$sQ\bar{q}\bar{q}$ ($q = u, d$; $Q = c, b$) tetraquarks in the chiral quark modelGang Yang^{1,*}, Jialun Ping^{2,†} and Jorge Segovia^{3,‡}¹*Department of Physics, Zhejiang Normal University, Jinhua 321004, China*²*Department of Physics and Jiangsu Key Laboratory for Numerical Simulation of Large Scale Complex Systems, Nanjing Normal University, Nanjing 210023, China*³*Departamento de Sistemas Físicos, Químicos y Naturales, Universidad Pablo de Olavide, E-41013 Sevilla, Spain*

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The low-lying $sQ\bar{q}\bar{q}$ ($q = u, d$, $Q = c, b$) tetraquark states with $J^P = 0^+, 1^+$, and 2^+ , and in the isoscalar and isovector sectors, are systematically investigated in the framework of the real- and complex-scaling range of a chiral quark model, whose parameters are fixed in advance describing hadron, hadron-hadron, and multi-quark phenomenology, and thus, all results presented here are pure predictions. Each tetraquark configuration compatible with the quantum numbers studied is taken into account; this includes meson-meson, diquark-antidiquark, and K-type arrangements of quarks with all possible color wave functions in the four-body sector. Among the different numerical techniques to solve the Schrödinger-like four-body bound-state equation, we use a variational method in which the trial wave function is expanded in complex-range Gaussian basis functions because of its simplicity and flexibility. Several compact bound states and narrow resonances are found in both charm-strange $cs\bar{q}\bar{q}$ and bottom-strange $bs\bar{q}\bar{q}$ tetraquark sectors, most of them as a product of the strong coupling between the different channels. In regard to the so-called $X_{0,1}(2900)$ structures recently discovered with good statistical significance by the LHCb Collaboration, whose properties point out to be $c\bar{s}u\bar{d}$ tetraquarks, we find that they are unstable in our formalism with several candidates in the single-channel computations. We distinguish four cases with $I(J^P) = 0(0^+)$ quantum numbers: three K-type configurations with masses at 2.89, 2.96, and 2.97 GeV, and one hidden-color coupled-channel calculation delivering a mass 2.86 GeV. With quantum numbers $I(J^P) = 0(1^+)$, only one candidate can be found in the hidden-color coupled-channel calculation with mass 2.94 GeV.

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The LHCb Collaboration using proton-proton collision data taken at $\sqrt{s} = 7, 8$, and 13 TeV, with an integrated luminosity of 9 fb^{-1} , has recently performed an amplitude analysis [1] and a model-independent study of structure [2] in $B^+ \rightarrow D^+ D^- K^+$ decays. In both research works, in order to obtain good agreement with the data, it is found to be necessary to include new spin-0 and spin-1 charm-strange resonances in the $D^- K^+$ channel with Breit-Wigner parameters [1]:

$$X_0(2900): M = (2866 \pm 7) \text{ MeV},$$

$$\Gamma = (57 \pm 13) \text{ MeV},$$

$$X_1(2900): M = (2904 \pm 5) \text{ MeV},$$

$$\Gamma = (110 \pm 12) \text{ MeV}.$$

With an overwhelming significance, these signals constitute the first clear observation of exotic hadrons that do not contain a heavy-quark-antiquark pair; even more, they might be the first experimental detection of tetraquark candidates with four different flavors of quarks: $u\bar{d}\bar{s}c$. It is worth mentioning that alternative models incorporating additional hadronic effects such as rescattering may also be able to accommodate these $D^- K^+$ structures (see, for instance, the nice review [3]). More detailed experimental investigations will require larger data samples and studies of additional decay modes.

The announcement of the LHCb Collaboration triggers many theoretical works using a wide variety of approaches. Before going through a brief review of them, it is fair to

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highlight that a couple of theoretical works indicating the possible existence of $ud\bar{s}\bar{c}$ structures were available in the literature before the release of the LHCb findings. At least, the lowest-lying state at a mass of 2.85 GeV was identified using a color-magnetic interaction model [4] and a coupled-channel unitary approach [5].

Concerning the $X_0(2900)$, many theoretical studies assign to this state total spin and parity quantum numbers $J^P = 0^+$. Within this assignment, an S -wave $\bar{D}^*K^*(D^*\bar{K}^*)$ molecular interpretation is shared by QCD sum rules [6,7], one-boson-exchange models [8,9], a quark delocalization color screening model [10], an effective field theory approach [11], and by analyzing strong decay processes [12]. However, some other theoretical studies explore the possibility of having a more compact tetraquark configuration for the $X_0(2900)$ state with either a dimeson or diquark-antidiquark configuration. For instance, Ref. [13] uses two-body chromomagnetic interactions to find that $X_0(2900)$ can be interpreted as a radial excited $0^+ ud\bar{s}\bar{c}$ tetraquark. Other examples are QCD sum rules [14–16] and nonrelativistic and relativized quark models [17–19]. All mentioned theoretical works have also delivered interpretations of the $X_1(2900)$. For instance, Ref. [13] concludes that the $X_1(2900)$ state is an orbitally excited compact tetraquark with $J^P = 1^-$, whereas a molecular interpretation within a chiral quark model is delivered in Ref. [20]. In any case, most theoretical works support the $J^P = 1^-$ assignment [9,21,22].

Note, however, that debates on the nature of the $X_0(2900)$ and $X_1(2900)$ exotic states are still intense, with other alternative interpretations not yet discarded. In an extended relativized quark model [23], the $X_0(2900)$ signal cannot be identified as a compact $ud\bar{s}\bar{c}$ tetraquark state with $J^P = 0^+$. By solving the Lippmann-Schwinger equation [24], $X_0(2900)$ can be interpreted as a $\bar{D}^*K^*$ hadronic molecule state but with quantum numbers $I(J^P) = 0(1^+)$. Within an effective field theory approach [25], the $X_1(2900)$ cannot be explained as a bound state of $\bar{D}_1K$, whereas a DK_1 bound state is possible at around 3.1 GeV. Moreover, these exotic states can also be induced by triangle singularities [26,27].

Additionally, decay and production properties have also been studied theoretically. Within the assumption that $X_0(2900)$ is an S -wave $\bar{D}^*K^*$ molecular state, its decay properties have been analyzed by using an effective field theory approach in Ref. [28]. The production of $X_{0,1}(2900)$ via weak decays of a B meson [29,30] and the hadronic effects on $X_{0,1}(2900)$ in heavy-ion collisions [31] have been also released.

The goal of the present manuscript is to study the charm-strange tetraquark candidates seen by the LHCb Collaboration using a QCD-inspired chiral quark model which has already been successfully applied in the description of other multiquark systems, e.g., hidden-charm pentaquarks $P_c^+(4312)$, $P_c^+(4380)$, $P_c^+(4440)$, and $P_c(4457)^+$ [32],

doubly charm pentaquarks [33], and doubly heavy tetraquarks $QQ\bar{q}\bar{q}$ ($q = u, d, s$; $Q = c, b$) [34,35]. Moreover, the formulation in the real- and complex-scaling method of the theoretical formalism has been discussed in detail in Ref. [36]. The complex-scaling method (CSM) allows us to distinguish three kinds of poles—bound, resonance, and scattering—and thus perform a complete analysis of the scattering singularities within the same formalism. Furthermore, the meson-meson, diquark-antidiquark, and K-type configurations plus their couplings shall be considered for the $sc\bar{q}\bar{q}$ tetraquark system. Finally, as a natural extension, the bottom-strange tetraquarks are also studied herein.

The manuscript is arranged as follows. In Sec. II the theoretical framework is presented; we briefly present the complex-range method applied to a chiral quark model and the $sQ\bar{q}\bar{q}$ ($q = u, d$; $Q = c, b$) tetraquark wave functions. Section III is devoted to the analysis and discussion of the obtained low-lying $sQ\bar{q}\bar{q}$ ($q = u, d$, $Q = c, b$) tetraquark states with $J^P = 0^+$, 1^+ , and 2^+ , and isospin $I = 0$ and 1. Finally, we summarize and give some prospects in Sec. IV.

II. THEORETICAL FRAMEWORK

A thorough review of the theoretical formalism used herein has been recently published in Ref. [36]. We shall, however, focus on the most relevant features of the chiral quark model and the numerical method concerning the charm- (bottom-) strange tetraquarks, viz., the $sQ\bar{q}\bar{q}$ system with Q denoting either a c or b quark, the strange quark written as an s quark and q denotes the light u and d quarks, isospin symmetry is assumed.

Within the so-called complex-range studies, the relative coordinate of a two-body interaction is rotated in the complex plane by an angle θ , i.e., $\vec{r}_{ij} \rightarrow \vec{r}_{ij}e^{i\theta}$. Therefore, the general form of the four-body Hamiltonian reads [37,38]

$$H(\theta) = \sum_{i=1}^4 \left(m_i + \frac{\vec{p}_i^2}{2m_i} \right) - T_{\text{CM}} + \sum_{j>i=1}^4 V(\vec{r}_{ij}e^{i\theta}), \quad (1)$$

where m_i is the quark mass, $\vec{p}_i$ is the quark's momentum, and T_{CM} is the center-of-mass kinetic energy. According to the so-called ABC theorem [39,40], the complex scaled Schrödinger equation

$$[H(\theta) - E(\theta)]\Psi_{JM}(\theta) = 0 \quad (2)$$

has (complex) eigenvalues which can be classified into three types, namely, the bound, resonance, and scattering states. In particular, bound states and resonances are independent of the rotated angle θ , with the first ones always fixed on the coordinate axis (there is no imaginary part of the eigenvalue) and the second ones located above the continuum threshold with a total decay width $\Gamma = -2\text{Im}(E)$.

The dynamics of the $sQ\bar{q}\bar{q}$ tetraquark system is driven by a two-body potential

$$V(\vec{r}_{ij}) = V_\chi(\vec{r}_{ij}) + V_{\text{CON}}(\vec{r}_{ij}) + V_{\text{OGE}}(\vec{r}_{ij}), \quad (3)$$

which takes into account the most relevant features of QCD at its low energy regime: dynamical chiral symmetry breaking, confinement, and the perturbative one-gluon exchange (OGE) interaction. Herein, the low-lying S -wave positive parity $sQ\bar{q}\bar{q}$ tetraquark states shall be investigated, and thus, the central and spin-spin terms of the potential are the only ones needed.

