Ground and excited state masses of Ω_c^0 , Ω_{cc}^+ and Ω_{ccc}^{++} baryons

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The masses of ground and excited states of Ω_c^0 , Ω_{cc}^+ , and Ω_{ccc}^{++} baryons are calculated within the framework of Regge phenomenology. Here our attempt is to assign a possible spin parity to these obtained masses. Our calculated masses in the (J, M^2) plane with both natural ($J^P = 1/2^+, 3/2^-, 5/2^+, \dots$) and unnatural ($J^P = 3/2^+, 5/2^-, 7/2^+, \dots$) spin parities are in agreement with the experimental observations where available and reasonably close to other theoretical predictions. Further, we fix the slope and intercept of each Regge line in the (n, M^2) plane and estimate the masses of higher excited states of these baryons. The obtained mass relations and the mass value predictions could provide useful information in future experimental searches and the spin-parity assignment of these states.

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I. INTRODUCTION

In 2017, the LHCb Collaboration observed five new narrow excited Ω_c^0 states— $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$ —decaying into $\Xi^+ K^-$ [1]. Except for $\Omega_c(3119)^0$, these states were further confirmed by the Belle Collaboration [2] in 2018. Many questions arose from this discovery, such as why only five states were observed and why they are so narrow. Many authors have addressed these questions in recent studies [3–15]. Since Ω_c^0 is composed of two strange quarks (ss) and one charm quark (c), if the ss diquark remains in its color triplet it has spin 1, as symmetry rules do not allow a spin-0 diquark. This spin combined, with the spin $1/2$ of the c quark, gives $S = 1/2$ or $3/2$. If we consider states with relative orbital momentum $L = 1$ and combine them with $S = 1/2$ and $3/2$, we obtain the total spins $J = 1/2$, $3/2$ and $J = 1/2$, $3/2$, $5/2$, respectively. All five of these states have negative parity. Also, these states are extremely narrow which indicates that it is difficult to pull apart the two s quarks in a diquark [4].

The study of heavy-light bound states provides an ideal platform to understand the dynamics of quantum chromodynamics (QCD) in the low-energy regime. The strong-interaction coupling is small for hadrons that contain a heavy quark (c or b), which makes it easier to understand QCD perturbatively than in systems that contain only a light quark [16]. Different experimental facilities have

provided new information on hadron spectroscopy, such as masses, decay widths, branching ratios, isospin mass splittings, spin, parity, polarization amplitudes, etc. It is very crucial to assign the spin parity of hadrons, which facilitates the determination of experimental properties. Experimentally, only the ground state Ω_c^0 and $\Omega_c(2770)^0$ have been observed with quantum numbers $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$, respectively [17], where J is the total spin and P denotes parity. The J^P values of the excited-state Ω_c^0 baryons are still missing, as shown in Table I.

Among the different research methods to study hadron spectroscopy, one of the prominent ways to determine the properties of hadronic states is mass spectrum analysis. There are various methods to depict the mass spectrum. The authors of Refs. [18–21] calculated the excited-state masses of singly, doubly, and triply heavy baryons using the nonrelativistic approach of the hypercentral constituent quark model. In Ref. [22] a mass analysis was carried out for the newly excited $\Omega_{c/b}$ baryons by proposing a new scheme of state classification called Jls mixing coupling, which goes beyond jj coupling. This scheme successfully predict the spin parity of these states. The ground-state and excited-state spectra of the Ω_c^0 baryon were estimated using lattice QCD with a dynamical quark field in Ref. [5], and the J^P assignments of the states $\Omega_c(3000)^0/\Omega_c(3050)^0$, $\Omega_c(3066)^0/\Omega_c(3090)^0$, and $\Omega_c(3119)^0$ were predicted as $1/2^-$, $3/2^-$ and $5/2^-$, respectively. The authors of Ref. [4] predicted the existence of five states with negative parity by assuming that these baryonic states were bound states of a c quark and a P wave ss diquark. Ebert *et al.* [23] determined the mass spectra of excited heavy baryons consisting of two light (u, d, s) and one heavy (c, b) quark, in a QCD framework motivated by the relativistic quark model. Robert *et al.* [24] used a nonrelativistic Hamiltonian quark

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TABLE I. Masses, widths, and J^P values of Ω_c^0 baryons reported by LHCb [1].

Resonance	Mass (MeV)	Width (MeV)	J^P
Ω_c^0	2695.2 ± 1.7	...	$\frac{1}{2}^+$
$\Omega_c(2770)^0$	2765.9 ± 2.0	...	$\frac{3}{2}^+$
$\Omega_c(3000)^0$	$3000.4 \pm 0.22 \pm 0.1$	$4.5 \pm 0.6 \pm 0.3$	$??$
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1$	$0.8 \pm 0.2 \pm 0.1$	$??$
$\Omega_c(3065)^0$	$3065.6 \pm 0.1 \pm 0.3$	$3.5 \pm 0.4 \pm 0.2$	$??$
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.9$	$8.7 \pm 0.1 \pm 0.8$	$??$
$\Omega_c(3120)^0$	$3119 \pm 0.3 \pm 0.9$	$1.1 \pm 0.8 \pm 0.4$	$??$

model to calculate the masses of heavy baryons. The authors of Ref. [25] estimated the masses and residues of these newly observed Ω_c^0 baryon states within QCD sum rules and tried to determine the nature of these states. So far, none of the doubly/triply Ω baryons have been experimentally detected. Many authors have predicted the masses of Ω_{cc}^+ and Ω_{ccc}^{*++} baryons using various theoretical and phenomenological models, like the relativistic quark model [26], Salpeter model [27], lattice QCD [28,29], the Hamiltonian model [30], and the hypercentral constituent quark model [18,19].

