in terms of a real symmetric rank- n spinor $T_{\alpha(n)}$ constrained by

$$\mathcal{D}^{\beta} T_{\alpha_1 \dots \alpha_{n-1} \beta} = \bar{\mathcal{D}}^{\beta} T_{\alpha_1 \dots \alpha_{n-1} \beta} = 0, \quad (9.17a)$$

$$\left(\frac{i}{2} \mathcal{D}^{\gamma} \bar{\mathcal{D}}_{\gamma} + m \right) T_{\alpha_1 \dots \alpha_n} = 0. \quad (9.17b)$$

It may be shown that

$$\left(\frac{i}{2} \mathcal{D}^{\gamma} \bar{\mathcal{D}}_{\gamma} \right)^2 T_{\alpha_1 \dots \alpha_n} = (\mathcal{D}^a \mathcal{D}_a + 2(n+2)|\mu|^2) T_{\alpha_1 \dots \alpha_n}. \quad (9.18)$$

One can also construct a massive model that leads directly to Eqs. (9.17), as an extension of the flat-space bosonic constructions of [51,52]. It is given by

$$\begin{aligned} \mathbb{S}_{\text{massive}}[\mathfrak{H}_{\alpha(n)}] &= -\frac{i^n}{2^{[n/2]+1}} \frac{\kappa}{m} \int d^3{}^4 z E \mathfrak{W}^{\alpha(n)}(\mathfrak{H}) \\ &\times \left\{ m + \frac{i}{2} \mathcal{D}^{\gamma} \bar{\mathcal{D}}_{\gamma} \right\} \mathfrak{H}_{\alpha(n)}. \end{aligned} \quad (9.19)$$

The action (9.19) is invariant under gauge transformations (9.15). To check gauge invariance, one makes use of the properties that $\mathfrak{W}_{\alpha(n)}(\mathfrak{H})$ is (i) gauge invariant; and (ii) transverse linear, $\bar{\mathcal{D}}^{\beta} \mathfrak{W}_{\beta \alpha_1 \dots \alpha_{n-1}} = \mathcal{D}^{\beta} \mathfrak{W}_{\beta \alpha_1 \dots \alpha_{n-1}} = 0$. The action (9.19) becomes superconformal in the $m \rightarrow \infty$ limit.

We now turn to describing the $(1,1) \rightarrow (1,0)$ AdS reduction of the massive models (9.16). The reduction of

the massless transverse (5.3) and longitudinal (6.2) formulations was achieved in Secs. V and VI, respectively. Thus, we need only carry out the reduction of the superconformal action. The superconformal gauge multiplet $\mathfrak{H}_{\alpha(2s)}$ was reduced in Sec. V.2. Upon reduction, the superfield $\mathfrak{H}_{\alpha(2s)}$ is equivalent to two unconstrained real $\mathcal{N} = 1$ superfields (5.8), defined modulo gauge transformations of the form (5.10). Since $\mathfrak{W}_{\alpha(2s)}$ is a real transverse linear superfield, it follows that $\mathfrak{W}_{\alpha(2s)}$ is equivalent to two real $\mathcal{N} = 1$ superfields, defined as

$$W_{\alpha(2s)}(H) = -\mathfrak{W}_{\alpha(2s)}(\mathfrak{H})|,$$

$$W_{\alpha(2s+1)}(H) = \frac{i}{2} \nabla_{(\alpha_1} \mathfrak{W}_{\alpha_2 \dots \alpha_{2s+1})}(\mathfrak{H})|. \quad (9.20)$$

Here each of the $\mathcal{N} = 1$ superfields is transverse linear and gauge invariant. Applying the reduction procedure to the superconformal action (9.14) yields two decoupled $\mathcal{N} = 1$ supersymmetric theories,

$$\begin{aligned} \mathbb{S}_{\text{SCS}}[\mathfrak{H}_{\alpha(2s)}] &= -\frac{(-1)^s}{2^{s+1}} \int d^3{}^2 z E \{ H^{\alpha(2s)} W_{\alpha(2s)} \\ &+ i H^{\alpha(2s+1)} W_{\alpha(2s+1)} \}. \end{aligned} \quad (9.21)$$

These two gauge-invariant theories can be identified as the $\mathcal{N} = 1$ superconformal actions in AdS₃ (9.11), for the cases $n = 2s$ and $n = 2s+1$, respectively. Recalling the reduction results (9.5a), it is apparent that both (9.16a) and (9.16b) decouple into two off-shell topologically massive $\mathcal{N} = 1$ theories: $\mathbb{S}_{(s+\frac{1}{2})}^{\parallel}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}|m]$ and $\mathbb{S}_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}|m]$. Thus, the $(1,1) \rightarrow (1,0)$ AdS reduction of (9.16) is only able to generate two of the gauge-invariant formulations for topologically massive $\mathcal{N} = 1$ supermultiplets, given by Eqs. (9.12) and (9.13).

In (2,0) AdS superspace, the following off-shell gauge-invariant massive models were proposed [29]:

$$\mathbb{S}_{(s+\frac{1}{2})}^{\text{II}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{Z}_{\alpha(2s-2)}|m] = \kappa \mathbb{S}_{\text{SCS}}[\mathfrak{H}_{\alpha(2s)}] + m^{2s-1} \mathbb{S}_{(s+\frac{1}{2})}^{\text{II}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{Z}_{\alpha(2s-2)}], \quad (9.22a)$$

$$\mathbb{S}_{(s+\frac{1}{2})}^{\text{III}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{V}_{\alpha(2s-2)}|m] = \kappa \mathbb{S}_{\text{SCS}}[\mathfrak{H}_{\alpha(2s)}] + m^{2s-1} \mathbb{S}_{(s+\frac{1}{2})}^{\text{III}}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{V}_{\alpha(2s-2)}]. \quad (9.22b)$$

Here

$$\mathbb{S}_{\text{SCS}}[\mathfrak{H}_{\alpha(2s)}] = -\frac{(-1)^s}{2^{s+1}} \int d^3{}^4 z E \mathfrak{H}^{\alpha(2s)} \mathfrak{W}_{\alpha(2s)}(\mathfrak{H}) \quad (9.23)$$

is the superconformal higher-spin action [23], with $\mathfrak{W}_{\alpha(2s)}(\mathfrak{H}) = \bar{\mathfrak{W}}_{\alpha(2s)}(\mathfrak{H})$ being the higher-spin super-Cotton tensor in (2,0) AdS superspace.

The (2,0) AdS analog of the model (9.19), which was described in [29], takes the form

$$\mathbb{S}_{\text{massive}}[\mathfrak{S}_{\alpha(n)}] = -\frac{i^n}{2^{[n/2]+1}} \frac{\kappa}{m} \int d^{3|4} z E \mathfrak{W}^{\alpha(n)}(\mathfrak{S}) \times \left\{ m + \frac{i}{2} \mathbf{D}^\gamma \bar{\mathbf{D}}_\gamma \right\} \mathfrak{S}_{\alpha(n)}, \quad (9.24)$$

where $\mathbf{D}_\alpha$ and $\bar{\mathbf{D}}_\alpha$ denote the spinor covariant derivatives of (2,0) AdS superspace. One can then obtain the following set of constraints describing an on-shell massive superspin- $(n+1)/2$ multiplet in (2,0) AdS superspace:

$$\mathbf{D}^\beta T_{\alpha_1 \dots \alpha_{n-1} \beta} = \bar{\mathbf{D}}^\beta T_{\alpha_1 \dots \alpha_{n-1} \beta} = 0, \quad (9.25a)$$

$$\left(\frac{i}{2} \mathbf{D}^\gamma \bar{\mathbf{D}}_\gamma + m \right) T_{\alpha_1 \dots \alpha_n} = 0. \quad (9.25b)$$

We note that the models (9.19) and (9.24) are not dual to each other since they have different types of supersymmetry, namely (1,1) and (2,0) AdS supersymmetry, respectively.

Recalling the reduction result (9.9), it is clear that (9.22a) decouples into two off-shell massive $\mathcal{N} = 1$ models: $S_{(s+\frac{1}{2})}^\parallel [H_{\alpha(2s+1)}, L_{\alpha(2s-2)} | m]$ and $S_{(s)}^\perp [H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)} | m]$. On the other hand, it follows directly from (9.10) that the reduction of (9.22b) leads to $S_{(s+\frac{1}{2})}^\perp [H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)} | m]$ and $S_{(s)}^\parallel [H_{\alpha(2s)}, V_{\alpha(2s-2)} | m]$. Thus, the $(2, 0) \rightarrow (1, 0)$ AdS reduction of (9.22) allows us to recover all four off-shell gauge-invariant formulations for topologically massive $\mathcal{N} = 1$ supermultiplets, which are described by (9.12) and (9.13).

