Reinvestigating the $B \rightarrow PP$ decays by including the contributions from ϕ_{B2}

Yueling Yang^{1,*}, Lan Lang¹, Xule Zhao¹, Jinshu Huang², and Junfeng Sun^{1,†}

¹*Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang 453007, China*

²*School of Physics and Electronic Engineering, Nanyang Normal University, Nanyang 473061, China*

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Considering the B mesonic distribution amplitude ϕ_{B2} , we reinvestigated the $B \rightarrow PP$ (where $P = \pi$ and K) decays with the perturbative QCD (pQCD) approach based on the k_T factorization for three scenarios. It is found that the contributions of ϕ_{B2} to form factors $F_0^{B \rightarrow P}(0)$ and branching ratios are comparable with those from the NLO corrections. The $B \rightarrow K\pi$ decays could be well explained by considering the ϕ_{B2} . Hence, when the nonleptonic B decays are studied with the pQCD approach, the ϕ_{B2} should be taken into account seriously.

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It is well known that many breakthrough discoveries have come from precise experiments. B physics is on the bleeding edge and one of hot topics of current particle physics, because of the renewed impetus from the successive CLEO, BABAR, Belle, LHCb, and Belle-II experiments. Various B meson decay modes with branching ratio larger than 10^{-6} have been extensively studied by the BABAR and Belle Collaborations with 0.56 ab^{-1} and 1.02 ab^{-1} data samples in the past years [1,2]. A few phenomena of inconsistencies between experimental measurements and theoretical expectations from the standard model (SM) are emerging. More and more B meson data are expected in the near future, about 50 ab^{-1} by the Belle-II detector at the e^+e^- SuperKEKB collider [3] and about 300 fb^{-1} by the LHCb detector at the High Luminosity LHC (HL-LHC) hadron collider [4]. Besides some new phenomena, the much more precise measurements of B meson weak decays will offer a much more rigorous test on SM. When looking for a smoking gun of new physics and settling the temporary differences between experimental and theoretical results, a more careful calculation on B meson decays within SM is very necessary and important. In this paper, we will reinvestigate the $B \rightarrow PP$ decays (here $P = \pi$ and K) based on the perturbative QCD approach within SM, by considering the contributions from B mesonic wave function ϕ_{B2} which usually attract less attention in previous calculation.

For clarity, we will sketch the phenomenological study of nonleptonic $B \rightarrow PP$ decays, although they have been

extensively studied, for example, in Refs. [5–13]. Because of our inadequate comprehension of the flavor mixing and possible glueball components, the final states of η and η' mesons are not considered here for the moment.

At the quark level, based on the operator product expansion and renormalization group (RG) method, the effective Hamiltonian responsible for $B \rightarrow PP$ decays is written as [14],

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left\{ V_{ub} V_{uq}^* \sum_{i=1}^2 C_i Q_i - V_{tb} V_{tq}^* \sum_{j=3}^{10} C_j O_j \right\} + \text{H.c.}, \quad (1)$$

where $G_F \simeq 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [1] is the Fermi weak coupling constant. With the Wolfenstein parametrization, the related Cabibbo-Kobayashi-Maskawa (CKM) factors are written as follows.

$$V_{ub} V_{ud}^* = A \lambda^3 (\rho - i\eta) \left(1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 \right) + \mathcal{O}(\lambda^8), \quad (2)$$

$$V_{tb} V_{td}^* = A \lambda^3 + A^3 \lambda^7 \left(\rho - i\eta - \frac{1}{2} \right) - V_{ub} V_{ud}^* + \mathcal{O}(\lambda^8), \quad (3)$$

$$V_{ub} V_{us}^* = A \lambda^4 (\rho - i\eta) + \mathcal{O}(\lambda^8), \quad (4)$$

$$V_{tb} V_{ts}^* = -A \lambda^2 \left(1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 \right) + \frac{1}{2} A^3 \lambda^6 - V_{ub} V_{us}^* + \mathcal{O}(\lambda^8), \quad (5)$$

and the latest values of the four Wolfenstein parameters (A , λ , ρ and η) from data with the CKMfitter method [1] are listed in Table I. The Wilson coefficients, C_i , are

*yangyueling@htu.edu.cn

†sunjunfeng@htu.edu.cn

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TABLE I. The values of the input parameters, where their central values will be regarded as the default inputs unless otherwise specified. The numbers in parentheses are errors.

CKM parameters [1]	$A = 0.790(17),$ $\bar{\rho} = 0.141(17),$	$\lambda = 0.22650(48),$ $\bar{\eta} = 0.357(11),$
Mass of the particles [1]	$m_{\pi^0} = 134.98 \text{ MeV},$ $m_{K^0} = 497.61 \text{ MeV},$ $m_{B_u} = 5279.34(12) \text{ MeV},$ $m_b = 4.78(6) \text{ GeV},$	$m_{\pi^\pm} = 139.57 \text{ MeV},$ $m_{K^\pm} = 493.68 \text{ MeV},$ $m_{B_d} = 5279.65(12) \text{ MeV},$ $m_s = 130.0(1.8) \text{ MeV} [15],$
Decay constants [1]	$f_\pi = 130.2(1.2) \text{ MeV},$ $f_K = 155.7(3) \text{ MeV},$ $f_B = 190.0(1.3) \text{ MeV},$	$f_{3\pi} = 0.45(15) \times 10^{-2} \text{ GeV}^2 [16],$ $f_{3K} = 0.45(15) \times 10^{-2} \text{ GeV}^2 [16],$
Lifetime [1]	$\tau_{B^\pm} = 1.638(4) \text{ ps},$	$\tau_{B^\pm} = 1.519(4) \text{ ps},$
Gegenbauer moments at the scale of $\mu = 1 \text{ GeV} [16]$	$a_1^\pi = 0,$ $a_1^K = 0.06(3),$	$a_2^\pi = 0.25(15),$ $a_2^K = 0.25(15).$

perturbatively calculable at the scale of $\mathcal{O}(m_W)$ and then evolved to the b quark decay scale $\mathcal{O}(m_b)$ with the RG equation [14]. The combinations of the well determined G_F , CKM factors and C_i could be regarded as the universal and effective couplings of the operators O_i . The tree operators $O_{1,2}$, QCD penguin operators $O_{3\sim 6}$ and electromagnetic penguin operators $O_{7\sim 10}$ are local four-quark interactions and expressed as follows.

$$O_1 = (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta u_\beta)_{V-A}, \quad (6)$$

$$O_2 = (\bar{u}_\alpha b_\beta)_{V-A} (\bar{q}_\beta u_\alpha)_{V-A}, \quad (7)$$

$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad (8)$$

$$O_4 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A}, \quad (9)$$

$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad (10)$$

$$O_6 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}, \quad (11)$$

$$O_7 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} Q_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \quad (12)$$

$$O_8 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} \frac{3}{2} Q_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A}, \quad (13)$$

$$O_9 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} Q_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \quad (14)$$

$$O_{10} = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} \frac{3}{2} Q_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A}, \quad (15)$$

where $(\bar{q}_1 q_2)_{V\pm A} \equiv \bar{q}_1 \gamma^\mu (1 \pm \gamma_5) q_2$; α and β is the color indices; $Q_{q'}$ is the electric charge of quark q' in the unit of $|e|$; and $q' \in \{u, d, c, s, b\}$.

The hadronic matrix elements (HMEs), $\langle O_i \rangle \equiv \langle P_1 P_2 | O_i | B \rangle$, describe the transformations from the quarks to hadrons. The calculation of HMEs is on the one hand very complicated due to the entanglements between perturbative and nonperturbative contributions, and on the other hand very sensitive to phenomenological models because of our limited knowledge of dynamics of hadronization and final state interactions. One of the main challenges is to calculate HMEs as properly as possible. Theoretically, the radiative corrections to HMEs should be appropriately included so that the strong phase angles closely related to CP violation could be obtained. For nonleptonic B decays, the HMEs are usually written as the product of the rescattering amplitudes of quarks (which are calculable order by order with perturbation theory in principle) and wave functions of participating hadrons (where nonperturbative contributions are housed) with the fashionable QCD-inspired phenomenological models, either the QCD factorization (QCDF) approach [17–22] based on the collinear approximation or the perturbative QCD (pQCD) approach [23–29] retaining the effects of transverse momentum k_T . Hadronic wave functions (WFs) or distribution amplitudes (DAs) are independent of specific process and determined from data, which enable evaluating HMEs to simplify greatly.

