Nondiagonal pair of Majorana particles at e^+e^- colliders

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We perform a comprehensive and model-independent analysis for characterizing the spin and dynamical structure of the production of a nondiagonal pair of Majorana particles with different masses and arbitrary spins $e^-e^+ \rightarrow X_2X_1$, followed by a sequential two-body decay $X_2 \rightarrow ZX_1$ of the heavier particle X_2 into a Z gauge boson and the lighter particle X_1 escaping undetected at high-energy e^+e^- colliders. Standard leptonic Z -boson decays, $Z \rightarrow \ell^-\ell^+$ with $\ell = e$ or μ , are employed for precisely diagnosing the Z polarization influenced by the production and decay processes. Based on helicity formalism and Wick helicity rotation for describing the correlated production-decay amplitudes and distributions, we work out the implications on the amplitudes and distributions of discrete CP symmetry and the Majorana condition that two particles are their own antiparticles. For the sake of a concrete illustration, an example of this type in the minimal supersymmetric Standard Model is investigated in detail.

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I. INTRODUCTION

The Standard Model (SM) [1–3] has been firmly confirmed as a self-consistent gauge theory with a weakly coupled sector for electroweak (EW) symmetry breaking with the discovery of a scalar boson [4,5] and the ever-increasing confidence of its compatibility with the SM Higgs boson [6,7] at the CERN Large Hadron Collider (LHC). Nevertheless, we highly expect new physics beyond the SM (BSM) to be revealed at the TeV scale (terascale), motivated by tiny but nonvanishing neutrino masses [8], matter dominance in our Universe [9,10], dark matter [11–13], and inflation [14], etc. Conceptually, the naturalness issue [15–17] has been the prime argument for the realization of new BSM physics at the weak scale of 246 GeV.

To much puzzlement, except for a SM-like Higgs boson, no new BSM particles have been so far observed in the LHC experiments around the terascale threshold. One plausible scenario for the LHC null search results is that all the strongly interacting colored BSM particles are too heavy to be directly produced at the LHC, and the EW BSM particles, although kinematically accessible, may not lead to tractable signals due to rather small production rate, uncharacteristic signature, and/or large SM backgrounds at hadron colliders. On the other hand, the future high-energy

e^+e^- colliders would be capable of discovering and diagnosing some new EW particles, as long as kinematically accessible, because of well-constrained event topology and very clean experimental environment.

In the present work, we study such a challenging but plausible scenario at an e^+e^- collider that two neutral BSM particles are kinematically accessible only with the combination of the diagonal pair of the lighter particles and the nondiagonal pair of two particles, while the diagonal pair of the heavier particles is kinematically inaccessible (see, for example, Ref. [18]). Particularly, two neutral particles are assumed to be their own antiparticles, but their spins are arbitrary. Such Majorana particles¹ are unavoidable in supersymmetric theories, guaranteeing that every known bosonic particle has a heavier fermionic partner and vice versa for each known fermion, and they are predicted also by various grand unified theories [19,20] and extra-dimensional models [21–24] and even in solid-state physics [25].

Referring to Refs. [26,27] as a few previous works for the processes of diagonal pair production, we focus on the analysis of the combined process of production of a nondiagonal pair of Majorana particles with different masses and arbitrary spins, followed by two sequential two-body decays as

$$\begin{aligned} e^- + e^+ &\rightarrow X_2 + X_1 \\ &\downarrow Z + X_1 \\ &\downarrow \ell^- + \ell^+, \end{aligned} \quad (1)$$

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¹Usually the term Majorana has been used for fermions with half-integer spin, but it will be employed for real bosons with integer spin as well.

where the mass splitting $\Delta m = m_2 - m_1$ of the particles X_2 and X_1 is larger than the Z -boson mass m_Z , i.e., $m_2 - m_1 > m_Z$, and the charged lepton ℓ is taken to be e or μ , allowing for the full reconstruction of the Z -boson momentum with great precision.²

The neutral particle X_1 is assumed to be stable and so it escapes undetected with no tractable signals as the lightest neutral supersymmetric particle in the minimal supersymmetric SM (MSSM) with R parity. Consequently, the combined process has a distinct V -shape signature of a charged-lepton pair of which the four momentum is balanced due to energy-momentum conservation with the missing four momentum carried away by two invisible X_1 particles

$$e^-e^+ \rightarrow \ell^-\ell^+ + \cancel{E}, \quad (2)$$

with $\cancel{E}$ denoting the invisible part and with the constraint $m_{\ell\ell} = m_Z$ for the invariant mass of two final leptons, signaling the presence of an intermediate on shell Z boson.

For a nonzero X_2 spin j_2 , the X_2 particle is produced generally in a polarized state in the process (1), especially, if the interactions are parity violating. The information on the polarization of the spin-1 gauge boson Z can be extracted through the angular distributions in its well-established leptonic decays, $Z \rightarrow \ell^-\ell^+$, with $\ell = e$ or μ . However, the Z momentum direction in the X_2 rest frame (X_2 RF), which is the most convenient for describing the decays analytically, is not identical to that in the e^+e^- c.m. frame (ee c.m.) directly reconstructed experimentally.

A proper Wick helicity rotation [28–30] needs to be incorporated for linking the Z polarization state with respect to the Z momentum direction in the ee c.m. to that with respect to the Z momentum direction in the X_2 RF.

The prime goal of the present work is to derive the fully analytic expressions of the correlated production-decay (polar-)angular correlations in a transparent and compact way, exploiting the helicity formalism and a Wick helicity rotation for probing the spins and dynamical properties of two Majorana particles in a general setting. These analytic results enable us to probe the dynamical features and symmetry properties of the correlated process in Eq. (1) efficiently by separating the complicated kinematical boost effects out clearly.

