Two-gluon production of ϕ and η' mesons in proton-proton collisions at high energies

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(Received 29 March 2021; accepted 8 May 2021; published 7 June 2021)

We discuss gluon-gluon mechanisms for production of mesons with hidden strangeness, such as η' and ϕ meson, in proton-proton collisions at large energies. The $g^*g^* \rightarrow \eta'$ and $g^*g^* \rightarrow \phi g$ mechanisms are considered only, and the corresponding cross sections are calculated in the k_t -factorization approach. The $F_{\gamma^*\gamma^* \rightarrow \eta'}$ and $F_{g^*g^* \rightarrow \eta'}$ form factors are calculated from the quark-antiquark η' light-cone wave function including quark/antiquark transverse momenta. The result for the two-photon transition form factor demonstrates that higher twists may survive even to large photon virtualities. The result is compared with the result of a recent leading-twist next-to-leading-order analysis, which uses phenomenological distribution amplitudes fitted to exclusive production of η' in $e^+e^- \rightarrow e^+e^-\eta'$ reaction. We calculate transverse momentum distributions of both η' and ϕ mesons in proton-proton collisions for Relativistic Heavy Ion Collider and LHC energies. The results are compared to experimental data whenever available. The results of the Lund string model are shown for comparison for η' production at LHC energies. It seems that the two-gluon fusion is not the dominant mechanism for both ϕ and η' production, although the situation for η' , especially at larger energies, is less clear due to lack of experimental data.

DOI: [10.1103/PhysRevD.103.114008](https://doi.org/10.1103/PhysRevD.103.114008)

I. INTRODUCTION

The mechanism of the particle production in proton-proton collisions was studied continuously for five decades. Pions and kaons are believed to be produced in the fragmentation process. The Lund string model is state of art in this context. On the other side, quarkonium production at high energies is studied considering the two-gluon process. Depending on C parity, we have either $g^*g^* \rightarrow \mathcal{Q}$ ($C = +1$) or $g^*g^* \rightarrow \mathcal{Q}g$ ($C = -1$) processes when limiting to color-singlet mechanisms. Color-octet processes are not under full theoretical control. Recently, our group showed that the production of η_c quarkonium can be understood assuming simple color-singlet $g^*g^* \rightarrow \eta_c$ fusion [1]. The situation with resonances, such as ρ^0 , $f_0(980)$, or $f_2(1270)$ is still different. We have shown that at large isoscalar meson transverse momenta the gluon-gluon fusion may be

an important mechanism [2,3]. At low transverse momenta, one has to include also the coalescence mechanism [3]. For fully heavy tetraquark production, the double-parton mechanism may be required [4].

The situation with hidden strangeness mesons was not discussed carefully in the literature. In PYTHIA [5], such mesons are produced within the perhaps Lund string model. Recently, there have been some works on the modification of strange hadron production. Effects of the rope hadronization on strangeness enhancement in pp collisions at the LHC are discussed, e.g., in Ref. [6].

Here, we wish to explore whether the two-gluon fusion may be also an important production mechanism. It was suggested that the hadronic production of the η' meson could be used to extract the two-gluon transition form factor [7,8]. The formalism of η' meson production in proton-proton collisions was discussed already some time ago, but no explicit calculation has been performed so far. In addition, it could not be compared to the data as the data was not available at that time (2007). In the meantime, the PHENIX Collaboration measured the η' production at $\sqrt{s} = 200$ GeV, but no comparison was made with theoretical results according to our knowledge. Here, we wish to study the situation more carefully and make comparison to realistic calculation of the gluon-gluon fusion.

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Exclusive $pp \rightarrow pp\eta'$ production is studied in Ref. [9] within the Kimber-Martin-Ryskin (KMR) perturbative approach, and the measured cross section could be explained at the relatively low $\sqrt{s} = 29.1$ GeV energy of the WA102 Collaboration experiment at CERN super proton synchrotron. On the other hand, soft Pomeron/Reggeon exchanges can be fitted to describe the experimental data [10].

The presence of the two-gluon component in η' may be relevant in the context of its production in the hadronic reaction. The gluon content in a meson may occur, e.g., via mixing with glueballs. Mixing of scalar glueball with the scalar-isoscalar “quarkonia” was discussed, e.g., in Ref. [11]. According to our knowledge, there was not such a discussion for η' . However, according to lattice QCD, a pseudoscalar glueball has a mass of about 2.6 GeV [12,13], so the mixing should be rather small. A low mass pseudoscalar glueball is discussed in Ref. [14], which mixes however rather with radial excitations. A review on experimental searches for the light pseudoscalar glueball can be found in Ref. [15]. On the other hand, an axial anomaly may “cause” the presence of gluons in the η' wave function [16]. No big gluonic content either in η or η' is found from the $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ radiative decays in Ref. [17]. The KLOE-2 Collaboration found $Z_g^2 \approx 0.11$ probability of the two-gluon component [18].

For comparison, the hadronization process for η' production has not been discussed carefully in the literature. Some discussion is presented, e.g., in Ref. [19], but in the context of e^+e^- collisions.

In this paper, we shall discuss possible consequences of the gluonic component in η' for its production in proton-proton collisions. We shall compare our results with both experimental data as well as with results of the Lund string model.