One consequence of the dynamical breaking of chiral symmetry is that Goldstone-boson-exchange interactions appear between constituent light quarks u , d , and s . Therefore, the chiral interaction can be written as [41]

$$V_\chi(\vec{r}_{ij}) = V_\pi(\vec{r}_{ij}) + V_\sigma(\vec{r}_{ij}) + V_K(\vec{r}_{ij}) + V_\eta(\vec{r}_{ij}) \quad (4)$$

given by

$$V_\pi(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\pi^2}{12m_i m_j} \frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} m_\pi \left[Y(m_\pi r_{ij}) - \frac{\Lambda_\pi^3}{m_\pi^3} Y(\Lambda_\pi r_{ij}) \right] (\vec{\sigma}_i \cdot \vec{\sigma}_j) \sum_{a=1}^3 (\lambda_i^a \cdot \lambda_j^a), \quad (5)$$

$$V_\sigma(\vec{r}_{ij}) = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} m_\sigma \left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right], \quad (6)$$

$$V_K(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_K^2}{12m_i m_j} \frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} m_K \left[Y(m_K r_{ij}) - \frac{\Lambda_K^3}{m_K^3} Y(\Lambda_K r_{ij}) \right] (\vec{\sigma}_i \cdot \vec{\sigma}_j) \sum_{a=4}^7 (\lambda_i^a \cdot \lambda_j^a), \quad (7)$$

$$V_\eta(\vec{r}_{ij}) = \frac{g_{ch}^2}{4\pi} \frac{m_\eta^2}{12m_i m_j} \frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2} m_\eta \left[Y(m_\eta r_{ij}) - \frac{\Lambda_\eta^3}{m_\eta^3} Y(\Lambda_\eta r_{ij}) \right] (\vec{\sigma}_i \cdot \vec{\sigma}_j) [\cos \theta_p (\lambda_i^8 \cdot \lambda_j^8) - \sin \theta_p], \quad (8)$$

where $Y(x) = e^{-x}/x$ is the standard Yukawa function. The physical η meson, instead of the octet one, is considered by introducing the angle θ_p . The λ^a are the SU(3) flavor Gell-Mann matrices. Taken from their experimental values, m_π , m_K , and m_η are the masses of the SU(3) Goldstone bosons. The value of m_σ is determined through the partial conservation of axial-vector current relation $m_\sigma^2 \simeq m_\pi^2 + 4m_{u,d}^2$ [42]. Finally, the chiral coupling constant g_{ch} is determined from the πNN coupling constant through

$$\frac{g_{ch}^2}{4\pi} = \frac{9}{25} \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}, \quad (9)$$

which assumes that flavor SU(3) is an exact symmetry only broken by the different mass of the strange quark.

Color confinement should be encoded in the non-Abelian character of QCD. It has been demonstrated by lattice-regularized QCD that multigluon exchanges produce an attractive linearly rising potential proportional to the distance between infinite-heavy quarks [43]. However, the spontaneous creation of light-quark pairs from the QCD vacuum may give rise at the same scale to a breakup of the created color flux tube [43]. These two observations can be described phenomenologically by

$$V_{\text{CON}}(\vec{r}_{ij}) = [-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta](\lambda_i^c \cdot \lambda_j^c), \quad (10)$$

where a_c , μ_c , and Δ are model parameters,¹ and the SU(3) color Gell-Mann matrices are denoted as λ^c . One can see in Eq. (10) that the potential is linear at short interquark distances with an effective confinement strength $\sigma = -a_c \mu_c (\lambda_i^c \cdot \lambda_j^c)$, while it becomes constant at large distances, $V_{\text{thr}} = (\Delta - a_c)(\lambda_i^c \cdot \lambda_j^c)$.

Beyond the chiral symmetry breaking scale, one expects the dynamics to be governed by QCD perturbative effects. In particular, the one-gluon exchange potential, which includes the so-called Coulomb and color-magnetism interactions, is the leading order contribution:

$$V_{\text{OGE}}(\vec{r}_{ij}) = \frac{1}{4} \alpha_s (\lambda_i^c \cdot \lambda_j^c) \left[\frac{1}{r_{ij}} - \frac{1}{6m_i m_j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{r_{ij} r_0^2(\mu_{ij})} \right], \quad (11)$$

where $r_0(\mu_{ij}) = \hat{r}_0/\mu_{ij}$ is a regulator which depends on the reduced mass of the $q\bar{q}$ pair, the Pauli matrices are denoted by $\vec{\sigma}$, and the contact term has been regularized as

$$\delta(\vec{r}_{ij}) \sim \frac{1}{4\pi r_0^2(\mu_{ij})} \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{r_{ij}}. \quad (12)$$

An effective scale-dependent strong coupling constant $\alpha_s(\mu_{ij})$ provides a consistent description of mesons and baryons from the light- to heavy-quark sectors. We use the frozen coupling constant of, for instance, Ref. [46]

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln\left(\frac{\mu_{ij}^2 + \mu_0^2}{\Lambda_0^2}\right)}, \quad (13)$$

in which α_0 , μ_0 , and Λ_0 are parameters of the model.

¹It is widely believed that confinement is flavor independent, and thus, it should be constrained by the light hadron spectra, despite our aim to determine the energy states in heavier quark sectors [44,45].

TABLE I. Model parameters.

Quark masses	$m_q (q = u, d)$ (MeV)	313
	m_s (MeV)	555
	m_c (MeV)	1752
	m_b (MeV)	5100
Goldstone bosons	$\Lambda_\pi = \Lambda_\sigma$ (fm $^{-1}$)	4.20
	$\Lambda_\eta = \Lambda_K$ (fm $^{-1}$)	5.20
	$g_{ch}^2/(4\pi)$	0.54
	$\theta_P(^{\circ})$	-15
Confinement	a_c (MeV)	430
	μ_c (fm $^{-1}$)	0.70
	Δ (MeV)	181.10
	α_0	2.118
OGE	Λ_0 (fm $^{-1}$)	0.113
	μ_0 (MeV)	36.976
	$\hat{r}_0$ (MeV fm)	28.170

The model parameters are listed in Table I. They have been fixed in advance, reproducing hadron [47–51], hadron-hadron [52–56], and multi-quark [32,57–60] phenomenology. Additionally, for later concern, Table II lists the theoretical and experimental (if available) masses of the $1S$ and $2S$ states of the $K^{(*)}$, $D^{(*)}$, and $B^{(*)}$ mesons.

Figure 1 shows six kinds of configurations for the $sQ\bar{q}\bar{q}$ tetraquark system. In particular, Fig. 1(a) is the meson-meson structure, Fig. 1(b) is the diquark-antidiquark one, and the other K-type configurations are from Figs 1(c)–1(f). All of them, and their couplings, are considered in our investigation. However, for the purpose of solving a manageable four-body problem, the K-type configurations are sometimes restricted. It is important to note that herein just one configuration would be enough for the calculation if all radial and orbital excited states were taken into account; however, this is obviously much less efficient, and thus, an economical way is to combine the different configurations in the ground state to perform the calculation.

TABLE II. Theoretical and experimental (if available) masses of $nL = 1S$ and $2S$ states of $K^{(*)}$, $D^{(*)}$, and $B^{(*)}$ mesons.

Meson	nL	M_{The} (MeV)	M_{Exp} (MeV)
K	$1S$	481	494
	$2S$	1468	...
K^*	$1S$	907	892
	$2S$	1621	...
D	$1S$	1897	1870
	$2S$	2648	...
D^*	$1S$	2017	2007
	$2S$	2704	...
B	$1S$	5278	5280
	$2S$	5984	...
B^*	$1S$	5319	5325
	$2S$	6005	...

FIG. 1. Six types of configurations in $sQ\bar{q}\bar{q}$ ($q = u, d$, $Q = c$, b) tetraquarks. Panel (a) is the meson-meson structure, panel (b) is the diquark-antidiquark one, and the other K-type structures are panels (c)–(f).

The multi-quark system's wave function at the quark level is an internal product of color, spin, flavor, and space terms. Concerning the color degree of freedom, the colorless wave function of a four-quark system in meson-meson configuration, as illustrated in Fig. 1(a), can be obtained by either two coupled color-singlet clusters $1 \otimes 1$,

$$\chi_1^c = \frac{1}{3}(\bar{r}r + \bar{g}g + \bar{b}b) \times (\bar{r}r + \bar{g}g + \bar{b}b), \quad (14)$$

or two coupled color-octet clusters $8 \otimes 8$,

$$\begin{aligned} \chi_2^c = & \frac{\sqrt{2}}{12}(3\bar{b}r\bar{r}b + 3\bar{g}r\bar{r}g + 3\bar{b}g\bar{g}b + 3\bar{g}b\bar{b}g + 3\bar{r}g\bar{g}r \\ & + 3\bar{r}b\bar{b}r + 2\bar{r}r\bar{r}r + 2\bar{g}g\bar{g}g + 2\bar{b}b\bar{b}b - \bar{r}r\bar{g}g \\ & - \bar{g}g\bar{r}r - \bar{b}b\bar{g}g - \bar{b}b\bar{r}r - \bar{g}g\bar{b}b - \bar{r}r\bar{b}b). \end{aligned} \quad (15)$$

The first color state is the so-called color-singlet channel and the second one is the named hidden-color case.