The paper is organized as follows. In Sec. II, we present the complete amplitudes of the production process and two

sequential two-body decays in a compact and general form and analyze the implications of the discrete CP symmetry and the Majorana condition on the amplitudes and production-decay correlations. Section III is devoted to a systematic derivation of all the angular correlations and a detailed analysis of the fully reconstructible polar-angle correlations. In Sec. IV, those model-independent theoretical results are demonstrated with one specific example of a nondiagonal pair of neutralinos [31–36], which are Majorana fermions in the MSSM. Finally, some conclusions are given in Sec. V.

II. PRODUCTION AND DECAY AMPLITUDES

In this section, first we present in a compact and transparent form the complete helicity amplitudes for the production of a nondiagonal pair of Majorana particles and for the two sequential two-body decays shown in Eq. (1), of which the kinematical configuration is depicted in detail in Fig. 1. A proper Wick helicity rotation and an azimuthal-angle adjustment are performed for linking the decay helicity amplitudes in the X_2 RF and ee c.m. Second, we remark on the general constraints on the amplitudes by CP invariance and by the Majorana condition.

FIG. 1. A diagrammatic description of kinematical configurations of the combined production-decay process (1). The X_2 polar and azimuthal angles, θ_2 and ϕ_2 , are defined with respect to the e^- momentum direction and a properly chosen x axis. The Z polar and azimuthal angles, θ_Z and ϕ_Z , are defined in the X_2 rest frame boosted back along the X_2 momentum direction from the ee c.m. The ℓ^- polar and azimuthal angles, θ_- and ϕ_- , are defined in the Z rest frame boosted back along the Z momentum direction in the ee c.m., respectively. The azimuthal angle ϕ_- denotes the relative angle between the eeZ plane and the $Z\ell\ell$ plane in the ee c.m.

²If $\Delta m < m_Z$, the particle X_2 decays directly into $X_1\ell^-\ell^+$ through several channels. These three-body decay processes are closely related to the production process $e^-e^+ \rightarrow X_2X_1$, especially for $\ell = e$, from a topological point of view. A detailed analysis of these combined production-decay processes involving a few sophisticated conceptual issues will be reported elsewhere.

A. Helicity amplitudes

We adopt the helicity formalism [28,37] for deriving the helicity amplitudes of the production process for a non-diagonal pair $X_2 X_1$ of Majorana particles

$$e^-(k, \sigma) + e^+(\bar{k}, \bar{\sigma}) \rightarrow X_2(p_2, \lambda_2) + X_1(p_1, \lambda_1) \quad (3)$$

in the $eec.m.$ The same formalism is applied to the two-body decay of the Majorana particle X_2 of mass m_2 and spin j_2 into an on shell spin-1 Z boson of mass m_Z and a Majorana particle X_1 of mass m_1 and spin j_1

$$X_2(p'_2, \lambda_2) \rightarrow Z(q_Z, \lambda_Z) + X_1(q_1, \sigma_1), \quad (4)$$

in the X_2 RF (see Refs. [29,38] for the neutralino two-body decay in the MSSM) and to the Z two-body leptonic decay

$$Z(q'_Z, \lambda'_Z) \rightarrow \ell^-(k_-, \tau_-) + \ell^+(k_+, \tau_+), \quad (5)$$

in the Z rest frame (ZRF). The four momentum and helicity of each particle are shown in parenthesis with each primed four momentum referring to the four momentum in the rest frame of its corresponding particle, X_2 or Z . One crucial point to be ensured in calculating the amplitudes of the correlated production-decay process is that the Z -boson polarization state in the X_2 RF is, in general, different from that in the ZRF directly reconstructible in the $eec.m.$

Explicitly, in the kinematical configuration depicted in Fig. 1, the helicity amplitude of the production process $e^- e^+ \rightarrow X_2 X_1$ can be written as

$$\begin{aligned} \mathcal{M}_{\sigma, \bar{\sigma}; \lambda_2, \lambda_1}^{e^- e^+ \rightarrow X_2 X_1}(\theta_2, \phi_2) \\ = \mathcal{P}_{\sigma, \bar{\sigma}; \lambda_2, \lambda_1}(\cos \theta_2) d_{\sigma - \bar{\sigma}, \lambda_2 - \lambda_1}^J(\theta_2) e^{i(\sigma - \bar{\sigma})\phi_2}, \end{aligned} \quad (6)$$

where $J = \max(|\sigma - \bar{\sigma}|, |\lambda_2 - \lambda_1|)$, and the angles θ_2 and ϕ_2 denote the scattering polar and azimuthal angles of the X_2 with respect to the e^- direction and a fixed x axis, of which the direction may be fixed for transverse e^- or e^+ beam polarizations, in the $eec.m.$ Finally the polar-angle-dependent function $d_{\sigma - \bar{\sigma}, \lambda_2 - \lambda_1}^J(\theta_2)$ is the Wigner d function in the convention of Rose [39].

The general theoretical analysis of the Z polarization in the two-body decay $X_2 \rightarrow ZX_1$ is the most transparent analytically in the X_2 RF. The decay helicity amplitude can be decomposed in terms of the decay polar and azimuthal angles, θ_Z and ϕ_Z , for the momentum direction of the Z boson in the X_2 RF

$$\begin{aligned} \mathcal{M}_{\lambda_2; \lambda_Z, \sigma_1}^{X_2 \rightarrow ZX_1}(\theta_Z, \phi_Z) \\ = \mathcal{C}_{\lambda_Z, \sigma_1} d_{\lambda_2, \lambda_Z - \sigma_1}^{j_2}(\theta_Z) e^{i\lambda_2 \phi_Z} \quad \text{with} \quad |\lambda_Z - \sigma_1| \leq j_2, \end{aligned} \quad (7)$$

where the azimuthal angle ϕ_Z is defined with respect to the plane formed by the e^- and X_2 momenta in the $eec.m.$