It is worth pointing out that there also exists an on-shell construction of gauge-invariant Lagrangian formulations for massive higher-spin $\mathcal{N} = 1$ supermultiplets in $\mathbb{R}^{2,1}$ and AdS_3 [53–55], extending previous works on the non-supersymmetric cases [56,57]. These framelike formulations are based on the gauge-invariant approach (also known as the Stueckelberg approach) to the dynamics of massive higher-spin fields, which were proposed by Zinoviev [58,59] and Metsaev [60]. As demonstrated in [53–55], in three dimensions it is possible to rewrite the corresponding Lagrangians in terms of a set of gauge-invariant curvatures, resulting in a more elegant formulation. An interesting open problem is to understand if there exists an off-shell uplift of these models.

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APPENDIX A: THE 3D NOTATION AND ADS IDENTITIES

Our 3D notation and conventions correspond to [17]. The 3D Minkowski metric is $\eta_{ab} = \text{diag}(-1, 1, 1)$ and the Levi-Civita tensor is normalized as $\epsilon^{012} = -\epsilon_{012} = 1$.

The $\text{SL}(2, \mathbb{R})$ invariant tensors

$$\epsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon^{\alpha\gamma} \epsilon_{\gamma\beta} = \delta^\alpha_\beta \quad (A1)$$

are used to raise and lower the spinor indices:

$$\psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta} \psi^\beta. \quad (A2)$$

We make use of real Dirac γ matrices, $\gamma_a := ((\gamma_a)_\alpha^\beta)$, which are defined by

$$(\gamma_a)_\alpha^\beta := \epsilon^{\beta\gamma} (\gamma_a)_{\alpha\gamma} = (-i\sigma_2, \sigma_3, \sigma_1). \quad (A3)$$

They have the following properties:

$$\gamma_a \gamma_b = \eta_{ab} \mathbb{1} + \epsilon_{abc} \gamma^c, \quad (A4a)$$

$$(\gamma^a)_{\alpha\beta} (\gamma_a)^{\rho\sigma} = -(\delta_\alpha^\rho \delta_\beta^\sigma + \delta_\alpha^\sigma \delta_\beta^\rho), \quad (A4b)$$

$$\epsilon_{abc} (\gamma^b)_{\alpha\beta} (\gamma^c)_{\gamma\delta} = \epsilon_{\gamma(\alpha} (\gamma_a)_{\beta)\delta} + \epsilon_{\delta(\alpha} (\gamma_a)_{\beta)\gamma}, \quad (A4c)$$

$$\text{tr}[\gamma_a \gamma_b \gamma_c \gamma_d] = 2\eta_{ab} \eta_{cd} - 2\eta_{ac} \eta_{db} + 2\eta_{ad} \eta_{bc}. \quad (A4d)$$

A three-vector x_a can be equivalently realized as a symmetric second-rank spinor $x_{\alpha\beta}$ defined as

$$x_{\alpha\beta} := (\gamma^a)_{\alpha\beta} x_a = x_{\beta\alpha}, \quad x_a = -\frac{1}{2} (\gamma_a)^{\alpha\beta} x_{\alpha\beta}. \quad (A5)$$

The relationships between Lorentz generators with two vector indices ($M_{ab} = -M_{ba}$), one vector index (M_a), and two spinor indices ($M_{\alpha\beta} = M_{\beta\alpha}$) are

$$\begin{aligned} M_{ab} &= -\epsilon_{abc} M^c, & M_a &= \frac{1}{2} \epsilon_{abc} M^{bc}, \\ M_{\alpha\beta} &= (\gamma^a)_{\alpha\beta} M_a, & M_a &= -\frac{1}{2} (\gamma_a)^{\alpha\beta} M_{\alpha\beta}. \end{aligned} \quad (A6)$$

These generators act on a vector V_c and a spinor Ψ_γ by the rules

$$M_{ab}V_c = 2\eta_{c[a}V_{b]}, \quad M_{\alpha\beta}\Psi_\gamma = \varepsilon_{\gamma(\alpha}\Psi_{\beta)}. \quad (\text{A7})$$

The (brackets) parentheses correspond to (anti)symmetrization of tensor or spinor indices, which encode a normalization factor, for example,

$$V_{[a_1 a_2 \dots a_n]} := \frac{1}{n!} \sum_{\pi \in S_n} \text{sgn}(\pi) V_{a_{\pi(1)} \dots a_{\pi(n)}},$$

$$V_{(\alpha_1 \dots \alpha_n)} := \frac{1}{n!} \sum_{\pi \in S_n} V_{\alpha_{\pi(1)} \dots \alpha_{\pi(n)}}, \quad (\text{A8})$$

with S_n being the symmetric group of n elements.

We provide a summary of essential identities for (1,1) AdS covariant derivatives, which we denote by $\nabla_A^I = (\nabla_a, \nabla_\alpha^I)$, where $I = \underline{1}, \underline{2}$. Making use of the algebra in the real basis (4.19), we can derive the following results:

$$\nabla_\alpha^{\underline{1}} \nabla_\beta^{\underline{1}} = i \nabla_{\alpha\beta} - 2i|\mu| M_{\alpha\beta} + \frac{1}{2} \varepsilon_{\alpha\beta} (\nabla^{\underline{1}})^2, \quad (\text{A9a})$$

$$\nabla^{\underline{1}\beta} \nabla_\alpha^{\underline{1}} \nabla_\beta^{\underline{1}} = 4i|\mu| \nabla_\alpha^{\underline{1}}, \quad [\nabla_\alpha^{\underline{1}}, \nabla_{\beta\gamma}] = 2|\mu| \varepsilon_{\alpha(\beta} \nabla_{\gamma)}^{\underline{1}}, \quad (\text{A9b})$$

$$\{(\nabla^{\underline{1}})^2, \nabla_\alpha^{\underline{1}}\} = 4i|\mu| \nabla_\alpha^{\underline{1}},$$

$$[\nabla_\alpha^{\underline{1}}, \square] = 2|\mu| \nabla_{\alpha\beta} \nabla^{\underline{1}\beta} + 3|\mu|^2 \nabla_\alpha^{\underline{1}}, \quad (\text{A9c})$$

$$(\nabla^{\underline{1}})^2 \nabla_\alpha^{\underline{1}} = 2i|\mu| \nabla_\alpha^{\underline{1}} + 2i \nabla_{\alpha\beta} \nabla^{\underline{1}\beta} - 4i|\mu| \nabla^{\underline{1}\beta} M_{\alpha\beta}, \quad (\text{A9d})$$

$$-\frac{1}{4} (\nabla^{\underline{1}})^2 (\nabla^{\underline{1}})^2 = \square - 2i|\mu| (\nabla^{\underline{1}})^2 + 2|\mu| \nabla^{\alpha\beta} M_{\alpha\beta} - 2|\mu|^2 M^{\alpha\beta} M_{\alpha\beta}, \quad (\text{A9e})$$

$$\nabla_\alpha^{\underline{2}} \nabla_\beta^{\underline{2}} = i \nabla_{\alpha\beta} + 2i|\mu| M_{\alpha\beta} + \frac{1}{2} \varepsilon_{\alpha\beta} (\nabla^{\underline{2}})^2, \quad (\text{A9f})$$

$$\nabla^{\underline{2}\beta} \nabla_\alpha^{\underline{2}} \nabla_\beta^{\underline{2}} = -4i|\mu| \nabla_\alpha^{\underline{2}}, \quad [\nabla_\alpha^{\underline{2}}, \nabla_{\beta\gamma}] = -2|\mu| \varepsilon_{\alpha(\beta} \nabla_{\gamma)}^{\underline{2}}, \quad (\text{A9g})$$

$$\{(\nabla^{\underline{2}})^2, \nabla_\alpha^{\underline{2}}\} = -4i|\mu| \nabla_\alpha^{\underline{2}},$$

$$[\nabla_\alpha^{\underline{2}}, \square] = -2|\mu| \nabla_{\alpha\beta} \nabla^{\underline{2}\beta} + 3|\mu|^2 \nabla_\alpha^{\underline{2}}, \quad (\text{A9h})$$

$$(\nabla^{\underline{2}})^2 \nabla_\alpha^{\underline{2}} = -2i|\mu| \nabla_\alpha^{\underline{2}} + 2i \nabla_{\alpha\beta} \nabla^{\underline{2}\beta} + 4i|\mu| \nabla^{\underline{2}\beta} M_{\alpha\beta}, \quad (\text{A9i})$$

$$-\frac{1}{4} (\nabla^{\underline{2}})^2 (\nabla^{\underline{2}})^2 = \square + 2i|\mu| (\nabla^{\underline{2}})^2 - 2|\mu| \nabla^{\alpha\beta} M_{\alpha\beta} - 2|\mu|^2 M^{\alpha\beta} M_{\alpha\beta}. \quad (\text{A9j})$$

APPENDIX B: COMPONENT STRUCTURE OF $\mathcal{N}=1$ SUPERSYMMETRIC HIGHER-SPIN ACTIONS IN AdS₃

In this Appendix, we study the component structure of the longitudinal and transverse formulations for both the half-integer and the integer superspin multiplets which are reviewed in Sec. III. In accordance with (4.42), any $\mathcal{N}=1$ supersymmetric action in AdS₃ can be reduced to components by the rule

$$S = \frac{1}{4} \int d^3x e(i\nabla^2 + 8|\mu|) L|_{\theta=0}. \quad (\text{B1})$$

In what follows, we will denote the torsion-free covariant derivative on AdS₃ by $\mathfrak{D}_a$. It is related to the vector covariant derivative ∇_a in (2.1) by the simple rule $\mathfrak{D}_a := \nabla_a|_{\theta=0}$, provided an appropriate Wess-Zumino gauge is chosen.