Wei *et al.* [31] derived some useful mass relations such as linear mass inequalities, quadratic mass inequalities, and quadratic mass equalities for hadrons using a quasi-linear Regge trajectory ansatz. Based on these relations, they determined the mass ranges of some mesons and baryons and predicted the J^P assignments of the $\Xi_c(2980)$, $\Xi_c(3055)$, $\Xi_c(3077)$, and $\Xi_c(3123)$ baryons. In Ref. [32] the same group extended this methodology to compute the masses of the ground states and excited states of other doubly and triply charmed baryons, such as Ω_{cc}^+ , Ξ_{cc}^{*++} , Ω_{cc}^{*+} , and Ω_{ccc}^{*++} . In the present work, we use the same approach of Regge phenomenology with the assumption of linear Regge trajectories. We obtain relations between the intercept, slope ratios, and baryon masses in both the (J, M^2) and (n, M^2) planes. Using these relations, we determine the mass spectra of the Ω_c^0 baryons and try to assign a possible spin parity to these five recently observed baryonic states. Further, we apply the same scheme for doubly and triply charmed omega baryons. These baryons are yet to be observed by experiments and therefore are not listed by the Particle Data Group (PDG) [17], which motivated us to study the mass spectra of Ω_c^0 , Ω_{cc}^+ , and Ω_{ccc}^{*++} baryons.

The remainder of this paper is organized as follows. In Sec. II we briefly explain the linear Regge trajectories and extract the quadratic mass relation in the (J, M^2) plane to calculate the ground-state and excited-state masses of Ω_c^0 baryons. Further, we extend this work and again derive some mass equations to evaluate the mass spectra for Ω_{cc}^+ and Ω_{ccc}^{*++} baryons. In addition, we estimate the masses of these baryons in the (n, M^2) plane by calculating the Regge

slopes and intercepts of particular Regge lines. A discussion of our results is given in Sec. III.

II. THEORETICAL FRAMEWORK

Regge theory is one of the simplest and most effective phenomenological approaches to study hadron spectroscopy. Various theories were developed to understand the Regge trajectory. Among them, one of the most straightforward explanations for linear Regge trajectories was proposed by Nambu [33,34]. He assumed that the uniform interaction of a quark-antiquark pair forms a strong flux tube, and at the end of the tube the light quarks rotate with the speed of light at radius R . The mass originating in this flux tube is estimated as [35]

$$M = 2 \int_0^R \frac{\sigma}{\sqrt{1 - v^2(r)}} dr = \pi \sigma R, \quad (1)$$

where σ represents the string tension, i.e., the mass density per unit length. Likewise, the angular momentum of this flux tube is evaluated as

$$J = 2 \int_0^R \frac{\sigma r v(r)}{\sqrt{1 - v^2(r)}} dr = \frac{\pi \sigma R^2}{2} + c'. \quad (2)$$

One can also write

$$J = \frac{M^2}{2\pi\sigma} + c'', \quad (3)$$

where c' and c'' are constants of integration. Hence, we can say that J and M^2 are linearly related to each other. The plots of Regge trajectories of hadrons in the (J, M^2) plane are usually called Chew-Frautschi plots [36]. Also, hadrons lying on the same Regge line possess the same internal quantum numbers. From Eq. (3), the most general form of linear Regge trajectories can be expressed as [31]

$$J = \alpha(M) = a(0) + \alpha' M^2, \quad (4)$$

where $a(0)$ and α' represent the intercept and slope of the trajectory, respectively. These parameters for different quark constituents of a baryon multiplet can be related by two relations [31,37–40]:

$$a_{iiq}(0) + a_{jjq}(0) = 2a_{ijq}(0), \quad (5)$$

$$\frac{1}{\alpha'_{iiq}} + \frac{1}{\alpha'_{jjq}} = \frac{2}{\alpha'_{ijq}}, \quad (6)$$

where i, j, q represent quark flavors, $m_i < m_j$, and q denotes an arbitrary light or heavy quark. Using Eqs. (4) and (5), we obtain

$$\alpha'_{iiq} M_{iiq}^2 + \alpha'_{jjq} M_{jjq}^2 = 2\alpha'_{ijq} M_{ijq}^2. \quad (7)$$

Combining Eqs. (6) and (7) and solving the quadratic equation, we obtain a pair of solutions as

$$\frac{\alpha'_{jjq}}{\alpha'_{iiq}} = \frac{1}{2M_{jjq}^2} \times \left[(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2) \pm \sqrt{(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2)^2 - 4M_{iiq}^2 M_{jjq}^2} \right]. \quad (8)$$

This is an important relation between the slope ratios and baryon masses. Equation (8) can also be expressed in terms of baryon masses by introducing k , where k can be any quark flavor,

$$\frac{\alpha'_{jjq}}{\alpha'_{iiq}} = \frac{\alpha'_{kkq}}{\alpha'_{iiq}} \times \frac{\alpha'_{jjq}}{\alpha'_{kkq}}. \quad (9)$$

We then have

$$\begin{aligned} & \frac{\left[(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2) + \sqrt{(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2)^2 - 4M_{iiq}^2 M_{jjq}^2} \right]}{2M_{jjq}^2} \\ &= \frac{\left[(4M_{ikq}^2 - M_{iiq}^2 - M_{kkq}^2) + \sqrt{(4M_{ikq}^2 - M_{iiq}^2 - M_{kkq}^2)^2 - 4M_{iiq}^2 M_{kkq}^2} \right] / 2M_{kkq}^2}{\left[(4M_{jkq}^2 - M_{jjq}^2 - M_{kkq}^2) + \sqrt{(4M_{jkq}^2 - M_{jjq}^2 - M_{kkq}^2)^2 - 4M_{jjq}^2 M_{kkq}^2} \right] / 2M_{kkq}^2}. \end{aligned} \quad (10)$$

This is a general relation in terms of baryon masses that can be used to predict the mass of any baryon state if we know all other masses.