Because the Z -momentum direction in the X_2 RF is different from that in the $eec.m.$, the helicity amplitudes in Eq. (7) need to be transformed by a proper Wick helicity rotation [28–30] for connecting the Z helicity state in the X_2 RF to that in the $eec.m.$ with a so-called Wick helicity rotation angle ω_Z satisfying

$$\cos \omega_Z = \frac{\beta_Z + \beta_2 \cos \theta_Z}{\sqrt{(1 + \beta_2 \beta_Z \cos \theta_Z)^2 - (1 - \beta_2^2)(1 - \beta_Z^2)}}, \quad (8)$$

$$\sin \omega_Z = \frac{\sqrt{1 - \beta_Z^2} \beta_2 \sin \theta_Z}{\sqrt{(1 + \beta_2 \beta_Z \cos \theta_Z)^2 - (1 - \beta_2^2)(1 - \beta_Z^2)}}, \quad (9)$$

where β_2 and β_Z are the X_2 speed in the $eec.m.$ and the Z speed in the X_2 RF, which are determined in terms of the $eec.m.$ energy $\sqrt{s}$ and the $X_{1,2}$ and Z -boson masses. The resulting decay helicity amplitude directly coupled with the Z -boson decay helicity amplitude reads³

$$\mathcal{A}_{\lambda_2; \lambda'_Z, \sigma_1}(\theta_Z, \phi_Z) = \sum_{\lambda_Z = \pm 1, 0} d_{\lambda'_Z, \lambda_Z}^1(\omega_Z) \mathcal{M}_{\lambda_2; \lambda_Z, \sigma_1}^{X_2 \rightarrow ZX_1}(\theta_Z, \phi_Z). \quad (10)$$

It is important to note that the Wick helicity rotation angle ω_Z along with the polar angle θ_Z is determined event by event, although the azimuthal angle ϕ_Z defined with respect to the $e^- X_2$ plane in the $eec.m.$ cannot be reconstructed due to the invisible X_1 .

Among various decay channels of the Z boson, the leptonic Z -boson decays $Z \rightarrow \ell^- \ell^+$, especially with $\ell = e$ and μ , provide a very clean and powerful means for reconstructing the Z -boson rest frame, independent of its production mechanisms, and for extracting the information on Z polarization. The helicity amplitude of the leptonic Z -boson decay can be written as

$$\mathcal{M}_{\lambda'_Z; \sigma_-, \sigma_+}^{Z \rightarrow \ell^- \ell^+}(\theta_-, \phi_-) = \mathcal{Z}_{\sigma_-, \sigma_+} d_{\lambda'_Z, \sigma_- - \sigma_+}^1(\theta_-) e^{i\lambda'_Z \phi_-}, \quad (11)$$

in terms of the polar and azimuthal angles, θ_- and ϕ_- , in the ZRF with the azimuthal angle defined with respect to the plane formed by the e^- and Z momenta in the $eec.m.$, which are determined fully with great precision. In terms of the normalized vector and axial-vector couplings $v_\ell = \sin^2 \theta_W - 1/4$ and $a_\ell = 1/4$ with the weak mixing angle θ_W , the reduced helicity amplitude $\mathcal{Z}_{\sigma_-, \sigma_+}$ in Eq. (11) is given by

$$\mathcal{Z}_{\sigma_-, \sigma_+} = -i\sqrt{2}g_Z m_Z (v_\ell + \Delta\sigma a_\ell) \delta_{\sigma_-, -\sigma_+}, \quad (12)$$

³We do not include another Wick helicity rotation connecting the X_1 helicity states in the $eec.m.$ and in the X_2 RF because its effects on any distributions are washed away completely with summing over the X_1 helicities, naturally taken for the invisible X_1 particle.

with $\Delta\sigma = \sigma_- - \sigma_+ = \pm 1$ and $g_Z = e/c_W s_W$ in terms of the positron electric charge e and the abbreviations, $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$, when the charged-lepton mass is ignored. However, it is necessary to adjust the azimuthal-angle phase factor of the helicity amplitude in Eq. (11) by an azimuthal angle γ_Z for compensating the mismatch between the ZX_2X_1 plane and the eeZ plane in the ee c.m. This leads to the Z-boson decay amplitude with an adjusted phase factor as

$$\mathcal{B}_{\lambda'_Z; \sigma_- \sigma_+}(\theta_-, \phi'_-) = \mathcal{Z}_{\sigma_- \sigma_+} d_{\lambda'_Z, \sigma_- \sigma_+}^1(\theta_-) e^{i\lambda'_Z \phi'_-}, \quad (13)$$

with the newly defined azimuthal angle $\phi'_- = \phi_- - \gamma_Z$. The angle γ_Z satisfies

$$\cos \gamma_Z = \frac{\cos \theta_2 \sin \theta'_Z + \sin \theta_2 \cos \theta'_Z \cos \phi_Z}{\sqrt{1 - (\cos \theta_2 \cos \theta'_Z - \sin \theta_2 \sin \theta'_Z \cos \phi_Z)^2}}, \quad (14)$$

$$\sin \gamma_Z = \frac{\sin \theta_2 \sin \phi_Z}{\sqrt{1 - (\cos \theta_2 \cos \theta'_Z - \sin \theta_2 \sin \theta'_Z \cos \phi_Z)^2}}, \quad (15)$$

where the angle θ'_Z is the Z-boson polar angle with respect to the X_2 -momentum direction in the ee c.m. This can be determined event by event through the relations⁴

$$\tan \theta'_Z = \frac{\beta_Z \sin \theta_Z}{\gamma_Z (\beta_2 + \beta_Z \cos \theta_Z)} \quad \text{and} \quad E'_Z = m_Z \gamma_Z \gamma_Z (1 + \beta_2 \beta_Z \cos \theta_Z), \quad (16)$$

with $\gamma_2 = (s + m_2^2 - m_1^2)/2m_2\sqrt{s}$, $\beta_2 = \sqrt{1 - 1/\gamma_2^2}$, $\gamma_Z = (m_2^2 - m_1^2 + m_Z^2)/2m_2 m_Z$ and $\beta_Z = \sqrt{1 - 1/\gamma_Z^2}$, and with the polar angle θ_Z determined by measuring the Z-boson energy E'_Z in the ee c.m. directly, as can be checked with the right expression in Eq. (16). However, the polar angle θ_2 and the azimuthal angle ϕ_Z cannot be directly measured event by event because of two invisible X_1 particles in the combined production-decay process (1).