WFs and/or DAs are the essential ingredients of the master formulas for evaluating HMEs with the QCDF and pQCD approaches. The B mesonic WFs are generally composed of two scalar functions [30,31] and written as follows with the convention of Refs. [32,33].

$$\begin{aligned}
 \langle 0 | \bar{q}_\alpha(z) b_\beta(0) | \bar{B}(p) \rangle &= +\frac{i}{4} f_B \int d^4 k e^{-ik \cdot z} \left\{ (\not{p} + m_B) \gamma_5 \left[\frac{\not{p}_+}{\sqrt{2}} \phi_B^+ + \frac{\not{p}_-}{\sqrt{2}} \phi_B^- \right] \right\}_{\beta\alpha} \\
 &= -\frac{i}{4} f_B \int d^4 k e^{-ik \cdot z} \left\{ (\not{p} + m_B) \gamma_5 \left[\phi_B^+ + \frac{\not{p}_-}{\sqrt{2}} (\phi_B^+ - \phi_B^-) \right] \right\}_{\beta\alpha} \\
 &= -\frac{i}{4} f_B \int d^4 k e^{-ik \cdot z} \left\{ (\not{p} + m_B) \gamma_5 \left(\phi_{B1} + \frac{\not{p}_-}{\sqrt{2}} \phi_{B2} \right) \right\}_{\beta\alpha}, \quad (16)
 \end{aligned}$$

where the coordinate of the light quark is on the light cone, i.e., $z^2 = 0$ and $z_+ = 0$. $n_+^\mu = (1, 0, 0)$ and $n_-^\mu = (0, 1, 0)$ are the light cone vectors. f_B is the decay constant. ϕ_B^+ and ϕ_B^- are respectively the leading- and subleading-twist WFs. The properties and relations of $\phi_B^\pm$ are listed as follows.

$$\int_0^1 dx \phi_B^\pm(x) = 1, \quad (17)$$

$$\phi_{B1} = \phi_B^+, \quad (18)$$

$$\phi_{B2} = \phi_B^+ - \phi_B^-, \quad (19)$$

$$\phi_B^+(x) + x \phi_B^-(x) = 0, \quad (20)$$

where x is the longitudinal momentum fraction carried by the light quark in the B meson. ϕ_B^+ and ϕ_B^- have different asymptotic behaviors as $x \rightarrow 0$, $\phi_B^+ \sim x$ but ϕ_B^- will not vanish. So they do not coincide, i.e., $\phi_B^+ \neq \phi_B^-$ or $\phi_{B2} \neq 0$. In many actual calculations of nonleptonic B decays, only the contributions of ϕ_{B1} are considered appropriately, while those of ϕ_{B2} are assumed to be power suppressed and almost completely neglected. However, studies of Refs. [32–36] have shown that contributions of ϕ_{B2} to the $B \rightarrow \pi$ transition formfactors with the pQCD approach could have a large proportion rather than negligible. For example, the share could reach up to $\sim 30\%$ for some specific cases [32,33]. Clearly, the contributions of ϕ_{B2} will have some impacts on branching ratios of B meson decays. We should pay due attention to contributions of ϕ_{B2} in pace with the improvements of measurement precision, which is one main motivation of this work. The contributions of ϕ_{B2} to the $B \rightarrow PP$ decays have been studied with the QCDF approach [37]. The study of Ref. [37] showed that ϕ_{B2} only contributed to nonfactorizable annihilation amplitudes, and is helpful in explaining pure annihilation B decays. Different from the QCDF case, ϕ_{B2} will contribute to both factorizable and nonfactorizable emission amplitudes with the pQCD approach, besides the nonfactorizable annihilation amplitudes. That is to say, ϕ_{B2} would have much more influence on nonleptonic B decays with the pQCD approach when compared with the QCDF approach. However, the contributions of ϕ_{B2} to the $B \rightarrow PP$ decays have not been studied with the pQCD approach, which is the focus of this paper.

One candidate of the most often used leading B mesonic WF ϕ_B^+ in earlier studies with the pQCD approach is written as [27]

$$\phi_B^+(x, b) = N x^2 \bar{x}^2 \exp \left\{ -\left(\frac{x m_B}{\sqrt{2} \omega_B} \right)^2 - \frac{1}{2} \omega_B^2 b^2 \right\}, \quad (21)$$

where b is the conjugate variable of the transverse momentum k_T . $\bar{x} = 1 - x$. ω_B is the shape parameter. N is the normalization constant.

$$\int_0^1 dx \phi_B^+(x, 0) = 1. \quad (22)$$

The corresponding sub-leading B mesonic WF ϕ_B^- [33] can be obtained by solving the equation of motion given by Eq. (20).

$$\begin{aligned}
 \phi_B^-(x, b) &= N \frac{2\omega_B^4}{m_B^4} \exp \left(-\frac{1}{2} \omega_B^2 b^2 \right) \\
 &\times \left\{ \sqrt{\pi} \frac{m_B}{\sqrt{2} \omega_B} \text{Erf} \left(\frac{m_B}{\sqrt{2} \omega_B}, \frac{x m_B}{\sqrt{2} \omega_B} \right) \right. \\
 &+ \left[1 + \left(\frac{m_B \bar{x}}{\sqrt{2} \omega_B} \right)^2 \right] \exp \left[-\left(\frac{x m_B}{\sqrt{2} \omega_B} \right)^2 \right] \\
 &\left. - \exp \left(-\frac{m_B^2}{2\omega_B^2} \right) \right\}. \quad (23)
 \end{aligned}$$

In addition, according to the convention of Refs. [16,38], WFs of the final pseudoscalars π^+ and K^+ are generally written as follows.

$$\begin{aligned}
 \langle M(p) | \bar{q}_\alpha(0) u_\beta(z) | 0 \rangle &= -\frac{i f_M}{4} \left\{ \gamma_5 \left[\not{p} \phi_M^a(x) + \mu_M \phi_M^p(x) \right. \right. \\
 &\left. \left. - \mu_M \left(\frac{\not{p} \not{z}}{p \cdot z} - 1 \right) \phi_M^i(x) \right] \right\}_{\beta\alpha}, \quad (24)
 \end{aligned}$$

where f_M is the decay constant. x is the longitudinal momentum fraction of the anti-quark. $\mu_M = 1.6 \pm 0.2 \text{ GeV}$ [16] is the chiral mass. DA ϕ_M^a is the leading twist (twist-2), and $\phi_M^{p,i}$ is the twist-3. Their explicit expressions are given in Ref. [16].

$$\phi_M^a(x) = 6x\bar{x}\{1 + a_1^M C_1^{3/2}(\xi) + a_2^M C_2^{3/2}(\xi)\}, \quad (25)$$

$$\begin{aligned} \phi_M^p(x) = & 1 + 3\rho_+^M - 9\rho_-^M a_1^M + 18\rho_+^M a_2^M \\ & + \frac{3}{2}(\rho_+^M + \rho_-^M)(1 - 3a_1^M + 6a_2^M) \ln(x) \\ & + \frac{3}{2}(\rho_+^M - \rho_-^M)(1 + 3a_1^M + 6a_2^M) \ln(\bar{x}) \\ & - \left(\frac{3}{2}\rho_-^M - \frac{27}{2}\rho_+^M a_1^M + 27\rho_-^M a_2^M\right) C_1^{1/2}(\xi) \\ & + (30\eta_M - 3\rho_-^M a_1^M + 15\rho_+^M a_2^M) C_2^{1/2}(\xi), \end{aligned} \quad (26)$$

$$\begin{aligned} \phi_M^t(x) = & \frac{3}{2}(\rho_-^M - 3\rho_+^M a_1^M + 6\rho_-^M a_2^M) \\ & - C_1^{1/2}(\xi)\{1 + 3\rho_+^M - 12\rho_-^M a_1^M + 24\rho_+^M a_2^M \\ & + \frac{3}{2}(\rho_+^M + \rho_-^M)(1 - 3a_1^M + 6a_2^M) \ln(x) \\ & + \frac{3}{2}(\rho_+^M - \rho_-^M)(1 + 3a_1^M + 6a_2^M) \ln(\bar{x})\} \\ & - 3\left(3\rho_+^M a_1^M - \frac{15}{2}\rho_-^M a_2^M\right) C_2^{1/2}(\xi), \end{aligned} \quad (27)$$

where the variable $\xi = x - \bar{x} = 2x - 1$. The normalization conditions are

$$\int_0^1 dx \phi_M^{a,p}(x) = 1, \quad (28)$$

$$\int_0^1 dx \phi_M^t(x) = 0. \quad (29)$$

Other parameters are expressed as [16]: $\rho_+^M = \frac{m_M^2}{\mu_K^2}$, $\rho_-^K \simeq \frac{m_s}{\mu_K}$, $\rho_-^\pi = 0$, and $\eta_M = \frac{f_{3M}}{f_M \mu_M}$. The Gegenbauer polynomials are written as follows.

$$C_1^{1/2}(\xi) = \xi, \quad (30)$$

$$C_1^{3/2}(\xi) = 3\xi, \quad (31)$$

$$C_2^{1/2}(\xi) = \frac{3}{2}\xi^2 - \frac{1}{2}, \quad (32)$$

$$C_2^{3/2}(\xi) = \frac{15}{2}\xi^2 - \frac{3}{2}. \quad (33)$$

The curves of the normalized DAs $\phi_B^+(x, 0)$ and $\phi_B^-(x, 0)$ for B meson in Eq. (21) and Eq. (23) are displayed in Fig. 1. It can be clearly seen from Fig. 1 that (1) DAs $\phi_B^\pm$ are very asymmetric, and peak at small x region. This fact is generally consistent with the plausible suspicion that the light quark shares a small momentum fraction in B meson. In addition, DAs $\phi_B^\pm$ vanish as $x \rightarrow 1$, and thus offer a

FIG. 1. The shape lines of the B mesonic WFs $\phi_B^\pm(x, 0)$ versus x (horizontal axis).

natural cutoff on the seemingly counterintuitive contributions from large x domain. (2) ϕ_B^- and ϕ_{B2} do not vanish as $x \rightarrow 0$, thus the integral $\int dx \frac{\phi_{B2}}{x}$ and $\int dx \frac{\phi_{B2}}{x^2}$ corresponding to the factorizable emission topologies (form factors) diverge at the endpoint $x = 0$, as discussed in Ref. [31] with the collinear approximation. This implies that, on the one hand, the contributions of ϕ_{B2} might be important at small x regions and should be given due consideration in calculation, although ϕ_B^- is subleading twist; on the other hand, it seems reasonable and necessary to retain the contributions of the transverse momentum to regulate the singularities at the endpoint with the pQCD approach.