II. SKETCH OF THE FORMALISM

The main color-singlet mechanism of ϕ meson production is illustrated in Fig. 1. In this case, ϕ is produced in association with an extra “hard” gluon due to C-parity conservation.

We calculate the dominant color-singlet $gg \rightarrow \phi g$ contribution, taking into account transverse momenta of initial gluons. In the k_t factorization, the next-to-leading-order differential cross section can be written as

$$\begin{aligned} \frac{d\sigma(pp \rightarrow \phi g X)}{dy_{J/\psi} dy_g d^2 p_{\phi,t} d^2 p_{g,t}} &= \frac{1}{16\pi^2 \hat{s}^2} \int \frac{d^2 q_{1t}}{\pi} \frac{d^2 q_{2t}}{\pi} |\mathcal{M}_{g^* g^* \rightarrow \phi g}^{\text{off-shell}}|^2 \\ &\times \delta^2(\vec{q}_{1t} + \vec{q}_{2t} - \vec{p}_{H,t} - \vec{p}_{g,t}) \\ &\times \mathcal{F}_g(x_1, q_{1t}^2, \mu^2) \mathcal{F}_g(x_2, q_{2t}^2, \mu^2), \end{aligned} \quad (2.1)$$

where $\mathcal{F}_g$ are unintegrated (or transverse-momentum-dependent) gluon distributions. The matrix elements were

FIG. 1. The leading-order diagram for direct ϕ meson production in the k_t -factorization approach.

calculated as done, e.g., for $J/\psi g$ production in Ref. [20]. The corresponding matrix element squared for the $gg \rightarrow \phi g$ is

$$|\mathcal{M}_{gg \rightarrow \phi g}|^2 \propto \alpha_s^3 |R(0)|^2. \quad (2.2)$$

Running coupling constants are used in the calculation. Different combinations of renormalization scales were tried. Finally, we decided to use

$$\alpha_s^3 \rightarrow \alpha_s(\mu_1^2) \alpha_s(\mu_2^2) \alpha_s(\mu_3^2), \quad (2.3)$$

where $\mu_1^2 = \max(q_{1t}^2, m_t^2)$, $\mu_2^2 = \max(q_{2t}^2, m_t^2)$, and $\mu_3^2 = m_t^2$, where here m_t is the ϕ transverse mass. The factorization scale in the calculation was taken as $\mu_F^2 = (m_t^2 + p_{t,g}^2)/2$.

The radial wave function at zero can be estimated from the decay of $\phi \rightarrow l^+ l^-$ as is usually done for $J/\psi(c\bar{c})$, see, e.g., Ref. [21],

$$\Gamma(\phi \rightarrow l^+ l^-) = 16\pi \frac{\alpha Q_s^2}{M_\phi^2} |\Psi_\phi(0)|^2 \left(1 - \frac{16\alpha_s}{3\pi}\right), \quad (2.4)$$

where Q_s is the fractional charge of the s quark. Then,

$$|\Psi_\phi(0)|^2 = \frac{\Gamma(\phi \rightarrow l^+ l^-)}{16\pi \alpha_{em} Q_s^2} \frac{M_\phi^2}{1 - 16\alpha_s/(3\pi)}. \quad (2.5)$$

In the evaluation, we use $\alpha_s = 0.3$. Using the branching fraction from particle data group book [22], we get $|\Psi(0)|$. By convention, $|R(0)|^2 = 4\pi |\Psi(0)|^2$. Assuming $\alpha_s = 0.3$, we get $|R(0)|^2 = 0.11 \text{ GeV}^3$. We shall use this value to estimate the cross section for production of the ϕ meson. For comparison for J/ψ (real quarkonium), one gets $|R(0)|^2 \approx 0.8 \text{ GeV}^3$.

Similarly, we perform calculation for S-wave η' meson production. Here, the lowest-order subprocess $gg \rightarrow \eta'$ is allowed by positive C -parity of η' mesons. In the k_t -factorization approach, the leading-order cross section for the η' meson production can be written as

$$\sigma_{pp \rightarrow \eta'} = \int dy d^2 p_t d^2 q_t \frac{1}{sx_1 x_2} \frac{1}{m_{t,\eta'}^2} |\overline{\mathcal{M}}_{g^* g^* \rightarrow \eta'}|^2 \times \mathcal{F}_g(x_1, q_{1t}^2, \mu_F^2) \mathcal{F}_g(x_2, q_{2t}^2, \mu_F^2) / 4, \quad (2.6)$$

which can be also used to calculate rapidity and transverse momentum distribution of the η' mesons. Above $\mathcal{F}_g$ are unintegrated (or transverse-momentum-dependent) gluon distributions, and $\mathcal{M}_{g^* g^* \rightarrow \eta'}$ is the $gg \rightarrow \eta'$ (off-shell) matrix element. In the last equation, $\vec{p}_t = \vec{q}_{1t} + \vec{q}_{2t}$ is the transverse momentum of the η' meson, and $\vec{q}_t = \vec{q}_{1t} - \vec{q}_{2t}$ is an auxiliary variable which is used in the integration. Furthermore, $m_{t,\eta'}$ is the so-called η' transverse mass, and $x_1 = \frac{m_{t,\eta'}}{\sqrt{s}} \exp(y)$, $x_2 = \frac{m_{t,\eta'}}{\sqrt{s}} \exp(-y)$. The factor $\frac{1}{4}$ is the Jacobian of transformation from $(\vec{q}_{1t}, \vec{q}_{2t})$ to $(\vec{p}_t, \vec{q}_t)$ variables. The situation is illustrated diagrammatically in Fig. 2.