1. Fronsdaal-type actions in AdS₃

We begin by reviewing the AdS₃ counterparts of the (Fang-)Fronsdaal actions in AdS₄ [61,62], mostly following the presentation in [37].

The Fang-Fronsdaal action in AdS₄ [62] is first order in derivatives. It can be generalized to AdS₃ using two different gauge-invariant actions, $S_{\text{FF}}^{(n,+)}$ and $S_{\text{FF}}^{(n,-)}$. Given an integer $n \geq 4$, such a model in AdS₃ is realized in terms of three real fields,

$$\mathcal{V}_{\text{FF}}^{(n,\pm)} = \{h_{\alpha(n)}, y_{\alpha(n-2)}, y_{\alpha(n-4)}\}, \quad (\text{B2})$$

which are defined modulo gauge transformations of the form

$$\delta h_{\alpha(n)} = \mathfrak{D}_{(\alpha_1 \alpha_2} \lambda_{\alpha_3 \dots \alpha_n)}, \quad (\text{B3a})$$

$$\delta y_{\alpha(n-2)} = \frac{1}{n} \mathfrak{D}_{(\alpha_1}{}^\beta \lambda_{\alpha_2 \dots \alpha_{n-2})\beta} \pm |\mu| \lambda_{\alpha(n-2)}, \quad (\text{B3b})$$

$$\delta y_{\alpha(n-4)} = \mathfrak{D}^{\beta\gamma} \lambda_{\beta\gamma\alpha(n-4)}. \quad (\text{B3c})$$

Here, the gauge parameter $\lambda_{\alpha(n-2)}$ is real and unconstrained. Modulo an overall constant, the corresponding gauge-invariant action is

$$S_{\text{FF}}^{(n,\pm)} = \frac{i^n}{2^{\lfloor n/2 \rfloor + 1}} \int d^3x e \left\{ h^{\beta\alpha(n-1)} \mathfrak{D}_{\beta}{}^{\gamma} h_{\gamma\alpha(n-1)} + 2(n-2) y^{\alpha(n-2)} \mathfrak{D}^{\beta\gamma} h_{\beta\gamma\alpha(n-2)} + 4(n-2) y^{\beta\alpha(n-3)} \mathfrak{D}_{\beta}{}^{\gamma} y_{\gamma\alpha(n-3)} \right. \\ \left. + \frac{2n(n-3)}{n-1} y^{\alpha(n-4)} \mathfrak{D}^{\beta\gamma} y_{\beta\gamma\alpha(n-4)} - \frac{(n-3)(n-4)}{(n-1)(n-2)} y^{\beta\alpha(n-5)} \mathfrak{D}_{\beta}{}^{\gamma} y_{\gamma\alpha(n-5)} \pm |\mu| \left((n-2) h^{\alpha(n)} h_{\alpha(n)} \right. \right. \\ \left. \left. - 4n(n-2) y^{\alpha(n-2)} y_{\alpha(n-2)} - \frac{n(n-3)}{n-1} y^{\alpha(n-4)} y_{\alpha(n-4)} \right) \right\}. \quad (\text{B4})$$

Only the action $S_{\text{FF}}^{(n,+)}$ was considered in [37]. Both actions $S_{\text{FF}}^{(n,+)}$ and $S_{\text{FF}}^{(n,-)}$ are the $d=3$ counterparts of the Fang-Fronsdal action provided n is odd, $n=2s+1$, with $s \geq 2$ an integer.

It must be noted that the action (B4) can be expressed in a more compact way in terms of a reducible frame field,

$$h_{\beta\gamma;\alpha(n-2)} := (\gamma^b)_{\beta\gamma} h_{b;\alpha(n-2)}, \quad (\text{B5})$$

which is defined modulo gauge transformations of the form

$$\delta h_{\beta\gamma;\alpha(n-2)} = \mathfrak{D}_{\beta\gamma} \lambda_{\alpha(n-2)} \pm (n-2) |\mu| (\varepsilon_{\beta(\alpha_1} \lambda_{\alpha_2 \dots \alpha_{n-2})\gamma} + \varepsilon_{\gamma(\alpha_1} \lambda_{\alpha_2 \dots \alpha_{n-2})\beta}). \quad (\text{B6})$$

The corresponding gauge-invariant action takes the form

$$S_{\text{FF}}^{(n,\pm)} = \frac{i^n}{2^{\lfloor n/2 \rfloor + 1}} \int d^3x e \{ h^{\beta\gamma;\alpha(n-2)} \mathfrak{D}_{\beta}{}^{\delta} h_{\gamma\delta;\alpha(n-2)} \pm (n-2) |\mu| (h^{\beta\gamma;\alpha(n-2)} h_{\beta\gamma;\alpha(n-2)} + 2h^{\beta\gamma;\alpha(n-3)}{}_{\beta} h_{\gamma\delta;\alpha(n-3)}{}^{\delta}) \}. \quad (\text{B7})$$

In the flat-space limit, the action (B7) reduces to the model considered by Tyutin and Vasiliev [63] (see also [37] for a review). Reference [57] also made use of the action (B7) in the framelike formulation, but only a particular choice of sign was considered. In order to relate the action (B7) to (B4), it is necessary to decompose the dynamical field $h_{\beta\gamma;\alpha(n-2)}$ into its irreducible components. The irreducible fields contained in $h_{\beta\gamma;\alpha(n-2)}$ can be defined as follows:

$$h_{\alpha(n)} := h_{(\alpha_1\alpha_2;\alpha_3 \dots \alpha_n)}, \quad (\text{B8a})$$

$$y_{\alpha(n-2)} := \frac{1}{n} h^{\beta}{}_{(\alpha_1;\alpha_2 \dots \alpha_{n-2})\beta}, \quad (\text{B8b})$$

$$y_{\alpha(n-4)} := h^{\beta\gamma}{}_{\beta\gamma\alpha(n-4)}. \quad (\text{B8c})$$

The gauge transformation laws for these fields, which follow from (B6), are given by those in (B3). Upon expressing the action (B7) in terms of the fields (B8), it can be shown that the resulting action coincides with (B4).

The Fronsdal action in AdS_4 [61] is second order in derivatives. It can be generalized to AdS_3 as follows. Given an integer $n \geq 4$, we introduce two real dynamical variables

$$\mathcal{V}_{\text{F}} = \{h_{\alpha(n)}, y_{\alpha(n-4)}\}, \quad (\text{B9})$$

which are defined modulo gauge transformations of the form

$$\delta h_{\alpha(n)} = \mathfrak{D}_{(\alpha_1\alpha_2} \lambda_{\alpha_3 \dots \alpha_n)}, \quad (\text{B10a})$$

$$\delta y_{\alpha(n-4)} = \frac{n-2}{n-1} \mathfrak{D}^{\beta\gamma} \lambda_{\beta\gamma\alpha(n-4)}, \quad (\text{B10b})$$

with the gauge parameter $\lambda_{\alpha(n-2)}$ being real unconstrained. Modulo an overall constant, the gauge-invariant action is given by

$$S_{\text{F}}^{(n)} = \frac{i^n}{2^{\lfloor n/2 \rfloor + 1}} \int d^3x e \left\{ h^{\alpha(n)} \mathcal{Q} h_{\alpha(n)} - \frac{n}{4} \mathfrak{D}_{\beta\gamma} h^{\beta\gamma\alpha(n-2)} \mathfrak{D}^{\delta\lambda} h_{\delta\lambda\alpha(n-2)} \right. \\ \left. - \frac{n-3}{2} y^{\alpha(n-4)} \mathfrak{D}^{\beta\gamma} \mathfrak{D}^{\delta\lambda} h_{\beta\gamma\delta\lambda\alpha(n-4)} - 2n(n-2) |\mu|^2 h^{\alpha(n)} h_{\alpha(n)} \right. \\ \left. - \frac{n-3}{n} \left(2y^{\alpha(n-4)} \mathcal{Q} y_{\alpha(n-4)} - 4(n^2+2) |\mu|^2 y^{\alpha(n-4)} y_{\alpha(n-4)} \right. \right. \\ \left. \left. + \frac{(n-4)(n-5)}{4(n-2)} \mathfrak{D}_{\beta\gamma} y^{\beta\gamma\alpha(n-6)} \mathfrak{D}^{\delta\lambda} y_{\delta\lambda\alpha(n-6)} \right) \right\}, \quad (\text{B11})$$

where $\mathcal{Q}$ is the following Casimir operator of the AdS group $\text{SO}(2, 2)$:

$$\mathcal{Q} := \mathfrak{D}^a \mathfrak{D}_a - 2|\mu|^2 M^{\beta\gamma} M_{\beta\gamma}, \quad [\mathcal{Q}, \mathfrak{D}_a] = 0. \quad (\text{B12})$$

The model (B11) is the $d = 3$ counterpart of the Fronsdal action provided n is even, $n = 2s$, with $s \geq 2$ an integer.