The line shapes of DAs $\phi_M^{a,p,t}(x)$ for π and K mesons in Eq. (25), Eq. (26) and Eq. (27) are shown in Fig. 2. The pionic DAs are totally symmetric with respect to the $x \leftrightarrow \bar{x}$ exchange. The $SU(3)$ breaking effects on kaonic DAs are considered. The quark-mass corrections modify the asymptotic behaviors of $\phi_M^{p,t}$ and induce the logarithmic endpoint singularities, as analyzed in Ref. [16].

With the above mesonic DAs, we can obtain the hadron transition formfactors and amplitudes of the $B \rightarrow PP$ decays with the pQCD approach. There are some conventions in our calculation. In the rest frame of B meson, the light-cone kinematic variables of participating particles in the heavy quark limit are defined as follows.

$$p_B = p_1 = (p_1^+, p_1^-, \vec{p}_{1T}) = \frac{m_B}{\sqrt{2}}(1, 1, 0), \quad (34)$$

$$p_M = p_2 = (p_2^+, p_2^-, \vec{p}_{2T}) = \frac{m_B}{\sqrt{2}}(0, 1, 0), \quad (35)$$

 FIG. 2. The shape lines of DAs $\phi_M^{a,p,t}$ for K in (a) and π in (b) versus x (horizontal axis).

$$p_{M'} = p_3 = (p_3^+, p_3^-, \vec{p}_{3T}) = \frac{m_B}{\sqrt{2}}(1, 0, 0), \quad (36)$$

$$k_1 = (x_1 p_1^+, 0, \vec{k}_{1T}), \quad (37)$$

$$k_2 = (0, x_2 p_2^-, \vec{k}_{2T}), \quad (38)$$

$$k_3 = (x_3 p_3^+, 0, \vec{k}_{3T}), \quad (39)$$

where k_1 and x_1 are respectively the momentum and longitudinal momentum fraction of light quark in the B meson; $k_{2,3}$ and $x_{2,3}$ are respectively the momentum and longitudinal momentum fraction of anti-quark in final hadrons. $\vec{k}_{iT}$ is the transverse momentum. It is clear that $p_1^2 = m_B^2$, $p_2^2 = m_M^2 = 0$ and $p_3^2 = m_{M'}^2 = 0$.

The form factors for the $B \rightarrow P$ transition are defined as [39]

$$\begin{aligned} & \langle M(p_2) | (\bar{q}b)_{V-A} | \bar{B}(p_1) \rangle \\ &= \left\{ (p_1 + p_2)^\mu - \frac{m_B^2 - m_M^2}{q^2} q^\mu \right\} F_1(q^2) \\ &+ \frac{m_B^2 - m_M^2}{q^2} q^\mu F_0(q^2), \end{aligned} \quad (40)$$

where $q = p_1 - p_2$. It is required that $F_0(0) = F_1(0)$ at the pole of $q^2 = 0$.

The lowest order Feynman diagrams for the $B \rightarrow M$ transition form factors are shown in Fig. 3. The form factors F_i are written as the convolution integrals of the quark scattering amplitudes $\mathcal{T}$ and hadron WFs Φ_i with the pQCD approach.

$$\begin{aligned} F_i &= \int dx_1 dx_2 db_1 db_2 \Phi_B(x_1, b_1) e^{-s_B} \\ &\times \mathcal{T}(x_1, x_2, b_1, b_2) \Phi_M(x_2, b_2) e^{-s_M}, \end{aligned} \quad (41)$$

where b_i is the conjugate variable of transverse momentum k_{iT} . The Sudakov factors e^{-s_B} and e^{-s_M} are introduced for WFs Φ_B and Φ_M , respectively. The Sudakov factor is a characteristic element and highly recommended by the pQCD approach to effectively regulate the nonperturbative contributions, so that a dominant share of form factor would come from hard gluon exchange, and the perturbative calculation would be reasonable and practicable. The expressions for form factors including the ϕ_{B2} contributions are listed in Appendix A. Our results of form factors are shown in Figs. 4–6 and Table II.

The dependences of form factor $F_0^{B \rightarrow \pi}(0)$ on some input parameters are shown in Fig. 4. It is seen clearly that (1) form factors obtained with the pQCD approach are sensitive to the shape parameter ω_B for B mesonic WFs. This phenomenon is basically analogical with that of Ref. [33]. (2) the effects of the chiral mass μ_M indicate the importance of the twist-3 contributions. It is shown in

 FIG. 3. Feynman diagrams contributing to the $\bar{B} \rightarrow M$ form factors, where the dots denote appropriate current interactions, and boxes denote quark scattering amplitudes.

FIG. 4. Contour plot of $F_0^{B \rightarrow \pi}(0)$. The values of form factors denoted by shades. The values in (a,b) and (c,d) are calculated without and with the contributions from ϕ_{B2} .

Ref. [33] that the contributions from twist-3 $\phi_{\pi}^{p,i}$ to $F_0^{B \rightarrow \pi}(0)$ could exceed 50% with appropriate parameters. (3) The contributions of WF ϕ_{B2} can enhance the form factors. By comparison of the branching ratios for $B \rightarrow PP$ decays with the experimental results, three optimal scenarios are obtained, when the ϕ_{B2} is considered.

Scenario 1 (S1): $\omega_B = 0.45$ GeV and $\mu_M = 1.7$ GeV for PDG data;

Scenario 2 (S2): $\omega_B = 0.43$ GeV and $\mu_M = 1.6$ GeV for Belle data;

Scenario 3 (S3): $\omega_B = 0.41$ GeV and $\mu_M = 1.5$ GeV for BABAR data.

The form factors are assumed to be perturbatively calculable with the pQCD approach, but reliable only for the large recoil transition, i.e., the small q^2 regions. Thus in this paper, the form factors with $q^2 \leq 4$ GeV² are calculated. The dependences of form factors $F_0^{B \rightarrow \pi}(q^2)$ and

$F_0^{B \rightarrow K}(q^2)$ on q^2 and are shown in Fig. 5. It is seen clearly that (1) the form factors F_0 increase monotonically and slowly with q^2 within the large recoil domains. (2) The lion's share of form factors is from B mesonic WFs ϕ_{B1} , and the share of ϕ_{B2} is relatively small. This is why the contributions from B mesonic WFs ϕ_{B2} were usually not considered in most of previous works. Our results in Table II show that the contributions from ϕ_{B2} to form factors $F_0^{B \rightarrow \pi}(0)$ and $F_0^{B \rightarrow K}(0)$ are about 17%, which is much larger than 7% from the next-to-leading order (NLO) contributions [40]. (3) More than half of the form factors is from the contributions of topology Fig. 3(a), about 67% shown in Table II.

In Fig. 6, more details about the contributions from ϕ_{B1} and ϕ_{B2} , from topology Fig. 3(a) and (b) to form factor $F_0^{B \rightarrow \pi}(0)$ at $q^2 = 0$ are displayed bin by bin with respect to the distributions of $\frac{\alpha_s}{\pi}$. It shows that about 90% of form

FIG. 5. Formfactors of $F_0^{B \rightarrow \pi}(q^2)$ and $F_0^{B \rightarrow K}(q^2)$ versus q^2 . The bands in (a,b) correspond to the contributions from WFs ϕ_{B1} and ϕ_{B2} . The bands in (c,d) correspond to the contributions from Fig. 3(a) and (b). The bands are calculated with $\omega_B = 0.41\text{--}0.45$ GeV and $\mu_M = 1.5\text{--}1.7$ GeV. The lines correspond the S2 scenario.

FIG. 6. The percentages of contributions to $F_0^{B \rightarrow \pi}(0)$ from different ranges of α_s/π for the S2 scenario. The numbers over histogram denote the total percentages. The histograms in (a) correspond to the contributions from WFs ϕ_{B1} and ϕ_{B2} . The histograms in (b) correspond to the contributions from Fig. 3(a) and (b).

factor comes from the region of $\frac{\alpha_s}{\pi} \leq 0.2$, where the contributions from ϕ_{B1} and ϕ_{B2} account for more than 70% and 15%, the contributions from Fig. 3(a) and (b) account for more than 55% and 30%, respectively. These results may imply that the quark scattering amplitudes are

dominated by hard gluon exchange, and the perturbative calculation of the form factor with the pQCD approach is feasible and reliable. An important and possible underlying mechanism is the way of choosing the hard scale as the maximum virtuality of quarks and gluons, see Eq. (A15),

TABLE II. The numerical values of form factors $F_0^{B \rightarrow \pi}$ and $F_0^{B \rightarrow K}$. The contributions from ϕ_{B1} , ϕ_{B2} , Fig. 3(a) and Fig. 3(b) are given in the corresponding rows. The uncertainties in parentheses arise from variations of $\omega_B \pm 0.01$ GeV, $\mu_M \pm 0.1$ GeV, and $a_2^M \pm 0.15$, respectively.