Combining the production helicity amplitude in Eq. (6) and two decay helicity amplitudes in Eqs.(10) and (13) adjusted by a Wick helicity rotation and an azimuthal rotation, we obtain the fully correlated production-decay helicity amplitude as

$$\mathcal{M}_{\sigma, \bar{\sigma}; \sigma_-, \sigma_+; \lambda_1, \sigma_1} = D_2(p_2^2) D_Z(q_Z^2) \sum_{\lambda_2, \lambda'_Z} \mathcal{M}_{\sigma, \bar{\sigma}; \lambda_2, \lambda_1}(\theta_2, \phi_2) \mathcal{A}_{\lambda_2; \lambda'_Z, \sigma_1}(\theta_Z, \phi_Z) \mathcal{B}_{\lambda'_Z; \sigma_-, \sigma_+}(\theta_-, \phi'_-), \quad (17)$$

with the adjusted azimuthal angle $\phi'_- = \phi_- - \gamma_Z$ and the X_2 and Z Breit-Wigner propagators $D_2 = 1/(p_2^2 - m_2^2 + im_2\Gamma_2)$ and $D_Z = 1/(q_Z^2 - m_Z^2 + im_Z\Gamma_Z)$.

B. CP symmetry and Majorana condition

Before going into a detailed description of the angular correlations in Sec. III, we study some general restrictions on the helicity amplitudes due to CP invariance and the Majorana condition that each of the neutral particles X_2 and X_1 is its own antiparticle, respectively.

Even in transitions involving weak interactions, the production and decay processes observe CP symmetry to a great extent while often violating P and C symmetries. So we discuss the consequences of the CP symmetry among discrete spacetime symmetries in the production and decay helicity amplitudes. For the production and decay processes involving two Majorana particles X_2 and X_1 , CP invariance leads to the following relations:

$$\mathcal{P}_{\sigma, \bar{\sigma}; \lambda_2, \lambda_1}(\cos \theta_2) = \eta_{CP}^P (-1)^J \mathcal{P}_{-\bar{\sigma}, -\sigma; -\lambda_2, -\lambda_1}(-\cos \theta_2), \quad (18)$$

$$\mathcal{C}_{\lambda_Z, \sigma_1} = \eta_{CP}^C \mathcal{C}_{-\lambda_Z, -\sigma_1}, \quad (19)$$

with the appropriate helicity-independent CP parities, η_{CP}^P and η_{CP}^C , consisting of intrinsic parities and particle spins. Note that these CP tests do not assume the absence of absorptive parts and rescattering effects at all.

Together with CPT invariance, valid in the absence of absorptive parts and/or rescattering effects, the Majorana condition that both of the two neutral particles X_2 and X_1 are their own antiparticles leads to the relations for the production and decay helicity amplitudes,

$$\mathcal{P}_{\sigma, \bar{\sigma}; \lambda_2, \lambda_1}(\cos \theta_2) = \eta_M^P (-1)^J \mathcal{P}_{-\bar{\sigma}, -\sigma; -\lambda_2, -\lambda_1}(-\cos \theta_2), \quad (20)$$

$$\mathcal{C}_{\lambda_Z, \sigma_1} = \eta_M^C \mathcal{C}_{-\lambda_Z, -\sigma_1}^*, \quad (21)$$

where the parity factors η_M^P and η_M^C are dependent on the intrinsic CPT parities and spins but independent of helicities.

⁴Initial-state radiation or beamstrahlung effects may render the θ'_Z determination uncertain. Therefore, practically those effects with usually known spectra have to be implemented for estimating the uncertainty in determining the polar angle.

III. CORRELATED ANGULAR DISTRIBUTIONS

The fully correlated production-decay amplitudes in Eq. (17) allow us to probe all the polarization phenomena with which the spins and interaction structures of the production and decay processes can be determined. In this section, we derive all the analytic expressions for the correlated angular distributions, which consist of three helicity-dependent parts.

The first process under attack is the production of a nondiagonal pair of Majorana particles $e^-e^+ \rightarrow X_2X_1$. Summing over the helicities of the invisible X_1 and incorporating the electron and positron 2×2 polarization density matrices ρ^- and ρ^+ , we can write the helicity-dependent differential cross section in the form

$$\frac{d\sigma}{d\Omega_2}(\lambda_2, \lambda'_2) = \frac{\kappa_{21}}{64\pi^2 s} \Sigma_{\lambda'_2}^{\lambda_2}, \quad (22)$$

where $d\Omega_2 = d\cos\theta_2 d\phi_2$, $\mu_{1,2} = m_{1,2}/\sqrt{s}$, and $\kappa_{21} = \lambda^{1/2}(1, \mu_2^2, \mu_1^2)$ with the Källén kinematical function $\lambda(1, x, y) = [1 - (x+y)^2][1 - (x-y)^2]$. The production tensor Σ in Eq. (22) reads

$$\Sigma_{\lambda'_2}^{\lambda_2} = \sum_{\sigma, \sigma'} \sum_{\bar{\sigma}, \bar{\sigma}'} \sum_{\lambda_1} \rho_{\sigma, \sigma'}^- \rho_{\bar{\sigma}, \bar{\sigma}'}^+ \mathcal{M}_{\sigma, \bar{\sigma}, \lambda_2, \lambda_1} \mathcal{M}_{\sigma', \bar{\sigma}', \lambda'_2, \lambda_1}^*, \quad (23)$$

with the implied summation over repeated indices $(\sigma, \sigma', \bar{\sigma}, \bar{\sigma}') = \pm 1/2 = \pm$ and $\lambda_1 = -j_1, \dots, j_1$. The $(2j_2 + 1) \times (2j_2 + 1)$ polarization density matrix of the produced X_2 is given by

$$\rho_{\lambda_2, \lambda'_2}^{X_2} = \Sigma_{\lambda'_2}^{\lambda_2} / \Sigma_{\sigma_2}^{\sigma_2}, \quad (24)$$

with the implied summation over the repeated X_2 helicity index $\sigma_2 = -j_2, \dots, j_2$.