The color wave functions associated with the diquark-antidiquark structure shown in Fig. 1(b) are the coupled color triplet-antitriplet clusters $3 \otimes \bar{3}$,

$$\begin{aligned} \chi_3^c = & \frac{\sqrt{3}}{6} (\bar{r}r\bar{g}g - \bar{g}r\bar{r}g + \bar{g}g\bar{r}r - \bar{r}g\bar{g}r + \bar{r}r\bar{b}b \\ & - \bar{b}r\bar{r}b + \bar{b}b\bar{r}r - \bar{r}b\bar{b}r + \bar{g}g\bar{b}b - \bar{b}g\bar{g}b \\ & + \bar{b}b\bar{g}g - \bar{g}b\bar{b}g), \end{aligned} \quad (16)$$

and the coupled color sextet-antisextet clusters $6 \otimes \bar{6}$,

$$\begin{aligned} \chi_4^c = & \frac{\sqrt{6}}{12} (2\bar{r}r\bar{r}r + 2\bar{g}g\bar{g}g + 2\bar{b}b\bar{b}b + \bar{r}r\bar{g}g + \bar{g}r\bar{r}g \\ & + \bar{g}g\bar{r}r + \bar{r}g\bar{g}r + \bar{r}r\bar{b}b + \bar{b}r\bar{r}b + \bar{b}b\bar{r}r \\ & + \bar{r}b\bar{b}r + \bar{g}g\bar{b}b + \bar{b}g\bar{g}b + \bar{b}b\bar{g}g + \bar{g}b\bar{b}g). \end{aligned} \quad (17)$$

Meanwhile, the colorless wave functions of the K-type structures shown in Figs. 1(c)–1(f) are

$$\begin{aligned} \chi_5^c = & \frac{1}{6\sqrt{2}} (\bar{r}r\bar{r}r + \bar{g}g\bar{g}g - 2\bar{b}b\bar{b}b) \\ & + \frac{1}{2\sqrt{2}} (\bar{r}b\bar{b}r + \bar{r}g\bar{g}r + \bar{g}b\bar{b}g + \bar{g}r\bar{r}g + \bar{b}g\bar{g}b + \bar{b}r\bar{r}b) \\ & - \frac{1}{3\sqrt{2}} (\bar{g}g\bar{r}r + \bar{r}r\bar{g}g) \\ & + \frac{1}{6\sqrt{2}} (\bar{b}b\bar{r}r + \bar{b}b\bar{g}g + \bar{r}r\bar{b}b + \bar{g}g\bar{b}b), \end{aligned} \quad (18)$$

$$\chi_6^c = \chi_1^c, \quad (19)$$

$$\chi_7^c = \chi_1^c, \quad (20)$$

$$\begin{aligned} \chi_8^c = & \frac{1}{4} \left(1 - \frac{1}{\sqrt{6}}\right) \bar{r}r\bar{g}g - \frac{1}{4} \left(1 + \frac{1}{\sqrt{6}}\right) \bar{g}g\bar{g}g - \frac{1}{4\sqrt{3}} \bar{r}g\bar{g}r \\ & + \frac{1}{2\sqrt{2}} (\bar{r}b\bar{b}r + \bar{g}b\bar{b}g + \bar{b}g\bar{g}b + \bar{g}r\bar{r}g + \bar{b}r\bar{r}b) \\ & + \frac{1}{2\sqrt{6}} (\bar{r}r\bar{b}b - \bar{g}g\bar{b}b + \bar{b}b\bar{g}g + \bar{g}g\bar{r}r - \bar{b}b\bar{r}r), \end{aligned} \quad (21)$$

$$\begin{aligned} \chi_9^c = & \frac{1}{2\sqrt{6}} (\bar{r}b\bar{b}r + \bar{r}r\bar{b}b + \bar{g}b\bar{b}g + \bar{g}g\bar{b}b + \bar{r}g\bar{g}r + \bar{r}r\bar{g}g \\ & + \bar{b}b\bar{g}g + \bar{b}g\bar{g}b + \bar{g}g\bar{r}r + \bar{g}r\bar{r}g + \bar{b}b\bar{r}r + \bar{b}r\bar{r}b) \\ & + \frac{1}{\sqrt{6}} (\bar{r}r\bar{r}r + \bar{g}g\bar{g}g + \bar{b}b\bar{b}b), \end{aligned} \quad (22)$$

$$\begin{aligned} \chi_{10}^c = & \frac{1}{2\sqrt{3}} (\bar{r}b\bar{b}r - \bar{r}r\bar{b}b + \bar{g}b\bar{b}g - \bar{g}g\bar{b}b + \bar{r}g\bar{g}r - \bar{r}r\bar{g}g \\ & - \bar{b}b\bar{g}g + \bar{b}g\bar{g}b - \bar{g}g\bar{r}r + \bar{g}r\bar{r}g - \bar{b}b\bar{r}r + \bar{b}r\bar{r}b), \end{aligned} \quad (23)$$

$$\chi_{11}^c = \chi_9^c, \quad (24)$$

$$\chi_{12}^c = -\chi_{10}^c. \quad (25)$$

As for the flavor degree of freedom, since the quark content of the investigated four-quark system is $sQ\bar{q}\bar{q}$, both the isoscalar $I = 0$ and isovector $I = 1$ sectors will be discussed. Note also that if the flavor wave function is denoted as χ_{I,M_I}^{fi} , the superscript $i = 1$ and 2 will refer to the $sc\bar{q}\bar{q}$ and $sb\bar{q}\bar{q}$ systems, respectively. The specific wave functions read as

$$\chi_{0,0}^{fi} = \frac{1}{\sqrt{2}} (\bar{u}s\bar{d}Q - \bar{d}s\bar{u}Q), \quad (26)$$

$$\chi_{1,0}^{fi} = \frac{1}{\sqrt{2}} (\bar{u}s\bar{d}Q + \bar{d}s\bar{u}Q), \quad (27)$$

where the third component of the isospin M_I is fixed to be zero for simplicity since the Hamiltonian does not have a flavor-dependent interaction which can distinguish the third component of the isospin quantum number.

We are going to consider S -wave ground states with spin ranging from $S = 0$ to 2. Therefore, the spin wave functions $\chi_{S,M_S}^{\sigma_i}$ are given by (M_S can be set to equal S without loss of generality)

$$\chi_{0,0}^{\sigma_{u_1}}(4) = \chi_{00}^{\sigma_0}\chi_{00}^{\sigma_0}, \quad (28)$$

$$\chi_{0,0}^{\sigma_{u_2}}(4) = \frac{1}{\sqrt{3}} (\chi_{11}^{\sigma_1}\chi_{1,-1}^{\sigma_1} - \chi_{10}^{\sigma_1}\chi_{10}^{\sigma_1} + \chi_{1,-1}^{\sigma_1}\chi_{11}^{\sigma_1}), \quad (29)$$

$$\begin{aligned} \chi_{0,0}^{\sigma_{u_3}}(4) = & \frac{1}{\sqrt{2}} \left(\left(\sqrt{\frac{2}{3}} \chi_{11}^{\sigma_1}\chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1} - \sqrt{\frac{1}{3}} \chi_{10}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1} \right) \chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1} \right. \\ & \left. - \left(\sqrt{\frac{1}{3}} \chi_{10}^{\sigma_1}\chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1} - \sqrt{\frac{2}{3}} \chi_{1,-1}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1} \right) \chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1} \right), \end{aligned} \quad (30)$$

$$\chi_{0,0}^{\sigma_{u_4}}(4) = \frac{1}{\sqrt{2}} (\chi_{00}^{\sigma_0}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1}\chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1} - \chi_{00}^{\sigma_0}\chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1}), \quad (31)$$

$$\chi_{1,1}^{\sigma_{w_1}}(4) = \chi_{00}^{\sigma_0}\chi_{11}^{\sigma_1}, \quad (32)$$

$$\chi_{1,1}^{\sigma_{w_2}}(4) = \chi_{11}^{\sigma_1}\chi_{00}^{\sigma_0}, \quad (33)$$

$$\chi_{1,1}^{\sigma_{w_3}}(4) = \frac{1}{\sqrt{2}} (\chi_{11}^{\sigma_1}\chi_{10}^{\sigma_1} - \chi_{10}^{\sigma_1}\chi_{11}^{\sigma_1}), \quad (34)$$

$$\begin{aligned} \chi_{1,1}^{\sigma_{w_4}}(4) = & \sqrt{\frac{3}{4}} \chi_{11}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1}\chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1} - \sqrt{\frac{1}{12}} \chi_{11}^{\sigma_1}\chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1} - \sqrt{\frac{1}{6}} \chi_{10}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1}, \end{aligned} \quad (35)$$

$$\chi_{1,1}^{\sigma_{w_5}}(4) = \left(\sqrt{\frac{2}{3}} \chi_{11}^{\sigma_1}\chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma_1} - \sqrt{\frac{1}{3}} \chi_{10}^{\sigma_1}\chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1} \right) \chi_{\frac{1}{2},\frac{1}{2}}^{\sigma_1}, \quad (36)$$

TABLE III. The values of the superscripts $u_1, \dots, u_4$ and $w_1, \dots, w_6$ that determine the spin wave function for each configuration of the $sQ\bar{q}\bar{q}$ tetraquark system.

	Dimeson	Diquark-antidiquark	K_1	K_2	K_3	K_4
u_1	1	3				
u_2	2	4				
u_3			5	7	9	11
u_4			6	8	10	12
w_1	1	4				
w_2	2	5				
w_3	3	6				
w_4			7	10	13	16
w_5			8	11	14	17
w_6			9	12	15	18

$$\chi_{1,1}^{\sigma_{w_6}}(4) = \chi_{00}^{\sigma} \chi_{\frac{1}{2},\frac{1}{2}}^{\sigma} \chi_{\frac{1}{2},\frac{1}{2}}^{\sigma}, \quad (37)$$

$$\chi_{2,2}^{\sigma_1}(4) = \chi_{11}^{\sigma} \chi_{11}^{\sigma}. \quad (38)$$

The superscripts $u_1, \dots, u_4$ and $w_1, \dots, w_6$ determine the spin wave function for each configuration of the $sQ\bar{q}\bar{q}$ tetraquark system, and their specific values are shown in Table III. Furthermore, the expressions above are obtained by considering the coupling of two subclusters whose spin wave functions are given by trivial SU(2) algebra, and the necessary basis reads as

$$\chi_{11}^{\sigma} = \chi_{\frac{3}{2},\frac{1}{2}}^{\sigma} \chi_{\frac{1}{2},\frac{1}{2}}^{\sigma}, \quad (39)$$

$$\chi_{1,-1}^{\sigma} = \chi_{\frac{3}{2},-\frac{1}{2}}^{\sigma} \chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma}, \quad (40)$$

$$\chi_{10}^{\sigma} = \frac{1}{\sqrt{2}} (\chi_{\frac{3}{2},\frac{1}{2}}^{\sigma} \chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma} + \chi_{\frac{3}{2},-\frac{1}{2}}^{\sigma} \chi_{\frac{1}{2},\frac{1}{2}}^{\sigma}), \quad (41)$$

$$\chi_{00}^{\sigma} = \frac{1}{\sqrt{2}} (\chi_{\frac{3}{2},\frac{1}{2}}^{\sigma} \chi_{\frac{1}{2},-\frac{1}{2}}^{\sigma} - \chi_{\frac{3}{2},-\frac{1}{2}}^{\sigma} \chi_{\frac{1}{2},\frac{1}{2}}^{\sigma}). \quad (42)$$