Transition	Case	S1	S2	S3
$F_0^{B \rightarrow \pi}(0)$	ϕ_{B1}	0.190(06)(08)(08)	0.186(06)(07)(08)	0.181(06)(07)(07)
	ϕ_{B2}	0.041(01)(00)(05)	0.039(01)(00)(04)	0.037(01)(00)(04)
	Fig. 3(a)	0.156(04)(03)(13)	0.152(04)(03)(12)	0.147(04)(03)(12)
	Fig. 3(b)	0.074(03)(05)(00)	0.073(03)(04)(00)	0.071(03)(04)(00)
	total	0.231(07)(07)(13)	0.224(07)(07)(12)	0.218(06)(06)(12)
$F_0^{B \rightarrow K}(0)$	ϕ_{B1}	0.235(08)(12)(09)	0.231(07)(11)(09)	0.227(07)(10)(09)
	ϕ_{B2}	0.051(01)(00)(05)	0.049(01)(00)(05)	0.047(01)(00)(05)
	Fig. 3(a)	0.193(05)(06)(13)	0.189(05)(05)(14)	0.185(04)(05)(13)
	Fig. 3(b)	0.093(04)(06)(00)	0.091(04)(05)(01)	0.089(04)(05)(01)
	total	0.286(09)(12)(14)	0.280(08)(11)(14)	0.274(08)(10)(14)

besides the suppression of the long-distance contributions from Sudakov factors.

The values of form factors in Table II are less than those of Refs. [32,33], due to different DAs models and different values of input parameters. As is shown in Fig. 4, the form factors decrease with the increase of shape parameter ω_B . A large shape parameter ω_B for B mesonic WFs is used in our calculation, compared with that in Ref. [33]. It should be pointed out that a relatively small value of form factor $F_0^{B \rightarrow \pi}(0)$ has recently been obtained by fitting the Bourrely-Lellouch-Caprini parametrization [41] with the available experimental data and theoretical information and then extrapolating to the point of $q^2 = 0$, for example, $0.254^{+0.023}_{-0.022}$ in Ref. [41],¹ 0.248 ± 0.082 in Ref. [42], 0.20 ± 0.14 in Ref. [43], 0.254 ± 0.081 in Ref. [44]. Our results of $F_0^{B \rightarrow \pi}(0)$ are basically consistent those of Refs. [41–44] within uncertainties. In addition, from the definition of form factor in Eq. (40), it is clear that there should be a relation between form factors and decay constants,

$$\frac{F_0^{B \rightarrow \pi}(0)}{F_0^{B \rightarrow K}(0)} \approx \frac{f_\pi}{f_K}. \quad (42)$$

The numbers in Table II hold this relation well. The small violation arises from the $SU(3)$ flavor breaking effects.

The Feynman diagrams for two-body nonleptonic B meson decays are shown in Fig. 7. The amplitudes $\mathcal{A}$ with the pQCD approach are usually divided into three parts: the short-distance contributions encoded in the Wilson coefficients C_i , the quark scattering amplitudes $\mathcal{T}_i$, and hadron WFs Φ_i . The general form of decay amplitude is

$$\mathcal{A}_i \propto \int \prod_j dx_j db_j C_i(t_i) \mathcal{T}_i(t_i, x_j, b_j) \Phi_j(x_j, b_j) e^{-S_j}. \quad (43)$$

¹ $F_0^{B \rightarrow \pi}(0) = f_+(0)$ is assumed.

In the rest frame of the B meson, the CP -averaged branching ratios are defined as:

$$\mathcal{B}r = \frac{\tau_B}{16\pi} \frac{p_{\text{cm}}}{m_B^2} \{ |\mathcal{A}(B \rightarrow f)|^2 + |\mathcal{A}(\bar{B} \rightarrow \bar{f})|^2 \}, \quad (44)$$

where τ_B is the lifetime of the B meson. p_{cm} is the common momentum of final states. The decay amplitudes including the ϕ_{B2} contributions are listed in Appendix B. For the charged B_u meson decays, the CP violating asymmetries arises from the interference between tree and penguin amplitudes. The direct CP violating asymmetry is defined as follows.

$$\begin{aligned} \mathcal{A}_{CP} &= \frac{\Gamma(B^- \rightarrow f) - \Gamma(B^+ \rightarrow \bar{f})}{\Gamma(B^- \rightarrow f) + \Gamma(B^+ \rightarrow \bar{f})} \\ &= \frac{|\mathcal{A}(B^- \rightarrow f)|^2 - |\mathcal{A}(B^+ \rightarrow \bar{f})|^2}{|\mathcal{A}(B^- \rightarrow f)|^2 + |\mathcal{A}(B^+ \rightarrow \bar{f})|^2}. \end{aligned} \quad (45)$$

For the neutral B_d meson decays into final state f with $f = \bar{f}$, the time-dependent CP violating asymmetry is defined as follows.

$$\begin{aligned} \mathcal{A}_{CP} &= \frac{\Gamma(\bar{B}^0 \rightarrow f) - \Gamma(B^0 \rightarrow \bar{f})}{\Gamma(\bar{B}^0 \rightarrow f) + \Gamma(B^0 \rightarrow \bar{f})} \\ &\approx S_f \sin(x\Gamma t) - C_f \cos(x\Gamma t), \end{aligned} \quad (46)$$

with the $y = \frac{\Delta\Gamma}{2\Gamma_B} \simeq 0$ approximation, where $x = \frac{\Delta m_B}{\Gamma_B} = 0.769(4)$ [1] is the $B^0 - \bar{B}^0$ oscillation parameter. $\Gamma_B = \frac{1}{\tau_B}$ is the full width of the B_d meson. C_f and S_f are the direct and mixing-induced CP asymmetries.

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad (47)$$

$$S_f = \frac{2\mathcal{I}m(\lambda_f)}{1 + |\lambda_f|^2}, \quad (48)$$

FIG. 7. Feynman diagrams contributing to the $\bar{B} \rightarrow MM'$ decays, where the dots denote appropriate interactions and boxes denote quark scattering amplitudes. (a) and (b) are factorizable emission diagrams. (c) and (d) are nonfactorizable emission diagrams. (e) and (f) are factorizable annihilation diagrams. (g) and (h) are nonfactorizable annihilation diagrams.

$$\lambda_f = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{\mathcal{A}(\bar{B}^0 \rightarrow f)}{\mathcal{A}(B^0 \rightarrow f)}. \quad (49)$$

The numerical results on the CP -averaged branching ratios together with experimental data are presented in Table III and IV, CP asymmetries in Table V and VI. Using the minimum χ^2 method,

$$\chi^2 = \sum_i \frac{(\mathcal{B}_r^{\text{theo.}} - \mathcal{B}_r^{\text{exp.}})^2}{\sigma_{\mathcal{B}_r^{\text{exp.}}}^2}, \quad (50)$$

three optimal scenarios (S1, S2, and S3) of parameters ω_B and μ_M are obtained when the contributions of ϕ_{B2} are considered. For the ten concerned B decay modes,

$\chi^2/\text{d.o.f} = 92.9/8, 73.1/8$ and $56.4/7$ correspond to data of PDG, Belle and BABAR, respectively. The agreement between theoretical and experimental results is illustrated by the χ^2 distribution in Fig. 8. The followings are our comments.

(1) From Table III and IV, it is seen that the contributions of WF ϕ_{B2} are more than 25% of total branching ratios, except for the pure annihilation $\bar{B}^0 \rightarrow K^+ K^-$ decay. That is because WF ϕ_{B2} contributes nothing to the factorizable annihilation amplitudes of Eqs. (C13)–(C18). From Table V and VI, it is seen that the contributions of WF ϕ_{B2} result in a small reduction of direct CP asymmetries.

(2) From appendix B, it is clearly seen that for the $B \rightarrow \pi\pi$ and $\bar{K}K$ decays, the CKM factors of the tree and penguin amplitudes are respectively $V_{ub}V_{ud}^*$ and $V_{tb}V_{td}^*$,

TABLE III. The CP -averaged branching ratios $\mathcal{B}r$ (in the unit of 10^{-6}) for the $B_u \rightarrow PP$ decays. The theoretical results are respectively calculated with parameters of S1, S2, S3 scenarios to compare with data of PDG, Belle and BABAR [1]. The theoretical uncertainties arise from variations of $\omega_B \pm 0.01$ GeV, $\mu_M \pm 0.1$ GeV, and $a_2^M \pm 0.15$, respectively.