If only the longitudinal polarizations⁵ of the e^- and e^+ beams and the electron chirality conservation related to the tiny electron mass [40] are imposed on the electron-positron current, the combined e^-e^+ polarization tensor is simplified as

$$\rho_{\sigma, \sigma'}^- \rho_{\bar{\sigma}, \bar{\sigma}'}^+ = \delta_{\sigma, \sigma'} \delta_{\bar{\sigma}, -\bar{\sigma}'} \delta_{\sigma', -\bar{\sigma}'} \frac{1}{4} [1 - P_L \bar{P}_L + \sigma(P_L - \bar{P}_L)], \quad (25)$$

with the degrees P_L and $\bar{P}_L$ of electron and positron longitudinal polarizations, respectively. Because of the Majorana condition (20), the polar-angle distribution set by the trace of the production tensor is forward-backward (FB) symmetric but the P -odd X_2 polarization components defined by the differences $\rho_{\lambda_2, \lambda_2}^{X_2} - \rho_{-\lambda_2, -\lambda_2}^{X_2}$ with $\lambda_2 = -j_2, \dots, j_2$ is FB antisymmetric.

In the narrow-width approximation, the produced X_2 particle decays on shell with good approximation. As pointed out before, it is necessary to include a Wick helicity rotation and an azimuthal-angle adjustment for calculating the helicity amplitude of the sequential decay chain of two two-body decays $X_2 \rightarrow ZX_1$ and $Z \rightarrow \ell^- \ell^+$. The correlated decay distribution including the matrix in Eq. (24) encoding X_2 polarization is given by

$$\frac{d\Gamma}{d\Omega_Z d\Omega_-} = \frac{3\kappa_Z}{256\pi^3 m_2} \text{B}(Z \rightarrow \ell^- \ell^+) \sum_{\lambda_2, \sigma_2} \sum_{\lambda'_2, \sigma'_2} \sum_{\sigma_1} \rho_{\lambda_2, \sigma_2}^{X_2} [\mathcal{A}_{\lambda_2; \lambda'_2, \sigma_1} \mathcal{A}_{\sigma_2; \sigma'_2, \sigma_1}^*] \rho_{\lambda'_2, \sigma'_2}^Z, \quad (26)$$

where $\kappa_Z = \lambda^{1/2}(1, m_1^2/m_2^2, m_Z^2/m_2^2)$, $d\Omega_Z = d\cos\theta_Z d\phi_Z$, and $d\Omega_- = d\cos\theta_- d\phi_-$ and the summation over all repeated helicity indices is taken. With the known $Z\ell\ell$ couplings in the SM, the normalized 3×3 Z -boson decay density matrix ρ^Z is given in terms of an asymmetry parameter $A_\ell = 2v_\ell a_\ell / (v_\ell^2 + a_\ell^2)$ by

$$\rho_{\lambda'_Z, \sigma'_Z}^Z(\theta_-, \phi'_-) = \frac{1}{4} \begin{pmatrix} 1 + c_-^2 + 2A_\ell c_- & \sqrt{2}(A_\ell + c_-)s_- e^{i\phi'_-} & s_-^2 e^{2i\phi'_-} \\ \sqrt{2}(A_\ell + c_-)s_- e^{-i\phi'_-} & 2s_-^2 & \sqrt{2}(A_\ell - c_-)s_- e^{i\phi'_-} \\ s_-^2 e^{-2i\phi'_-} & \sqrt{2}(A_\ell - c_-)s_- e^{-i\phi'_-} & 1 + c_-^2 - 2A_\ell c_- \end{pmatrix}, \quad (27)$$

in the $(+1, 0, -1)$ helicity basis of the Z boson with the abbreviations, $c_- = \cos\theta_-$ and $s_- = \sin\theta_-$, and with the adjusted azimuthal angle $\phi'_- = \phi_- - \gamma_Z$. We emphasize once more that the azimuthal angle γ_Z depends on the X_2 polar angle and Z polar and azimuthal angles and so it is not straightforward to construct the ϕ'_- distribution.

In contrast, the θ_- distribution can be measured rather precisely. Integrating the distribution over the lepton azimuthal angle ϕ_- casts the density matrix into a diagonal form

$$\rho_{\lambda_Z, \lambda_Z}^Z = \frac{1}{4} \text{diag}(1 + c_-^2 + 2A_\ell c_-, 2s_-^2, 1 + c_-^2 - 2A_\ell c_-) \quad \text{with } \lambda_Z = +1, 0, -1, \quad (28)$$

⁵Transversely polarized beams are not considered in the present work because their effects will be washed out after integrating the distributions over the production azimuthal angle.

depending on the reconstructible polar angle θ_- . Furthermore, integrating the correlated distribution over the azimuthal angle ϕ_Z also washes out the effects due to the

off-diagonal components of the X_2 polarization density matrix ρ^{X_2} and leads to the correlated polar-angle distribution given by

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta_Z d\cos\theta_-} &= \frac{3\kappa_Z \mathcal{B}(Z \rightarrow \ell\ell)}{64\pi m_2} \sum_{\lambda_2=-j_2}^{j_2} \rho_{\lambda_2, \lambda_2}^{X_2} \sum_{\lambda'_Z} \sum_{\lambda_Z, \sigma_Z} [d_{\lambda'_Z, \lambda_Z}^1(\omega_Z) d_{\lambda'_Z, \sigma_Z}^1(\omega_Z)] \\ &\times \sum_{\sigma_1=-j_1}^{j_1} \mathcal{C}_{\lambda_Z, \sigma_1} \mathcal{C}_{\sigma_Z, \sigma_1}^* [d_{\lambda_2, \lambda_Z-\sigma_1}^{j_2}(\theta_Z) d_{\lambda_2, \sigma_Z-\sigma_1}^{j_2}(\theta_Z)] \rho_{\lambda'_Z, \lambda'_Z}^Z(\theta_-), \end{aligned} \quad (29)$$

with the X_2 polarization density matrix ρ^{X_2} depending on the X_2 production mechanism and with the constraints $|\lambda_Z - \sigma_1| \leq j_2$ and $|\sigma_Z - \sigma_1| \leq j_2$ on the summation over the X_1 helicities as well as the Z helicities ± 1 and 0 .