A. Masses of Ω_c^0 baryons

In the present work we use Eq. (10) to evaluate the ground-state masses of the Ω_c^0 baryon. Ω_c^0 is composed of two strange quarks and one charmed quark (ssc). Keeping this configuration in mind, we put $i = u$, $j = s$, $q = c$, $k = u$ in Eq. (10) and we get

$$\begin{aligned} & [(M_{\Sigma_c^{++}} + M_{\Omega_c^0})^2 - 4M_{\Xi_c^+}^2] \\ &= \sqrt{(4M_{\Xi_c^+}^2 - M_{\Sigma_c^{++}}^2 - M_{\Omega_c^0}^2)^2 - 4M_{\Omega_c^0}^2 M_{\Sigma_c^{++}}^2}, \end{aligned} \quad (11)$$

Substituting the masses of Ξ_c^+ ($J^P = 1/2^+, 3/2^+$) and Σ_c^{++} ($J^P = 1/2^+, 3/2^+$) from the PDG [17] into Eq. (11), we obtain the ground-state masses of the Ω_c^0 baryon, such as 2.702 GeV for $J^P = \frac{1}{2}^+$ and 2.772 GeV for $J^P = \frac{3}{2}^+$. From Eq. (8) we can write

$$\begin{aligned} \frac{\alpha'_{ssc}}{\alpha'_{uuc}} &= \frac{1}{2M_{\Omega_c^0}^2} \times \left[(4M_{\Xi_c^+}^2 - M_{\Sigma_c^{++}}^2 - M_{\Omega_c^0}^2) \right. \\ &\quad \left. + \sqrt{(4M_{\Xi_c^+}^2 - M_{\Sigma_c^{++}}^2 - M_{\Omega_c^0}^2)^2 - 4M_{\Sigma_c^{++}}^2 M_{\Omega_c^0}^2} \right]. \end{aligned} \quad (12)$$

Putting the values of the masses into the above equation, we get $\alpha'_{ssc}/\alpha'_{uuc}$. We can determine the slope by using Eq. (4),

$$\alpha' = \frac{(J+1) - J}{M_{J+1}^2 - M_J^2}.$$

For $\Sigma_c^{++}(uuc)$ it is

$$\alpha'_{uuc} = \frac{1}{M_{\Sigma_c^{++}(\frac{3}{2}^+)}^2 - M_{\Sigma_c^{++}(\frac{1}{2}^+)}^2}, \quad (13)$$

and we get $\alpha'_{ssc} = 0.50127 \text{ GeV}^{-2}$ for $J^P = \frac{1}{2}^+$. Similarly, we get $\alpha'_{ssc} = 0.60658 \text{ GeV}^{-2}$ for $J^P = \frac{3}{2}^+$.

From Eq. (4) we have

$$M_{J+1} = \sqrt{M_J^2 + \frac{1}{\alpha'}}. \quad (14)$$

Using this equation, we calculate the masses of the excited states of the Ω_c^0 baryon, which are presented in Tables II and III for natural and unnatural parity states, respectively. We compare our results with other theoretical predictions and experimental observations where available.

TABLE II. Masses of excited states of the Ω_c^0 baryon (in GeV) in the (J, M^2) plane with natural parities $P = (-1)^{J-\frac{1}{2}}$.

States	$1^2S_{\frac{1}{2}}$	$1^2P_{\frac{3}{2}}$	$1^2D_{\frac{5}{2}}$	$1^2F_{\frac{7}{2}}$	$1^2G_{\frac{9}{2}}$	$1^2H_{\frac{11}{2}}$
Present	2.702	3.049	3.360	3.645	3.909	4.156
LHCb [1]		3.050				
Belle [2]		3.050				
PDG [17]	2.695	3.050				
Ref. [20]	2.696	2.968	3.251	3.519	3.777	
Ref. [21]	2.695	3.024	3.299	3.565		
Ref. [23]	2.698	3.054	3.297	3.498	3.685	
Ref. [24]	2.718	2.986	3.196			
Ref. [30]	2.731	3.033				
Ref. [41]	2.698	3.026	3.218			
Ref. [42]	2.718	3.056	3.273			

TABLE III. Masses of excited states of the Ω_c^0 baryon (in GeV) in the (J, M^2) plane with unnatural parities $P = (-1)^{J+\frac{1}{2}}$.

States	$1^4S_{\frac{3}{2}}$	$1^4P_{\frac{3}{2}}$	$1^4D_{\frac{3}{2}}$	$1^4F_{\frac{3}{2}}$	$1^4G_{\frac{1}{2}}$	$1^4H_{\frac{1}{2}}$
Present	2.772	3.055	3.314	3.555	3.779	3.991
PDG [17]	2.765					
Ref. [20]	2.766	2.962	3.241	3.503	3.756	
Ref. [21]	2.765	3.010	3.276	3.532		
Ref. [23]	2.776	3.051	3.283	3.485	3.665	
Ref. [24]	2.776	3.014	3.206			
Ref. [30]	2.779	3.057				
Ref. [41]	2.765	3.022	3.237			
Ref. [42]	2.766	3.014				

B. Masses of Ω_{cc}^+ and Ω_{ccc}^{*++}

The values of $\alpha'_{jjq}/\alpha'_{iiq}$ from Eq. (8) should be real numbers. So, we can write Eq. (8) as

$$|4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2| \geq 2M_{iiq}M_{jjq}, \quad (15)$$

and this inequality relation can be simplified to

$$2M_{ijq} \geq (M_{iiq} + M_{jjq}). \quad (16)$$

Many theoretical studies have shown that the slopes of Regge trajectories decrease with increasing quark masses [37,38,43–46]. For $m_j > m_i$, we can write $\alpha'_{jjq}/\alpha'_{iiq} < 1$. Therefore, Eq. (8) is

$$\begin{aligned} \frac{\alpha'_{jjq}}{\alpha'_{iiq}} &= \frac{1}{2M_{jjq}^2} \times \left[(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2) \right. \\ &\quad \left. + \sqrt{(4M_{ijq}^2 - M_{iiq}^2 - M_{jjq}^2)^2 - 4M_{iiq}^2M_{jjq}^2} \right] < 1, \end{aligned} \quad (17)$$