As for ϕ production, the running coupling constants are used. Different combinations of scales are tried. In QCD calculations, usually the largest scale related to a vertex decides about the argument of the running strong coupling constant. In our opinion, for the $g^* g^* \rightarrow \eta'$, the best choice is

$$\alpha_s^2 \rightarrow \alpha_s(\mu_1^2) \alpha_s(\mu_2^2), \quad (2.7)$$

where $\mu_1^2 = \max(q_{1t}^2, m_t^2)$ and $\mu_2^2 = \max(q_{2t}^2, m_t^2)$. Above, m_t is the transverse mass of the η' meson. Equation (2.7) is a standard prescription typically used for production of

FIG. 2. The leading-order diagram for η' meson production in the k_t -factorization approach.

quarkonia with $C = +1$ (χ_c or η_c) via gluon-gluon fusion. A similar prescription is used for Higgs boson production. For η' meson and for not too small meson transverse momenta ($p_t > 2$ GeV) one typically has $q_{1t} > m_{\eta'}$, $q_{2t} > m_{\eta'}$ i.e., gluon virtualities decide in (2.7). We estimate the uncertainties related to the choice of the renormalization scales to be less than 30%. As will be discussed in the results section, this is much less than the agreement with experimental transverse momentum distribution of η' .

The factorization scale(s) for the η' meson production are fixed traditionally as $\mu_F^2 = m_t^2$.

The $g^* g^* \rightarrow \eta'$ coupling has a relatively simple one-term form,

$$T_{\mu\nu}(q_1, q_2) = F_{g^* g^* \eta'}(q_1, q_2) \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta, \quad (2.8)$$

where the $F_{g^* g^* \eta'}(q_1, q_2)$ object is known as the two-gluon transition form factor. The matrix element to be used in the k_t -factorization approach is then

$$\mathcal{M}^{ab} = \frac{q_{1,\perp}^\mu q_{2,\perp}^\nu}{|\mathbf{q}_1||\mathbf{q}_2|} T_{\mu\nu}. \quad (2.9)$$

In contrast to the convention for the two-photon transition form factor, the strong coupling constants are usually absorbed into the two-gluon form factor definition.

The matrix element squared for the $gg \rightarrow \eta'$ subprocess is

$$|\mathcal{M}_{gg \rightarrow \eta'}|^2 \propto F_{g^* g^* \eta'}^2(q_{1t}^2, q_{2t}^2) \propto \alpha_s^2 F_{\gamma^* \gamma^* \eta'}^2(q_{1t}^2, q_{2t}^2), \quad (2.10)$$

where $F_{g^* g^* \eta'}^2(q_{1t}^2, q_{2t}^2)$ and $F_{\gamma^* \gamma^* \eta'}^2(q_{1t}^2, q_{2t}^2)$ are two-gluon and two-photon transition form factors of the η' meson, respectively. It was discussed, e.g., in Ref. [23], in leading-twist collinear approximation. Such an approach is valid for $Q_1^2 = q_{1t}^2 \gg 0$ and $Q_2^2 = q_{2t}^2 \gg 0$. Here, we need such a transition form factor also for $Q_1^2, Q_2^2 \sim 0$. There is a simple relation between the two-gluon and two-photon form factors for the quark-antiquark systems (see, e.g., Refs. [1–3]). The η' meson may have also the two-gluon component in its Fock decomposition [16]. The form factor found there can be approximately parametrized as

$$\bar{Q}^2 F_{g^* g^* \eta'}^2(Q_1^2, Q_2^2) \approx 0.2 \pm 0.1 \text{ GeV}, \quad (2.11)$$

where $\bar{Q}^2 = (Q_1^2 + Q_2^2)/2$. A better approach would be to use their Eqs. (5.13)–(5.16) with parameters given there. The result from Ref. [23] is

$$F(\bar{Q}^2, \omega) = 4\pi\alpha_s \frac{f_P}{Q^2} \frac{\sqrt{n_f}}{N_c} A(\omega). \quad (2.12)$$

In the factorized (in $\bar{Q}^2$ and ω) formula,

$$A(\omega) = A_{q\bar{q}}(\omega) + \frac{N_c}{2n_f} A_{gg}(\omega), \quad (2.13)$$

where

$$A_{q\bar{q}}(\omega) = \int_0^1 dx \Phi_1(x, \mu_F^2) \frac{1}{1 - \omega^2(1 - 2x)^2}, \quad (2.14)$$

$$A_{gg}(\omega) = \int_0^1 dx \frac{\Phi_g(x, \mu_F^2)}{x\bar{x}} \frac{1 - 2x}{1 - \omega^2(1 - 2x)^2} \quad (2.15)$$

and Φ_1 and Φ_g are singlet and gluon distribution functions, respectively. Above,

$$\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2}. \quad (2.16)$$

Φ_1 and Φ_g undergo QCD evolution [23], which is included also in the present paper.

