Spin- $\frac{3}{2}$ doubly charmed baryon contribution to the magnetic moments of the spin- $\frac{1}{2}$ doubly charmed baryons

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We have systematically investigated the magnetic moments of spin- $\frac{1}{2}$ doubly charmed baryons in the framework of the heavy baryon chiral perturbation theory. In this paper, one loop corrections with intermediate spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryon states are considered. The numerical results are calculated to next-to-leading order: $\mu_{\Xi_{cc}^{++}} = 0.35\mu_N$, $\mu_{\Xi_{cc}^{+}} = 0.62\mu_N$, $\mu_{\Omega_{cc}^{+}} = 0.41\mu_N$. Our results may be useful for future experiment and chiral extrapolation of the lattice QCD.

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I. INTRODUCTION

The doubly charmed baryon was first claimed by SELEX Collaboration in the decay mode $\Xi_{cc}^{+} \rightarrow \Lambda_c^{+} K^{-} \pi^{+}$ with the mass $M_{\Xi_{cc}^{+}} = 3519 \pm 1 \text{ MeV}$ [1]. Since then, many experimental collaborations have shown great interest to the doubly charmed baryons. Although other experimental collaborations like FOCUS [2], BABAR [3], and Belle [4] did not find any evidence for doubly charmed baryons, researchers have never stopped studying on the doubly charmed baryons. In 2017, LHCb collaboration reported the discovery of Ξ_{cc}^{++} in the mass spectrum of $\Lambda_c^{+} K^{-} \pi^{+} \pi^{+}$ with the mass $M_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14 \text{ MeV}$ [5].

In the past decade, the masses and decay properties of double charmed baryons have been studied extensively in literature [6–43]. It is very important to investigate the baryon electromagnetic form factors, especially the magnetic moments as the electromagnetic properties give information about the internal structures and shape deformations, which provide valuable insight in describing the inner structures of hadrons and understanding the mechanism of strong interactions at low-energy. With nonrelativistic quark model, Lichtenberg first investigated the doubly charmed baryons magnetic moments in Ref. [44]. With relativistic quark model, the magnetic moments have also been evaluated in Refs. [45,46]. Besides the quark

models, magnetic moments have been studied in different theoretical models and approaches [47–54], however it is difficult to include the chiral corrections. In fact, chiral perturbation theory (ChPT) [55] and heavy baryon chiral perturbation theory (HBChPT) [56–59] are quite helpful to analyze the low-energy interactions order by order. To consider the chiral corrections, the magnetic moments of spin- $\frac{1}{2}$ doubly charmed baryons have been investigated with HBChPT in Ref. [60]. In Ref. [61], the electromagnetic form factors of the doubly charmed baryons have been studied in covariant chiral perturbation theory within the extended on-mass-shell (EOMS) scheme. However, the spin- $\frac{3}{2}$ doubly charmed baryon was not included explicitly in the analysis of Refs [60,61].

In this work, we will investigate the magnetic moments of spin- $\frac{1}{2}$ doubly charmed baryons in HBChPT. We explicitly consider both the spin- $\frac{3}{2}$ and spin- $\frac{1}{2}$ doubly charmed baryon intermediate states in the loop calculation because the mass splitting between the octet and decuplet baryons is small. Moreover, the spin- $\frac{3}{2}$ doubly charmed baryon generally strongly couple to the spin- $\frac{1}{2}$ doubly charmed baryon. We use quark model to determine the corresponding low energy constants (LECs) and calculate the magnetic moments order by order. The analytical and numerical results are calculated to next-to-leading order.

This paper is organized as follows. In Sec. II, we discuss the electromagnetic form factors of spin- $\frac{1}{2}$ doubly charmed baryons. We introduce the effective chiral Lagrangians of spin- $\frac{3}{2}$ and spin- $\frac{1}{2}$ doubly charmed baryons in Sec. III. We calculate the magnetic moments of the spin- $\frac{1}{2}$ doubly charmed baryons order by order in Sec. IV and present our numerical results in Sec. V. A short summary is given in Sec. VI. We collect some useful formulas in the Appendix.