	Mode	$B^- \rightarrow \pi^- \pi^0$	$B^- \rightarrow \pi^- \bar{K}^0$	$B^- \rightarrow \pi^0 K^-$	$B^- \rightarrow K^0 K^-$
S1	PDG	5.5 ± 0.4	23.7 ± 0.8	12.9 ± 0.5	1.31 ± 0.17
	$\phi_{B1} + \phi_{B2}$	$3.04^{+0.18+0.16+0.27}_{-0.17-0.16-0.25}$	$24.12^{+1.73+3.69+2.50}_{-1.60-3.32-2.36}$	$13.13^{+0.93+1.95+1.11}_{-0.87-1.76-1.07}$	$1.95^{+0.13+0.34+0.18}_{-0.13-0.30-0.16}$
	ϕ_{B1}	$2.16^{+0.14+0.15+0.14}_{-0.13-0.15-0.13}$	$17.89^{+1.36+2.97+1.30}_{-1.26-2.65-1.24}$	$9.71^{+0.73+1.57+0.58}_{-0.68-1.41-0.56}$	$1.45^{+0.11+0.27+0.07}_{-0.10-0.25-0.06}$
S2	Belle	5.86 ± 0.46	23.97 ± 0.89	12.62 ± 0.64	1.11 ± 0.20
	$\phi_{B1} + \phi_{B2}$	$3.23^{+0.19+0.18+0.29}_{-0.18-0.18-0.27}$	$23.87^{+1.72+3.83+2.51}_{-1.60-3.43-2.38}$	$13.03^{+0.93+2.03+1.12}_{-0.87-1.83-1.07}$	$1.88^{+0.13+0.35+0.17}_{-0.12-0.31-0.15}$
	ϕ_{B1}	$2.27^{+0.15+0.17+0.15}_{-0.14-0.16-0.14}$	$17.64^{+1.36+3.08+1.30}_{-1.25-2.74-1.25}$	$9.60^{+0.74+1.64+0.59}_{-0.68-1.46-0.57}$	$1.38^{+0.10+0.29+0.06}_{-0.10-0.25-0.05}$
S3	BABAR	5.02 ± 0.54	23.9 ± 1.5	13.6 ± 0.9	1.61 ± 0.45
	$\phi_{B1} + \phi_{B2}$	$3.42^{+0.21+0.20+0.32}_{-0.19-0.20-0.30}$	$23.49^{+1.71+3.96+2.51}_{-1.58-3.53-2.38}$	$12.87^{+0.93+2.11+1.12}_{-0.86-1.89-1.07}$	$1.79^{+0.12+0.36+0.15}_{-0.12-0.32-0.13}$
	ϕ_{B1}	$2.39^{+0.16+0.18+0.16}_{-0.14-0.18-0.15}$	$17.30^{+1.35+3.19+1.30}_{-1.24-2.81-1.24}$	$9.44^{+0.73+1.70+0.59}_{-0.67-1.51-0.57}$	$1.30^{+0.10+0.29+0.05}_{-0.09-0.26-0.04}$

TABLE IV. The numerical values of the CP -averaged branching ratios (in the unit of 10^{-6}) for the $B_d \rightarrow PP$ decays. Other legends are the same as those of Table III.

	Mode	$\bar{B}^0 \rightarrow \pi^+ \pi^-$	$\bar{B}^0 \rightarrow \pi^0 \pi^0$	$\bar{B}^0 \rightarrow \pi^+ K^-$
S1	PDG	5.12 ± 0.19	1.59 ± 0.26	19.6 ± 0.5
	$\phi_{B1} + \phi_{B2}$	$5.52^{+0.34+0.37+0.47}_{-0.32-0.36-0.43}$	$0.24^{+0.02+0.04+0.01}_{-0.02-0.03-0.01}$	$20.07^{+1.44+3.13+2.05}_{-1.33-2.81-1.94}$
	ϕ_{B1}	$3.81^{+0.25+0.31+0.23}_{-0.23-0.30-0.20}$	$0.19^{+0.01+0.03+0.02}_{-0.01-0.03-0.01}$	$15.00^{+1.14+2.53+1.07}_{-1.05-2.25-1.02}$
S2	Belle	5.04 ± 0.28	1.31 ± 0.27	20.00 ± 0.69
	$\phi_{B1} + \phi_{B2}$	$5.82^{+0.36+0.41+0.50}_{-0.34-0.39-0.46}$	$0.24^{+0.02+0.04+0.01}_{-0.02-0.04-0.01}$	$19.81^{+1.43+3.24+2.05}_{-1.33-2.89-1.95}$
	ϕ_{B1}	$3.99^{+0.26+0.34+0.24}_{-0.25-0.33-0.21}$	$0.18^{+0.01+0.03+0.02}_{-0.01-0.03-0.01}$	$14.76^{+1.14+2.62+1.07}_{-1.05-2.32-1.02}$
S3	BABAR	5.5 ± 0.5	1.83 ± 0.25	19.1 ± 0.8
	$\phi_{B1} + \phi_{B2}$	$6.12^{+0.39+0.45+0.54}_{-0.36-0.43-0.50}$	$0.23^{+0.02+0.04+0.01}_{-0.02-0.04-0.01}$	$19.45^{+1.42+3.34+2.04}_{-1.31-2.97-1.94}$
	ϕ_{B1}	$4.16^{+0.28+0.37+0.26}_{-0.26-0.36-0.23}$	$0.18^{+0.01+0.03+0.02}_{-0.01-0.03-0.01}$	$14.43^{+1.13+2.70+1.06}_{-1.04-2.38-1.01}$
	Mode	$\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$	$\bar{B}^0 \rightarrow K^+ K^-$	$\bar{B}^0 \rightarrow K^0 \bar{K}^0$
S1	PDG	9.9 ± 0.5	0.078 ± 0.015	1.21 ± 0.16
	$\phi_{B1} + \phi_{B2}$	$9.11^{+0.66+1.48+1.15}_{-0.61-1.32-1.08}$	$0.107^{+0.003+0.005+0.062}_{-0.003-0.004-0.048}$	$1.43^{+0.10+0.25+0.05}_{-0.09-0.23-0.03}$
	ϕ_{B1}	$6.81^{+0.52+1.19+0.61}_{-0.48-1.05-0.57}$	$0.089^{+0.003+0.004+0.047}_{-0.003-0.004-0.037}$	$1.06^{+0.08+0.21+0.03}_{-0.07-0.19-0.01}$
S2	Belle	9.68 ± 0.68	0.10 ± 0.09	1.26 ± 0.20
	$\phi_{B1} + \phi_{B2}$	$8.95^{+0.65+1.53+1.15}_{-0.61-1.36-1.08}$	$0.109^{+0.003+0.005+0.065}_{-0.003-0.004-0.050}$	$1.37^{+0.10+0.26+0.03}_{-0.09-0.24-0.02}$
	ϕ_{B1}	$6.67^{+0.52+1.22+0.60}_{-0.48-1.08-0.57}$	$0.091^{+0.003+0.004+0.050}_{-0.003-0.004-0.039}$	$1.01^{+0.07+0.22+0.04}_{-0.07-0.19-0.02}$
S3	BABAR	10.1 ± 0.7		1.08 ± 0.30
	$\phi_{B1} + \phi_{B2}$	$8.75^{+0.64+1.57+1.15}_{-0.60-1.38-1.07}$	$0.111^{+0.003+0.005+0.068}_{-0.003-0.004-0.052}$	$1.30^{+0.09+0.27+0.02}_{-0.08-0.24-0.01}$
	ϕ_{B1}	$6.50^{+0.51+1.26+0.60}_{-0.47-1.10-0.57}$	$0.093^{+0.003+0.004+0.053}_{-0.003-0.004-0.040}$	$0.94^{+0.07+0.22+0.05}_{-0.06-0.20-0.03}$

and have the same order of magnitude $\propto \lambda^3$. For the $B \rightarrow \pi K$ decays, the tree amplitudes being proportional to the CKM factor $V_{ub}V_{us}^*$ are suppressed by λ^2 compared with the penguin amplitudes being proportional to the CKM factor $V_{tb}V_{ts}^*$. In addition, the theoretical and experimental results in Table III and IV show that branching ratios for $B \rightarrow \pi K$ decays are in general larger than those for $B \rightarrow \pi\pi$ and $\bar{K}K$ decays. These facts confirm previous studies [6,10] that penguin contributions are dynamically enhanced and essential for explaining the $B \rightarrow \pi K$ decays. What is more, our studies show that the nonfactorizable annihilation amplitudes mainly from

WF ϕ_{B1} rather than ϕ_{B2} provide large strong phases for the $B \rightarrow \pi K$ decays, as analyzed in Ref. [10].

(3) From Fig. 8, it is seen that (i) for the $B \rightarrow \pi K$ decays, when the contributions of WF ϕ_{B2} are included, theoretical results of branching ratio can give a satisfactory explanation on experimental data. Compared the numbers in Table III and IV with the NLO results of Refs. [11,12,45] (see Table VII), it is seen that the contributions of ϕ_{B2} to branching ratios at the leading order (LO) is roughly equivalent to the NLO corrections without the participation of ϕ_{B2} . (ii) The consideration of WF ϕ_{B2} cannot well settle the so-called CP asymmetries “ $K\pi$ ” puzzle, i.e., the

TABLE V. The CP asymmetries $\mathcal{A}_{CP}$ (in the unit of 10^{-2}) for $B \rightarrow PP$ decays. Other legends are the same as those of Table III.