Among the different methods to solve the Schrödinger-like four-body bound-state equation, we use the Rayleigh-Ritz variational principle which is one of the most extended tools to solve eigenvalue problems because of its simplicity and flexibility. Moreover, we use the complex-range method, and thus, the spatial wave function is written as follows:

$$\psi_{LM_L}(\theta) = [[\phi_{n_1 l_1}(\vec{\rho} e^{i\theta}) \phi_{n_2 l_2}(\vec{\lambda} e^{i\theta})]_l \phi_{n_3 l_3}(\vec{R} e^{i\theta})]_{LM_L}, \quad (43)$$

where the internal Jacobi coordinates are defined as

$$\vec{\rho} = \vec{x}_1 - \vec{x}_2, \quad (44)$$

$$\vec{\lambda} = \vec{x}_3 - \vec{x}_4, \quad (45)$$

$$\vec{R} = \frac{m_1 \vec{x}_1 + m_2 \vec{x}_2}{m_1 + m_2} - \frac{m_3 \vec{x}_3 + m_4 \vec{x}_4}{m_3 + m_4} \quad (46)$$

for the meson-meson configuration of Fig. 1(a) and as

$$\vec{\rho} = \vec{x}_1 - \vec{x}_3, \quad (47)$$

$$\vec{\lambda} = \vec{x}_2 - \vec{x}_4, \quad (48)$$

$$\vec{R} = \frac{m_1 \vec{x}_1 + m_3 \vec{x}_3}{m_1 + m_3} - \frac{m_2 \vec{x}_2 + m_4 \vec{x}_4}{m_2 + m_4} \quad (49)$$

for the diquark-antidiquark structure of Fig. 1(b). The remaining K-type configurations shown in Figs. 1(c)–1(f) are [i, j, k, l take values according Figs. 1(c)–1(f)]

$$\vec{\rho} = \vec{x}_i - \vec{x}_j, \quad (50)$$

$$\vec{\lambda} = \vec{x}_k - \frac{m_i \vec{x}_i + m_j \vec{x}_j}{m_i + m_j}, \quad (51)$$

$$\vec{R} = \vec{x}_l - \frac{m_i \vec{x}_i + m_j \vec{x}_j + m_k \vec{x}_k}{m_i + m_j + m_k}. \quad (52)$$

It becomes obvious now that the center-of-mass kinetic term T_{CM} can be completely eliminated for a nonrelativistic system defined in any of the above sets of relative coordinates.

A crucial aspect of the Rayleigh-Ritz variational method is the basis expansion of the trial wave function. We are going to use the Gaussian expansion method [61] in which each relative coordinate is expanded in terms of Gaussian basis functions whose sizes are taken in geometric progression. This method has proven to be very efficient in solving the bound-state problem of multiquark systems [32–34,59], and the details on how the geometric progression is fixed can be found in, e.g., Ref. [32]. Therefore, the form of the orbital wave functions ϕ 's in Eq. (43) is

$$\phi_{nlm}(\vec{r} e^{i\theta}) = N_{nl} (r e^{i\theta})^l e^{-\nu_n (r e^{i\theta})^2} Y_{lm}(\hat{r}). \quad (53)$$

Since only S -wave states of charm- (bottom-) strange tetraquarks are investigated in this work, no laborious Racah algebra is needed while computing the matrix elements. In this case, the value of the spherical harmonic function is just a constant, viz., $Y_{00} = \sqrt{1/4\pi}$.

Finally, the complete wave function that fulfills the Pauli principle is written as

$$\Psi_{JM_J, I, i, j, k}(\theta) = \mathcal{A} [[\psi_L(\theta) \chi_S^{\sigma_i}(4)]_{JM_J} \chi_I^f \chi_k^c], \quad (54)$$

where $\mathcal{A}$ is the antisymmetry operator of $sQ\bar{q}\bar{q}$ tetraquarks which takes into account the fact of having two identical light antiquarks, ($\bar{q}\bar{q}$). Its definition according to Fig. 1 is

$$\mathcal{A} = 1 - (13). \quad (55)$$

This is necessary since the complete wave function of the four-quark system is constructed from two sub-clusters: meson-meson, diquark-antidiquark, and K-type configurations.

III. RESULTS

In the present calculation, we investigate all possible S-wave charm- (bottom-) strange tetraquarks by taking into account dimeson, diquark-antidiquark, and K-type configurations. In our approach, an $sQ\bar{q}\bar{q}$ tetraquark state has positive parity assuming that the angular momenta l_1 , l_2 , and l_3 in Eq. (43) are all equal to zero. Accordingly, the total angular momentum J coincides with the total spin S and can take the values of 0, 1, and 2. In addition, the value of isospin I can be either 0 or 1 considering the quark content of the $sQ\bar{q}\bar{q}$ system.

Tables IV–XXIII list our calculated results of the lowest-lying $sc\bar{q}\bar{q}$ and $sb\bar{q}\bar{q}$ tetraquark states. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column; when possible, the experimental value of the noninteracting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin (χ_i^s), flavor (χ_i^f), and color (χ_i^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy; moreover, their dominant components and the distances between any two quarks of the system are also discussed. When the CSM is used in the complete coupled-channel calculation, we show in Figs. 2–13 the distribution of complex eigenenergies, and, therein, the obtained bound and resonance states are indicated inside circles.

Let us proceed now to describe in detail our theoretical findings for each sector of $sQ\bar{q}\bar{q}$ tetraquarks, with Q either a c or a b quark.

A. The $sc\bar{q}\bar{q}$ tetraquarks

There are two bound states with quantum numbers $I(J^P) = 0(0^+)$ and $0(1^+)$, respectively. Moreover, several narrow resonances whose masses cluster around 3.1 and 3.5 GeV are found in the remaining $I(J^P)$ channels studied herein. Each isoscalar and vector sectors with total spin and parity $J^P = 0^+, 1^+, \text{ and } 2^+$ shall be discussed individually below.

1. The $I(J^P) = 0(0^+)$ sector

Two meson-meson channels DK and D^*K^* in both color-singlet and hidden-color configurations, two

FIG. 2. The complete coupled-channel calculation of the $sc\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 0(0^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

diquark-antidiquark channels $(sc)(\bar{q}\bar{q})$ and $(sc)^*(\bar{q}\bar{q})^*$, along with K-type configurations are individually studied in Table IV. One can see that the formation of a bound state is not possible in each single-channel calculation. The two singlet-color meson-meson states are located above their respective thresholds with masses at 2.38 and 2.92 GeV, respectively. The other tetraquark configurations—hidden-color meson-meson, diquark-antidiquark, and K-type channels—present states in an energy region which ranges from 2.7 to 3.7 GeV. If a coupled-channel calculation among each type of configuration is performed, a loosely bound state is found in the singlet-color meson-meson channel with a mass of 2374 MeV and a binding energy of -4 MeV. Furthermore, we observe a strong coupling effect in the complete coupled-channel calculation which helps in obtaining a tightly bound state whose mass and binding energy are 2342 and -36 MeV, respectively.

We have investigated the composition of this state in Table V. The dominant structure is of K-type contributing 55% to the total wave function, but traces of singlet-color DK (14%) and D^*K^* (10%) components are also found. Table VI indicates that all quark-quark (antiquark-quark) pairs are separated about the same distance with a value of around 1 fm. All of these features indicate that this state might be a compact $sc\bar{q}\bar{q}$ tetraquark.

In a further step, the complete coupled-channel calculation is performed using the complex-range method, and its output is shown in Fig. 2. Therein, with a rotated angle ranging from 0° to 6° , the bound state is fixed at 2342 MeV of the real axis, and we mark it with an orange circle. One can also see in Fig. 2 that the scattering states of $D(1S)K(1S)$, $D^*(1S)K^*(1S)$, and $D(2S)K(1S)$ are well presented within an interval of 2.3–3.3 GeV. No resonance pole is found around 2.9 GeV within the complete coupled-channel

TABLE IV. Lowest-lying $sc\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 0(0^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the noninteracting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_i^{\sigma_i}$), flavor ($\chi_i^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_i^{\sigma_i}; \chi_i^{f_j}; \chi_k^c$ [$i; j; k$]	M	Mixed
$(DK)^1(2364)$	1	[1; 1; 3]	2378	
$(D^*K^*)^1(2899)$	2	[2; 1; 3]	2924	2374
$(DK)^8$	3	[1; 1; 4]	3198	
$(D^*K^*)^8$	4	[2; 1; 4]	3014	2861
$(sc)(\bar{q}\bar{q})$	5	[3; 1; 1]	2735	
$(sc)^*(\bar{q}\bar{q})^*$	6	[4; 1; 2]	2997	2632
K_1	7	[5; 1; 5]	2899	
	8	[5; 1; 6]	3048	
	9	[6; 1; 5]	3163	
	10	[6; 1; 6]	2630	2561
K_2	11	[7; 1; 7]	2964	
	12	[7; 1; 8]	3011	
	13	[8; 1; 7]	2491	
	14	[8; 1; 8]	3183	2460
K_3	15	[9; 1; 9]	2978	
	16	[9; 1; 10]	3755	
	17	[10; 1; 9]	3777	
	18	[10; 1; 10]	2871	2722
K_4	19	[11; 1; 11]	2988	
	20	[12; 1; 12]	2739	2611
Complete coupled channels:				2342
				$E_B = -36$

calculation; nevertheless, several good candidates for the $X_0(2900)$ signal reported by the LHCb Collaboration are found in, for instance, single-channel calculations of K-type configurations: 2.89 GeV in K_1 , 2.96 GeV in K_2 , and

TABLE V. Relative strengths of various meson-meson, diquark-antidiquark, and K-type components in the wave function of the $I(J^P) = 0(0^+)$ $sc\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed. The superscripts 1 and 8 are for singlet-color and hidden-color channels, respectively.

$(DK)^1$	$(D^*K^*)^1$	$(DK)^8$	$(D^*K^*)^8$	$(sc)(\bar{q}\bar{q})$
13.5%	9.9%	2.8%	2.6%	4.7%
$(sc)^*(\bar{q}\bar{q})^*$	K_1	K_2	K_3	K_4
0.8%	19.4%	35.0%	6.7%	4.6%

TABLE VI. The distance, in fm, between any two quarks of the $I(J^P) = 0(0^+)$ $sc\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed.