The above relation gives another inequality equation,

$$2M_{ijq}^2 < M_{iiq}^2 + M_{jjq}^2. \quad (18)$$

Thus, from Eqs. (16) and (18) we have

$$\frac{M_{iiq} + M_{jjq}}{2} < M_{ijq} < \sqrt{\frac{M_{iiq}^2 + M_{jjq}^2}{2}}. \quad (19)$$

This relation can be used to calculate the upper and lower limits for baryon masses M_{ijq} . Further, to estimate the deviation of Eq. (19) we introduce a parameter called δ which is used to replace the inequalities with equal signs. For baryons it is denoted by $\delta_{ij,q}^b$ and is given by

$$\delta_{ij,q}^b = M_{iiq}^2 + M_{jjq}^2 - 2M_{ijq}^2, \quad (20)$$

where i , j , and q represent the arbitrary light or heavy quarks.

Now from Eqs. (5) and (6) we can write

$$a_{iiq}(0) - a_{ijq}(0) = a_{ijq}(0) - a_{jjq}(0), \quad (21)$$

$$\frac{1}{\alpha'_{iiq}} - \frac{1}{\alpha'_{ijq}} = \frac{1}{\alpha'_{ijq}} - \frac{1}{\alpha'_{jjq}}. \quad (22)$$

Based on these equations, we introduce two parameters,

$$\lambda_x = a_{nnn}(0) - a_{nnx}(0), \quad \gamma_x = \frac{1}{\alpha'_{nnx}} - \frac{1}{\alpha'_{nnn}}, \quad (23)$$

where n represents a light nonstrange quark (u or d) and x denotes i , j , or q . From Eqs. (21)–(23) we have

$$\begin{aligned} a_{ijq}(0) &= a_{nnn}(0) - \lambda_i - \lambda_j - \lambda_q, \\ \frac{1}{\alpha'_{ijq}} &= \frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_q. \end{aligned} \quad (24)$$

In baryon multiplets for nnn and ijq states, from Eq. (4) we can write

$$\begin{aligned} J &= a_{nnn}(0) + \alpha'_{nnn}M_{nnn}^2, \\ J &= a_{ijq}(0) + \alpha'_{ijq}M_{ijq}^2. \end{aligned} \quad (25)$$

Solving Eqs. (24) and (25), we have

$$M_{ijq}^2 = (\alpha'_{nnn}M_{nnn}^2 + \lambda_i + \lambda_j + \lambda_q) \left(\frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_q \right). \quad (26)$$

Therefore, from Eqs. (20) and (26) we have

$$\begin{aligned} \delta_{ij,q}^b &= M_{iiq}^2 + M_{jjq}^2 - 2M_{ijq}^2 \\ &= (\alpha'_{nnn}M_{nnn}^2 + 2\lambda_i + \lambda_q) \left(\frac{1}{\alpha'_{nnn}} + 2\gamma_i + \gamma_q \right) \\ &\quad + (\alpha'_{nnn}M_{nnn}^2 + 2\lambda_j + \lambda_q) \left(\frac{1}{\alpha'_{nnn}} + 2\gamma_j + \gamma_q \right) \\ &\quad - 2(\alpha'_{nnn}M_{nnn}^2 + \lambda_i + \lambda_j + \lambda_q) \left(\frac{1}{\alpha'_{nnn}} + \gamma_i + \gamma_j + \gamma_q \right) \\ &= 2(\lambda_i - \lambda_j)(\gamma_i - \gamma_j). \end{aligned} \quad (27)$$

So, we can say that $\delta_{ij,q}^b$ is independent of quark flavor q . Hence, by keeping i and j fixed and changing the q quark, using Eq. (20) we have the following relations for $\frac{3}{2}^+$ states.

(I) $i = u$, $j = s$, $q = u$, c :

$$\begin{aligned} \delta_{us}^{(3/2)^+} &= M_{\Delta}^2 + M_{\Xi^*}^2 - 2M_{\Sigma^*}^2 \\ &= M_{\Sigma_c^*}^2 + M_{\Omega_c^*}^2 - 2M_{\Xi_c^*}^2. \end{aligned} \quad (28)$$

(II) $i = u, j = c, q = u, s:$

$$\begin{aligned}\delta_{uc}^{(3/2)^+} &= M_{\Delta}^2 + M_{\Xi_{cc}^*}^2 - 2M_{\Sigma_c^*}^2 \\ &= M_{\Sigma_c^*}^2 + M_{\Omega_{cc}^*}^2 - 2M_{\Xi_c^*}^2.\end{aligned}\quad (29)$$

(III) $i = s, j = c, q = u, c:$

$$\begin{aligned}\delta_{sc}^{(3/2)^+} &= M_{\Xi^*}^2 + M_{\Xi_{cc}^*}^2 - 2M_{\Xi_c^*}^2 \\ &= M_{\Omega_c^*}^2 + M_{\Omega_{cc}^*}^2 - 2M_{\Omega_{cc}^*}^2.\end{aligned}\quad (30)$$

Solving Eqs. (28) and (29), we have

$$(M_{\Omega_{cc}^*}^2 - M_{\Xi_{cc}^*}^2) + (M_{\Xi^*}^2 - M_{\Sigma^*}^2) = (M_{\Omega_c^0}^2 - M_{\Sigma_c^{*++}}^2). \quad (31)$$