$\mathcal{A}_{CP}$	$\pi^- \pi^0$	$\pi^- \bar{K}^0$	$\pi^0 K^-$	$K^0 K^-$	$\pi^+ K^-$
PDG	3 ± 4	-1.7 ± 1.6	3.7 ± 2.1	4 ± 14	-8.3 ± 0.4
$\phi_{B1} + \phi_{B2}$	-0.004	$-0.67^{+0.02+0.04+0.14}_{-0.02-0.05-0.11}$	$-6.25^{+0.25+0.30+1.51}_{-0.26-0.33-1.56}$	$14.78^{+0.27+1.05+3.33}_{-0.27-0.90-3.87}$	$-7.33^{+0.29+0.41+1.98}_{-0.30-0.44-2.04}$
ϕ_{B1}	-0.02	$-0.74^{+0.02+0.04+0.14}_{-0.02-0.05-0.10}$	$-6.73^{+0.30+0.36+1.74}_{-0.31-0.40-1.88}$	$14.83^{+0.26+1.10+3.28}_{-0.26-0.93-3.71}$	$-8.14^{+0.33+0.49+2.38}_{-0.34-0.54-2.54}$
Belle	2.5 ± 4.4	-1.1 ± 2.2	4.3 ± 2.4	1.4 ± 16.8	-6.9 ± 1.6
$\phi_{B1} + \phi_{B2}$	-0.01	$-0.69^{+0.02+0.05+0.15}_{-0.02-0.05-0.12}$	$-6.05^{+0.25+0.30+1.43}_{-0.26-0.33-1.49}$	$15.28^{+0.28+1.21+3.53}_{-0.28-1.03-4.08}$	$-7.18^{+0.28+0.41+1.91}_{-0.29-0.45-1.97}$
ϕ_{B1}	-0.02	$-0.75^{+0.02+0.05+0.15}_{-0.02-0.05-0.12}$	$-6.51^{+0.29+0.36+1.66}_{-0.30-0.40-1.80}$	$15.39^{+0.27+1.10+3.55}_{-0.27-1.08-3.98}$	$-7.98^{+0.33+0.50+2.31}_{-0.34-0.54-2.47}$
BABAR	3 ± 8	-2.9 ± 4.0	3 ± 4	10 ± 26	-10.7 ± 1.7
$\phi_{B1} + \phi_{B2}$	-0.01	$-0.70^{+0.02+0.05+0.16}_{-0.02-0.06-0.13}$	$-5.86^{+0.24+0.30+1.36}_{-0.25-0.32-1.41}$	$15.90^{+0.29+1.41+3.78}_{-0.29-1.18-4.34}$	$-7.03^{+0.28+0.41+1.84}_{-0.29-0.44-1.89}$
ϕ_{B1}	-0.03	$-0.77^{+0.02+0.05+0.16}_{-0.02-0.06-0.13}$	$-6.30^{+0.29+0.36+1.57}_{-0.30-0.40-1.72}$	$16.12^{+0.29+1.56+3.89}_{-0.29-1.27-4.31}$	$-7.83^{+0.33+0.50+2.24}_{-0.34-0.55-2.40}$

TABLE VI. The CP asymmetries (in the unit of 10^{-2}) for $B_d \rightarrow PP$ decays. Other legends are the same as those of Table III.

	$C_{\pi^+\pi^-}$	$S_{\pi^+\pi^-}$	$C_{\pi^0\pi^0}$	$S_{\pi^0\pi^0}$
PDG	-32 ± 4	-65 ± 4	-33 ± 22	
$\phi_{B1} + \phi_{B2}$	$-17.85^{+0.55+0.05+3.84}_{-0.54-0.02-3.42}$	$-81.91^{+0.17+1.57+2.43}_{-0.17-1.52-1.96}$	$38.34^{+1.39+1.78+5.37}_{-1.37-1.57-5.40}$	$89.03^{+0.49+1.13+0.97}_{-0.51-1.38-1.21}$
ϕ_{B1}	$-21.52^{+0.71+0.13+4.96}_{-0.69-0.10-4.52}$	$-80.15^{+0.15+1.65+2.94}_{-0.16-1.60-2.30}$	$39.06^{+1.60+1.86+4.49}_{-1.58-1.62-4.17}$	$85.90^{+0.53+1.32+0.35}_{-0.54-1.59-0.25}$
Belle	-33 ± 7	-64 ± 9	-14 ± 37	
$\phi_{B1} + \phi_{B2}$	$-16.75^{+0.51+0.05+3.52}_{-0.52-0.02-3.11}$	$-83.12^{+0.16+1.55+2.34}_{-0.16-1.50-1.91}$	$37.29^{+1.41+1.91+5.29}_{-1.40-1.68-5.09}$	$88.69^{+0.50+1.31+0.83}_{-0.51-1.63-1.09}$
ϕ_{B1}	$-20.24^{+0.66+0.12+4.57}_{-0.68-0.09-4.13}$	$-81.46^{+0.15+1.64+2.86}_{-0.15-1.59-2.28}$	$37.65^{+1.63+2.01+4.49}_{-1.62-1.75-3.86}$	$85.41^{+0.54+1.54+0.34}_{-0.55-1.88-0.28}$
BABAR	-25 ± 8	-68 ± 10	-43 ± 26	
$\phi_{B1} + \phi_{B2}$	$-15.72^{+0.48+0.05+3.23}_{-0.49-0.02-2.82}$	$-84.33^{+0.15+1.53+2.24}_{-0.15-1.48-1.85}$	$36.32^{+1.43+2.08+5.21}_{-1.42-1.80-4.74}$	$88.11^{+0.50+1.55+0.70}_{-0.51-1.96-1.00}$
ϕ_{B1}	$-19.04^{+0.63+0.12+4.20}_{-0.64-0.09-3.76}$	$-82.79^{+0.14+1.62+2.77}_{-0.14-1.58-2.24}$	$36.33^{+1.66+2.22+4.47}_{-1.63-1.89-3.51}$	$84.65^{+0.55+1.83+0.37}_{-0.56-2.27-0.37}$
	$C_{\pi^0\bar{K}^0}$	$S_{\pi^0\bar{K}^0}$	$C_{K^+K^-}$	$S_{K^+K^-}$
PDG	0 ± 13	58 ± 17		
$\phi_{B1} + \phi_{B2}$	$-0.54^{+0.01+0.05+0.25}_{-0.01-0.05-0.21}$	$68.74^{+0.02+0.16+0.20}_{-0.02-0.13-0.16}$	$-81.17^{+0.45+0.79+0.13}_{-0.45-0.70-2.04}$	$-28.21^{+0.94+1.59+0.95}_{-0.93-1.58-0.53}$
ϕ_{B1}	$-0.85^{+0.01+0.06+0.36}_{-0.01-0.06-0.31}$	$69.29^{+0.03+0.18+0.13}_{-0.03-0.15-0.14}$	$-82.90^{+0.46+0.67+0.27}_{-0.45-0.59-1.24}$	$-28.35^{+1.02+1.25+0.35}_{-1.01-1.27-1.24}$
Belle	-14 ± 14	67 ± 32		
$\phi_{B1} + \phi_{B2}$	$-0.60^{+0.01+0.05+0.27}_{-0.00-0.06-0.23}$	$68.86^{+0.02+0.18+0.20}_{-0.02-0.15-0.16}$	$-81.26^{+0.44+0.88+0.11}_{-0.43-0.78-1.96}$	$-27.97^{+0.94+1.66+0.24}_{-0.92-1.65-0.12}$
ϕ_{B1}	$-0.93^{+0.01+0.07+0.39}_{-0.01-0.07-0.34}$	$69.40^{+0.04+0.21+0.15}_{-0.04-0.18-0.16}$	$-83.11^{+0.44+0.76+0.25}_{-0.43-0.67-1.24}$	$-27.65^{+1.01+1.35+0.75}_{-0.99-1.37-1.80}$
BABAR	13 ± 13	55 ± 20		
$\phi_{B1} + \phi_{B2}$	$-0.67^{+0.00+0.06+0.30}_{-0.00-0.07-0.25}$	$68.99^{+0.02+0.21+0.20}_{-0.02-0.18-0.16}$	$-81.24^{+0.42+0.97+0.08}_{-0.41-0.87-1.81}$	$-27.80^{+0.93+1.71+0.35}_{-0.92-1.70-0.56}$
ϕ_{B1}	$-1.02^{+0.01+0.08+0.43}_{-0.00-0.09-0.36}$	$69.53^{+0.04+0.24+0.16}_{-0.04-0.20-0.17}$	$-83.19^{+0.41+0.86+0.25}_{-0.40-0.76-1.24}$	$-27.06^{+1.00+1.45+1.18}_{-0.99-1.46-2.44}$

discrepancy between theoretical and experimental results of $\mathcal{A}_{CP}(B^- \rightarrow \pi^0 K^-) - \mathcal{A}_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$. The studies of Refs. [12,45] showed that the NLO corrections including the glauher effects could flip the sign of $\mathcal{A}_{CP}(B^- \rightarrow \pi^0 K^-)$. It should be noted that the NLO and NLOG theoretical uncertainties of branching ratios are still large, and the current measurement accuracy of $\mathcal{A}_{CP}(B^- \rightarrow \pi^0 K^-)$ needs to be improved.

(4) From Fig. 8 and Table III and IV, it is seen that for the $B \rightarrow \pi\pi$ decays, the pQCD results of branching ratios deviate from the current experimental measurement. The contributions of WF ϕ_{B2} can enhance the branching ratios and reduce these deviations. Compared the numbers in Table III and IV with the NLO results of Refs. [11,12,46,47] (see Table VIII), it is seen that (i) for the $\bar{B}^0 \rightarrow \pi\pi$ decays, the LO contributions of ϕ_{B2} to branching ratios is roughly equivalent to the NLO corrections without the participation of ϕ_{B2} . (ii) Besides the large theoretical uncertainties, the pQCD results, including either WF ϕ_{B2} or the NLO contributions, cannot well explain data on branching ratio for the $\bar{B}^0 \rightarrow \pi^0 \pi^0$ decay and CP asymmetries for the $\bar{B}^0 \rightarrow \pi^+ \pi^-$ decay.