$r_{\bar{q}\bar{q}}$	$r_{\bar{q}s}$	$r_{\bar{q}c}$	r_{sc}
1.08	0.84	0.82	0.98

2.97 GeV in K_3 , and the hidden-color coupled-channel calculation with a tentative mass of 2.86 GeV.

2. The $I(J^P) = 0(1^+)$ sector

There are 30 channels in this case which include three color-singlet meson-meson configurations, another three in the hidden-color meson-meson ones, three more in the diquark-antidiquark arrangement, and 21 K-type configurations. Table VII shows the obtained masses in each channel which are always above the lowest meson-meson threshold. The three dimeson structures in color-singlet configuration have masses at 2.50, 2.80, and 2.92 GeV, respectively. Furthermore, the masses of the hidden-color meson-meson configurations are ~ 3.2 GeV, diquark-antidiquark channels are characterized with masses around 3.1 GeV, except for the $(sc)(\bar{q}\bar{q})$ channel with a mass of 2.77 GeV, and, for the K-type configurations, each calculated channel presents a mass lying in an interval of 2.5–3.3 GeV.

If one considers the coupling between channels of the same kind of tetraquark configuration, the lowest masses are located at 2497, 2935, 2697, 2636, 2554, 2815, and 2668 MeV, respectively. Particularly interesting is the weakly bound state of the color-singlet dimeson configuration. This result is quite similar to the one obtained in the $I(J^P) = 0(0^+)$ $cs\bar{q}\bar{q}$ tetraquark sector. Meanwhile, a deeper binding energy $E_B = -31$ MeV is obtained when a complete coupled-channel calculation is performed; the predicted bound state has a mass $M = 2467$ MeV.

We have investigated the composition of the last state in Table VIII. The overwhelming dominant structure is of K-type contributing approximately 85% to the total wave function. Table IX indicates that all quark-quark (quark-antiquark) pairs are separated about the same distance with a value of around 1 fm. All of these features indicate that this state might be a compact $sc\bar{q}\bar{q}$ tetraquark.

In a further step, the complete coupled-channel calculation is performed using the complex-range method, and its output is shown in Fig. 3. Therein, with a rotated angle ranging from 0° to 6° , the bound state is fixed at 2467 MeV of the real axis, and we mark it with an orange circle. One can also see in Fig. 3 that the scattering states of $D^*(1S)K(1S)$, $D(1S)K^*(1S)$, $D^*(1S)K^*(1S)$, and $D^*(2S)K(1S)$ are well presented within an interval of 2.4–3.4 GeV. No resonance pole is found around 2.9 GeV within the complete coupled-channel calculation; nevertheless, a good candidate for the $X_1(2900)$ signal reported by the LHCb Collaboration is

TABLE VII. Lowest-lying $sc\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 0(1^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the noninteracting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_J^{\sigma_i}$), flavor ($\chi_I^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_J^{\sigma_i}; \chi_I^{f_j}; \chi_k^c$ [$i; j; k$]	M	Mixed
$(D^*K)^1(2501)$	1	[1; 1; 3]	2498	
$(DK^*)^1(2762)$	2	[2; 1; 3]	2804	
$(D^*K^*)^1(2899)$	3	[3; 1; 3]	2924	2497
$(D^*K)^8$	4	[1; 1; 4]	3224	
$(DK^*)^8$	5	[2; 1; 4]	3269	
$(D^*K^*)^8$	6	[3; 1; 4]	3232	2935
$(sc)(\bar{q}\bar{q})^*$	7	[4; 1; 1]	2766	
$(sc)^*(\bar{q}\bar{q})$	8	[5; 1; 2]	3180	
$(sc)^*(\bar{q}\bar{q})^*$	9	[6; 1; 2]	3068	2697
K_1	10	[7; 1; 5]	3275	
	11	[7; 1; 6]	3158	
	12	[8; 1; 5]	2971	
	13	[8; 1; 6]	3049	
	14	[9; 1; 5]	3184	
	15	[9; 1; 6]	2689	2636
K_2	16	[10; 1; 7]	2932	
	17	[10; 1; 8]	2659	
	18	[11; 1; 7]	3016	
	19	[11; 1; 8]	3269	
	20	[12; 1; 7]	2571	
	21	[12; 1; 8]	3209	2554
K_3	22	[13; 1; 9]	3069	
	23	[13; 1; 10]	2945	
	24	[14; 1; 9]	3062	
	25	[14; 1; 10]	2951	
	26	[15; 1; 9]	3171	
	27	[15; 1; 10]	3788	2815
K_4	28	[16; 1; 11]	3188	
	29	[17; 1; 11]	3037	
	30	[18; 1; 12]	2762	2668
Complete coupled channels:				2467
			$E_B = -31$	

found in the hidden-color coupled-channel calculation with a tentative mass of 2.94 GeV.

3. The $I(J^P) = 0(2^+)$ state

Table X shows that only one D^*K^* meson-meson structure (in both color-singlet and hidden-color configurations), one $(sc)^*(\bar{q}\bar{q})^*$ diquark-antidiquark arrangement,

TABLE VIII. Relative strengths of various meson-meson, diquark-antidiquark, and K-type components in the wave function of the $I(J^P) = 0(1^+)$ $sc\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed. The superscripts 1 and 8 are for singlet-color and hidden-color channels, respectively.

$(D^*K)^1$	$(DK^*)^1$	$(D^*K^*)^1$	$(D^*K)^8$	$(DK^*)^8$
2.9%	0.8%	0.7%	3.4%	2.6%
$(D^*K^*)^8$	$(sc)(\bar{q}\bar{q})^*$	$(sc)^*(\bar{q}\bar{q})$	$(sc)^*(\bar{q}\bar{q})^*$	K_1
2.2%	1.3%	0.2%	0.6%	21.7%
K_2	K_3	K_4		
12.4%	49.7%	1.5%		

TABLE IX. The distance, in fm, between any two quarks of the $I(J^P) = 0(1^+)$ $sc\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed.

$r_{\bar{q}\bar{q}}$	$r_{\bar{q}s}$	$r_{\bar{q}c}$	r_{sc}
1.15	0.89	0.90	1.06

and seven K-type configurations contribute to the $I(J^P) = 0(2^+)$ $sc\bar{q}\bar{q}$ tetraquark state. First, for this highest spin state, no bound state is found in either each single-channel calculation or in any coupled-channel ones. The mass of the color-singlet D^*K^* channel is 2924 MeV, which is just at the theoretical noninteracting meson-meson threshold value. The other exotic configurations—hidden-color dimeson, diquark-antidiquark, and K-type—are all located above 3.0 GeV. Even when a complete coupled-channel

FIG. 3. The complete coupled-channel calculation of the $sc\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 0(1^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

TABLE X. Lowest-lying $sc\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 0(2^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the noninteracting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_j^{\sigma_j}$), flavor ($\chi_l^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_j^{\sigma_j}; \chi_l^{f_j}; \chi_k^c$ [$i; j; k$]	M
$(D^*K^*)^1(2899)$	1	[1; 1; 3]	2924
$(D^*K^*)^8$	2	[1; 1; 3]	3308
$(sc)^*(\bar{q}\bar{q})^*$	3	[1; 1; 3]	3182
K_1	4	[1; 1; 3]	3295
	5	[1; 1; 3]	3182
K_2	6	[1; 1; 3]	3041
	7	[1; 1; 3]	3295
K_3	8	[1; 1; 3]	3175
	9	[1; 1; 3]	3800
K_4	10	[1; 1; 3]	3216
Complete coupled channels:			2924

computation is performed, the scattering nature of states remains unchanged.

We can also notice that the nature of the $I(J^P) = 0(2^+)$ $sc\bar{q}\bar{q}$ tetraquark seems different with respect to the $0(0^+)$ and $0(1^+)$ cases. Figure 4 shows the distribution of

FIG. 4. The complete coupled-channel calculation of the $sc\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 0(2^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

complex energies in the complete coupled-channel calculation; the scattering nature of the $D^*(1S)K^*(1S)$, $D^*(2S)K^*(1S)$, and $D^*(1S)K^*(2S)$ states can be clearly identified. Moreover, the $D^*(2S)K^*(1S)(3611)$ and $D^*(1S)K^*(2S)(3638)$ structures are almost degenerated. In the complex plane of Fig. 4, a fixed dot is found at 3493 MeV with a width of 2.6 MeV, indicating the existence of a $D^*(1S)K^*(1S)$ molecular state.

4. The $I(J^P) = 1(0^+)$ sector

Table XI lists our results for the isovector $sc\bar{q}\bar{q}$ tetraquark with quantum numbers $J^P = 0^+$. As in the case of the $I(J^P) = 0(0^+)$ state, 20 channels are under investigation, and not one shows a bound state. The masses of the color-singlet meson-meson configurations are 2378 and 2924 MeV, respectively. The other channels are generally in a mass region between 3.0 and 3.7 GeV, except for two

TABLE XI. Lowest-lying $sc\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 1(0^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the noninteracting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_j^{\sigma_j}$), flavor ($\chi_l^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_j^{\sigma_j}; \chi_l^{f_j}; \chi_k^c$ [$i; j; k$]	M	Mixed
$(DK)^1(2364)$	1	[1; 1; 3]	2378	
$(D^*K^*)^1(2899)$	2	[2; 1; 3]	2924	2378
$(DK)^8$	3	[1; 1; 4]	3314	
$(D^*K^*)^8$	4	[2; 1; 4]	3294	3014
$(sc)(\bar{q}\bar{q})$	5	[3; 1; 1]	3214	
$(sc)^*(\bar{q}\bar{q})^*$	6	[4; 1; 2]	3086	3035
K_1	7	[5; 1; 5]	3257	
	8	[5; 1; 6]	3219	
	9	[6; 1; 5]	3288	
	10	[6; 1; 6]	2785	2773
K_2	11	[7; 1; 7]	3076	
	12	[7; 1; 8]	3279	
	13	[8; 1; 7]	2617	
	14	[8; 1; 8]	3322	2604
K_3	15	[9; 1; 9]	3666	
	16	[9; 1; 10]	3085	
	17	[10; 1; 9]	3207	
	18	[10; 1; 10]	3781	3012
K_4	19	[11; 1; 11]	3072	
	20	[12; 1; 12]	3253	3016
Complete coupled channels:				2378

FIG. 5. The complete coupled-channel calculation of the $sc\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 1(0^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

channels, K_1 and K_2 configurations, which present masses of 2785 and 2617 MeV. Additionally, this output is not changed when we perform coupled-channel calculations in either configuration's sector and considering all channels.