2. Dual formulation of the Fronsdal-type action in Minkowski space

Here we introduce a one-parameter family of dual formulations for the flat-space counterpart of the model (B11). These results will be important for our analysis in the next subsection.

Let us consider the flat-space limit of the model (B11)

$$S_F^{(n)} = \frac{i^n}{2^{[n/2]+1}} \int d^3x \left\{ \frac{1}{2} \partial^{\beta\gamma} h^{\alpha(n)} \partial_{\beta\gamma} h_{\alpha(n)} - \frac{n}{4} \partial_{\beta\gamma} h^{\beta\gamma\alpha(n-2)} \partial^{\delta\lambda} h_{\delta\lambda\alpha(n-2)} \right. \\ \left. + \frac{n-3}{n} \left(\frac{n}{2} \partial^{\beta\gamma} y^{\alpha(n-4)} \partial^{\delta\lambda} h_{\beta\gamma\delta\lambda\alpha(n-4)} - \partial^{\beta\gamma} y^{\alpha(n-4)} \partial_{\beta\gamma} y_{\alpha(n-4)} - \frac{(n-4)(n-5)}{4(n-2)} \partial_{\beta\gamma} y^{\beta\gamma\alpha(n-6)} \partial^{\delta\lambda} y_{\delta\lambda\alpha(n-6)} \right) \right\}, \quad (\text{B13})$$

where $\partial_{\alpha\beta}$ are the partial derivatives of three-dimensional Minkowski space. This action is invariant under the gauge transformations

$$\delta_\lambda h_{\alpha(n)} = \partial_{(\alpha_1 \alpha_2} \lambda_{\alpha_3 \dots \alpha_n)}, \quad (\text{B14a})$$

$$\delta_\lambda y_{\alpha(n-4)} = \frac{n-2}{n-1} \partial^{\beta\gamma} \lambda_{\beta\gamma\alpha(n-4)}. \quad (\text{B14b})$$

Both fields $h_{\alpha(n)}$ and $y_{\alpha(n-4)}$ appear in (B13) with derivatives,

$$S_F^{(n)} = \frac{i^n}{2^{[n/2]+1}} \int d^3x \mathcal{L}(\partial_{\beta\gamma} h_{\alpha(n)}, \partial_{\beta\gamma} y_{\alpha(n-4)}), \quad (\text{B15})$$

and therefore a duality transformation may be performed upon each of them. Here we will only dualize $y_{\alpha(n-4)}$ and keep $h_{\alpha(n)}$ intact.⁹

We may think of $\partial_{\beta\gamma} y_{\alpha(n-4)}$ as a longitudinal tensor field, by analogy with a longitudinal vector field. Our dual formulation for (B15) is obtained by introducing the following first-order action:

$$S_{\text{first-order}} = \frac{i^n}{2^{[n/2]+1}} \int d^3x \{ \mathcal{L}(\partial_{\beta\gamma} h_{\alpha(n)}, \mathcal{H}_{\beta\gamma;\alpha(n-4)}) \\ - (n-3) \mathcal{H}^{\beta\gamma;\alpha(n-4)} F_{\beta\gamma;\alpha(n-4)} \}, \quad (\text{B16})$$

where we have introduced the Lagrangian

$$\mathcal{L}(\partial_{\beta\gamma} h_{\alpha(n)}, \mathcal{H}_{\beta\gamma;\alpha(n-4)}) = \frac{1}{2} \partial^{\beta\gamma} h^{\alpha(n)} \partial_{\beta\gamma} h_{\alpha(n)} - \frac{n}{4} \partial_{\beta\gamma} h^{\beta\gamma\alpha(n-2)} \partial^{\delta\lambda} h_{\delta\lambda\alpha(n-2)} \\ + \frac{n-3}{n} \left(\frac{n}{2} \mathcal{H}^{\beta\gamma;\alpha(n-4)} \partial^{\delta\lambda} h_{\beta\gamma\delta\lambda\alpha(n-4)} - \mathcal{H}^{\beta\gamma;\alpha(n-4)} \mathcal{H}_{\beta\gamma;\alpha(n-4)} \right. \\ \left. + A \mathcal{H}^{\beta\gamma;\delta\lambda\alpha(n-6)} \mathcal{H}_{\delta\lambda;\beta\gamma\alpha(n-6)} + B \mathcal{H}_{\beta\gamma;\beta\gamma\alpha(n-6)} \mathcal{H}^{\delta\lambda;\delta\lambda\alpha(n-6)} \right), \quad (\text{B17})$$

which involves the real coefficients A and B constrained by

$$A + B = -\frac{(n-4)(n-5)}{4(n-2)}. \quad (\text{B18})$$

The field $\mathcal{H}_{\beta\gamma;\alpha(n-4)}$ in (B16) is unconstrained and the Lagrange multiplier $F_{\beta\gamma;\alpha(n-4)}$ is given by

⁹The gauge transformation law (B14a) allows us to interpret $h_{\alpha(n)}$ as a conformal spin- $\frac{n}{2}$ gauge field, while (B14b) is compatible with the interpretation of $y_{\alpha(n-4)}$ as a conformal compensator (following the modern supergravity terminology). Performing a duality transformation on $y_{\alpha(n-4)}$ is equivalent to the introduction of an alternative conformal compensator.

$$F_{\beta\gamma;\alpha(n-4)} = \partial_{(\beta} \delta \Phi_{\gamma)\delta;\alpha(n-4)} \Rightarrow \partial^{\beta\gamma} F_{\beta\gamma;\alpha(n-4)} = 0, \quad (\text{B19})$$

for some unconstrained prepotential $\Phi_{\gamma\delta;\alpha(n-4)}$. Varying the first-order action (B16) with respect to $\Phi^{\gamma\delta;\alpha(n-4)}$ gives

$$\partial_{(\delta} \beta \mathcal{H}_{\gamma)\beta;\alpha(n-4)} = 0 \Rightarrow \mathcal{H}_{\beta\gamma;\alpha(n-4)} = \partial_{\beta\gamma} y_{\alpha(n-4)}, \quad (\text{B20})$$

and hence, $S_{\text{first-order}}$ reduces to the original action (B15), in accordance with (B13) and (B18). On the other hand, we can integrate out $\mathcal{H}_{\beta\gamma;\alpha(n-4)}$ using its equation motion,

$$\frac{\partial}{\partial \mathcal{H}^{\beta\gamma;\alpha(n-4)}} \mathcal{L}(\partial_{\delta\lambda} h_{\alpha(n)}, \mathcal{H}_{\delta\lambda;\alpha(n-4)}) + F_{\beta\gamma;\alpha(n-4)} = 0, \quad (\text{B21})$$

which is equivalent to

$$\mathcal{H}_{\beta\gamma;\alpha(n-4)} - \frac{B}{2}(\varepsilon_{\beta(\alpha_1}\varepsilon_{|\gamma|\alpha_2}\mathcal{H}^{\delta\lambda}_{\alpha_3\cdots\alpha_{n-4})\delta\lambda} + \varepsilon_{\gamma(\alpha_1}\varepsilon_{|\beta|\alpha_2}\mathcal{H}^{\delta\lambda}_{\alpha_3\cdots\alpha_{n-4})\delta\lambda}) - A\mathcal{H}_{(\alpha_1\alpha_2;\alpha_3\cdots\alpha_{n-4})\beta\gamma} = \frac{n}{4}\partial^{\delta\lambda}h_{\delta\lambda\beta\gamma\alpha(n-4)} - \frac{n}{2}F_{\beta\gamma;\alpha(n-4)}. \quad (\text{B22})$$

This equation allows us to express $\mathcal{H}_{\beta\gamma;\alpha(n-4)}$ in terms of $h_{\alpha(n)}$ and $F_{\beta\gamma;\alpha(n-4)}$:

$$\begin{aligned} \mathcal{H}_{\beta\gamma;\alpha(n-4)} = & \frac{n}{4(1-A)}\{\partial^{\delta\lambda}h_{\delta\lambda\beta\gamma\alpha(n-4)} - 2F_{\beta\gamma;\alpha(n-4)} + c_1(A)(\varepsilon_{\beta(\alpha_1}F^{\delta}_{\alpha_2;\alpha_3\cdots\alpha_{n-4})\gamma\delta} + \varepsilon_{\gamma(\alpha_1}F^{\delta}_{\alpha_2;\alpha_3\cdots\alpha_{n-4})\beta\delta}) \\ & + c_2(A)(\varepsilon_{\beta(\alpha_1}\varepsilon_{|\gamma|\alpha_2}F^{\delta\lambda}_{\alpha_3\cdots\alpha_{n-4})\delta\lambda} + \varepsilon_{\gamma(\alpha_1}\varepsilon_{|\beta|\alpha_2}F^{\delta\lambda}_{\alpha_3\cdots\alpha_{n-4})\delta\lambda})\}, \end{aligned} \quad (\text{B23})$$

where we have defined

$$c_1(A) = \frac{2A(n-4)}{(n+2(A-2))}, \quad (\text{B24a})$$

$$c_2(A) = -\frac{(n-4)(2A(2A+B(n-3)) + (n-4)(B-A)(n+2(A-2)))}{(n+2(A-2))((n-4)(n-5-B(n-3)) - 2A)}. \quad (\text{B24b})$$

Substituting (B23) into $S_{\text{first-order}}$ results in a dual action

$$\begin{aligned} S_A^{(n)}[h_{\alpha(n)}, \Phi_{\gamma\delta;\alpha(n-4)}] \\ = \frac{i^n}{2^{[n/2]+1}} \int d^3x \mathcal{L}_{\text{dual}}(\partial_{\beta\gamma}h_{\alpha(n)}, F_{\beta\gamma;\alpha(n-4)}). \end{aligned} \quad (\text{B25})$$

We have obtained a one-parameter family of dual actions.