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II. ELECTROMAGNETIC FORM FACTORS

For spin- $\frac{1}{2}$ doubly charmed baryons, the matrix elements of the electromagnetic current read,

$$\langle \Psi(p') | J_\mu | \Psi(p) \rangle = e \bar{u}(p') \mathcal{O}_\mu(p', p) u(p), \quad (1)$$

with

$$\mathcal{O}_\mu(p', p) = \frac{1}{M_H} \left[P_\mu G_E(q^2) + \frac{i \sigma_{\mu\nu} q^\nu}{2} G_M(q^2) \right], \quad (2)$$

where $P = \frac{1}{2}(p' + p)$, $q = p' - p$, M_H is doubly charmed baryon mass.

In the heavy baryon limit, the doubly charmed baryon field B can be decomposed into the large component H and the small component L .

$$B = e^{-iM_H v \cdot x} (H + L), \quad (3)$$

$$H = e^{iM_H v \cdot x} \frac{1 + \not{v}}{2} B, \quad L = e^{iM_H v \cdot x} \frac{1 - \not{v}}{2} B, \quad (4)$$

where M_H is the spin- $\frac{1}{2}$ doubly charmed baryon mass and $v_\mu = (1, \vec{0})$ is the velocity of the baryon. Now the doubly charmed baryon matrix elements of the electromagnetic current J_μ can be parametrized as

$$\langle H(p') | J_\mu | H(p) \rangle = e \bar{u}(p') \mathcal{O}_\mu(p', p) u(p). \quad (5)$$

The tensor $\mathcal{O}_\mu$ can be parameterized in terms of electric and magnetic form factors.

$$\mathcal{O}_\mu(p', p) = v_\mu G_E(q^2) + \frac{[S^\mu, S^\nu] q^\nu}{M} G_M(q^2), \quad (6)$$

where $G_E(q^2)$ is the electric form factor and $G_M(q^2)$ is the magnetic form factor. When $q^2 = 0$, we obtain the charge (Q) and magnetic moment (μ_H),

$$Q = G_E(0), \quad \mu_H = \frac{e}{2M_H} G_M(0). \quad (7)$$

III. CHIRAL LAGRANGIANS

A. The strong interaction chiral Lagrangians

To calculate the chiral corrections to the magnetic moment of doubly charmed baryon, we follow the basic definitions of the pseudoscalar mesons and the spin- $\frac{1}{2}$ baryon chiral effective Lagrangians in Refs. [59,62] to construct the relevant chiral Lagrangians. The pseudoscalar meson fields read

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \quad (8)$$

The chiral connection and axial vector field read [59,63]:

$$\Gamma_\mu = \frac{1}{2} [u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger], \quad (9)$$

$$u_\mu \equiv \frac{1}{2} i [u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger], \quad (10)$$

where

$$u^2 = U = \exp(i\phi/f_0). \quad (11)$$

We use the pseudoscalar meson decay constants $f_\pi \approx 92.4$ MeV, $f_K \approx 113$ MeV and $f_\eta \approx 116$ MeV.

The doubly charmed baryon fields read.

$$\Psi = \begin{pmatrix} \Xi_{cc}^{*++} \\ \Xi_{cc}^{*+} \\ \Omega_{cc}^{*+} \end{pmatrix}, \quad \Psi^{*\mu} = \begin{pmatrix} \Xi_{cc}^{*++} \\ \Xi_{cc}^{*+} \\ \Omega_{cc}^{*+} \end{pmatrix}^\mu \Rightarrow \begin{pmatrix} ccu \\ ccd \\ ccs \end{pmatrix}^\mu, \quad (12)$$

where Ψ and $\Psi^{*\mu}$ are spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryon fields, respectively. The leading order pseudoscalar meson and baryon interaction Lagrangians read

$$\begin{aligned} \hat{\mathcal{L}}_0^{(1)} = & \bar{\Psi}(i\mathcal{D} - M_H)\Psi \\ & + \bar{\Psi}^{*\mu} [-g_{\mu\nu}(i - M_T) + i(\gamma_\mu D_\nu + \gamma_\nu D_\mu) \\ & - \gamma_\mu(i\mathcal{D} + M_T)\gamma_\nu] \Psi^{*\nu}, \end{aligned} \quad (13)$$