A. $F_{\gamma^*\gamma^*\rightarrow\eta'}$ form factor

In Ref. [24], we show how to calculate the transition form factor from the light-cone $Q\bar{Q}$ wave function of the η_c quarkonium. Here, we shall follow the same idea but for light quark and light antiquark system. The flavor wave function of the η' meson can be approximated as [25]

$$|\eta'\rangle \approx \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}). \quad (2.17)$$

The spatial wave function could be calculated, e.g., in potential models. The momentum wave function can then be obtained as a Fourier transform of the spatial one. We shall not follow this path in the present study. Instead, we shall take a simple, but reasonable, parametrization of the respective light-cone wave function. In principle, each component in (2.17) may have different spatial as well as momentum wave function. Here, for simplicity, we shall

assume one effective wave function for each flavor component. We shall take the simple parametrization of the momentum wave function,

$$u(p) \propto \exp(p^2/(2\beta)). \quad (2.18)$$

The light-cone wave function is obtained then via Terentev's transformation (see, e.g., Ref. [24]). We shall use the normalization of the light-cone wave function as

$$\int_0^1 \frac{dz}{z(1-z)} \frac{d^2k}{16\pi^3} |\phi(z, k_t)|^2 = 1. \quad (2.19)$$

Above,

$$\phi(z, k_t) \propto \sqrt{M_{q\bar{q}}} \exp(-p^2/(2\beta^2)), \quad (2.20)$$

and the so-called Terentev prescription, relating the rest-frame and light-cone variables, is used:

$$p^2 = \frac{1}{4}(M_{q\bar{q}}^2 - 4m_{\text{eff}}^2). \quad (2.21)$$

Above, $M_{q\bar{q}}$ is the invariant mass of the $q\bar{q}$ system.

The parameters in the above equations, m_{eff} [hidden in $\phi(z, k_t)$] and β , are in principle free. Here, we shall take

$$m_{\text{eff}} = (2/3)m_q + (1/3)m_s, \quad (2.22)$$

where m_q and m_s are constituent masses of light (u,d) and strange quarks, respectively. Therefore, $m_{\text{eff}} \sim 0.4$ GeV. The weights are from the flavor wave function (2.17). We shall try a few different β values in the range (0.4–0.6) GeV. The normalization constant can be then obtained from the light-cone wave function normalization.

Having fixed light-cone wave function, one can calculate electromagnetic $\gamma^*\gamma^* \rightarrow \eta'$ transition form factor as

$$F(Q_1^2, Q_2^2) = -\frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2)\sqrt{N_c}4m_{\text{eff}} \cdot \int \frac{dz d^2k}{z(1-z)16\pi^3} \psi(z, k) \times \left\{ \frac{1-z}{(\mathbf{k} - (1-z)\mathbf{q}_2)^2 + z(1-z)q_1^2 + m_{\text{eff}}^2} + \frac{z}{(\mathbf{k} + z\mathbf{q}_2)^2 + z(1-z)q_1^2 + m_{\text{eff}}^2} \right\}. \quad (2.23)$$

The $F(0,0)$ is known and can be calculated from the radiative decay width [26].

The present *BABAR* data [26] are not sufficiently precise to get the parameters of our model (m_{eff} and β). They could be adjusted in future to precise experi-

mental data for the $e^+e^- \rightarrow e^+e^-\eta'$ reaction from Belle 2.

When neglecting transverse momenta of photons and quark and antiquark in the meson formula (2.23) can be reduced to a single integral,

$$F(Q_1^2, Q_2^2) = \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2)f_{\eta'} \cdot \int_0^1 dz \left\{ \frac{(1-z)\phi(z)}{(1-z)^2 Q_1^2 + z(1-z)Q_2^2 + m_{\text{eff}}^2} + \frac{z\phi(z)}{z^2 Q_1^2 + z(1-z)Q_2^2 + m_{\text{eff}}^2} \right\} \quad (2.24)$$

where the so-called distribution amplitudes $\phi(z) \propto \int d^2k \Psi(z, \mathbf{k})$ and so-called decay constant $f_{\eta'}$ are introduced (for more details, see, e.g., Ref. [24]).

We shall use also a simple parametrization of the transition form factor called the nonfactorized monopole for brevity:

$$F^{nf, \text{monopole}}(Q_1^2, Q_2^2) = F(0, 0) \frac{\Lambda^2}{\Lambda^2 + Q_1^2 + Q_2^2}. \quad (2.25)$$

This two-parameter formula can be correctly normalized at $Q_1^2 = 0$ and $Q_2^2 = 0$ [26]. It also has correct asymptotic dependence on $\bar{Q}^2 = (Q_1^2 + Q_2^2)/2$. This is very similar to the approach presented long ago by Brodsky and Lepage [27] in the case of a neutral pion.