As described above, the one-parameter first-order action (B16) is equivalent to the original model (B15). The action (B16) is invariant under the gauge λ transformations (B14a), which act on $\mathcal{H}_{\beta\gamma;\alpha(n-4)}$ and $\Phi_{\beta\gamma;\alpha(n-4)}$ as follows:

$$\delta_\lambda \mathcal{H}_{\beta\gamma;\alpha(n-4)} = \frac{n-2}{n-1} \partial_{\beta\gamma} \partial^{\delta\rho} \lambda_{\delta\rho\alpha(n-4)}, \quad (\text{B26a})$$

$$\begin{aligned} \delta_\lambda \Phi_{\beta\gamma;\alpha(n-4)} \\ = -\frac{2}{n(n-1)}((2n-5-2B(n-2))\partial_{(\beta}{}^\delta \lambda_{\gamma)\alpha(n-4)\delta} \\ - (n-4-2B(n-2))\partial_{(\alpha_1}{}^\delta \lambda_{\alpha_2\cdots\alpha_{n-4})\beta\gamma\delta}). \end{aligned} \quad (\text{B26b})$$

The dual action (B25) is also invariant under the gauge ρ transformation,

$$\delta_\rho \Phi_{\gamma\delta;\alpha(n-4)} = \partial_{\gamma\delta} \rho_{\alpha(n-4)} \Rightarrow \delta_\rho F_{\beta\gamma;\alpha(n-4)} = 0, \quad (\text{B27a})$$

$$\delta_\rho h_{\alpha(n-4)} = 0. \quad (\text{B27b})$$

We emphasize that the parameters A and B are constrained by (B18).

3. Transverse formulation of the superspin- $(s + \frac{1}{2})$ multiplet

The transverse formulation of the massless superspin- $(s + \frac{1}{2})$ multiplet is described by the action (3.10), which is invariant under the gauge transformations (3.9). The gauge freedom (3.9) can be used to impose the following Wess-Zumino gauge:

$$\begin{aligned} H_{\alpha(2s+1)}| &= 0, & \nabla^\beta H_{\beta\alpha(2s)}| &= 0, \\ \Upsilon_{\beta;\alpha(2s-2)}| &= 0, & \nabla^\beta \Upsilon_{\beta;\alpha(2s-2)}| &= 0. \end{aligned} \quad (\text{B28})$$

The residual gauge freedom preserving the gauge conditions (B28) is given by

$$\nabla_{(\alpha_1} \zeta_{\alpha_2\cdots\alpha_{2s+1})}| = 0, \quad (\text{B29a})$$

$$\nabla^2 \zeta_{\alpha(2s)}| = -\frac{2is}{s+1}(\nabla_{(\alpha_1}{}^\beta \zeta_{\alpha_2\cdots\alpha_{2s})\beta} + 2(s+1)|\mu|\zeta_{\alpha(2s)}|), \quad (\text{B29b})$$

$$\nabla_\beta \eta_{\alpha(2s-2)}| = \nabla_{(\beta} \eta_{\alpha(2s-2))}| = -\frac{1}{2s+1} \nabla^\gamma \zeta_{\beta\gamma\alpha(2s-2)}|, \quad (\text{B29c})$$

$$\nabla^2 \eta_{\alpha(2s-2)}| = -\frac{i}{2s+1} \nabla^{\beta\gamma} \zeta_{\beta\gamma\alpha(2s-2)}|. \quad (\text{B29d})$$

This implies that there are three real independent gauge parameters at the component level, which we choose as

$$\begin{aligned}\xi_{\alpha(2s)} &:= \zeta_{\alpha(2s)}|, & \lambda_{\alpha(2s-1)} &:= -\frac{is}{2s+1}\nabla^\beta \zeta_{\beta\alpha(2s-1)}|, \\ \rho_{\alpha(2s-2)} &:= -\eta_{\alpha(2s-2)}|.\end{aligned}\quad (\text{B30})$$

The next task is to identify the remaining independent component fields of $H_{\alpha(2s+1)}$ and $\Upsilon_{\beta;\alpha(2s-2)}$ in the Wess-Zumino gauge (B28).

Let us consider the fermionic sector. We begin by decomposing the superfield $\Upsilon_{\beta;\alpha(2s-2)}$ into irreducible components

$$\Upsilon_{\beta;\alpha(2s-2)} := Y_{\beta\alpha(2s-2)} + \sum_{k=1}^{2s-2} \varepsilon_{\beta\alpha_k} Z_{\alpha_1 \dots \alpha_k \dots \alpha_{2s-2}}, \quad (\text{B31})$$

where we have introduced the irreducible superfields

$$Y_{\alpha(2s-1)} := \Upsilon_{(\alpha_1; \alpha_2 \dots \alpha_{2s-1})}, \quad (\text{B32a})$$

$$Z_{\alpha(2s-3)} := \frac{1}{2s-1} \Upsilon_{\beta; \alpha(2s-3)}^\beta. \quad (\text{B32b})$$

We find that the remaining independent fermionic fields are given by

$$h_{\alpha(2s+1)} := \frac{i}{4} \nabla^2 H_{\alpha(2s+1)}|, \quad (\text{B33a})$$

$$y_{\alpha(2s-1)} := \frac{i}{8} \nabla^2 Y_{\alpha(2s-1)}|, \quad (\text{B33b})$$

$$y_{\alpha(2s-3)} := \frac{is}{2} (2s-1) \nabla^2 Z_{\alpha(2s-3)}|. \quad (\text{B33c})$$

Their gauge transformations are

$$\delta h_{\alpha(2s+1)} = \mathfrak{D}_{(\alpha_1 \alpha_2} \lambda_{\alpha_3 \dots \alpha_{2s+1})}, \quad (\text{B34a})$$

$$\delta y_{\alpha(2s-1)} = \frac{1}{2s+1} \mathfrak{D}_{(\alpha_1}^\beta \lambda_{\alpha_2 \dots \alpha_{2s-1})\beta} + |\mu| \lambda_{\alpha(2s-1)}, \quad (\text{B34b})$$

$$\delta y_{\alpha(2s-3)} = \mathfrak{D}^{\beta\gamma} \lambda_{\beta\gamma\alpha(2s-3)}. \quad (\text{B34c})$$

Upon the component reduction of the action (3.10), we find that the fermionic sector coincides with the Fang-Fronsdal-type action, $S_{\text{FF}}^{(2s+1,+)}$ given by (B4).

We now turn to the bosonic sector. To start with, we point out that the fourth condition in (B28) is equivalent to

$$\nabla_\beta \Upsilon_{\gamma;\alpha(2s-2)}| = \nabla_\gamma \Upsilon_{\beta;\alpha(2s-2)}|. \quad (\text{B35})$$

We then choose the following independent bosonic fields:

$$h_{\alpha(2s+2)} := -\nabla_{(\alpha_1} H_{\alpha_2 \dots \alpha_{2s+2})}|, \quad (\text{B36a})$$

$$\Phi_{\beta\gamma;\alpha(2s-2)} := \nabla_\beta \Upsilon_{\gamma;\alpha(2s-2)}| = \nabla_{(\beta} \Upsilon_{\gamma);\alpha(2s-2)}|. \quad (\text{B36b})$$