$$\hat{\mathcal{L}}_{\text{int}}^{(1)} = \frac{\tilde{g}_A}{2} \bar{\Psi} \not{\gamma}_5 \Psi + \frac{\tilde{g}_C}{2} [\bar{\Psi}^{*\mu} u_\mu \Psi + \bar{\Psi} u_\mu \Psi^{*\mu}], \quad (14)$$

where M_H is the spin- $\frac{1}{2}$ doubly charmed baryon mass, M_T is the spin- $\frac{3}{2}$ doubly charmed baryon mass,

$$\begin{aligned} D_\mu \Psi &= \partial_\mu \Psi + \Gamma_\mu \Psi, \\ D^\nu \Psi^{*\mu} &= \partial^\nu \Psi^{*\mu} + \Gamma^\nu \Psi^{*\mu}. \end{aligned} \quad (15)$$

In the framework of HBChPT, we denote the large component of the spin- $\frac{3}{2}$ doubly charmed baryon as T_μ . The leading order nonrelativistic pseudoscalar meson and baryon Lagrangians read

$$\mathcal{L}_0^{(1)} = \bar{H}(i v \cdot D) H - i \bar{T}^\mu (v \cdot D - \delta) T_\mu, \quad (16)$$

$$\mathcal{L}_{\text{int}}^{(1)} = \tilde{g}_A \bar{H} S_\mu u^\mu H + \frac{\tilde{g}_C}{2} [\bar{T}^\mu u_\mu H + \bar{H} u_\mu T^\mu], \quad (17)$$

where S_μ is the covariant spin-operator, $\delta = M_T - M_H$ is the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryon mass splitting. The ϕHH coupling $\tilde{g}_A$ and ϕTH coupling $\tilde{g}_C$ can be estimated in the quark model in Sec. V. For the pseudo-scalar mesons masses, we use $m_\pi = 0.140$ GeV, $m_K = 0.494$ GeV, and $m_\eta = 0.550$ GeV. We use the nucleon masses $M_B = 0.938$ GeV and the spin- $\frac{1}{2}$ doubly charmed baryon mass $M_H = 3.62$ GeV.

B. The electromagnetic chiral Lagrangians at $\mathcal{O}(p^2)$

The lowest order $\mathcal{O}(p^2)$ Lagrangian contributes to the magnetic moments of the doubly charmed baryons at the tree level

$$\mathcal{L}_{\mu_H}^{(2)} = a_1 \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu] \hat{F}_{\mu\nu}^+ H + a_2 \frac{-i}{4M_B} \bar{H}[S^\mu, S^\nu] H \text{Tr}(F_{\mu\nu}^+), \quad (18)$$

where the coefficients $a_{1,2}$ are the LECs. The chirally covariant QED field strength tensor $F_{\mu\nu}^\pm$ is defined as

$$F_{\mu\nu}^\pm = u^\dagger F_{\mu\nu}^R u \pm u F_{\mu\nu}^L u^\dagger, \quad (19)$$

$$F_{\mu\nu}^R = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \quad (19)$$

$$F_{\mu\nu}^L = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu], \quad (20)$$

where $r_\mu = l_\mu = -e Q_H A_\mu$ and $Q_H = \text{diag}(2, 1, 1)$. The operator $\hat{F}_{\mu\nu}^+ = F_{\mu\nu}^+ - \frac{1}{3} \text{Tr}(F_{\mu\nu}^+)$ is traceless and transforms as the adjoint representation. Recall that the direct product $3 \otimes \bar{3} = 1 \oplus 8$. Therefore, there are two independent interaction terms in the $\mathcal{O}(p^2)$ Lagrangians for the magnetic moments of the doubly charmed baryons.