The so-called vector meson dominance model (factorized monopole)

$$F^{VDM}(Q_1^2, Q_2^2) = F(0, 0) \frac{m_V^2}{m_V^2 + Q_1^2} \frac{m_V^2}{m_V^2 + Q_2^2} \quad (2.26)$$

has incorrectly strong $\bar{Q}^2$ dependence [26]. We shall compare results obtained with the form factor calculated with the light-cone wave function (2.23) with the parametrization (2.25) for $\Lambda = 1$ GeV. Results of such a calculation will be treated as a reference ones for other approaches.

The effect of internal transverse momenta of quarks and antiquarks in a meson was discussed long ago [28], postulating some wave function of π^0 in the impact parameter space and including suppression due to the so-called Sudakov form factor.

In Ref. [29], the authors tried to adjust the coefficient of the lowest-order Gegenbauer polynomials to describe the *BABAR* data [26] for two virtual photons within the leading-twist collinear approximation. However, the corresponding error bars on expansion coefficients are very large.

The two-gluon transition form factor is closely related to two-photon transition form factor, provided the meson is of the quark-antiquark type, i.e., its wave function is as in Eq. (2.17); see the triangle diagrams in Fig. 3. Replacing virtual photons by virtual gluons, the electromagnetic coupling constants by the strong coupling constants, correcting by fractional charges of quarks in the electromagnetic case and by color factors being consistent with the unintegrated gluon distributions:

FIG. 3. The assumed mechanisms of the $\gamma^* \gamma^* \rightarrow \eta'$ (left) and $g^* g^* \rightarrow \eta'$ (right) couplings.

$$|F_{g^* g^* \rightarrow \eta'}(Q_1^2, Q_2^2)|^2 = |F_{\gamma^* \gamma^* \rightarrow \eta'}(Q_1^2, Q_2^2)|^2 \frac{g_s^2}{g_{em}^2} \frac{1}{4N_c(N_c^2 - 1)} \frac{1}{(\langle e_q^2 \rangle)^2}. \quad (2.27)$$

Above, g_{em}^2 must be taken, provided it is included in the definition of $F_{\gamma^* \gamma^* \rightarrow \eta'}$ transition form factor. Usually, it is not. The relation (2.27) assumes simplified structure of the meson (η' in our case). Assuming such a relation for the η_c meson leads to a fairly good agreement of the transverse momentum distribution of η_c produced in proton-proton collisions with the LHCb data [1]. In the case of η' , this relation neglects the possible gluon component in the wave function. This will be discussed explicitly in the present paper.

In Fig. 4, we show $q\bar{q}$ and gg distribution amplitudes from Ref. [23] for different evolution scales. Such distribution amplitudes can be used to calculate $F_{g^* g^* \rightarrow \eta'}$ [see Eq. (2.13)] needed in calculating η' production in proton-proton collisions.

FIG. 4. $q\bar{q}$ (upper curves) and gg (lower curves) distribution amplitudes for three different scales: 1 (solid), 10 (dashed), and 100 GeV^2 (dotted).

III. RESULTS

In this section, we present our results for ϕ and η' meson production. In the calculations below, for both ϕ and η' hadroproduction, we shall use the KMR unintegrated gluon distribution [30].

A. ϕ production

In this subsection, we show the cross section for ϕ meson production for $\sqrt{s} = 200$ GeV, $\sqrt{s} = 2.76$ TeV, and $\sqrt{s} = 8$ TeV (see Fig. 5). Our results are shown together with the PHENIX [31] and ALICE [32,33] experimental data.

For each considered case, the result of calculation is below the experimental data. This suggests that the gluon-gluon fusion is not the dominant production mechanism of ϕ meson production. The fragmentation mechanism is considered in Refs. [34,35], and it may be the dominant mechanism of ϕ meson production.

B. $F_{\gamma^*\gamma^*\rightarrow\eta'}$ form factor

Before presenting our results for the η' production, we wish to show our results for the $F_{\gamma^*\gamma^*\rightarrow\eta'}$ form factor.

We will start with our results obtained from the LCWF (light cone wave function) for $F_{\gamma\gamma\rightarrow\eta'}(0,0)$. In Fig. 6, we present $F_{\gamma\gamma\rightarrow\eta'}(0,0)$ as a function of β and m_{eff} . There is a small dependence on the parameters. The experimental value is

$$F_{\gamma^*\gamma^*\rightarrow\eta'}(0,0) = \sqrt{\frac{4\Gamma_{\eta'\rightarrow 2\gamma}}{\pi\alpha_{em}^2 m_{\eta'}^3}} = 0.342 \pm 0.006 \text{ GeV}^{-1}. \quad (3.1)$$

A broad range of β and m_{eff} is allowed, taking into account the simplicity of our approach.

Most of the studies on transition form factors has concentrated on the case of only one virtual photon. In Fig. 7, we show $Q^2 F_{\gamma^*\gamma^*\rightarrow\eta'}(Q^2)$ for different values of

FIG. 5. Invariant cross section for ϕ production at $\sqrt{s} = 200$ GeV, 2.76 TeV, and 8 TeV. We show the experimental data of the PHENIX Collaboration [31], ALICE Collaboration [32], and results from Ref. [33]. Here, $\Psi(0)$ was calculated from Eq. (2.5).