Similarly, the corresponding relation for $\frac{1}{2}^+$ baryonic states is

$$(M_{\Omega_c^+}^2 - M_{\Xi_{cc}^{*++}}^2) + (M_{\Xi^0}^2 - M_{\Sigma^+}^2) = (M_{\Omega_c^0}^2 - M_{\Sigma_c^{*++}}^2). \quad (32)$$

Using Eqs. (11) and (32), we have the expression

$$\begin{aligned}&(4M_{\Xi_c^+}^2 - M_{\Omega_c^+}^2 + M_{\Xi_{cc}^{*++}}^2 - 2M_{\Sigma_c^{*++}}^2 - M_{\Xi^0}^2 + M_{\Sigma^+}^2) - 2M_{\Sigma_c^{*++}} \sqrt{M_{\Omega_c^+}^2 - M_{\Xi_{cc}^{*++}}^2 + M_{\Xi^0}^2 - M_{\Sigma^+}^2 + M_{\Sigma_c^{*++}}^2} \\ &= -\sqrt{(4M_{\Xi_c^+}^2 - M_{\Omega_c^+}^2 + M_{\Xi_{cc}^{*++}}^2 - 2M_{\Sigma_c^{*++}}^2 - M_{\Xi^0}^2 + M_{\Sigma^+}^2)^2 - 4M_{\Sigma_c^{*++}}^2 (M_{\Omega_c^+}^2 - M_{\Xi_{cc}^{*++}}^2 + M_{\Xi^0}^2 - M_{\Sigma^+}^2 + M_{\Sigma_c^{*++}}^2)}.\end{aligned}\quad (33)$$

Using the above expression, we calculate the ground-state masses of the Ω_{cc}^+ baryon. Inserting the masses $M_{\Xi_c^+}$, $M_{\Sigma_c^{*++}}$, $M_{\Xi_{cc}^{*++}}$, and M_{Ξ^0} , M_{Σ^+} from the PDG [17] and $M_{\Omega_c^0}$ (calculated above) into Eq. (33), we get $M_{\Omega_{cc}^+} = 3.752$ GeV for $J^P = \frac{1}{2}^+$. Similarly, we get $M_{\Omega_{cc}^+} = 3.816$ GeV for $J^P = \frac{3}{2}^+$. Since the quark configuration of Ω_{cc}^+ is scc , using Eq. (8) we have

$$\frac{\alpha'_{scc}}{\alpha'_{uus}} = \frac{1}{2M_{\Omega_{cc}^+}^2} \times \left[(4M_{\Xi_c^+}^2 - M_{\Sigma^+}^2 - M_{\Omega_{cc}^+}^2) + \sqrt{(4M_{\Xi_c^+}^2 - M_{\Sigma^+}^2 - M_{\Omega_{cc}^+}^2)^2 - 4M_{\Sigma^+}^2 M_{\Omega_{cc}^+}^2} \right]. \quad (34)$$

Further, using Eqs. (34) and (14), we calculate the excited-state masses of the Ω_{cc}^+ baryon in the same way as for Ω_c^0 . Our predicted masses are labeled “Present” for both natural and unnatural parity states in Tables IV and V, respectively, where our predictions are reasonably close to other theoretical predictions.

Similarly, for the triply charmed omega baryon (Ω_{ccc}^{*++}), using Eqs. (30) and (33) we have

$$\begin{aligned}&(3M_{\Xi_{cc}^{*++}}^2 - M_{\Omega_{ccc}^{*++}}^2 + 6M_{\Xi_c^{*++}}^2 - 4M_{\Sigma_c^{*++}}^2 - M_{\Omega_c^0}^2 - M_{\Xi^0}^2 + 2M_{\Sigma^+}^2) \\ &- 2M_{\Sigma_c^{*++}} \sqrt{2M_{\Omega_{ccc}^{*++}}^2 - 6M_{\Xi_{cc}^{*++}}^2 + 2M_{\Omega_c^0}^2 + 2M_{\Xi^0}^2 + 4(M_{\Xi_c^{*++}}^2 + M_{\Sigma_c^{*++}}^2 - M_{\Sigma^+}^2)} \\ &= -\sqrt{(3M_{\Xi_{cc}^{*++}}^2 - M_{\Omega_{ccc}^{*++}}^2 + 6M_{\Xi_c^{*++}}^2 - 4M_{\Sigma_c^{*++}}^2 - M_{\Omega_c^0}^2 - M_{\Xi^0}^2 + 2M_{\Sigma^+}^2)^2 - 8M_{\Sigma_c^{*++}}^2 [M_{\Omega_{ccc}^{*++}}^2 - 3M_{\Xi_{cc}^{*++}}^2 + M_{\Omega_c^0}^2 + M_{\Xi^0}^2 + 2(M_{\Xi_c^{*++}}^2 + M_{\Sigma_c^{*++}}^2 - M_{\Sigma^+}^2)]}.\end{aligned}\quad (35)$$

Since the ground-state ($J^P = \frac{3}{2}^+$) mass of Ξ_{cc}^{*++} has not been confirmed by the PDG yet, we take $M_{\Xi_{cc}^{*++}} = 3.695$ GeV [32], and inserting all other masses from the latest PDG [17] into Eq. (35) we obtain $M_{\Omega_{ccc}^{*++}} = 4.841$ GeV for $J^P = \frac{3}{2}^+$. Excited-state masses for the Ω_{ccc}^{*++} baryon are calculated in a similar manner as for Ω_c^0 and Ω_{cc}^+ , but with different quark constituents, i.e., i, j , and q . Our calculated masses of the excited Ω_{ccc}^{*++} are presented in Table VI. Our results are in accordance with other theoretical predictions.