IV. FORMALISM UP TO ONE-LOOP LEVEL

We follow the standard power counting scheme as in Ref. [60], the chiral order D_χ of a given diagram is given by [64]

$$D_\chi = 4N_L - 2I_M - I_B + \sum_n n N_n, \quad (21)$$

where N_L is the number of loops, I_M is the number of internal pion lines, I_B is the number of internal baryon lines and N_n is the number of the vertices from the n th order Lagrangians. The chiral order of magnetic moments μ_H is $(D_\chi - 1)$ based on Eq. (7).

FIG. 1. The $\mathcal{O}(p^2)$ tree level diagrams where the doubly charmed baryon is denoted by the solid line. The dot represent second-order coupling.

FIG. 2. The one-loop diagrams where spin- $\frac{1}{2}$ (spin- $\frac{3}{2}$) doubly charmed baryon is denoted by the single (double) solid line. The dashed and wiggly lines represent the pseudoscalar meson and photon respectively.

We assume the exact isospin symmetry with $m_u = m_d$ throughout this work. The tree-level Lagrangians in Eq. (18) contribute to the doubly charmed baryon magnetic moments at $\mathcal{O}(p^1)$ as shown in Fig. 1. The Clebsch-Gordan coefficients for the various doubly charmed baryons are collected in Table II. All tree level doubly charmed baryon magnetic moments are given in terms of a_1, a_2 .

There are two Feynman diagrams contributing to the doubly charmed baryon magnetic moments to next-to-leading order at one-loop level as shown in Fig. 2. The intermediate states in diagrams (a) and (b) are spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryons respectively. The photon vertex is from the meson photon interaction term while the meson vertex is from the interaction terms in Eq. (17).

We collect our numerical results of the doubly charmed baryon magnetic moments to the next-to-leading order in Table II. We also compare the numerical results of the magnetic moments when the chiral expansions are truncated at $\mathcal{O}(p^1)$ and $\mathcal{O}(p^2)$ respectively in Table III.

Summing the one-loop level contributions to the doubly charmed baryon magnetic moments in Fig. 2, considering both the spin- $\frac{3}{2}$ and spin- $\frac{1}{2}$ doubly charmed baryon intermediate states, the loop corrections to the doubly charmed baryon magnetic moments can be expressed as

$$\mu_H^{(2, \text{loop})} = \sum_{\phi=\pi, K} \left(-\frac{\tilde{g}_A^2 m_\phi M_N \beta_a^\phi}{64\pi f_\phi^2} + \frac{\beta_b^\phi \tilde{g}_C^2 (-\delta [\log(\frac{m_\phi^2}{\lambda^2}) - 1] + 2\sqrt{m_\phi^2 - \delta^2} \arccos(\frac{\delta}{m_\phi}))}{192\pi^2 f_\phi^2} \right) \quad (22)$$

TABLE I. The coefficients of the loop corrections to the doubly charmed baryon magnetic moments from Figs. 2(a) and 2(b).

Baryons	β_a^π	β_a^K	β_b^π	β_b^K
Ξ_{cc}^{++}	2	2	2	2
Ξ_{cc}^+	-2	0	-2	0
Ω_{cc}^+	0	-2	0	-2

TABLE II. The doubly charmed baryon magnetic moments to the next-to-leading order(in unit of μ_N).

Baryons	$\mathcal{O}(p^1)$ tree	$\mathcal{O}(p^2)$ loop (a)	$\mathcal{O}(p^2)$ loop (b)
Ξ_{cc}^{++}	$\frac{2}{3}a_1 + 4a_2$	$-0.51\tilde{g}_A^2$	$0.20\tilde{g}_C^2$
Ξ_{cc}^+	$-\frac{1}{3}a_1 + 4a_2$	$0.15\tilde{g}_A^2$	$-0.07\tilde{g}_C^2$
Ω_{cc}^+	$-\frac{1}{3}a_1 + 4a_2$	$0.36\tilde{g}_A^2$	$-0.12\tilde{g}_C^2$

where $\lambda = 4\pi f_\pi$ is the renormalization scale. Here, the coefficients β_{a-b}^ϕ arise from the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryon intermediate states respectively in Fig. 2. We collect their explicit expressions in Table I.