The complete coupled-channel calculation has been extended to the complex range. Although bound states are unavailable, one narrow resonance is found. Figure 5 shows the distribution of complex energies within the energy range of 2.3–3.3 GeV. The $D(1S)K(1S)$, $D^*(1S)K^*(1S)$, and $D(2S)K(1S)$ scattering states are clearly shown. Moreover, one can find an unchanged pole which lies between the $D^*(1S)K^*(1S)$ and $D(2S)K(1S)$ threshold lines, and it is extremely close to the real axis. The calculated mass and width are 3089 and 0.3 MeV, respectively. The dominant channel is $D^*(1S)K^*(1S)$ for this resonance pole, and it is far from the $D(1S)K(1S)$ threshold. In addition, due to the quite narrow width, this resonance state should be stable against two-meson strong decay processes.

5. The $I(J^P) = 1(1^+)$ sector

The numerical analysis of this case is as tedious as the $I(J^P) = 0(1^+)$ case because 30 channels must still be explored. From Table XII, we could conclude that no bound state is obtained when the three kinds of real-range calculations (single-channel analysis, coupled-channel computation in each configuration of quarks, and complete coupled-channel investigation) are performed. Theoretical masses of D^*K , DK^* , and D^*K^* in color-singlet channels are located at 2498, 2804, and 2924 MeV, respectively. The remaining channels—hidden color, diquark-antidiquark, and K-types—are generally at around 3.2 GeV, except for several channels of K-types that can be found at about 2.7 and 3.7 GeV.

TABLE XII. Lowest-lying $sc\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 1(1^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the noninteracting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin (χ_J^s), flavor (χ_I^f), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_J^s; \chi_I^f; \chi_k^c$ [$i; j; k$]	M	Mixed
$(D^*K)^1(2501)$	1	[1; 1; 3]	2498	
$(DK^*)^1(2762)$	2	[2; 1; 3]	2804	
$(D^*K^*)^1(2899)$	3	[3; 1; 3]	2924	2498
$(D^*K)^8$	4	[1; 1; 4]	3288	
$(DK^*)^8$	5	[2; 1; 4]	3271	
$(D^*K^*)^8$	6	[3; 1; 4]	3279	3043
$(sc)(\bar{q}\bar{q})^*$	7	[4; 1; 1]	3210	
$(sc)^*(\bar{q}\bar{q})$	8	[5; 1; 2]	3154	
$(sc)^*(\bar{q}\bar{q})^*$	9	[6; 1; 2]	3112	3068
K_1	10	[7; 1; 5]	3122	
	11	[7; 1; 6]	3124	
	12	[8; 1; 5]	3260	
	13	[8; 1; 6]	3200	
	14	[9; 1; 5]	3267	
	15	[9; 1; 6]	2806	2793
K_2	16	[10; 1; 7]	3042	
	17	[10; 1; 8]	3288	
	18	[11; 1; 7]	3047	
	19	[11; 1; 8]	3174	
	20	[12; 1; 7]	2666	
	21	[12; 1; 8]	3295	2644
K_3	22	[13; 1; 9]	3191	
	23	[13; 1; 10]	3128	
	24	[14; 1; 9]	3183	
	25	[14; 1; 10]	3129	
	26	[15; 1; 9]	3779	
	27	[15; 1; 10]	3156	3051
K_4	28	[16; 1; 11]	3126	
	29	[17; 1; 11]	3082	
	30	[18; 1; 12]	3245	3046
Complete coupled channels:				2498

In a complex-range investigation of the complete coupled-channel calculation, where the angle θ is varied from 0° to 6° , the energy dots are presented in Fig. 6. Four scattering states $D^*(1S)K(1S)$, $D(1S)K^*(1S)$, $D^*(1S)K^*(1S)$, and $D^*(2S)K(1S)$ are well shown within an energy interval of 2.4–3.4 GeV. An extremely narrow resonance pole is found at 3091 MeV, and its width is just 0.5 MeV. This state should also be stable against two-meson strong decays, and the dominant channel is $D^*(1S)K^*(1S)$.

FIG. 6. The complete coupled-channel calculation of the $sc\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 1(1^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

6. The $I(J^P) = 1(2^+)$ sector

The real- and complex-range results for the highest spin and isospin $sc\bar{q}\bar{q}$ state are shown in Table XIII and Fig. 7, respectively. The only dimeson channel is the D^*K^* one, and it is a scattering state with a theoretical mass of 2924 MeV. The states of the hidden-color, diquark-antidiquark, and K-type configurations are all above 3.0 GeV. Moreover, the scattering nature of the D^*K^* channel remains in a complete coupled-channel calculation.

The $D^*(1S)K^*(1S)$ scattering state along with two almost degenerate radial excited states $D^*(2S)K^*(1S)$ (3611) and $D^*(1S)K^*(2S)$ (3638) are well presented in Fig. 7. Therein, a resonance pole is found, and its mass and width are 3511 and 2.1 MeV, respectively. Notice herein that a state of similar nature is found in the $0(2^+)$ $sc\bar{q}\bar{q}$ tetraquark sector.

B. The $sb\bar{q}\bar{q}$ tetraquarks

A natural continuation of the investigation performed above is the analysis of the $sb\bar{q}\bar{q}$ bound-state problem with spin parity $J^P = 0^+, 1^+, \text{ and } 2^+$, and isospin $I = 0$ and 1. A similar situation to the one discussed in the charm-strange sector is found for the bottom-strange tetraquark system. There are two bound states with quantum numbers $I(J^P) = 0(0^+)$ and $0(1^+)$. In addition, resonances are also found in the former channels but also in the $I(J^P) = 0(2^+)$, $1(0^+)$, and $1(2^+)$ ones. Further details can be found below.

1. The $I(J^P) = 0(0^+)$ sector

Table XIV summarizes all contributing channels; there are 20 and include meson-meson, diquark-antidiquark, and K-type structures. A loosely bound state of BK is obtained in the color-singlet channel calculation, and the predicted mass is 5755 MeV, which means a binding energy of

TABLE XIII. Lowest-lying $sc\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 1(2^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the non-interacting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_J^{\sigma_i}$), flavor ($\chi_I^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_J^{\sigma_i}; \chi_I^{f_j}; \chi_k^c$ [$i; j; k$]	M
$(D^*K^*)^1$ (2899)	1	[1; 1; 3]	2924
$(D^*K^*)^8$	2	[1; 1; 3]	3139
$(sc)^*(\bar{q}\bar{q})^*$	3	[1; 1; 3]	3160
K_1	4	[1; 1; 3]	3149
	5	[1; 1; 3]	3151
K_2	6	[1; 1; 3]	3057
	7	[1; 1; 3]	3161
K_3	8	[1; 1; 3]	3782
	9	[1; 1; 3]	3162
K_4	10	[1; 1; 3]	3156
Complete coupled channels:			2924

−2 MeV. The color-singlet B^*K^* channel is a scattering state with a mass just located at the value of theoretical threshold, 6223 MeV. In addition, the remaining channels of the diquark-antidiquark and K-type configurations are generally located in the interval of 6.0–7.0 GeV.

FIG. 7. The complete coupled-channel calculation of the $sc\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 1(2^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

If a coupled-channel calculation is performed in each individual quark configuration, the bound state of BK is further pushed down and its coupled mass is 5752 MeV. When all 20 channels listed in Table XIV are considered in a coupled-channel calculation in the real range, the lowest mass at 5710 MeV indicates a binding energy of -49 MeV. Therefore, a strong coupling effect is confirmed in the $I(J^P) = 0(0^+)$ $sc\bar{q}\bar{q}$ tetraquark sector, as it is the case in the $sc\bar{q}\bar{q}$ one. We have investigated the composition of this state in Table XV. The dominant structure is of K-type contributing 44% to the total wave function, but singlet-color BK (23%) and B^*K^* (18%) components are, when combined, equally important. Table XVI indicates that all quark-quark (quark-antiquark) pairs are separated about the same distance with a value below 1 fm. Our interpretation is that this state looks like a compact tetraquark structure.

TABLE XIV. Lowest-lying $sb\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 0(0^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the non-interacting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin (χ_j^s), flavor (χ_l^f), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_j^s; \chi_l^f; \chi_k^c$ [$i; j; k$]	M	Mixed
$(BK)^1(5774)$	1	[1; 1; 3]	5755	
$(B^*K^*)^1(6217)$	2	[2; 1; 3]	6223	5752
$(BK)^8$	3	[1; 1; 4]	6519	
$(B^*K^*)^8$	4	[2; 1; 4]	6378	6217
$(sb)(\bar{q}\bar{q})$	5	[3; 1; 1]	6044	
$(sb)^*(\bar{q}\bar{q})^*$	6	[4; 1; 2]	6367	5965
K_1	7	[5; 1; 5]	6228	
	8	[5; 1; 6]	6341	
	9	[6; 1; 5]	6475	
	10	[6; 1; 6]	5957	5890
K_2	11	[7; 1; 7]	6235	
	12	[7; 1; 8]	6375	
	13	[8; 1; 7]	5784	
	14	[8; 1; 8]	6511	5773
K_3	15	[9; 1; 9]	6351	
	16	[9; 1; 10]	7049	
	17	[10; 1; 9]	7093	
	18	[10; 1; 10]	6185	6076
K_4	19	[11; 1; 11]	6306	
	20	[12; 1; 12]	6036	5925
Complete coupled channels:				5710
				$E_B = -49$

TABLE XV. Relative strengths of various meson-meson, diquark-antidiquark, and K-type components in the wave function of the $I(J^P) = 0(0^+)$ $sb\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed. The superscripts 1 and 8 are for singlet-color and hidden-color channels, respectively.

$(BK)^1$	$(B^*K^*)^1$	$(BK)^8$	$(B^*K^*)^8$	$(sb)(\bar{q}\bar{q})$
22.7%	18.0%	1.4%	4.0%	5.6%
$(sb)^*(\bar{q}\bar{q})^*$	K_1	K_2	K_3	K_4
2.3%	9.9%	20.9%	12.7%	2.5%

TABLE XVI. The distance, in fm, between any two quarks of the $I(J^P) = 0(0^+)$ $sb\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed.