IV. A SPECIFIC EXAMPLE

As a concrete example of the correlated production-decay process (1), we consider the production of a non-diagonal pair of two lighter neutralinos $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^0$ among the four neutralinos, all of which are mixtures of $U(1)_Y$ and $SU(2)_L$ gauginos $\tilde{B}$ and $\tilde{W}_3$ and two Higgsinos $\tilde{H}_1^0$ and $\tilde{H}_2^0$ and are spin-1/2 Majorana fermions in the MSSM. In this example, we assume that the two-body decay $\tilde{\chi}_2^0 \rightarrow Z\tilde{\chi}_1^0$ is kinematically allowed; i.e., the second neutralino mass is greater than the sum of the first neutralino mass and the Z -boson mass [38]. For notational convenience and consistency, we set $\tilde{\chi}_2^0 = X_2$ and $\tilde{\chi}_1^0 = X_1$ in the following.

Generally, the production process $e^-e^+ \rightarrow X_2X_1$ has the contributions from t - and u -channel selectron exchanges as well as an s -channel Z exchange. Nevertheless, for a simple demonstration without too much loss of generality in the context of the present work, we assume all the selectron-exchange contributions to be decoupled due to sufficiently large selectron masses, as in the context of the so-called split supersymmetry scenario [41,42], while maintaining only the s -channel Z contribution. In this case, for both the production process $e^-e^+ \rightarrow X_2X_1$ and two-body decay $X_2 \rightarrow ZX_1$, it is sufficient to consider, in addition to the

standard $Z\ell\ell$ vertices, the X_2X_1Z vertices whose expressions are given in terms of a complex coupling by

$$\langle X_{2R}|Z|X_{1R}\rangle = \langle X_{1R}|Z|X_{2R}\rangle^* = +g_Z \mathcal{Q}, \quad (30)$$

$$\langle X_{2L}|Z|X_{1L}\rangle = \langle X_{1L}|Z|X_{2L}\rangle^* = -g_Z \mathcal{Q}^*, \quad (31)$$

for the right and left chiral modes with $g_Z = e/c_W s_W$ and the normalized coupling $\mathcal{Q} = (N_{13}N_{23}^* - N_{14}N_{24}^*)/2$ in terms of the unitary 4×4 matrix N rotating the gauge eigenstate basis to the mass eigenstate basis for diagonalizing the neutralino mass matrix [18]. Therefore, the axial-vector and vector couplings are purely real and purely imaginary, respectively.

The production transition amplitude for the process $e^-e^+ \rightarrow X_2X_1$ can be expressed as a sum of two-current products as follows:

$$\begin{aligned} \mathcal{T}(e^-e^+ \rightarrow X_2X_1) &= g_Z^2 D_Z(s) \sum_{a,b=\pm} \mathcal{Q}_{ab} [\bar{v}(e^+) \gamma_\mu P_a u(e^-)] [\bar{u}(X_2) \gamma^\mu P_b v(X_1)], \end{aligned} \quad (32)$$

in terms of four bilinear charges, defined by the chiralities of the associated electron and neutralino currents with $P_\pm = (1 \pm \gamma_5)/2$. Explicitly, the normalized bilinear charges are

$$\mathcal{Q}_{++} = c_+ \mathcal{Q}^*, \quad \mathcal{Q}_{+-} = -c_+ \mathcal{Q}, \quad \mathcal{Q}_{-+} = c_- \mathcal{Q}^*, \quad \mathcal{Q}_{--} = -c_- \mathcal{Q}, \quad (33)$$

with the normalized $Z\ell\ell$ right and left chiral couplings $c_+ = s_W^2$ and $c_- = s_W^2 - 1/2$. Ignoring the electron mass, the electron and positron helicities are opposite to each other in all amplitudes so that the reduced production helicity amplitudes $T_{\sigma, -\sigma; \lambda_2, \lambda_1} = g_Z^2 s D_Z(s) \langle \sigma; \lambda_2, \lambda_1 \rangle$ with $\sigma, \lambda_2, \lambda_1 = \pm 1/2 = \pm$ are written in a compact form as

$$\begin{aligned} \langle +; \pm, \pm \rangle &= \sqrt{2} c_+ \left[\mu_+ \sqrt{1 - \mu_-^2} \text{Im}(\mathcal{Q}) \pm \mu_- \sqrt{1 - \mu_+^2} \text{Re}(\mathcal{Q}) \right], \\ \langle -; \pm, \pm \rangle &= \sqrt{2} c_- \left[\mu_+ \sqrt{1 - \mu_-^2} \text{Im}(\mathcal{Q}) \pm \mu_- \sqrt{1 - \mu_+^2} \text{Re}(\mathcal{Q}) \right], \end{aligned} \quad (34)$$

$$(35)$$

$$\langle +; \pm, \mp \rangle = 2c_+ \left[\sqrt{1 - \mu_-^2} i \text{Im}(\mathcal{Q}) \mp \sqrt{1 - \mu_+^2} \text{Re}(\mathcal{Q}) \right], \quad (36)$$

$$\langle -; \pm, \mp \rangle = 2c_- \left[\sqrt{1 - \mu_-^2} i \text{Im}(\mathcal{Q}) \mp \sqrt{1 - \mu_+^2} \text{Re}(\mathcal{Q}) \right], \quad (37)$$

with the normalized dimensionless factors $\mu_{\pm} = (m_2 \pm m_1)/\sqrt{s}$. We note that CP is violated if both the real and imaginary parts of the complex factor $\mathcal{Q}$ are nonzero, as can be checked with the relation in Eq. (18). On the other hand, the Majorana condition in Eq. (20) is satisfied with $J = 1$ and the overall intrinsic parity of $\eta_M^P = +1$.