FIG. 6. $F_{\gamma^*\gamma^*\rightarrow\eta'}(0,0)$ as a function of β and m_{eff} . FIG. 7. $Q^2 F(Q^2)$ as a function of one photon virtuality. We show results for $m_{\text{eff}} = 0.4$ GeV and for different values of $\beta = 0.4, 0.5, 0.6$ GeV (from bottom to top). For comparison, we show experimental data from Refs. [37–39].

model parameters $\beta = 0.4, 0.5, 0.6$ GeV. In the leading-twist approach, without QCD evolution of distribution amplitudes, one should get a constant at large photon virtualities. In our approach, this happens at extremely large virtualities. Below $Q^2 < 50$ GeV, our model clearly contains higher twists. In the collinear leading-twist approach, the rise of $Q^2 F_{\gamma^*\gamma^*\rightarrow\eta'}(Q^2)$ is caused by the evolution in one-photon virtuality (see, e.g., Ref. [36]).

In Fig. 8, we show the two-photon η' form factor as a function of both photon virtualities. The form factor drops quickly from $F_{\gamma^*\gamma^*\rightarrow\eta'}(0,0)$ in the region $Q_1^2 < 10$ GeV², $Q_2^2 < 10$ GeV². The change beyond this region is rather mild. We show our result obtained using the light-cone wave function (left panel) and for comparison the result from Ref. [29] (right panel). The leading-twist result is reliable only for larger virtualities. Both the results are similar for larger virtualities.

To better visualize our result, we show in Fig. 9 also the ratio

$$R(Q_1^2, Q_2^2) = F^{LC}(Q_1^2, Q_2^2)/F^{nf, \text{monopole}}(Q_1^2, Q_2^2) \quad (3.2)$$

and similar one obtained when F^{LC} is replaced by (2.24).

We observe that the form factor calculated from (2.23) deviates only slightly from the simple parametrization (2.25). The two parametrizations almost coincide in the broad range of (Q_1^2, Q_2^2) . A similar result is obtained when using Eq. (2.24) with asymptotic distribution amplitude.

In Fig. 10, we show the two-photon transition form factor as a function of asymmetry parameter ω [see Eq. (2.16)] for different values of $\bar{Q}^2$ specified in the figure caption. In contrast to the nonfactorizable monopole form factor (2.25), we get some dependence on asymmetry parameter ω . This dependence is somewhat similar to the result of Ref. [29]. In contrast to Ref. [29], our dependence on ω is not universal, i.e., different for different values of $\bar{Q}^2$.

The two-photon form factor will be transformed to two-gluon form factor, and the latter will be used in the

FIG. 8. $F_{\gamma^*\gamma^*\rightarrow\eta'}(Q_1^2, Q_2^2)$ obtained with the light-cone wave function for $m_{\text{eff}} = 0.4$ GeV and $\beta = 0.5$ GeV (left panel) and the leading-twist result from Ref. [29] (right panel).

FIG. 9. $R(Q_1^2, Q_2^2)$ [see Eq. (3.2)] with the $\gamma^*\gamma^* \rightarrow \eta'$ form factor calculated with η' light-cone wave function (left panel). In this calculation, $m_{\text{eff}} = 0.4$ GeV and $\beta = 0.5$ GeV are used for example. In the right panel, we show a similar ratio obtained from Eq. (2.24) with asymptotic distribution amplitude.

calculation of η' production. For this purpose, first a grid for $F_{\gamma^*\gamma^* \rightarrow \eta'}$ in the $(\bar{Q}^2, \omega)$ plane is prepared. The grid is then used in the interpolation when calculating differential distributions of η' meson in proton-proton collisions.

C. η' production

In this subsection, we discuss the η' production considering the simple gluon-gluon fusion mechanism illustrated in Fig. 2.

FIG. 10. The $\gamma^*\gamma^* \rightarrow \eta'$ form factor as a function of ω for fixed values of $\bar{Q}^2$ (from top to bottom: 2, 5, 10, 20, 50, 100 GeV^2) calculated with η' light-cone wave function. In this calculation, $m_{\text{eff}} = 0.4$ GeV, and $\beta = 0.5$ GeV.

What are typical gluon transverse momenta for the hadronic $g^*g^* \rightarrow \eta'$ process? In Fig. 11, we show the distribution of the cross section integrated over η' transverse momenta in the (q_{1t}, q_{2t}) . Both small and large gluon virtualities enter into the p_t -integrated cross section.

In Fig. 12, we show similar distributions as above but for two different regions of meson transverse momentum p_t . For large p_t rather large gluon transverse momenta are involved.

Figure 13 shows a similar distributions, but in the (q_{1t}^2, q_{2t}^2) space used usually for presentation of transition form factors. One can observe that at large transverse momentum p_t one is sensitive to the region of Q_1^2 very small and Q_2^2 very large or Q_1^2 very large and Q_2^2 very small. These are regions relevant for the leading-twist collinear

FIG. 11. Two-dimensional map in (q_{1t}, q_{2t}) for the full range of η' transverse momentum. Here, $\sqrt{s} = 200$ GeV, and the KMR unintegrated gluon distribution function was used.