$r_{\bar{q}\bar{q}}$	$r_{\bar{q}s}$	$r_{\bar{q}b}$	r_{sb}
0.97	0.78	0.75	0.85

Figure 8 shows the distribution of complex energies when the complete coupled-channel calculation is extended to the complex plane. Six scattering states, i.e., $B(1S)K(1S)$, $B^*(1S)K^*(1S)$, $B(2S)K(1S)$, $B(1S)K(2S)$, $B^*(2S)K^*(1S)$, and $B^*(1S)K^*(2S)$, are well identified in the energy interval of 5.7–7.0 GeV; there is a degeneration between $B^*(2S)K^*(1S)$ (6912) and $B^*(1S)K^*(2S)$ (6940). Two singularities deserve to be highlighted in Fig. 8: the bound state at a real-axis fixed mass of 5710 MeV and a narrow resonance whose parameters are $M = 6850$ MeV and $\Gamma = 2.0$ MeV. The last one is independent of the rotated angle θ , which is varied from 0° to 6° , and it can be identified as a $B(1S)K(2S)$ resonance state because its

FIG. 8. The complete coupled-channel calculation of the $sb\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 0(0^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

location is far from the $B(1S)K(1S)$, $B^*(1S)K^*(1S)$, and $B(2S)K(1S)$ threshold lines.

2. The $I(J^P) = 0(1^+)$ sector

Thirty channels shown in Table XVII are under investigation, namely, three meson-meson channels in both color-singlet and hidden-color configurations, three

TABLE XVII. Lowest-lying $sb\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 0(1^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the non-interacting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_i^{\sigma_i}$), flavor ($\chi_i^{f_i}$), and color (χ_i^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_i^{\sigma_i}; \chi_i^{f_i}; \chi_i^c$ [$i; j; k$]	M	Mixed
$(B^*K)^1(5819)$	1	[1; 1; 3]	5797	
$(BK^*)^1(6172)$	2	[2; 1; 3]	6185	
$(B^*K^*)^1(6217)$	3	[3; 1; 3]	6226	5795
$(B^*K)^8$	4	[1; 1; 4]	6530	
$(BK^*)^8$	5	[2; 1; 4]	6587	
$(B^*K^*)^8$	6	[3; 1; 4]	6547	6246
$(sb)(\bar{q}\bar{q})^*$	7	[4; 1; 1]	6057	
$(sb)^*(\bar{q}\bar{q})$	8	[5; 1; 2]	6511	
$(sb)^*(\bar{q}\bar{q})^*$	9	[6; 1; 2]	6419	5989
K_1	10	[7; 1; 5]	6588	
	11	[7; 1; 6]	6471	
	12	[8; 1; 5]	6256	
	13	[8; 1; 6]	6341	
	14	[9; 1; 5]	6483	
	15	[9; 1; 6]	5980	5919
K_2	16	[10; 1; 7]	6221	
	17	[10; 1; 8]	6470	
	18	[11; 1; 7]	6255	
	19	[11; 1; 8]	6592	
	20	[12; 1; 7]	5820	
	21	[12; 1; 8]	6522	5812
K_3	22	[13; 1; 9]	6413	
	23	[13; 1; 10]	6252	
	24	[14; 1; 9]	6412	
	25	[14; 1; 10]	6254	
	26	[15; 1; 9]	6501	
	27	[15; 1; 10]	7092	6109
K_4	28	[16; 1; 11]	6499	
	29	[17; 1; 11]	6325	
	30	[18; 1; 12]	6046	5947
Complete coupled channels:			5754	
			$E_B = -46$	

diquark-antidiquark structures, and 21 K-type arrangements. In the single-channel calculation, we obtain a bound state with binding energy -3 MeV and total mass 5797 MeV. The other channels are generally above 6.0 GeV, except for two cases, a K_1 channel (5980 MeV) and a K_2 one (5820 MeV).

The coupled-channel calculation of color-singlet meson-meson structures delivers a slightly deeper bound state with a mass 5795 MeV and $E_B = -5$ MeV. When the complete coupled-channel computation is performed in the real range, the binding energy changes to -46 MeV, and thus, a deeply bound state with mass equal to 5754 MeV is obtained. It appears that the increase in binding energy is due to a strong coupling effect between channels. We have investigated the composition of this state in Table XVIII. The dominant structure is of K-type contributing 74% to the total wave function, but traces of singlet-color $B^{(*)}K^{(*)}$ (12%) can be found. Table XIX indicates that all quark-quark (antiquark-quark) pairs are separated about the same distance with a value below 1 fm. Our interpretation is that this state looks like a compact tetraquark structure.

Figure 9 shows our findings using the complex-range method to the complete coupled-channel calculation. The scattering states of $B^*(1S)K(1S)$, $B(1S)K^*(1S)$, $B^*(1S)K^*(1S)$, $B^*(2S)K(1S)$, and $B^*(1S)K(2S)$ are clearly identified in the energy region of 5.7–6.9 GeV, and there are two almost degenerate states: $B(1S)K^*(1S)$ (6185) and $B^*(1S)K^*(1S)$ (6226). The tightly bound state at 5754 MeV is independent of the rotated angle, which is varied from 0° to 6° . Two resonances are found whose parameters are (6313, 12.5) MeV and (6754, 1.2) MeV, respectively; they

TABLE XVIII. Relative strengths of various meson-meson, diquark-antidiquark, and K-type components in the wave function of the $I(J^P) = 0(0^+)$ $sb\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed. The superscripts 1 and 8 are for singlet-color and hidden-color channels, respectively.

$(B^*K)^1$	$(BK^*)^1$	$(B^*K^*)^1$	$(B^*K)^8$	$(BK^*)^8$
4.4%	2.7%	3.0%	0.4%	0.5%
$(B^*K^*)^8$	$(sc)(\bar{q}\bar{q})^*$	$(sc)^*(\bar{q}\bar{q})$	$(sc)^*(\bar{q}\bar{q})^*$	K_1
0.7%	1.1%	0.2%	0.2%	2.6%
K_2	K_3	K_4		
5.6%	78.0%	0.5%		

TABLE XIX. The distance, in fm, between any two quarks of the $I(J^P) = 0(1^+)$ $sb\bar{q}\bar{q}$ tetraquark bound state obtained when a complete coupled-channel calculation is performed.

$r_{\bar{q}\bar{q}}$	$r_{\bar{q}s}$	$r_{\bar{q}b}$	r_{sb}
0.99	0.79	0.77	0.87

FIG. 9. The complete coupled-channel calculation of the $sb\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 0(1^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

can be identified as $B^*(1S)K^*(1S)$ and $B^*(2S)K(1S)$ resonances.

3. The $I(J^P) = 0(2^+)$ sector

From Table XX, we can see that the ten contributing channels are all above 6.2 GeV, and the lowest mass is

TABLE XX. Lowest-lying $sb\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 0(2^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the noninteracting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_J^{\sigma_i}$), flavor ($\chi_I^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_J^{\sigma_i}; \chi_I^{f_j}; \chi_k^c$ [$i; j; k$]	M
$(B^*K^*)^1(6217)$	1	[1; 1; 3]	6226
$(B^*K^*)^8$	2	[1; 1; 3]	6611
$(sb)^*(\bar{q}\bar{q})^*$	3	[1; 1; 3]	6505
K_1	4	[1; 1; 3]	6596
	5	[1; 1; 3]	6481
K_2	6	[1; 1; 3]	6267
	7	[1; 1; 3]	6604
K_3	8	[1; 1; 3]	6497
	9	[1; 1; 3]	7086
K_4	10	[1; 1; 3]	6510
Complete coupled channels:			6226

FIG. 10. The complete coupled-channel calculation of the $sb\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 0(2^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

6226 MeV, which is just the theoretical value of the B^*K^* threshold. Moreover, this fact is not changed when a complete coupled-channel calculation is performed in the real range.

In the complex-scaling investigation, the scattering states of $B^*(1S)K^*(1S)$, $B^*(2S)K^*(1S)$, and $B^*(1S)K^*(2S)$ are well identified when the rotated angle is varied from 0° to 6° . They are located within an energy interval of 6.2–7.2 GeV (see Fig. 10). It is worth highlighting that a pole is obtained between the threshold lines of $B^*(1S)K^*(1S)$ and the two almost degenerate $B^*(2S)K^*(1S)$ (6912) and $B^*(1S)K^*(2S)$ (6940). Accordingly, it can be identified as a $B^*(1S)K^*(1S)$ resonance with mass and width 6754 and 1.2 MeV, respectively.

4. The $I(J^P) = 1(0^+)$ sector

Table XXI lists the calculated masses of the $1(0^+)$ $sb\bar{q}\bar{q}$ tetraquark state. As in the charm-strange sector, no bound state is found in each single channel and in the different variants of the coupled-channel calculations. The lowest mass obtained is 5759 MeV, which is just the theoretical value of the BK threshold. The remaining channels are generally located within the energy interval of 6.1–7.0 GeV, except for one K_2 channel which presents a mass of 5850 MeV.

When the complete coupled-channel calculation is performed using the complex-range method, the scattering nature of $B(1S)K(1S)$, $B^*(1S)K^*(1S)$, $B(2S)K(1S)$, $B(1S)K(2S)$, $B^*(2S)K^*(1S)$, and $B^*(1S)K^*(2S)$ is clearly shown in Fig. 11. There is a dense distribution of complex energies in a region between 6.9 and 7.0 GeV. One fixed resonance pole obviously emerges therein; its resonance parameters are $M = 6876$ MeV and $\Gamma = 4.2$ MeV.