C. Masses of Ω_c^0 , Ω_{cc}^+ , and Ω_{ccc}^{*++} baryons in the (n, M^2) plane

The general equation for linear Regge trajectories in the (n, M^2) plane can be expressed as

$$n = \beta_0 + \beta M^2, \quad (36)$$

where $n = 1, 2, 3, \dots$ is the radial principal quantum number, and β and β_0 are the slope and intercept of the trajectories. The Regge slope (β) and Regge intercept (β_0) are assumed to be the same for all baryon multiplets lying on the single Regge line. We calculate these parameters (β and β_0), and then use them to estimate the excited-states masses ($N^{2S+1}L_J$, where N, L, S denote the radial excited quantum number, orbital quantum number, and intrinsic spin, respectively) of Ω_c^0 , Ω_{cc}^+ , and Ω_{ccc}^{*++} baryons lying on these Regge trajectories. For the Ω_c^0 baryon, using the slope equation $\beta_{(S)} = 1/(M_{\Omega_c(2S)}^2 - M_{\Omega_c(1S)}^2)$, where $M_{\Omega_c(1S)} = 2.702$ GeV (calculated above) and taking

TABLE IV. Masses of excited states of the Ω_{cc}^+ baryon (in GeV) in the (J, M^2) plane with natural parities $P = (-1)^{J-\frac{1}{2}}$.

States	$1^2S_{\frac{1}{2}}$	$1^2P_{\frac{3}{2}}$	$1^2D_{\frac{5}{2}}$	$1^2F_{\frac{7}{2}}$	$1^2G_{\frac{9}{2}}$	$1^2H_{\frac{11}{2}}$
Present	3.752	3.975	4.186	4.387	4.579	4.763
Ref. [18]	3.650	3.972	4.141	4.296		
Ref. [24]	3.815	4.052	4.202			
Ref. [26]	3.719					
Ref. [30]	3.832	4.086	4.264			
Ref. [31]	3.650		4.174			
Ref. [32]	3.650	3.910	4.153	4.383		
Ref. [47]	3.778	4.102				
Ref. [48]	3.710					
Ref. [49]	3.637					
Ref. [50]	3.732	3.986				
Ref. [51]	3.804					

 TABLE V. Masses of excited states of the Ω_{cc}^+ baryon (in GeV) in the (J, M^2) plane with unnatural parities $P = (-1)^{J+\frac{1}{2}}$.

states	$1^4S_{\frac{3}{2}}$	$1^4P_{\frac{5}{2}}$	$1^4D_{\frac{7}{2}}$	$1^4F_{\frac{9}{2}}$	$1^4G_{\frac{11}{2}}$	$1^4H_{\frac{13}{2}}$
Present	3.816	4.094	4.354	4.599	4.832	5.055
Ref. [18]	3.810	3.958	4.122	4.274		
Ref. [24]	3.876	4.152	4.230			
Ref. [29]	3.822					
Ref. [31]	3.808		4.313			
Ref. [32]	3.809	4.058	4.294	4.516		
Ref. [48]	3.760					
Ref. [49]	3.762					
Ref. [50]	3.765					
Ref. [51]	3.850					
Ref. [52]	3.730	4.134	4.204			
Ref. [53]	3.800					

 TABLE VI. Masses of excited states of the Ω_{ccc}^{*++} baryon (in GeV) in the (J, M^2) plane with unnatural parities $P = (-1)^{J+\frac{1}{2}}$.

states	$1^4S_{\frac{3}{2}}$	$1^4P_{\frac{5}{2}}$	$1^4D_{\frac{7}{2}}$	$1^4F_{\frac{9}{2}}$	$1^4G_{\frac{11}{2}}$	$1^4H_{\frac{13}{2}}$
Present	4.841	5.081	5.310	5.530	5.741	5.945
Ref. [24]	4.965		5.331			
Ref. [26]	4.803					
Ref. [31]	4.818		5.302			
Ref. [32]	4.834		5.301			
Ref. [50]	4.773					
Ref. [51]	4.930					
Ref. [54]	4.827					
Ref. [55]	4.758		5.300			
Ref. [56]	4.761		5.396			
Ref. [57]	4.880					

 TABLE VII. Masses of excited states of the Ω_c^0 baryon (in GeV) in the (n, M^2) plane with natural parity $P = (-1)^{J-\frac{1}{2}}$. The masses from Ref. [21] are taken as input.

$N^{2S+1}L_J$	Present	[23]	[20]	[24]	[55]	[30]
$1^2S_{\frac{1}{2}}$	2.702	2.698	2.696	2.718	2.699	2.731
$2^2S_{\frac{1}{2}}$	3.164 [21]	3.088	3.165	3.152	3.159	3.227
$3^2S_{\frac{1}{2}}$	3.566	3.489	3.540			3.292
$4^2S_{\frac{1}{2}}$	3.928	3.814	3.895			
$5^2S_{\frac{1}{2}}$	4.259	4.102	4.238			
$6^2S_{\frac{1}{2}}$	4.566		4.569			
$1^2P_{\frac{3}{2}}$	3.049	3.054	2.968	2.986		3.033
$2^2P_{\frac{3}{2}}$	3.408 [21]	3.433	3.334			3.056
$3^2P_{\frac{3}{2}}$	3.732	3.752	3.687			
$4^2P_{\frac{3}{2}}$	4.031	4.036	4.029			
$5^2P_{\frac{3}{2}}$	4.309		4.362			
$1^2D_{\frac{5}{2}}$	3.360	3.297	3.251			
$2^2D_{\frac{5}{2}}$	3.680 [21]	3.626	3.604			
$3^2D_{\frac{5}{2}}$	3.974	3.752	3.947			
$4^2D_{\frac{5}{2}}$	4.248		4.282			
$5^2D_{\frac{5}{2}}$	4.505					