With the low energy counter terms and loop contributions (22), we obtain the magnetic moments,

$$\mu_H = \{\mu_H^{(1)}\} + \{\mu_H^{(2,\text{loop})}\} \quad (23)$$

where $\mu_H^{(1)}$ is the tree-level magnetic moments from Eq. (18).

V. NUMERICAL RESULTS AND DISCUSSIONS

In ChPT, the low energy constants (LECs) should be fitted through experimental inputs. As there are not any experimental data of the doubly charmed baryon magnetic moments so far, in this paper, we adopt the same strategy as in Ref. [65]. We use quark model to determine the leading-order tree level magnetic moments. In other words, our calculation is actually the chiral corrections to the quark model.

We collect our numerical results of the doubly charmed baryon magnetic moments to the next-to-leading order in Table II. We also compare the numerical results of the magnetic moments when the chiral expansions are truncated at $\mathcal{O}(p^1)$ and $\mathcal{O}(p^2)$ respectively in Table III.

At the leading order $\mathcal{O}(p^1)$, as the charge matrix Q_H is not traceless, there are two unknown LECs $a_{1,2}$ in Eq. (18). Notice the second column in Table II, the a_1 parts are proportional to the light quark charge within the doubly charmed baryon and the a_2 parts are the same for all three doubly charmed baryons and arise solely from the two charm quarks.

At the quark level, the flavor and spin wave functions of the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryons Ξ_{ccq} and Ξ_{ccq}^* read:

TABLE III. The doubly charmed baryon magnetic moments when the chiral expansion is truncated at $\mathcal{O}(p^1)$ and $\mathcal{O}(p^2)$, respectively (in unit of μ_N).

Baryons ($\delta = 0.1$ GeV)	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$ loop	$\mathcal{O}(p^2)$ total
Ξ_{cc}^{++}	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_u = -0.12$	0.47	0.35
Ξ_{cc}^+	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_d = 0.81$	-0.19	0.62
Ω_{cc}^+	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_s = 0.69$	-0.28	0.41

$$\begin{aligned} \left| \Xi_{ccq}; s_3 = \frac{1}{2} \right\rangle = \frac{1}{3\sqrt{2}} [& 2c\uparrow c\uparrow q\downarrow - c\uparrow c\downarrow q\uparrow - c\downarrow c\uparrow q\uparrow \\ & + 2c\uparrow q\downarrow c\uparrow - c\downarrow q\uparrow c\uparrow - c\downarrow q\downarrow c\downarrow \\ & + 2q\downarrow c\uparrow c\uparrow - q\downarrow c\downarrow c\downarrow - q\uparrow c\downarrow c\uparrow], \quad (24) \end{aligned}$$

$$\left| \Xi_{ccq}^*; s_3 = \frac{1}{2} \right\rangle = \frac{1}{\sqrt{3}} [c\uparrow c\uparrow q\downarrow + c\uparrow c\downarrow q\uparrow + c\downarrow c\uparrow q\uparrow], \quad (25)$$

where the arrows denote the third-components of the spin. q can be u, d and s quark. The magnetic moments in the quark model are the matrix elements of the following operator,

$$\vec{\mu} = \sum_i \mu_i \vec{\sigma}^i, \quad (26)$$

where μ_i is the magnetic moment of the quark:

$$\mu_i = \frac{e_i}{2m_i}, \quad i = u, d, s, c. \quad (27)$$

We adopt the $m_u = m_d = 336$ MeV, $m_s = 540$ MeV, $m_c = 1660$ MeV as the constituent quark masses and give the results in the second column in Table III.

Up to $\mathcal{O}(p^2)$, we need to include both the tree-level magnetic moments and the $\mathcal{O}(p^2)$ loop corrections. We use the quark model to estimate the leading-order tree level transition magnetic moments. Thus, there exist only two LECs: $\tilde{g}_A$ and $\tilde{g}_C$ at this order. $\tilde{g}_A = -0.5$ has been estimated in Ref. [60]. Similarly, we can also obtain the ϕHT coupling $\tilde{g}_C$.