FIG. 12. Two-dimensional map in (q_{1t}, q_{2t}) for two different ranges of η' transverse momentum: $4 \text{ GeV} < p_t < 6 \text{ GeV}$ (left panel) and $9 \text{ GeV} < p_t < 11 \text{ GeV}$ (right panel). Here, $\sqrt{s} = 200 \text{ GeV}$, and the KMR UGDF was used.

FIG. 13. Two-dimensional map in (q_{1t}^2, q_{2t}^2) for $4.5 \text{ GeV} < p_t < 5.5 \text{ GeV}$ (left panel) and $9.5 \text{ GeV} < p_t < 10.5 \text{ GeV}$ (right panel). Here, $\sqrt{s} = 200 \text{ GeV}$. In this calculation, the KMR UGDF and the light-cone wave function with $\beta = 0.5 \text{ GeV}$ are used.

approach to two-gluon transition form factor. This is also the region of the phase space where the meson light-cone approach gives a small relative enhancement compared to the naive monopole parametrizations (see Fig. 9). We observe that at $p_t \sim 10 \text{ GeV}$ the gluon virtualities $Q_1^2 > 20 \text{ GeV}^2$ or $Q_2^2 > 20 \text{ GeV}^2$ are clearly in the domain of leading-twist approach [23].

This is shown better in Fig. 14, where we display distribution in $\bar{Q}^2$. Only large $\bar{Q}^2$ occur for $p_t > 10 \text{ GeV}$. This situation is generic, independent of the form factor used.

Finally, in Fig. 15, we show the invariant cross section for the η' meson production for the Relativistic Heavy Ion Collider (RHIC) energy $\sqrt{s} = 200 \text{ GeV}$ relevant for the PHENIX experiment [31]. In the left panel, we show different results: (a) with nonfactorized monopole form factor (solid line), (b) with the form factor calculated from the LCWF with $\beta = 0.5 \text{ GeV}$ (dashed line), (c) leading-twist parametrization (2.11) of the $F_{g^*g^* \rightarrow \eta'}$ result from Ref. [23] (dot-dashed line), (d) without $F_{g^*g^* \rightarrow \eta'}$, except of normalization constant (dotted line).

The result obtained with the leading-twist parametrization (2.11) can be taken seriously only for $p_t > 5 \text{ GeV}$, when Q_1^2 or Q_2^2 are bigger than 5 GeV^2 . We also used the formalism of collinear distribution amplitudes from Ref. [23], including their QCD evolution, to calculate $F_{g^*g^* \rightarrow \eta'}(q_{1t}^2, q_{2t}^2)$ from evolved $q\bar{q}$ and gg distribution amplitudes. The evolution scale of distribution amplitudes is taken as $\mu^2 = \bar{Q}^2 + \mu_0^2$, where $\mu_0^2 = 1 \text{ GeV}^2$ is used in our calculation. The corresponding result is shown by the red thick solid line. The line is below other lines in the region where the experimental data exist. For comparison, we show also result with $q\bar{q}$ alone (red thick dashed line) and gg alone (red thick dotted line) in Eq. (2.13). Both the results are much bigger than the result when both components are included coherently. Clearly, a strong destructive interference effect of both contributions is observed. The opposite sign of the gg distribution amplitude would cause constructive interference in Eq. (2.13).

In all cases, we get smaller cross section than measured by the PHENIX Collaboration at the RHIC. This suggests that the gluon-gluon fusion is probably not the dominant mechanism of η' production, at least in the measured region

FIG. 14. Distribution in $Q_{ave}^2 = \bar{Q}^2$ for the two distinct cases from the previous figure: $p_t = 5 \pm 0.5$ GeV (solid line) and $p_t = 10 \pm 0.5$ GeV (dashed line). Here, $\sqrt{s} = 200$ GeV. In this calculation, the KMR UGDF and the light-cone wave function with $\beta = 0.5$ GeV are used.

of transverse momenta. A natural candidate is the fragmentation process, which was not discussed in the literature so far in the context of η' production.

Neglecting the form factor at all leads to overestimation of the cross section at large transverse momenta of η' (see the dotted line in the left panel of Fig. 15). Such a result one could expect in the transverse momentum dependent gluon distributions (TMD) approach [40] in which the incoming gluons should be taken on mass shell. The present result

therefore show shortcomings of the TMD approach in the context of meson production.

To better illustrate the role of the initial gg component in the approach with distribution amplitudes in Fig. 16, we show the final (including QCD evolution) result with different initial gg component: as in Ref. [23] (plus), with opposite sign (minus) and with initial gg component put to zero. The final results are quite different. We conclude that the η' transverse momentum distribution is very sensitive to the unknown nonperturbative gg distribution amplitude.

So far, we used only one unintegrated gluon distribution. In Fig. 17, we compare results obtained using different UGDFs. The result obtained with the Jung-Hautmann UGDF is similar to that obtained with the KMR UGDF. The Golec-Biernat-Wuesthoff (GBW) UGDF gives sizeable cross section only at low η' transverse momenta as it does not include higher-order perturbative effects.