Upon the component reduction of the action (3.10), we obtain the bosonic action

$$\begin{aligned}S_{\text{bos}} = & \left(-\frac{1}{2} \right)^s \int d^3 x e \left\{ -\frac{1}{4} h^{\alpha(2s+2)} \mathcal{Q} h_{\alpha(2s+2)} + \frac{3}{16} \mathfrak{D}_{\beta\gamma} h^{\beta\gamma\alpha(2s)} \mathfrak{D}^{\delta\rho} h_{\delta\rho\alpha(2s)} + \frac{1}{4} (2s-1) F^{(\beta\gamma;\alpha(2s-2))} \mathfrak{D}^{\delta\lambda} h_{\beta\gamma\delta\lambda\alpha(2s-2)} \right. \\ & - \frac{1}{4} (2s-1) F^{\beta\gamma;\alpha(2s-2)} F_{\beta\gamma;\alpha(2s-2)} + \frac{1}{2} (s-1)(2s-1) F^{\beta\gamma;\alpha(2s-3)}_\beta F_{\delta\gamma;\alpha(2s-3)}^\delta \\ & - \frac{1}{2} (2s-1) |\mu| (h^{\alpha(2s+2)} \mathfrak{D}_{(\alpha_1}^\beta h_{\alpha_2 \dots \alpha_{2s+2})\beta} - (s-1) \Phi^{\beta\gamma;\alpha(2s-2)} \mathfrak{D}^{\delta\lambda} h_{\delta\lambda\beta\gamma\alpha(2s-2)} - 2 F^{\beta\gamma;\alpha(2s-2)} \Phi_{\beta\gamma;\alpha(2s-2)} \\ & + 8(s-1) F^{\beta\gamma;\alpha(2s-3)}_\beta \Phi_{\delta\gamma;\alpha(2s-3)}^\delta - 2(s-1)(2s-3) F_{\beta\gamma;\alpha(2s-4)}^{\beta\gamma\alpha(2s-4)} \Phi^{\delta\lambda}_{\delta\lambda\alpha(2s-4)}) \\ & + |\mu|^2 (s-1)(2s-1) \left(\frac{3s}{(s-1)(2s-1)} h^{\alpha(2s+2)} h_{\alpha(2s+2)} + (s+1) \Phi^{\beta\gamma;\alpha(2s-2)} \Phi_{\beta\gamma;\alpha(2s-2)} \right. \\ & \left. \left. + 2(2s^2 - s + 3) \Phi^{\beta\gamma;\alpha(2s-3)}_\beta \Phi_{\gamma\delta;\alpha(2s-3)}^\delta + (s-2)(2s-3) \Phi_{\beta\gamma;\alpha(2s-4)}^{\beta\gamma\alpha(2s-4)} \Phi^{\delta\lambda}_{\delta\lambda\alpha(2s-4)} \right) \right\}, \quad (\text{B37})\end{aligned}$$

where we have introduced the field strength $F_{\beta\gamma;\alpha(2s-2)}$,

$$F_{\beta\gamma;\alpha(2s-2)} := \mathfrak{D}_{(\beta}^\delta \Phi_{\gamma)\delta;\alpha(2s-2)}. \quad (\text{B38})$$

The bosonic action (B37) is invariant under the gauge transformations

$$\delta h_{\alpha(2s+2)} = \mathfrak{D}_{(\alpha_1 \alpha_2} \xi_{\alpha_3 \dots \alpha_{2s+2})}, \quad (\text{B39a})$$

$$\begin{aligned}\delta \Phi_{\beta\gamma;\alpha(2s-2)} = & \mathfrak{D}_{\beta\gamma} \rho_{\alpha(2s-2)} + 2(s-1) |\mu| (\varepsilon_{\beta(\alpha_1} \rho_{\alpha_2 \dots \alpha_{2s-2})\gamma} + \varepsilon_{\gamma(\alpha_1} \rho_{\alpha_2 \dots \alpha_{2s-2})\beta}) - \frac{1}{(2s+1)(s+1)} ((s+2) \mathfrak{D}_{(\beta}^\delta \xi_{\gamma)\delta\alpha(2s-2)} \\ & + (s-1) \mathfrak{D}_{(\alpha_1}^\delta \xi_{\alpha_2 \dots \alpha_{2s-2})\beta\gamma\delta} + 2(s+1)(2s+1) |\mu| \xi_{\beta\gamma\alpha(2s-2)}). \quad (\text{B39b})\end{aligned}$$

From (B39b), it can be shown that the field strength transforms as

$$\begin{aligned} \delta F_{\beta\gamma;\alpha(2s-2)} = & -2(s-1)|\mu|(\epsilon_{(\alpha_1|(\beta}\mathfrak{D}_{\gamma)}|\delta\rho_{\alpha_2\ldots\alpha_{2s-2})\delta} + \mathfrak{D}_{(\beta|(\alpha_1}\rho_{\alpha_2\ldots\alpha_{2s-2})|\gamma)} - 4|\mu|\epsilon_{(\beta|(\alpha_1}\rho_{\alpha_2\ldots\alpha_{2s-2})|\gamma)}) \\ & - \frac{1}{(2s+1)(s+1)} \left(\frac{1}{2}(s+2)\mathfrak{D}_{\beta\gamma}\mathfrak{D}^{\delta\rho}\xi_{\delta\rho\alpha(2s-2)} + (2s+1)\mathfrak{Q}\xi_{\gamma\beta\alpha(2s-2)} \right. \\ & + (s-1)\mathfrak{D}_{(\beta|(\alpha_1}\mathfrak{D}^{\delta\rho}\xi_{\alpha_2\ldots\alpha_{2s-2})|\gamma)}\delta\rho + 2(s+1)(2s+1)|\mu|\mathfrak{D}_{(\beta}^{\delta}\xi_{\gamma)\alpha(2s-2)}\delta \\ & \left. - 4(s-1)(s+1)(2s+1)|\mu|^2\xi_{\beta\gamma\alpha(2s-2)} \right). \end{aligned} \quad (\text{B40})$$

Thus, the field strength $F_{\beta\gamma;\alpha(2s-2)}$ is invariant under ρ -gauge transformations (B39b), $\delta_\rho F_{\beta\gamma;\alpha(2s-2)} = 0$, only in the case $|\mu| \rightarrow 0$.

The complete component action of our transverse half-integer model takes the form

$$\begin{aligned} S_{(s+\frac{1}{2})}^\perp[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}] \\ = S_{\text{bos}}[h_{\alpha(2s+2)}, \Phi_{\beta\gamma;\alpha(2s-2)}] \\ + S_{\text{FF}}^{(2s+1,+)}[h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}]. \end{aligned} \quad (\text{B41})$$

By applying the duality transformation described in Sec. B 2, it can be shown that the flat-space counterpart of (B37) is the dual formulation (B25) of the Fronsdal action $S_{\text{F}}^{(2s+2)}$ (B13) provided we fix the coefficients A and B as

$$A = -\frac{1}{2}(s-1), \quad B = \frac{3}{4s}(s-1). \quad (\text{B42})$$

That is, the dual action given in (B25) coincides with the bosonic action (B37) in Minkowski space

$$S_{\text{bos}}[h_{\alpha(2s+2)}, \Phi_{\beta\gamma;\alpha(2s-2)}]||_{|\mu|=0} = S_{\text{A}}^{(2s+2)}[h_{\alpha(2s+2)}, \Phi_{\beta\gamma;\alpha(2s-2)}]. \quad (\text{B43})$$

However, it must be noted that this duality does not hold in the presence of a nonvanishing AdS curvature, since (B37) cannot be written solely in terms of the field strength $F_{\beta\gamma;\alpha(2s-2)}$.

4. Longitudinal formulation of the superspin- $(s + \frac{1}{2})$ multiplet

The longitudinal formulation of the massless superspin- $(s + \frac{1}{2})$ multiplet is described by the action (3.6), which is invariant under the gauge transformations (3.5). The component structure of this model was studied in [37] only in the flat superspace limit. Here we extend these results to AdS_3 .

The gauge freedom (3.9) can be used to impose the following Wess-Zumino gauge:

$$H_{\alpha(2s+1)}| = 0, \quad \nabla^\beta H_{\beta\alpha(2s)}| = 0. \quad (\text{B44})$$

The residual gauge freedom that preserves the conditions (B44) is described by

$$0 = \nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s+1})}|, \quad (\text{B45a})$$

$$\nabla^2\zeta_{\alpha(2s)}| = -\frac{2is}{s+1}(\nabla_{(\alpha_1}{}^\beta\zeta_{\alpha_2\ldots\alpha_{2s})\beta} + 2(s+1)|\mu|\zeta_{\alpha(2s)}|). \quad (\text{B45b})$$

These conditions imply that there are only two independent gauge parameters, which we define as follows:

$$\xi_{\alpha(2s)} := \zeta_{\alpha(2s)}|, \quad \lambda_{\alpha(2s-1)} := -\frac{is}{2s+1}\nabla^\beta\zeta_{\beta\alpha(2s-1)}|. \quad (\text{B46})$$

Upon imposing the gauge (B44), we are left with the following component fields:

$$h_{\alpha(2s+1)} := \frac{i}{4}\nabla^2 H_{\alpha(2s+1)}|, \quad (\text{B47a})$$

$$h_{\alpha(2s+2)} := -\nabla_{(\alpha_1} H_{\alpha_2\ldots\alpha_{2s+2})}|, \quad (\text{B47b})$$

$$y_{\alpha(2s-2)} := -4L_{\alpha(2s-2)}|, \quad (\text{B47c})$$

$$y_{\alpha(2s-1)} := (2s+1)i\nabla_{(\alpha_1} L_{\alpha_2\ldots\alpha_{2s-1})}|, \quad (\text{B47d})$$

$$y_{\alpha(2s-3)} := 2i\nabla^\beta L_{\beta\alpha(2s-3)}|, \quad (\text{B47e})$$

$$F_{\alpha(2s-2)} := \frac{i}{4}\nabla^2 L_{\alpha(2s-2)}|. \quad (\text{B47f})$$