TABLE XXI. Lowest-lying $sb\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 1(0^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the non-interacting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_I^{\sigma_i}$), flavor ($\chi_I^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_I^{\sigma_i}; \chi_I^{f_j}; \chi_k^c$ [$i; j; k$]	M	Mixed
$(BK)^1(5774)$	1	[1; 1; 3]	5759	
$(B^*K^*)^1(6217)$	2	[2; 1; 3]	6226	5759
$(BK)^8$	3	[1; 1; 4]	6622	
$(B^*K^*)^8$	4	[2; 1; 4]	6606	6353
$(sb)(\bar{q}\bar{q})$	5	[3; 1; 1]	6548	
$(sb)^*(\bar{q}\bar{q})^*$	6	[4; 1; 2]	6390	6362
K_1	7	[5; 1; 5]	6563	
	8	[5; 1; 6]	6514	
	9	[6; 1; 5]	6581	
K_2	10	[6; 1; 6]	6096	6084
	11	[7; 1; 7]	6290	
	12	[7; 1; 8]	6598	
K_3	13	[8; 1; 7]	5850	5848
	14	[8; 1; 8]	6625	
	15	[9; 1; 9]	7011	
K_4	16	[9; 1; 10]	6390	
	17	[10; 1; 9]	6540	
	18	[10; 1; 10]	7085	6340
K_5	19	[11; 1; 11]	6365	
	20	[12; 1; 12]	6551	6321
Complete coupled channels:			5759	

The dominant channel of this narrow resonance state is $B(1S)K(2S)$.

5. The $I(J^P) = 1(1^+)$ sector

Thirty channels contribute to the $sb\bar{q}\bar{q}$ tetraquark system with $1(1^+)$ quantum numbers. The three meson-meson channels B^*K , BK^* , and B^*K^* are all scattering states, and the lowest mass 5.8 GeV is just the value of the noninteracting B^*K threshold. The other exotic quark arrangements are generally above 6.2 GeV, except for one K_2 channel at 5.88 GeV. In a further step, when coupled-channel calculations are performed within the real and complex range, neither bound nor resonance states are available. In particular, five scattering states $B^*(1S)K(1S)$, $B(1S)K^*(1S)$, $B^*(1S)K^*(1S)$, $B^*(2S)K(1S)$, and $B^*(1S)K(2S)$ are presented in Fig. 12, and one can see that bound and resonance poles do not emerge.

FIG. 11. The complete coupled-channel calculation of the $sb\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 1(0^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

FIG. 12. The complete coupled-channel calculation of the $sb\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 1(1^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

6. The $I(J^P) = 1(2^+)$ sector

Table XXIII shows all the channels that can contribute to the mass of the highest spin and isospin state of the $sb\bar{q}\bar{q}$ tetraquark. Again, no bound state is found in the single- and each-sector-coupled-channel calculations, with the lowest mass indicating the value of the B^*K^* threshold, 6226 MeV. Our results of the complete coupled-channel calculation in the complex range are shown in Fig. 13. One can see that a $B^*(1S)K^*(2S)$ resonance state with mass 7175 MeV and width 8.1 MeV is obtained.

FIG. 13. The complete coupled-channel calculation of the $sb\bar{q}\bar{q}$ tetraquark system with $I(J^P) = 1(2^+)$ quantum numbers. We use the complex-scaling method of the chiral quark model varying θ from 0° to 6° .

TABLE XXII. Lowest-lying $sb\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 1(1^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the non-interacting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_J^{\sigma_i}$), flavor ($\chi_I^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_J^{\sigma_i}; \chi_I^{f_j}; \chi_k^c$ [$i; j; k$]	M	Mixed
$(B^*K)^1(5819)$	1	[1; 1; 3]	5800	
$(BK^*)^1(6172)$	2	[2; 1; 3]	6185	
$(B^*K^*)^1(6217)$	3	[3; 1; 3]	6226	5800
$(B^*K)^8$	4	[1; 1; 4]	6611	
$(BK^*)^8$	5	[2; 1; 4]	6583	
$(B^*K^*)^8$	6	[3; 1; 4]	6601	6363
$(sb)(\bar{q}\bar{q})^*$	7	[4; 1; 1]	6546	
$(sb)^*(\bar{q}\bar{q})$	8	[5; 1; 2]	6465	
$(sb)^*(\bar{q}\bar{q})^*$	9	[6; 1; 2]	6411	6374
K_1	10	[7; 1; 5]	6426	
	11	[7; 1; 6]	6428	
	12	[8; 1; 5]	6564	
	13	[8; 1; 6]	6507	

(Table continued)

TABLE XXII. (Continued)

Channel	Index	$\chi_J^{\sigma_i}; \chi_I^{f_j}; \chi_k^c$ [$i; j; k$]	M	Mixed
K_2	14	[9; 1; 5]	6573	
	15	[9; 1; 6]	6104	6092
	16	[10; 1; 7]	6273	
	17	[10; 1; 8]	6603	
	18	[11; 1; 7]	6290	
	19	[11; 1; 8]	6507	
K_3	20	[12; 1; 7]	5877	
	21	[12; 1; 8]	6614	5876
	22	[13; 1; 9]	6517	
	23	[13; 1; 10]	6434	
	24	[14; 1; 9]	6515	
	25	[14; 1; 10]	6434	
K_4	26	[15; 1; 9]	7096	
	27	[15; 1; 10]	6469	6360
	28	[16; 1; 11]	6429	
	29	[17; 1; 11]	6369	
	30	[18; 1; 12]	6548	6333
Complete coupled channels:			5800	

TABLE XXIII. Lowest-lying $sb\bar{q}\bar{q}$ tetraquark states with $I(J^P) = 1(2^+)$ calculated within the real-range formulation of the chiral quark model. The allowed meson-meson, diquark-antidiquark, and K-type configurations are listed in the first column. When possible, the experimental value of the non-interacting meson-meson threshold is labeled in parentheses. Each channel is assigned an index in the second column; it reflects a particular combination of spin ($\chi_J^{\sigma_i}$), flavor ($\chi_I^{f_j}$), and color (χ_k^c) wave functions that are shown explicitly in the third column. The theoretical mass obtained in each channel is shown in the fourth column, and the coupled result for each kind of configuration is presented in the fifth column. When a complete coupled-channel calculation is performed, the last row of the table indicates the lowest-lying mass and binding energy (unit: MeV).

Channel	Index	$\chi_J^{\sigma_i}; \chi_I^{f_j}; \chi_k^c$ [$i; j; k$]	M
$(B^*K^*)^1(6217)$	1	[1; 1; 3]	6226
$(B^*K^*)^8$	2	[1; 1; 3]	6449
$(sb)^*(\bar{q}\bar{q})^*$	3	[1; 1; 3]	6450
K_1	4	[1; 1; 3]	6437
	5	[1; 1; 3]	6439
K_2	6	[1; 1; 3]	6298
	7	[1; 1; 3]	6458
K_3	8	[1; 1; 3]	7093
	9	[1; 1; 3]	6454
K_4	10	[1; 1; 3]	6441
Complete coupled channel:s			6226

TABLE XXIV. Summary of the bound states and resonance structures found in the $sQ\bar{q}\bar{q}$ ($q = u, d$, $Q = c, b$) tetraquark sectors. The first column shows the isospin, total spin, and parity quantum numbers of each singularity. The second column refers to the dominant configuration; the obtained poles are presented with the following notation: $E = M + i\Gamma$ in the last column, and binding energy E_B is given in parentheses for the bound states (unit: MeV).

$I(J^P)$	Dominant channel	Pole
$0(0^+)$	$sc\bar{q}\bar{q}$	2342($E_B = -36$)
	$sb\bar{q}\bar{q}$	5710($E_B = -49$)
	$B(1S)K(2S)$	$6850 + i2.0$
$0(1^+)$	$sc\bar{q}\bar{q}$	2467($E_B = -31$)
	$sb\bar{q}\bar{q}$	5754($E_B = -46$)
	$B^*(1S)K^*(1S)$	$6313 + i12.5$
$0(2^+)$	$B^*(2S)K(1S)$	$6754 + i1.2$
	$D^*(1S)K^*(1S)$	$3493 + i2.6$
	$B^*(1S)K^*(1S)$	$6754 + i1.2$
$1(0^+)$	$D^*(1S)K^*(1S)$	$3089 + i0.3$
	$B(1S)K(2S)$	$6876 + i4.2$
$1(1^+)$	$D^*(1S)K^*(1S)$	$3091 + i0.5$
$1(2^+)$	$D^*(1S)K^*(1S)$	$3511 + i2.1$
	$B^*(1S)K^*(2S)$	$7175 + i8.1$

IV. SUMMARY

The $sQ\bar{q}\bar{q}$ ($q = u, d$, $Q = c, b$) tetraquarks with spin parity $J^P = 0^+$, 1^+ , and 2^+ and in the isoscalar and isovector sectors are systemically investigated by means of the real- and complex-scaling range of the chiral quark model, along with a high-efficiency numerical approach, Gaussian expansion method. The model, which contains one-gluon exchange, linear-screened confining, and Goldstone-boson exchanges between light-quark interactions, is successfully applied to the description of hadron, hadron-hadron, and multi-quark phenomenology. We consider in our calculations all possible tetraquark arrangements allowed by quantum numbers: singlet- and hidden-color meson-meson configurations, diquark-antidiquark arrangements with their allowed color triplet-antitriplet

and sextet-antisextet wave functions, and four K-type structures.

Several bound states and resonance structures are found in both charm-strange $cs\bar{q}\bar{q}$ and bottom-strange $bs\bar{q}\bar{q}$ tetraquark sectors. Table XXIV summarizes our theoretical findings. We collect quantum numbers, dominant configuration, and pole position in the complex plane for each singularity. We provide below a brief review of them.

Tightly bound charm- and bottom-strange tetraquarks are found in the $I(J^P) = 0(0^+)$ and $0(1^+)$ sectors. Their binding energies are around -40 MeV, and they are interpreted as compact structures with all interquark distances $\lesssim 1$ fm. Resonances of charm-strange tetraquarks are located within the energy interval of $3.1\text{--}3.5$ GeV; their total decay widths are usually less than 3 MeV, and thus they are extremely narrow. Resonances of bottom-strange tetraquarks are located in the energy region of $6.3\text{--}7.1$ GeV with decay widths smaller than 13 MeV.

The development of this work is motivated by the recent LHCb Collaboration observation of two charmed-strange structures whose properties point out to be $ud\bar{c}\bar{s}$ tetraquarks. They were named $X_{0,1}(2900)$ and, within our formalism, are unstable with several candidates in the single-channel computations. We find four cases with $I(J^P) = 0(0^+)$ quantum numbers: K_1 with mass at 2.89 GeV, K_2 at 2.96 GeV, K_3 at 2.97 GeV, and the hidden-color coupled-channel calculation delivering a mass of 2.86 GeV. With quantum numbers $I(J^P) = 0(1^+)$, only one candidate can be found in the hidden-color coupled-channel calculation with mass of 2.94 GeV.

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