The same complex factor $\mathcal{Q}$ appearing in Eq. (33) enables us to describe the two-body decay $X_2 \rightarrow ZX_1$ fully. The reduced decay helicity amplitudes in the X_2 RF, which are independent of the X_2 helicity due to angular momentum conservation, read

$$\mathcal{C}_{\pm\pm} = \sqrt{2}g_Z \left[\sqrt{m_-^2 - m_Z^2} \text{Im}(\mathcal{Q}) \mp i \sqrt{m_+^2 - m_Z^2} \text{Re}(\mathcal{Q}) \right], \quad (38)$$

$$\mathcal{C}_{0\pm} = g_Z \left[\frac{m_+}{m_Z} \sqrt{m_-^2 - m_Z^2} \text{Im}(\mathcal{Q}) \mp i \frac{m_-}{m_Z} \sqrt{m_+^2 - m_Z^2} \text{Re}(\mathcal{Q}) \right], \quad (39)$$

for $\lambda_Z = \pm 1, 0$ and $\sigma_1 = \pm 1/2 = \pm$ with the convention $m_{\pm} = m_2 \pm m_1$ introduced for notational convenience. The remaining reduced helicity amplitudes $\mathcal{C}_{\pm\mp}$ are vanishing due to angular momentum conservation. Furthermore, all the angular-dependent parts are encoded solely in Wigner d functions. The CP relation in Eq. (19) is violated again if the coupling $\mathcal{Q}$ is neither purely real nor purely imaginary. Note that the Majorana condition (21) is valid with the combined intrinsic parity of $\eta_M^C = +1$.

Since the lightest neutralino escapes undetected and the heavier neutralino decays into the invisible lightest

neutralino and a Z boson, the production angle θ_2 cannot be determined unambiguously for nonasymptotic energies.

To describe the electron and positron polarizations in a general setting, the reference frame must be fixed. The electron-momentum direction can be used to define the z axis. If the electron beam is transversely polarized, the direction of transverse polarization is set to be the x axis. In any case, because the azimuthal angle ϕ_2 of the X_2 momentum cannot be reconstructed with the invisible X_1 , we consider only the longitudinally polarized electron and positron beams. The polarized differential production cross section is given in terms of the degrees of electron and positron longitudinal polarizations P_L and $\bar{P}_L$ by

$$\frac{d\sigma}{d\cos\theta_2} = \frac{\pi\alpha_Z^2\kappa_{21}}{4} s |D_Z(s)|^2 [(1 - P_L\bar{P}_L)\Sigma_{\text{unp}} + (P_L - \bar{P}_L)\Sigma_{LL}], \quad (40)$$

with $\kappa_{21} = \lambda^{1/2}(1, m_2^2/s, m_1^2/s)$ and $\alpha_Z = g_Z^2/4\pi$. The coefficients Σ_{unp} and Σ_{LL} depend on the polar angle θ_2 and the e^-e^+ c.m. energy but not on the azimuthal angle ϕ_2 any more. Their expressions are given in terms of the chiral complex factor $\mathcal{Q}$ by

$$\Sigma_{\text{unp}} = (c_+^2 + c_-^2) \{ [1 - (\mu_2^2 - \mu_1^2)^2 + \lambda_{21} \cos^2\theta_2] |\mathcal{Q}|^2 - 4\mu_2\mu_1 \text{Re}(\mathcal{Q}^2) \}, \quad (41)$$

$$\Sigma_{LL} = (c_+^2 - c_-^2) \{ [1 - (\mu_2^2 - \mu_1^2)^2 + \lambda_{21} \cos^2\theta_2] |\mathcal{Q}|^2 - 4\mu_2\mu_1 \text{Re}(\mathcal{Q}^2) \}, \quad (42)$$

with $\mu_{1,2} = m_{1,2}/\sqrt{s}$. We note that the coefficients Σ_{unp} and Σ_{LL} are FB symmetric with respect to the polar angle θ_2 , as guaranteed by the Majorana condition, and as a matter of fact they are proportional to each other, rendering the normalized angular distribution independent of the beam polarizations. In any case, the e^- and e^+ beam polarizations can be employed for increasing the production rate.

The chiral structure of the neutralinos can be also inferred from the polarization of the neutralinos. The degree of longitudinal X_2 polarization for longitudinally polarized electron and positron beams is given in a simple factorized form as

$$P_L(\theta_2) = \mathcal{P}_{ee}(P_L, \bar{P}_L) \frac{[(1 - \mu_2^2 - \mu_1^2)|\mathcal{Q}|^2 - 2\mu_2\mu_1 \text{Re}(\mathcal{Q}^2)] \cos\theta_2}{[1 - (\mu_2^2 - \mu_1^2)^2 + \kappa_{21}^2 \cos^2\theta_2] |\mathcal{Q}|^2 - 4\mu_2\mu_1 \text{Re}(\mathcal{Q}^2)}, \quad (43)$$

with the effective e^-e^+ longitudinal-polarization factor $\mathcal{P}_{ee}$ given by

$$\mathcal{P}_{ee}(P_L, \bar{P}_L) = \frac{(1 - P_L\bar{P}_L)A_e + P_L - \bar{P}_L}{1 - P_L\bar{P}_L + (P_L - \bar{P}_L)A_e}, \quad (44)$$

with $A_e = (c_+^2 - c_-^2)/(c_+^2 + c_-^2) = 2v_e a_e/(v_e^2 + a_e^2) \simeq -0.16$ [43]. Consequently, the longitudinal beam polarizations changes the overall size, but it does not affect the angular distribution of the X_2 longitudinal polarization. Furthermore, the longitudinal polarization is FB antisymmetric

with respect to the polar angle θ_2 so that the X_2 particle is unpolarized on average after integrating over the production polar-angle θ_2 .