(5) For the $\bar{B}^0 \rightarrow \bar{K}K$ decays, the pQCD results with the contributions of WF ϕ_{B2} are in good agreement with data of Belle and BABAR within uncertainties. But there are signs of tension between the pQCD result with the contributions of WF ϕ_{B2} and experimental data for the $B^- \rightarrow K^0 K^-$ decay. In addition, from Table VI, there is a particularly

interesting phenomenon that the direct CP asymmetries C_f is in general larger than the mixing-induced CP asymmetries S_f for the $B \rightarrow PP$ decays, but the opposite is true for the pure annihilation $\bar{B}^0 \rightarrow K^+ K^-$ decay.

(6) As pointed out earlier, the form factors and branching ratios are sensitive to the shape parameter ω_B

FIG. 8. The χ^2 distribution of branching ratios for three optimal scenarios.

TABLE VII. The previous LO and NLO pQCD results for the $B \rightarrow \pi K$ decays, where NLO and NLOG denote without and with the Glauber effects, the unit of branching ratios and direct CP asymmetries are respectively 10^{-6} and 10^{-2} .

Mode	LO [11]	NLO [11]	NLOG ^a [12]	NLO [45]	NLOG [45]
$Br(B^- \rightarrow \pi^- \bar{K}^0)$	17.0	$24.5^{+13.6}_{-8.1}$	21.1	$27.2^{+9.3}_{-6.7}$	$24.1^{+8.3}_{-6.0}$
$Br(B^- \rightarrow \pi^0 K^-)$	10.2	$13.9^{+10.0}_{-5.6}$	12.9	$15.3^{+5.2}_{-3.8}$	$14.0^{+4.7}_{-3.5}$
$Br(\bar{B}^0 \rightarrow \pi^+ K^-)$	14.2	$20.9^{+15.6}_{-8.3}$	17.7	$23.3^{+7.8}_{-5.7}$	$21.7^{+7.4}_{-5.3}$
$Br(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0)$	5.7	$9.1^{+5.6}_{-3.3}$	7.2	$10.2^{+3.4}_{-2.5}$	$9.3^{+3.2}_{-2.3}$
$\mathcal{A}_{CP}(B^- \rightarrow \pi^0 K^-)$	-8	-1^{+3}_{-5}	10	$-0.8^{+1.3}_{-1.4}$	2.1 ± 1.6
$\mathcal{A}_{CP}(\bar{B}^0 \rightarrow \pi^+ K^-)$	-12	-9^{+6}_{-8}	-11	-7.6 ± 1.7	-8.1 ± 1.7

^aThe Glauber phases $S_e = S_{e1} = S_{e2} = -\frac{\pi}{2}$ is assumed.TABLE VIII. The previous LO and NLO pQCD results for the $B \rightarrow \pi\pi$ decays. Other legends are the same as those of Table VII.

Mode	LO [11]	NLO [11]	NLOG ^b [12]	LO [46]	NLO [46]	NLO [47]	NLOG [47]
$Br(B^- \rightarrow \pi^- \pi^0)$	3.5	$4.0^{+3.4}_{-1.9}$	6.6	3.54	$4.27^{+1.85}_{-1.47}$	$3.35^{+1.10}_{-0.80}$	$4.45^{+1.43}_{-1.05}$
$Br(\bar{B}^0 \rightarrow \pi^+ \pi^-)$	7.0	$6.5^{+6.7}_{-3.8}$	6.4	7.46	$7.67^{+3.47}_{-2.64}$	$6.19^{+2.12}_{-1.52}$	$5.39^{+1.88}_{-1.33}$
$Br(\bar{B}^0 \rightarrow \pi^0 \pi^0)$	0.12	$0.29^{+0.50}_{-0.20}$	1.2	0.12	$0.23^{+0.19}_{-0.15}$	$0.29^{+0.11}_{-0.07}$	$0.61^{+0.21}_{-0.17}$
$C_f(\bar{B}^0 \rightarrow \pi^+ \pi^-)$	-14	-18^{+20}_{-12}	-17	-27	-12^{+4}_{-6}		
$S_f(\bar{B}^0 \rightarrow \pi^+ \pi^-)$	-34	-43^{+100}_{-56}	-43	-28	-40^{+5}_{-4}		

^bThe Glauber phases $S_e = S_{e1} = S_{e2} = -\frac{\pi}{2}$ is assumed. FIG. 9. The χ^2 distribution of branching ratios between Belle data and pQCD results with and without the participation of ϕ_{B2} for different ω_B , where $\mu_P = 1.6$ GeV is fixed.

for B mesonic WFs. It is noticed that a relatively large value of the parameter ω_B is optimized for three scenarios by using the minimum χ^2 method. An appropriate choice of ω_B could enhance the contributions from ϕ_{B1} to branching ratios and the effects of ϕ_{B2} could be partly covered, which might be one reason why the contributions of ϕ_{B2} are often not considered seriously in many previous studies. To illustrate the unique role of ϕ_{B2} , the good consistencies between Belle data and pQCD results with different ω_B are shown in Fig. 9. It is seen that (i) the contributions of ϕ_{B2} could be substituted by an appropriate parameter ω_B for some cases; (ii) a more comprehensive agreement of branching ratios between pQCD calculations and experimental data is improved by the participation of ϕ_{B2} . In addition, although it is difficult to resolve the CP puzzles with the LO contributions from ϕ_{B2} , the effects of the NLO contributions from ϕ_{B2} on the CP violation might be significant.

In summary, the B mesonic WF ϕ_{B2} can contribute to emission amplitudes and nonfactorizable annihilation amplitudes with the pQCD approach. The enhancements from ϕ_{B2} to hadronic transition form factors and branching ratios for the nonleptonic $B \rightarrow PP$ decays are comparable with those from the NLO corrections without taking the ϕ_{B2} into account. By considering ϕ_{B2} , the $B \rightarrow K\pi$ decays could be well explained at the LO levels. However, the LO contributions from ϕ_{B2} cannot simultaneously settle the branching ratios and CP asymmetries for the $\bar{B}^0 \rightarrow \pi\pi$ decays. A more careful study of the NLO corrections and other effects should be considered in the future. In addition, the contributions from ϕ_{B2} result in a small reductions on the CP asymmetries. All in all, the oft-ignored ϕ_{B2} in

previous studies should be given due attention in order to match the improvement of theoretical and experimental results.

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APPENDIX A: THE FORM FACTORS FOR $B \rightarrow P$ TRANSITIONS

Besides the definition of Eq. (40), another definition of form factors is

$$\begin{aligned} \langle M(p_2) | (\bar{q}b)_{V-A} | \bar{B}(p_1) \rangle &= p_1^\mu \tilde{f}_1 + p_2^\mu \tilde{f}_2 \\ &= (p_1 + p_2)^\mu f_+ + q^\mu f_- . \end{aligned} \quad (A1)$$

The relations among form factors are

$$F_1 = f_+ = \frac{1}{2}(\tilde{f}_1 + \tilde{f}_2), \quad (A2)$$

$$F_0 = \frac{1}{2}\tilde{f}_1 \left(1 + \frac{q^2}{m_B^2}\right) + \frac{1}{2}\tilde{f}_2 \left(1 - \frac{q^2}{m_B^2}\right). \quad (A3)$$

Using the pQCDF formula of Eq. (41), the form factors can be written as follows.

$$\tilde{f}_i = 2 \frac{\pi C_F}{N_c} m_B^2 f_B f_M (\tilde{f}_i^a + \tilde{f}_i^b), \quad (A4)$$

$$\begin{aligned} \tilde{f}_1^a &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db_1 \int_0^\infty db_2 \alpha_s(t_a) S_t(x_2) H_{ab}(\alpha_g, \beta_a, b_1, b_2) \\ &\quad \times r_M e^{-S_B} e^{-S_M} [\phi_{B1}(x_1, b_1) - \phi_{B2}(x_1, b_1)] [\phi_M^p(x_2) - \phi_M^t(x_2)], \end{aligned} \quad (A5)$$

$$\begin{aligned} \tilde{f}_2^a &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db_1 \int_0^\infty db_2 \alpha_s(t_a) S_t(x_2) H_{ab}(\alpha_g, \beta_a, b_1, b_2) \\ &\quad \times e^{-S_B} e^{-S_M} \left\{ \phi_{B1}(x_1, b_1) \left[\phi_M^a(x_2) (1 + x_2 \eta) - 2r_M x_2 \phi_M^p(x_2) + 2r_M \phi_M^t(x_2) \left(\frac{1}{\eta} - x_2 \right) \right] \right. \\ &\quad \left. - \phi_{B2}(x_1, b_1) \left[r_M \phi_M^t(x_2) \left(\frac{1}{\eta} - x_2 \right) + \phi_M^a(x_2) - r_M \phi_M^p(x_2) \left(\frac{1}{\eta} + x_2 \right) \right] \right\}, \end{aligned} \quad (A6)$$

$$\begin{aligned} \tilde{f}_1^b &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db_1 \int_0^\infty db_2 \alpha_s(t_b) S_t(x_1) H_{ab}(\alpha_g, \beta_b, b_2, b_1) \\ &\quad \times e^{-S_B} e^{-S_M} x_1 \{ \phi_{B1}(x_1, b_1) [\phi_M^a(x_2) \eta - 2r_M \phi_M^p(x_2)] + \phi_{B2}(x_1, b_1) 2r_M \phi_M^p(x_2) \}, \end{aligned} \quad (A7)$$

$$\begin{aligned} \tilde{f}_2^b &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db_1 \int_0^\infty db_2 \alpha_s(t_b) S_t(x_1) H_{ab}(\alpha_g, \beta_b, b_2, b_1) \\ &\quad \times e^{-S_B} e^{-S_M} \left\{ \phi_{B1}(x_1, b_1) \left[2r_M \phi_M^p(x_2) \left(1 + \frac{x_1}{\eta} \right) - x_1 \phi_M^a(x_2) \right] - \phi_{B2}(x_1, b_1) \phi_M^p(x_2) \frac{2r_M x_1}{\eta} \right\}, \end{aligned} \quad (A8)$$

where $N_c = 3$ is the color number. The color factor $C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$. α_s is the QCD coupling constant. $r_M = \frac{\mu_M}{m_B}$ and $\eta = 1 - \frac{q^2}{m_B^2}$. The parametrization of $S_t(x)$ can be found in Ref. [29]. Other parameters are written as follows.