$M_{\Omega_c(2S)} = 3.164$ GeV from Ref. [21] for the $J^P = 1/2^+$ state, we get $\beta_{(S)} = 0.3689$ GeV⁻². Using the equations

$$\begin{aligned} 1 &= \beta_{0(S)} + \beta_{(S)} M_{\Omega_c(1S)}^2, \\ 2 &= \beta_{0(S)} + \beta_{(S)} M_{\Omega_c(2S)}^2, \end{aligned} \quad (37)$$

we have $\beta_{0(S)} = -1.6939$. In the same way, we can write

$$\begin{aligned} 1 &= \beta_{0(P)} + \beta_{(P)} M_{\Omega_c(1P)}^2, \\ 2 &= \beta_{0(P)} + \beta_{(P)} M_{\Omega_c(2P)}^2, \\ 1 &= \beta_{0(D)} + \beta_{(D)} M_{\Omega_c(1D)}^2, \\ 2 &= \beta_{0(D)} + \beta_{(D)} M_{\Omega_c(2D)}^2. \end{aligned} \quad (38)$$

With the help of $\beta_{(S)}$ and $\beta_{0(S)}$, we calculate the masses of the excited Ω_c^0 baryon for $n = 3, 4, 5, \dots$. Tables VII and VIII show our calculated radial and orbital excited-state masses of the Ω_c^0 baryon in the (n, M^2) plane for natural and unnatural parity states, respectively. In a similar way, we estimate the mass spectra for Ω_{cc}^+ (see Tables IX and X) and Ω_{ccc}^{*++} (see Table XI) baryons, also in the (n, M^2) plane.

TABLE VIII. Masses of excited states of the Ω_c^0 baryon (in GeV) in the (n, M^2) plane with unnatural parity $P = (-1)^{J+\frac{1}{2}}$. The masses from Ref. [21] are taken as input.

$N^{2S+1}L_J$	Present	[23]	[20]	[24]	[55]	[30]
$1^4S_{\frac{3}{2}}$	2.772	2.768	2.766	2.776	2.768	2.779
$2^4S_{\frac{3}{2}}$	3.197 [21]	3.123	3.208	3.190	3.202	3.257
$3^4S_{\frac{3}{2}}$	3.571	3.510	3.564			3.285
$4^4S_{\frac{3}{2}}$	3.910	3.830	3.911			
$5^4S_{\frac{3}{2}}$	4.222	4.114	4.248			
$6^4S_{\frac{3}{2}}$	4.513					
$1^4P_{\frac{5}{2}}$	3.055	3.051	2.962	3.041		3.057
$2^4P_{\frac{5}{2}}$	3.393 [21]	3.427	3.328			3.477
$3^4P_{\frac{5}{2}}$	3.700	3.744	3.681			3.620
$4^4P_{\frac{5}{2}}$	3.983	4.028	4.023			
$5^4P_{\frac{5}{2}}$	4.248					
$1^4D_{\frac{7}{2}}$	3.314	3.283	3.241			
$2^4D_{\frac{7}{2}}$	3.656 [21]	3.611	3.594			
$3^4D_{\frac{7}{2}}$	3.968		3.938			
$4^4D_{\frac{7}{2}}$	4.258		4.273			
$5^4D_{\frac{7}{2}}$	4.529		4.594			

TABLE X. Masses of excited states of the Ω_{cc}^+ baryon (in GeV) in the (n, M^2) plane with unnatural parity $P = (-1)^{J+\frac{1}{2}}$. The masses from Ref. [18] are taken as input.

$N^{2S+1}L_J$	Present	[47]	[30]	[24]	[58]
$1^4S_{\frac{3}{2}}$	3.816	3.872	3.883	3.876	3.772
$2^4S_{\frac{3}{2}}$	4.096 [18]	4.174	4.263	4.188	4.142
$3^4S_{\frac{3}{2}}$	4.358		4.265		
$4^4S_{\frac{3}{2}}$	4.605				
$5^4S_{\frac{3}{2}}$	4.839				
$6^4S_{\frac{3}{2}}$	5.063				
$1^4P_{\frac{5}{2}}$	4.094		4.220	4.152	
$2^4P_{\frac{5}{2}}$	4.247 [18]				
$3^4P_{\frac{5}{2}}$	4.394				
$4^4P_{\frac{5}{2}}$	4.537				
$5^4P_{\frac{5}{2}}$	4.676				
$1^4D_{\frac{7}{2}}$	4.354				
$2^4D_{\frac{7}{2}}$	4.391 [18]				
$3^4D_{\frac{7}{2}}$	4.427				
$4^4D_{\frac{7}{2}}$	4.464				
$5^4D_{\frac{7}{2}}$	4.500				

TABLE IX. Masses of excited states of the Ω_{cc}^+ baryon (in GeV) in the (n, M^2) plane with natural parity $P = (-1)^{J-\frac{1}{2}}$. The masses from Ref. [18] are taken as input.

$N^{2S+1}L_J$	Present	[47]	[30]	[24]	[32]
$1^2S_{\frac{1}{2}}$	3.752	3.778	3.832	3.815	3.650
$2^2S_{\frac{1}{2}}$	4.041 [18]	4.075	4.227	4.180	
$3^2S_{\frac{1}{2}}$	4.310	4.321	4.295		
$4^2S_{\frac{1}{2}}$	4.564				
$5^2S_{\frac{1}{2}}$	4.804				
$6^2S_{\frac{1}{2}}$	5.033				
$1^2P_{\frac{3}{2}}$	3.975	4.102	4.086	4.052	3.910
$2^2P_{\frac{3}{2}}$	4.259 [18]	4.345	4.201	4.140	
$3^2P_{\frac{3}{2}}$	4.525				
$4^2P_{\frac{3}{2}}$	4.776				
$5^2P_{\frac{3}{2}}$	5.015				
$1^2D_{\frac{5}{2}}$	4.186		4.264	4.202	4.153
$2^2D_{\frac{5}{2}}$	4.407 [18]				
$3^2D_{\frac{5}{2}}$	4.617				
$4^2D_{\frac{5}{2}}$	4.818				
$5^2D_{\frac{5}{2}}$	5.011				

TABLE XI. Masses of the excited states of the Ω_{ccc}^{++} baryon (in GeV) in the (n, M^2) plane with unnatural parity $P = (-1)^{J+\frac{1}{2}}$. The masses from Ref. [19] are taken as input.