After the θ_2 integration is taken, the normalized correlated polar-angle distribution in Eq. (29), which is independent of the production mechanism, reads

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_Z d\cos\theta_-} = \frac{3}{4} \sum_{\lambda'_Z, \lambda'_Z} W_{\lambda'_Z, \lambda'_Z}(\omega_Z) \rho_{\lambda'_Z, \lambda'_Z}^Z(\theta_-), \quad (45)$$

where the so-called Wick distribution functions $W_{\lambda'_Z, \lambda'_Z}(\omega_Z)$ appearing in the second expression is defined as [30]

$$W_{\lambda'_Z, \lambda'_Z}(\omega_Z) = \sum_{\lambda_Z, \sigma_1} [d_{\lambda'_Z, \lambda_Z}^1(\omega_Z)]^2 |\mathcal{C}_{\lambda_Z, \sigma_1}|^2 / \sum_{\lambda_Z, \sigma_1} |\mathcal{C}_{\lambda_Z, \sigma_1}|^2$$

with $|\lambda_Z - \sigma_1| \leq 1/2$, (46)

of which the sum is normalized to unity. The Majorana condition (21) on the reduced decay helicity amplitudes guarantees $W_{++}(\omega_Z) = W_{--}(\omega_Z)$ leading to the absence of the parity-violating distribution linear in $\cos\theta_-$. Consequently, like the production polar-angle distribution, the decay θ_- distribution is forward-backward symmetric. Explicitly, the normalized two-dimensional correlated polar-angle distribution independent of the magnitude of the complex factor Q is given by

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_Z d\cos\theta_-} = \frac{1}{4} \left[1 - \frac{1}{4} \eta_Q (3\cos^2\omega_Z - 1)(3\cos^2\theta_- - 1) \right], \quad (47)$$

where the α_Q -dependent coefficient η_Q is defined as

$$\eta_Q = \frac{1}{2} \frac{[(m_2 + m_1)^2 - m_Z^2][(m_2 - m_1)^2 - m_Z^2]}{[(m_2 + m_1)^2 + 2m_Z^2][(m_2 - m_1)^2 - m_Z^2] + 12m_2 m_1 m_Z^2 \cos^2\alpha_Q}, \quad (48)$$

in terms of the phase angle α_Q of $Q = |Q|(\cos\alpha_Q + i\sin\alpha_Q)$. If the lepton polar-angle dependence is not taken into account, the θ_Z distribution is simply isotropic, i.e., independent of the polar angle θ_Z . On the other hand, the θ_- dependence is sensitive to the boost factor β_2 of the decaying particle X_2 . For instance, if the particle X_2 is at rest, the vanishing Wick helicity rotation angle ω_Z renders the θ_- distribution maximally dependent on the coefficient η_Q .

For an explicit numerical illustration, we set the following values for the X_2 and X_1 masses:

$$m_2 = 300 \quad \text{and} \quad m_1 = 100 \text{ GeV}, \quad (49)$$

while varying the e^-e^+ c.m. energy $\sqrt{s} = E_{\text{c.m.}}$ from 0.4 to 1.0 TeV. The left side of Fig. 2 shows the dependence of the coefficient η_Q on the phase α_Q . By measuring the coefficient, we can determine the phase α_Q up to a twofold discrete ambiguity. Unless α_Q is 0, $\pi/2$, or π , i.e., unless Q is purely real or imaginary, CP is violated in the neutralino system. The right side of Fig. 2 shows the integral of $P_2(\cos\omega_Z) = (3\cos^2\omega_Z - 1)/2$ over the polar angle θ_Z as a function of the e^-e^+ c.m. energy $\sqrt{s} = E_{\text{c.m.}}$. For a simple illustration, the $E_{\text{c.m.}}$ range is taken from 0.4 TeV (the threshold energy of $m_2 + m_1$) to 1.0 TeV. The integral value is monotonically decreasing, implying that the sensitivity to the complex factor Q is maximal at the production threshold. In contrast, the production cross

FIG. 2. Left: the coefficient η_Q is shown as a function of the phase α_Q . Right: the integral $[P_2(\cos\omega_Z)]$ of the second Legendre polynomial P_2 of $\cos\omega_Z$ is shown as a function of the e^-e^+ c.m. energy $\sqrt{s} = E_{\text{c.m.}}$. For this simple illustration, the range of $\sqrt{s} = E_{\text{c.m.}}$ is taken to be from 0.4 to 1.0 TeV.

section is increasing with the c.m. energy near the threshold. So, there exists a specific value of $E_{\text{c.m.}}$ above the threshold for the optimal sensitivity to $\mathcal{Q}$.

V. CONCLUSIONS

In this paper, we have made a general and systematic model-independent study of correlated distributions connected to the production process $e^-e^+ \rightarrow X_2X_1$ of two Majorana particles X_2 and X_1 with different masses and arbitrary spins, followed by two sequential two-body decays, $X_2 \rightarrow ZX_1$ and $Z \rightarrow \ell^-\ell^+$ with $\ell = e, \mu$, with invisible X_1 . The constraints due to CP invariance and the Majorana condition were discussed. Formally, a proper Wick helicity rotation and an azimuthal-angle adjustment were taken into account for combining the production and decay helicity amplitudes derived in the most compact form from an analytic point of view and a few general properties of the combined production-decay process involving two Majorana particles. Then, a specific example with the nondiagonal pair of two lighter neutralinos was investigated for demonstrating the validity of all the worked-out general properties in a concrete and detailed manner.

In relation to this work, we are at present analyzing the general structure of the ZX_2X_1 interaction vertices of a Z boson and two Majorana particles X_2 and X_1 with any integer and/or half-integer spins. Furthermore, we plan to probe that of the interaction vertices of three Majorana particles with arbitrary spin combinations by utilizing not only the fully correlated polar and azimuthal angular distributions but also the invariant Z -boson mass distribution covering the situation when the Z boson is off shell. This research project, of which a partial outcome has already been reported in Ref. [44] and the full result will be reported soon elsewhere, is a natural extension of several previous works [45–50].

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