At quark level, the pion-quark interaction reads

$$\mathcal{L}_{\text{quark}} = \frac{g_0}{2} (\bar{u}\gamma^\mu \gamma_5 \partial_\mu \pi^0 u - \bar{d}\gamma^\mu \gamma_5 \partial_\mu \pi^0 d) \quad (28)$$

where g_0 is the quark level coupling constant. The matrix elements both at hadron level and quark level,

$$\langle H_1 | i\mathcal{L}_{H_1 H_2 \pi^0} | H_2; \pi^0 \rangle = \langle H_1 | i\mathcal{L}_{\text{quark}} | H_2; \pi^0 \rangle, \quad (29)$$

where $H_{1,2}$ are the hadrons. Considering the π^0 coupling at hadron level

$$\mathcal{L}_{\Xi_{cc}^{*++}\Xi_{cc}^{++}\pi^0} = -\frac{1}{2F_0} \frac{\tilde{g}_C}{2} \sqrt{\frac{2}{3}} \bar{\Xi}_{cc}^{*++} \partial_\mu \pi^0 \Xi_{cc}^{*++}. \quad (30)$$

we obtain the $\Xi_{cc}^{*++}\Xi_{cc}^{++}\pi^0$ matrix elements at the hadron level,

$$\langle \Xi_{cc}^{*++} | i\mathcal{L}_{\Xi_{cc}^{*++}\Xi_{cc}^{++}\pi^0} | \Xi_{cc}^{*++}; \pi^0 \rangle \sim -\frac{1}{2} \frac{\tilde{g}_C}{2} \sqrt{\frac{2}{3}} q_3, \quad (31)$$

and at the quark level,

$$\langle \Xi_{cc}^{*++} | i\mathcal{L}_{\text{quark}} | \Xi_{cc}^{*++}; \pi^0 \rangle \sim -\frac{2\sqrt{2}}{6} q_3. \quad (32)$$

Compare with the axial charge of the nucleon,

$$\frac{\frac{1}{2} \frac{\tilde{g}_C}{2} \sqrt{\frac{2}{3}}}{-\frac{2\sqrt{2}}{6}} = \frac{\frac{1}{2} g_A}{\frac{5}{6}}. \quad (33)$$

We obtain the ϕTH coupling $\tilde{g}_C = -\frac{4\sqrt{3}}{5} g_A = -1.75$.

With $\tilde{g}_A$ and $\tilde{g}_C$, we also need the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryon mass splitting $\delta = M_T - M_H$ to obtain the numerical results of the $\mathcal{O}(p^2)$ doubly charmed baryon magnetic moments. As the spin- $\frac{3}{2}$ doubly charmed baryons have not been observed in the experiments, the masses of the spin- $\frac{3}{2}$ doubly charmed baryons remain unknown. There are independent determinations of mass splittings in nonrelativistic lattice QCD [14], lattice QCD [15,17,18] and pNRQCD [19], the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryon mass splitting δ vary from 40 MeV to 120 MeV.

FIG. 3. The variations of doubly charmed baryon magnetic moments μ (in unit of μ_N) with δ (in unit of GeV).

TABLE IV. Comparison of the decuplet to octet baryon transition magnetic moments in literature including quark model (QM) [44], relativistic three-quark model (RTQM) [45], relativistic quark model (RQM) [46], MIT bag model [47], confining logarithmic potential (CLP) [49], skyrmion description [50], nonrelativistic quark model (NQM) [52], lattice QCD(LQCD) [54] and chiral perturbation theory (ChPT) [60](in unit of μ_N).

Baryons	Ξ_{cc}^{*++}	Ξ_{cc}^{++}	Ω_{cc}^{++}
QM [44]	-0.124	0.806	0.688
RTQM [45]	$0.13^{+0.52}_{-0.38}$	$0.72^{+0.52}_{+0.20}$	$0.67^{+0.53}_{+0.14}$
RQM [46]	$-0.10^{+0.40}_{-0.22}$	$0.86^{+0.08}_{-0.18}$	$0.72^{+0.06}_{-0.06}$
MIT bag model [47]	0.17	0.86	0.84
CLP(Set 1) [49]	-0.154	0.778	0.657
CLP(Set 2) [49]	-0.172	0.790	0.663
Skyrmion(Set 1) [50]	-0.47	0.98	0.59
Skyrmion(Set 2) [50]	-0.47	0.89	0.64
NQM [52]	$-0.208^{+0.035}_{-0.086}$	$0.785^{+0.050}_{-0.030}$	$0.635^{+0.012}_{-0.015}$
LQCD [54]	...	$0.425^{+0.029}_{-0.029}$	$0.413^{+0.024}_{-0.024}$
ChPT[60]	-0.25	0.85	0.78
This work	0.35	0.62	0.41