$N^{2S+1}L_J$	Present	[24]	[32]	[31]	[55]
$1^4S_{\frac{3}{2}}$	4.841	4.965	4.834	4.818	4.758
$2^4S_{\frac{3}{2}}$	5.300 [19]	5.313			
$3^4S_{\frac{3}{2}}$	5.722				
$4^4S_{\frac{3}{2}}$	6.115				
$5^4S_{\frac{3}{2}}$	6.484				
$6^4S_{\frac{3}{2}}$	6.834				
$1^4P_{\frac{5}{2}}$	5.081				
$2^4P_{\frac{5}{2}}$	5.553 [19]				
$3^4P_{\frac{5}{2}}$	5.987				
$4^4P_{\frac{5}{2}}$	6.393				
$5^4P_{\frac{5}{2}}$	6.774				
$1^4D_{\frac{7}{2}}$	5.310	5.331	5.301	5.302	5.300
$2^4D_{\frac{7}{2}}$	5.961 [19]				
$3^4D_{\frac{7}{2}}$	6.547				
$4^4D_{\frac{7}{2}}$	7.085				
$5^4D_{\frac{7}{2}}$	7.585				

III. RESULT AND DISCUSSION

In the framework of Regge phenomenology, we have obtained the radial and orbital excited states of singly, doubly, and triply charmed omega baryons. In the present work we have focused on studying the masses of the Ω_c^0 baryon by obtaining the mass relations derived from Regge phenomenology. Ground-state as well as excited-state masses were obtained successfully in both the (J, M^2) and (n, M^2) planes for the following baryons.

- (i) Ω_c^0 baryon: Our calculated masses in the (J, M^2) plane for the Ω_c^0 baryon are shown in Tables II and III for natural and unnatural parity states, respectively. First, we compared our predicted ground-state masses with experimentally available data [1,2,17] and other theoretical predictions [20,21,23,24,30,41,42]. Our results are very close to those of Ref. [17] with a slight mass difference of 7–8 MeV, and they are also in good agreement with Refs. [20,21,23,41]. Then, the excited-state masses were compared with other theoretical outcomes. For $1^2P_{\frac{3}{2}}$ and $1^4P_{\frac{3}{2}}$ states our results are in accordance with Refs. [21,23,30,41,42] with a mass difference in the range 5–50 MeV, and for $1^2D_{\frac{3}{2}} - 1^2F_{\frac{7}{2}}$ and $1^4D_{\frac{7}{2}} - 1^4F_{\frac{7}{2}}$ states our predictions are consistent with Refs. [20,23] with a mass difference of 30–65 MeV. Similarly, in the (n, M^2) plane our calculated results are shown in Tables VII and VIII for natural and unnatural parity states, respectively. Our predicted masses are in agreement with Refs. [20,23,24]. The experimentally observed state $\Omega_c(3050)^0$ with mass 3.050 GeV is close to our prediction 3.049 GeV, so we assigned $\Omega_c(3050)^0$ as a $1P$ state with $J^P = 3/2^-$ for $S = 1/2$. The other four states— $\Omega_c(3000)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$ —were not identified in this work (they may belong to the remaining $1P$ states, such as $1^2P_{\frac{1}{2}}$, $1^4P_{\frac{1}{2}}$, and $1^4P_{\frac{3}{2}}$ [4]). We obtained masses for $1^2S_{\frac{1}{2}}$, $1^2P_{\frac{3}{2}}$, $1^2D_{\frac{3}{2}}$,... and $1^4S_{\frac{3}{2}}$, $1^4P_{\frac{3}{2}}$, $1^4D_{\frac{7}{2}}$,... states for natural and unnatural parity, respectively, in the (J, M^2) plane lying on Regge lines.
- (ii) Ω_{ccc}^+ baryon: Tables IV and V show our estimated results for natural and unnatural parity states,

respectively, in the (J, M^2) plane. Our calculated ground-state ($1^2S_{\frac{1}{2}}$) mass is in good agreement with the predictions of Refs. [24,26,47,48,50,51] with a mass difference of 25–65 MeV, and for $1^2P_{\frac{3}{2}} - 1^2F_{\frac{7}{2}}$ states our results are in accordance with Refs. [24,31,32,50] with a mass difference of 20–70 MeV (see Table IV). Similarly, for the $1^4S_{\frac{3}{2}}$ state our predicted mass is very close to the results of Refs. [29,31,32,53], with a slight mass difference of 7–16 MeV, and for $1^4P_{\frac{3}{2}} - 1^4F_{\frac{7}{2}}$ states our results are in accordance with other theoretical outcomes (see Table V). In the same way, we compared our calculated radial and orbital excited-state masses evaluated in the (n, M^2) plane for both natural and unnatural parity states and our results are consistent with other theoretical and phenomenological studies (see Tables IX and X).

- (iii) Ω_{ccc}^{*++} baryon: For the triply charmed omega baryon, we calculated the ground-state and excited-state masses in both the (J, M^2) and (n, M^2) planes, as shown in Tables VI and XI, respectively. The masses of the ground-state Ω_{ccc}^{*++} vary in the range 4.750–4.950 GeV in other theoretical references (Table VI). Our predicted mass for the ($1^4S_{\frac{3}{2}}$) state shows a few-MeV difference with the results of Refs. [26,31,32,54]. For the excited state, very few results are available from previous theoretical predictions [24,31,32,55], and our calculated masses are in accordance with them.

We have successfully employed the Regge phenomenological approach to calculate the masses of singly, doubly, and triply charmed Ω baryons. This study will definitely help future experimental studies at LHCb, Belle II, and the upcoming facility PANDA [59] to identify these baryonic states from resonances.

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