What about larger energies? The number of η' per event as a function of η' transverse momentum is shown in Fig. 18 for $\sqrt{s} = 8$ TeV. We show the results for the nonfactorized monopole (2.25) two-photon transition form factor (solid line), for light-cone wave function with $\beta = 0.5$ GeV (dashed line), with the simple leading-twist parametrization (2.11) of the two-gluon transition form factor, and of the collinear approach with evolution of distribution amplitudes [see Eq. (2.12)]. For comparison, we show also the result from the Lund string model. The two-gluon mechanism gives a much smaller cross section than that from the Lund string model. So, even at the LHC, we do not find any region of the phase space (y, p_t) where the two-gluon fusion is the dominant mechanism of η' production.

FIG. 15. Invariant cross section as a function of meson transverse momentum. Here, $\sqrt{s} = 200$ GeV, and the KMR UGDF was used in the calculation. In the left panel are results for the nonfactorized monopole, LCWF with $\beta = 0.5$ GeV, and simple leading twist parametrization. In the right panel, we show results obtained using distribution amplitudes from Ref. [23]. We show the full result as well as the result when only $q\bar{q}$ or only gg components in (2.13) are included.

FIG. 16. Invariant cross section as a function of meson transverse momentum in the approach with distribution amplitudes and different initial Φ_{gg} . Here, $\sqrt{s} = 200$ GeV, and the KMR UGDF was used in the calculation.

FIG. 17. Invariant cross section as a function of meson transverse momentum. Here, $\sqrt{s} = 200$ GeV. We show results with the KMR (solid line), Jung-Hautmann (dashed line), and GBW (dot-dashed line) UGDFs. In this calculation, the form factor based on the LCWF with $\beta = 0.5$ GeV is used for illustration.

IV. CONCLUSIONS

In this study, we have considered production of two (ϕ and η') isoscalar mesons with hidden strangeness via gluon-gluon fusion in proton-proton collisions for different collision energies relevant for the RHIC and the LHC. The calculations have been performed within the

FIG. 18. Number of η' mesons per event as a function of meson transverse momentum. Here, $\sqrt{s} = 8$ TeV, and the KMR UGDF was used in the calculation. The result of the Lund string model simulations is shown as “data points” for comparison.

k_t -factorization approach with the KMR UGDF, which is known to include effectively higher-order corrections [41,42].

For the ϕ production, we have extended the calculation performed earlier for the $J/\psi g$ production by using an effective spatial wave function at the origin $R_{s\bar{s}}(0)$, which can be estimated from the decay $\phi \rightarrow e^+e^-$ by adjusting it to experimental branching ratio. Having found the parameter, we have compared results of our calculation with the PHENIX and ALICE experimental data. In both cases, the calculated cross section stays below the experimental data by 2 (PHENIX) and by 1 (ALICE) order of magnitude. This shows that another mechanism is more important. The fragmentation of $s/\bar{s} \rightarrow \phi$ is a natural candidate.

Inspired by the successful description of η_c production in proton-proton collisions [1], here we have considered the $g^*g^* \rightarrow \eta'$ fusion with off-shell initial gluons. The coupling can be described by the two-gluon nonperturbative transition form factor. For the quark-antiquark states, the latter object is closely related to the two-photon transition form factor, studied theoretically and measured by the CLEO, L3, and BABAR collaborations.

The two-photon form factor has been calculated using a light-cone wave function for different values of model parameters. The result obtained with the form factor has been compared with a simple nonfactorized monopole parametrization as well as the results obtained recently by Kroll and Passek-Kumericki in the leading-twist collinear next-to-leading-order approach.

The two-photon form factors have been translated to the two-gluon ones, assuming the dominance of the quark-antiquark components in the Fock η' wave function expansion. Then, it was used in the k_T -factorization approach to calculate the cross section for η' production in pp collisions. The results have been compared with the PHENIX experimental data. In spite of the expectation of the community, the calculated cross section is definitely smaller than the measured one obtained by the PHENIX Collaboration. The situation may improve at larger energies, but the relevant cross section at the LHC was not measured so far. We have presented our predictions for the LHC and have compared our two-gluon fusion result with the result from the PYTHIA generator. For $\sqrt{s} = 8$ TeV, the cross section from the Lund string model is much above that for the two-gluon fusion mechanism. Respective data from the ALICE Collaboration would be very important to clarify the situation.

ACKNOWLEDGMENTS

A. S. is indebted to Wolfgang Schäfer for long-standing collaboration on quarkonia production. We are also indebted to Francois Fillion-Gourdeau for pointing to us Ref. [43], Kornelija Passek-Kumericki for providing us the two-photon η' form factor from their leading-twist analysis in Ref. [29] and interesting discussion on transition form factors, Arvind Khuntia for providing us experimental data of the ALICE Collaboration for ϕ production at $\sqrt{s} = 8$ TeV presented in his Ph.D. thesis, Jacek Biernat and Jacek Otwinowski for providing us results of the Lund string model generator PYTHIA, Francesco Giacosa for a discussion on gluonic components in mesons, and Jacek Okołowicz for careful reading of this manuscript. This study was partially supported by the Polish National Science Center Grant No. UMO-2018/31/B/ST2/03537 and by the Center for Innovation and Transfer of Natural Sciences and Engineering Knowledge in Rzeszów.

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