We adopt $\delta = 0.1$ GeV approximatively in Table III, and show the variations of doubly charmed baryon magnetic moments with the spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ doubly charmed baryon mass splitting δ in Fig. 3.

Compared to the results in Ref. [60], it is interesting to notice that after taking the spin- $\frac{3}{2}$ doubly charmed baryon contribution into consideration, the $\mathcal{O}(p^2)$ loop corrections for the Ξ_{cc}^{*++} magnetic moment change from $-0.13 \mu_N$ to $0.47 \mu_N$. The reason for this is that the ϕTH coupling $\tilde{g}_C = -1.75$ is much bigger than the ϕHH coupling $\tilde{g}_A = -0.5$ in Table II. Thus, the spin- $\frac{3}{2}$ doubly charmed baryon contribution is magnified. The total Ξ_{cc}^{*++} magnetic moment changes from $-0.25 \mu_N$ to $0.35 \mu_N$. At present, the magnetic moment of Ξ_{cc}^{*++} has not been measured in the experiment, our calculation shows that the magnetic moment of Ξ_{cc}^{*++} may be positive due to the spin- $\frac{3}{2}$ doubly charmed baryon contribution in the chiral loop corrections. We are looking forward to further progresses in experiment.

In Table IV, we compare our results obtained in the HBChPT with those from other model calculations such as quark model (QM) [44], relativistic three-quark model (RTQM) [45], relativistic quark model (RQM) [46], MIT bag model [47], confining logarithmic potential (CLP) [49], skyrmion description [50], nonrelativistic quark model (NQM) [52], lattice QCD(LQCD)[54] and chiral perturbation theory (ChPT) [60]. We have shown the errors on the different model predictions for appropriate comparisons.

VI. CONCLUSIONS

In short summary, we have investigated the magnetic moments for the spin- $\frac{1}{2}$ doubly charmed baryons to the next-to-leading order in the framework of HBChPT. The

spin- $\frac{3}{2}$ doubly charmed baryons were included as an explicit degree of freedom and its contribution to the spin- $\frac{1}{2}$ doubly charmed baryon magnetic moments evaluated. We use quark model to determine the leading-order magnetic moments and obtain the numerical results to next-to-leading order. Our calculation shows that $\mu_{\Xi_{cc}^{++}} = 0.35\mu_N$, $\mu_{\Xi_{cc}^{+}} = 0.62\mu_N$, $\mu_{\Omega_{cc}^{+}} = 0.41\mu_N$.

Our analysis indicates that after taking the spin- $\frac{3}{2}$ doubly charmed baryon contribution into consideration, the Ξ_{cc}^{++} magnetic moment changes a lot from $-0.25\mu_N$ to $0.35\mu_N$. Our calculation shows that the magnetic moment of Ξ_{cc}^{++} may be positive due to the spin- $\frac{3}{2}$ doubly charmed baryon contribution in the chiral loop corrections. We are looking

forward to further progresses and hope our results may be useful for future experimental measurement of the doubly charmed baryons magnetic moments. Our analytical results may be useful to the possible chiral extrapolation of the lattice simulations.