Upon component reduction of the higher-spin model (3.6), we find that the theory decouples into bosonic and fermionic sectors. The bosonic action is given by

$$\begin{aligned}
S_{\text{bos}} = & \left(-\frac{1}{2}\right)^s \int d^3x e \left\{ -\frac{1}{4} h^{\alpha(2s+2)} \mathcal{Q} h_{\alpha(2s+2)} + \frac{1}{8} (s+1) \mathfrak{D}_{\beta\gamma} h^{\beta\gamma\alpha(2s)} \mathfrak{D}^{\delta\lambda} h_{\delta\lambda\alpha(2s)} \right. \\
& + \frac{1}{8} (2s-1) y^{\alpha(2s-2)} \mathfrak{D}^{\beta\gamma} \mathfrak{D}^{\delta\lambda} h_{\beta\gamma\delta\lambda\alpha(2s-2)} + \frac{1}{16s} (2s-1)(s+1) y^{\alpha(2s-2)} \mathcal{Q} y_{\alpha(2s-2)} \\
& + \frac{1}{s} (s-1)(2s-1) F^{\beta\alpha(2s-3)} \mathfrak{D}_{\beta}^{\gamma} y_{\gamma\alpha(2s-3)} + \frac{4}{s} (s+1)(2s-1) F^{\alpha(2s-2)} F_{\alpha(2s-2)} \\
& + \frac{1}{4} (s-1)(2s-1) |\mu| y^{\beta\alpha(2s-3)} \mathfrak{D}_{\beta}^{\gamma} y_{\gamma\alpha(2s-3)} + 2(s-1)(2s-1) |\mu| F^{\alpha(2s-2)} y_{\alpha(2s-2)} \\
& \left. + 2s(s+1) |\mu|^2 h^{\alpha(2s+2)} h_{\alpha(2s+2)} - \frac{1}{2} (2s-1)(s+1) |\mu|^2 y^{\alpha(2s-2)} y_{\alpha(2s-2)} \right\}. \quad (\text{B48})
\end{aligned}$$

The bosonic field $F_{\alpha(2s-2)}$ is auxiliary, so upon elimination via its equation of motion

$$F_{\alpha(2s-2)} = -\frac{1}{8(s+1)} (s-1) \mathfrak{D}_{(\alpha_1}^{\beta} y_{\alpha_2 \dots \alpha_{2s-2})\beta} - \frac{s}{4} |\mu| y_{\alpha(2s-2)}, \quad (\text{B49})$$

we find that the resulting action coincides with the massless spin- $(s+1)$ action, $S_F^{(2s+2)}$, which is obtained from (B11) by setting $n = 2s + 2$.

It can be shown that the fermionic action emerging from the reduction procedure coincides with the Fang-Fronsdal-type action $S_{\text{FF}}^{(2s+1,-)}$ (B4). Therefore, our component actions take the form

$$S_{(s+\frac{1}{2})}^{\parallel} [H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] = S_F^{(2s+2)} [h_{\alpha(2s+2)}, y_{\alpha(2s-2)}] + S_{\text{FF}}^{(2s+1,-)} [h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}]. \quad (\text{B50})$$

Comparing (B50) with (B41), it is seen that the fermionic sector resulting from the reduction of the longitudinal model is given by $S_{\text{FF}}^{(2s+1,+)} [h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}]$. In flat space, the two bosonic actions, $S_F^{(2s+2)} [h_{\alpha(2s+2)}, y_{\alpha(2s-2)}]$ and $S_{\text{bos}} [h_{\alpha(2s+2)}, \Phi_{\beta\gamma;\alpha(2s-2)}]$, are related to each other by a duality transformation as described in Sec. B.2. The fermionic actions are now given by $S_{\text{FF}}^{(2s+1)}$, and thus they coincide identically.

5. Transverse formulation of the superspin- s multiplet

The transverse formulation of the massless superspin- s multiplet is described by the action (3.17) and is invariant under gauge transformations (3.16). The gauge freedom (3.16) can be used to impose the following Wess-Zumino gauge conditions:

$$H_{\alpha(2s)}| = 0, \quad \nabla^{\beta} H_{\beta\alpha(2s-1)}| = 0, \quad \Psi_{\beta;\alpha(2s-2)}| = 0, \quad \nabla^{\beta} \Psi_{\beta;\alpha(2s-2)}| = 0. \quad (\text{B51})$$

The residual gauge symmetry preserving (B51) is given by

$$\nabla_{(\alpha_1} \zeta_{\alpha_2 \dots \alpha_{2s})}| = 0, \quad (\text{B52a})$$

$$\nabla^2 \zeta_{\alpha(2s-1)}| = -\frac{2i(2s-1)}{2s+1} (\nabla_{(\alpha_1}^{\beta} \zeta_{\alpha_2 \dots \alpha_{2s-1})\beta} + (2s+1) |\mu| \zeta_{\alpha(2s-1)}|), \quad (\text{B52b})$$

$$\nabla_{\beta} \eta_{\alpha(2s-2)}| = \nabla_{(\beta} \eta_{\alpha(2s-2))}| = -i \zeta_{\beta\alpha(2s-2)}|, \quad (\text{B52c})$$

$$\nabla^2 \eta_{\alpha(2s-2)}| = -i \nabla^{\beta} \zeta_{\beta\alpha(2s-2)}|. \quad (\text{B52d})$$

Thus, there are three real independent gauge parameters, which we define as

$$\xi_{\alpha(2s-1)} := \zeta_{\alpha(2s-1)}|, \quad \rho_{\alpha(2s-2)} := -\eta_{\alpha(2s-2)}|, \quad (\text{B53a})$$

$$\lambda_{\alpha(2s-2)} := -\frac{2s-1}{4s} \nabla^{\beta} \zeta_{\beta\alpha(2s-2)}|. \quad (\text{B53b})$$

We now wish to find the remaining independent component fields of $H_{\alpha(2s)}$ and $\Psi_{\beta;\alpha(2s-2)}$ in the Wess-Zumino gauge (B51). The last Wess-Zumino gauge condition (B51) yields the relation

$$\nabla_\beta \Psi_{\gamma;\alpha(2s-2)} = \nabla_\gamma \Psi_{\beta;\alpha(2s-2)}. \quad (\text{B54})$$

Thus, we choose to describe the bosonic sector in terms of the independent fields

$$X_{\alpha(2s)} := \frac{i}{4} \nabla^2 H_{\alpha(2s)}|, \quad (\text{B55a})$$

$$\Phi_{\beta\gamma;\alpha(2s-2)} := \nabla_\beta \Psi_{\gamma;\alpha(2s-2)}| = \nabla_{(\beta} \Psi_{\gamma);\alpha(2s-2)}|. \quad (\text{B55b})$$

Upon the component reduction of the action (3.10), we obtain the bosonic action

$$\begin{aligned} S_{\text{bos}} = & -2(2s-1) \left(-\frac{1}{2} \right)^s \int d^3x e \left\{ \frac{s-1}{2s-1} X^{\alpha(2s)} X_{\alpha(2s)} + F^{\beta\gamma;\alpha(2s-2)} X_{\beta\gamma\alpha(2s-2)} \right. \\ & + \frac{1}{2} F^{\beta\gamma;\alpha(2s-2)} F_{\beta\gamma;\alpha(2s-2)} + \frac{1}{2s} (s-1) F^{\beta\gamma;\alpha(2s-3)}{}_\beta F_{\gamma\delta;\alpha(2s-3)}{}^\delta \\ & + 2(s-1) |\mu| \Phi^{\beta\gamma;\alpha(2s-2)} X_{\beta\gamma\alpha(2s-2)} + (2s-1) |\mu| F^{\beta\gamma;\alpha(2s-2)} \Phi_{\beta\gamma;\alpha(2s-2)} \\ & + 2(s-1) |\mu| F^{\beta\gamma;\alpha(2s-3)}{}_\beta \Phi_{\gamma\delta;\alpha(2s-3)}{}^\delta + \frac{2}{s} (s-1)^2 |\mu| F^{\beta\gamma;\alpha(2s-3)}{}_\beta \Phi_{\delta(\gamma;\alpha(2s-3))}{}^\delta \\ & + 2s(s-1) |\mu|^2 \Phi^{\beta\gamma;\alpha(2s-2)} \Phi_{\beta\gamma;\alpha(2s-2)} + 2s(s-1) |\mu|^2 \Phi^{\beta\gamma;\alpha(2s-3)}{}_\beta \Phi_{\gamma\delta;\alpha(2s-3)}{}^\delta \\ & \left. + \frac{2}{s} (s-1)^2 (s-2) |\mu|^2 \Phi^{\beta\gamma;\alpha(2s-3)}{}_\beta \Phi_{\delta(\gamma;\alpha(2s-3))}{}^\delta \right\}, \quad (\text{B56}) \end{aligned}$$

where we have introduced the field strength $F_{\beta\gamma;\alpha(2s-2)}$,

$$F_{\beta\gamma;\alpha(2s-2)} := \mathfrak{D}_{(\beta}{}^\delta \Phi_{\gamma)\delta;\alpha(2s-2)}. \quad (\text{B57})$$

It can be shown that the action (B56) is invariant under the gauge transformations

$$\delta X_{\alpha(2s)} = \mathfrak{D}_{(\alpha_1 \alpha_2} \lambda_{\alpha_3 \dots \alpha_{2s})}, \quad (\text{B58a})$$

$$\delta \Phi_{\beta\gamma;\alpha(2s-2)} = \mathfrak{D}_{\beta\gamma} \rho_{\alpha(2s-2)} + 4(s-1) |\mu| \varepsilon_{(\beta | (\alpha_1 \rho_{\alpha_2 \dots \alpha_{2s-2}) | \gamma)} + \frac{4}{2s-1} (s-1) \varepsilon_{(\beta | (\alpha_1 \lambda_{\alpha_2 \dots \alpha_{2s-2}) | \gamma)}. \quad (\text{B58b})$$

The field $X_{\alpha(2s)}$ is auxiliary, so upon elimination via its equation of motion

$$X_{\alpha(2s)} = -\frac{2s-1}{2(s-1)} (F_{\alpha(2s)} + 2(s-1) |\mu| \Phi_{(\alpha_1 \alpha_2; \alpha_3 \dots \alpha_{2s})}), \quad (\text{B59})$$

we obtain

$$\begin{aligned} S_{\text{bos}} = & \frac{2s-1}{s(2s-2)} \left(-\frac{1}{2} \right)^s \int d^3x e \left\{ s F^{\beta\gamma;\alpha(2s-2)} F_{\beta\gamma;\alpha(2s-2)} + 2s(s-1) F^{\beta\gamma;\alpha(2s-3)}{}_\beta F_{\gamma\delta;\alpha(2s-3)}{}^\delta \right. \\ & + (2s-3)(s-1) F^{\beta\gamma;\alpha(2s-4)} F^{\delta\lambda;}_{\delta\lambda\alpha(2s-4)} - 4(s-1)^2 |\mu|^2 (s \Phi^{\beta\gamma;\alpha(2s-2)} \Phi_{\beta\gamma;\alpha(2s-2)} + 2 \Phi^{\beta\gamma;\alpha(2s-3)}{}_\beta \Phi_{\delta\gamma;\alpha(2s-3)}{}^\delta \\ & \left. - (2s-3) \Phi_{\beta\gamma; \beta\gamma\alpha(2s-4)} \Phi^{\delta\lambda;}_{\delta\lambda\alpha(2s-4)}) \right\}. \quad (\text{B60}) \end{aligned}$$

We can express the field $\Phi_{\beta\gamma;\alpha(2s-2)}$ in terms of its irreducible components

$$h_{\alpha(2s)} := \Phi_{(\alpha_1\alpha_2;\alpha_3\cdots\alpha_{2s})}, \quad (\text{B61a})$$

$$y_{\alpha(2s-2)} := \Phi_{(\alpha_1;\alpha_2\cdots\alpha_{2s-2})}^\beta, \quad (\text{B61b})$$

$$y_{\alpha(2s-4)} := \frac{2(s-1)}{2s-1} \Phi_{\beta\gamma;\alpha(2s-4)}^{\beta\gamma}. \quad (\text{B61c})$$

The gauge freedom (B58b) can be used to gauge away $y_{\alpha(2s-2)}$. It follows that the remaining fields, (B61a) and (B61c), have the corresponding gauge transformations

$$\delta h_{\alpha(2s)} = \mathfrak{D}_{(\alpha_1\alpha_2}\rho_{\alpha_3\cdots\alpha_{2s})}, \quad (\text{B62a})$$

$$\delta y_{\alpha(2s-4)} = \frac{2(s-1)}{2s-1} \mathfrak{D}^{\beta\gamma}\rho_{\beta\gamma\alpha(2s-4)}. \quad (\text{B62b})$$

Hence, by rewriting (B60) in terms of the irreducible fields (B61a) and (B61c), the resulting action proves to coincide (up to an overall factor) with the massless spin- s action (B11).

The study of the fermionic sector requires the decomposition of the reducible superfield $\Psi_{\beta;\alpha(2s-2)}$ into irreducible parts. This procedure is completely analogous to that of the prepotential $\Upsilon_{\beta;\alpha(2s-2)}$ (B31). In this case, we find that the remaining independent fermionic fields are given by

$$h_{\alpha(2s+1)} := -i\nabla_{(\alpha_1}H_{\alpha_2\cdots\alpha_{2s+1})}, \quad (\text{B63a})$$

$$y_{\alpha(2s-1)} := \frac{i}{4}\nabla^2 Y_{\alpha(2s-1)}, \quad (\text{B63b})$$

$$z_{\alpha(2s-3)} := \frac{i}{2}(2s-1)\nabla^2 Z_{\alpha(2s-3)}. \quad (\text{B63c})$$

Upon the component reduction of the action (3.17), it can be shown that the fermionic action obtained coincides with the Fang-Fronsdal-type action, $S_{\text{FF}}^{(2s+1,-)}$ (B4). As a result, we find that the action corresponding to the transverse formulation of the massless superspin- s multiplet in AdS_3 decomposes into

$$\begin{aligned} S_{(s)}^\perp[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}] \\ = \frac{2s-1}{s-1} S_{\text{F}}^{(2s)}[h_{\alpha(2s)}, y_{\alpha(2s-4)}] \\ + S_{\text{FF}}^{(2s+1,-)}[h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}]. \end{aligned} \quad (\text{B64})$$

6. Longitudinal formulation of the superspin- s multiplet

The longitudinal formulation of the massless superspin- s multiplet is described by the action (3.14) and is invariant

under gauge transformations (3.13). This formulation corresponds to the massless first-order model, whose component reduction in the flat-superspace limit has been studied in [37].

The gauge freedom (3.13) can be used to impose the following Wess-Zumino gauge:

$$H_{\alpha(2s)}| = 0, \quad \nabla^\beta H_{\beta\alpha(2s-1)}| = 0, \quad V_{\alpha(2s-2)}| = 0. \quad (\text{B65})$$

The residual gauge symmetry that preserves the gauge transformations (B65) are

$$0 = \nabla_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s})}|, \quad (\text{B66a})$$

$$0 = \nabla^\beta \zeta_{\beta\alpha(2s-2)}|, \quad (\text{B66b})$$

$$\begin{aligned} \nabla^2 \zeta_{\alpha(2s-1)}| &= -\frac{2i}{2s+1}(2s-1)(\nabla_{(\alpha_1}\beta\zeta_{\beta\alpha_2\cdots\alpha_{2s-1})} \\ &+ (2s+1)|\mu|\zeta_{\alpha(2s-1)}|). \end{aligned} \quad (\text{B66c})$$

The relations (B66) indicate that there is only one independent gauge parameter, which we choose to define as

$$\xi_{\alpha(2s-1)} := \zeta_{\alpha(2s-1)}|. \quad (\text{B67})$$

Thus, we are left with the remaining independent component fields in the gauge (B44):

$$h_{\alpha(2s+1)} := -i\nabla_{(\alpha_1}H_{\alpha_2\cdots\alpha_{2s+1})}|, \quad (\text{B68a})$$

$$h_{\alpha(2s)} := \frac{i}{4}\nabla^2 H_{\alpha(2s)}|, \quad (\text{B68b})$$

$$y_{\alpha(2s-2)} := \frac{i}{4}\nabla^2 V_{\alpha(2s-2)}|, \quad (\text{B68c})$$

$$y_{\alpha(2s-1)} := \frac{i}{2}\nabla_{(\alpha_1}V_{\alpha_2\cdots\alpha_{2s-1})}|, \quad (\text{B68d})$$

$$y_{\alpha(2s-3)} := -2is\nabla^\beta V_{\beta\alpha(2s-3)}|. \quad (\text{B68e})$$

It is easy to see that the gauge transformation laws for the bosonic fields are

$$\delta h_{\alpha(2s)} = 0, \quad \delta y_{\alpha(2s-2)} = 0. \quad (\text{B69})$$

Upon reduction of the higher-spin model (3.6), it can be shown that these bosonic fields appear without derivatives, and hence, they are pure auxiliary fields. Indeed, we find that

$$S_{(s)}^{\parallel}[H_{\alpha(2s)}, V_{\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int d^3x e \{ h^{\alpha(2s)} h_{\alpha(2s)} + 2s(2s-1) y^{\alpha(2s-2)} y_{\alpha(2s-2)} \} \\ + S_{\text{FF}}^{(2s+1,+)}[h_{\alpha(2s+1)}, y_{\alpha(2s-1)}, y_{\alpha(2s-3)}]. \quad (\text{B70})$$

Unlike the reduction of the transverse superspin- s multiplet described by (B64), here the fermionic action comes with a positive sign in the $|\mu|$ -dependent terms, $S_{\text{FF}}^{(2s+1,+)}$.

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