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APPENDIX: INTEGRALS AND LOOP FUNCTIONS

We collect some common integrals and loop functions in this appendix.

$$\Delta = i \int \frac{d^d l \lambda^{4-d}}{(2\pi)^d} \frac{1}{l^2 - m^2} = 2m^2 \left(L(\lambda) + \frac{1}{32\pi^2} \ln \frac{m^2}{\lambda^2} \right), \quad (\text{A1})$$

$$L(\lambda) = \frac{\lambda^{d-4}}{16\pi^2} \left[\frac{1}{d-4} - \frac{1}{2} (\ln 4\pi + 1 + \Gamma'(1)) \right], \quad (\text{A2})$$

$$I_0(q^2) = i \int \frac{d^d l \lambda^{4-d}}{(2\pi)^d} \frac{1}{(l^2 - m^2 + i\epsilon)((l+q)^2 - m^2 + i\epsilon)}, \quad r = \sqrt{|1 - 4m^2/q^2|}$$

$$= \begin{cases} -\frac{1}{16\pi^2} (1 - \ln \frac{m^2}{\lambda^2} - r \ln |\frac{1+r}{1-r}|) + 2L(\lambda) & (q^2 < 0) \\ -\frac{1}{16\pi^2} (1 - \ln \frac{m^2}{\lambda^2} - 2r \arctan \frac{1}{r}) + 2L(\lambda) & (0 < q^2 < 4m^2), \\ -\frac{1}{16\pi^2} (1 - \ln \frac{m^2}{\lambda^2} - r \ln |\frac{1+r}{1-r}| + i\pi r) + 2L(\lambda) & (q^2 > 4m^2) \end{cases} \quad (\text{A3})$$

$$i \int \frac{d^d l \lambda^{4-d}}{(2\pi)^d} \frac{[1, l_\alpha, l_\alpha l_\beta]}{(l^2 - m^2 + i\epsilon)(\omega + v \cdot l + i\epsilon)} = [J_0(\omega), v_\alpha J_1(\omega), g_{\alpha\beta} J_2(\omega) + v_\alpha v_\beta J_3(\omega)], \quad \omega = v \cdot r + \delta \quad (\text{A4})$$

$$J_0(\omega) = \begin{cases} \frac{-\omega}{8\pi^2} (1 - \ln \frac{m^2}{\lambda^2}) + \frac{\sqrt{\omega^2 - m^2}}{4\pi^2} (\operatorname{arccosh} \frac{\omega}{m} - i\pi) + 4\omega L(\lambda) & (\omega > m) \\ \frac{-\omega}{8\pi^2} (1 - \ln \frac{m^2}{\lambda^2}) + \frac{\sqrt{m^2 - \omega^2}}{4\pi^2} \arccos \frac{-\omega}{m} + 4\omega L(\lambda) & (\omega^2 < m^2), \\ \frac{-\omega}{8\pi^2} (1 - \ln \frac{m^2}{\lambda^2}) - \frac{\sqrt{\omega^2 - m^2}}{4\pi^2} \operatorname{arccosh} \frac{-\omega}{m} + 4\omega L(\lambda) & (\omega < -m) \end{cases} \quad (\text{A5})$$

$$J_1(\omega) = -\omega J_0(\omega) + \Delta, \quad (\text{A6})$$

$$J_2(\omega) = \frac{1}{d-1} [(m^2 - \omega^2) J_0(\omega) + \omega \Delta], \quad (\text{A7})$$

$$J_3(\omega) = -\omega J_1(\omega) - J_2(\omega), \quad (\text{A8})$$

$$\begin{aligned}
L_0(\omega) &= i \int \frac{d^d l \lambda^{4-d}}{(2\pi)^d} \frac{1}{(l^2 - m^2 + i\epsilon)((l+q)^2 - m^2 + i\epsilon)(\omega + v \cdot l + i\epsilon)}, \quad v \cdot q = 0 \\
&= \begin{cases} \frac{-1}{8\pi^2} \frac{1}{\sqrt{\omega^2 - m^2}} (\operatorname{arccosh} \frac{\omega}{m} - i\pi) & (\omega > m) \\ \frac{1}{8\pi^2} \frac{1}{\sqrt{m^2 - \omega^2}} \arccos \frac{-\omega}{m} & (\omega^2 < m^2) \\ \frac{1}{8\pi^2} \frac{1}{\sqrt{\omega^2 - m^2}} \operatorname{arccosh} \frac{-\omega}{m} & (\omega < -m) \end{cases} \quad (\text{A9})
\end{aligned}$$

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