$$H_{ab}(\alpha, \beta, b_i, b_j) = b_i b_j K_0(b_i \sqrt{\alpha}) \{ \theta(b_i - b_j) \times K_0(b_j \sqrt{\beta}) I_0(b_j \sqrt{\beta}) + (b_i \leftrightarrow b_j) \}, \quad (\text{A9})$$

$$S_B = s(x_1, b_1, p_1^+) + 2 \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q, \quad (\text{A10})$$

$$S_M = s(x_2, b_2, p_2^-) + s(\bar{x}_2, b_2, p_2^-) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q, \quad (\text{A11})$$

$$\alpha_g = x_1 x_2 \eta m_B^2, \quad (\text{A12})$$

$$\beta_a = x_2 \eta m_B^2, \quad (\text{A13})$$

$$\beta_b = x_1 \eta m_B^2, \quad (\text{A14})$$

$$t_{a(b)} = \max\left(\sqrt{\beta_{a(b)}}, \frac{1}{b_1}, \frac{1}{b_2}\right), \quad (\text{A15})$$

where I_0 and K_0 are Bessel functions. The expression of $s(x, b, Q)$ can be found in Ref. [25]. $\gamma_q = -\frac{\alpha_s}{\pi}$ is the quark anomalous dimension. α_g and $\beta_{a(b)}$ are the virtualities of gluon and quarks, respectively.

APPENDIX B: THE AMPLITUDES FOR $B \rightarrow \pi\pi, \pi K, K\bar{K}$ DECAYS

The amplitude of B meson nonleptonic weak decay is written as

$$\mathcal{A}(\bar{B} \rightarrow M_1 M_2) = \langle M_1 M_2 | \mathcal{H}_{\text{eff}} | \bar{B} \rangle, \quad (\text{B1})$$

where $\mathcal{H}_{\text{eff}}$ is given in Eq. (1).

The explicit expressions for specific final states are written as

$$\begin{aligned} \mathcal{A}(B_u^- \rightarrow \pi^- \pi^0) = & \frac{G_F}{2} V_{ub} V_{ud}^* \{ a_2 \mathcal{A}_{ab}^{LL}(\pi^-, \pi^0) + C_1 \mathcal{A}_{cd}^{LL}(\pi^-, \pi^0) + a_1 \mathcal{A}_{ab}^{LL}(\pi^0, \pi^-) + C_2 \mathcal{A}_{cd}^{LL}(\pi^0, \pi^-) \} \\ & - \frac{G_F}{2} V_{tb} V_{td}^* \left\{ \frac{3}{2} (a_9 - a_7) \mathcal{A}_{ab}^{LL}(\pi^-, \pi^0) + \frac{1}{2} a_{10} \mathcal{A}_{ab}^{LL}(\pi^-, \pi^0) + \frac{1}{2} a_8 \mathcal{A}_{ab}^{SP}(\pi^-, \pi^0) \right. \\ & + \frac{1}{2} C_9 \mathcal{A}_{cd}^{LL}(\pi^-, \pi^0) + \frac{3}{2} C_{10} \mathcal{A}_{cd}^{LL}(\pi^-, \pi^0) + \frac{3}{2} C_8 \mathcal{A}_{cd}^{LR}(\pi^-, \pi^0) + \frac{1}{2} C_7 \mathcal{A}_{cd}^{SP}(\pi^-, \pi^0) \\ & \left. + a_{10} \mathcal{A}_{ab}^{LL}(\pi^0, \pi^-) + a_8 \mathcal{A}_{ab}^{SP}(\pi^0, \pi^-) + C_9 \mathcal{A}_{cd}^{LL}(\pi^0, \pi^-) + C_7 \mathcal{A}_{cd}^{SP}(\pi^0, \pi^-) \right\}, \quad (\text{B2}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_u^- \rightarrow \pi^- \bar{K}^0) = & \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \{ a_1 \mathcal{A}_{ef}^{LL}(\bar{K}, \pi) + C_2 \mathcal{A}_{gh}^{LL}(\bar{K}, \pi) \} \\ & - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ \left(a_4 - \frac{1}{2} a_{10} \right) \mathcal{A}_{ab}^{LL}(\pi, \bar{K}) + \left(a_6 - \frac{1}{2} a_8 \right) \mathcal{A}_{ab}^{SP}(\pi, \bar{K}) \right. \\ & + \left(C_3 - \frac{1}{2} C_9 \right) \mathcal{A}_{cd}^{LL}(\pi, \bar{K}) + \left(C_5 - \frac{1}{2} C_7 \right) \mathcal{A}_{cd}^{SP}(\pi, \bar{K}) + (a_4 + a_{10}) \mathcal{A}_{ef}^{LL}(\bar{K}, \pi) + (a_6 + a_8) \mathcal{A}_{ef}^{SP}(\bar{K}, \pi) \\ & \left. + (C_3 + C_9) \mathcal{A}_{gh}^{LL}(\bar{K}, \pi) + (C_5 + C_7) \mathcal{A}_{gh}^{SP}(\bar{K}, \pi) \right\}, \quad (\text{B3}) \end{aligned}$$

$$\begin{aligned} \mathcal{A}(B_u^- \rightarrow K^- \pi^0) = & \frac{G_F}{2} V_{ub} V_{us}^* \{ a_1 \mathcal{A}_{ab}^{LL}(\pi, \bar{K}) + a_2 \mathcal{A}_{ab}^{LL}(\bar{K}, \pi) + a_1 \mathcal{A}_{ef}^{LL}(\bar{K}, \pi) + C_2 \mathcal{A}_{cd}^{LL}(\pi, \bar{K}) + C_1 \mathcal{A}_{cd}^{LL}(\bar{K}, \pi) \\ & + C_2 \mathcal{A}_{gh}^{LL}(\bar{K}, \pi) \} - \frac{G_F}{2} V_{tb} V_{ts}^* \left\{ (a_4 + a_{10}) \mathcal{A}_{ab}^{LL}(\pi, \bar{K}) + (a_6 + a_8) \mathcal{A}_{ab}^{SP}(\pi, \bar{K}) \right. \\ & + (C_3 + C_9) \mathcal{A}_{cd}^{LL}(\pi, \bar{K}) + (C_5 + C_7) \mathcal{A}_{cd}^{SP}(\pi, \bar{K}) + \frac{3}{2} (a_9 - a_7) \mathcal{A}_{ab}^{LL}(\bar{K}, \pi) + (a_4 + a_{10}) \mathcal{A}_{ef}^{LL}(\bar{K}, \pi) \\ & + (a_6 + a_8) \mathcal{A}_{ef}^{SP}(\bar{K}, \pi) + \frac{3}{2} C_{10} \mathcal{A}_{cd}^{LL}(\bar{K}, \pi) + \frac{3}{2} C_8 \mathcal{A}_{cd}^{LR}(\bar{K}, \pi) \\ & \left. + (C_3 + C_9) \mathcal{A}_{gh}^{LL}(\bar{K}, \pi) + (C_5 + C_7) \mathcal{A}_{gh}^{SP}(\bar{K}, \pi) \right\}, \quad (\text{B4}) \end{aligned}$$

$$\begin{aligned}
\mathcal{A}(B_u^- \rightarrow K^- K^0) = & \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \{a_1 \mathcal{A}_{ef}^{LL}(K, \bar{K}) + C_2 \mathcal{A}_{gh}^{LL}(K, \bar{K})\} \\
& - \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ \left(a_4 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ab}^{LL}(\bar{K}, K) + \left(a_6 - \frac{1}{2} a_8\right) \mathcal{A}_{ab}^{SP}(\bar{K}, K) + \left(C_3 - \frac{1}{2} C_9\right) \mathcal{A}_{cd}^{LL}(\bar{K}, K) \right. \\
& + \left(C_5 - \frac{1}{2} C_7\right) \mathcal{A}_{cd}^{SP}(\bar{K}, K) + (a_4 + a_{10}) \mathcal{A}_{ef}^{LL}(K, \bar{K}) + (a_6 + a_8) \mathcal{A}_{ef}^{SP}(K, \bar{K}) \\
& \left. + (C_3 + C_9) \mathcal{A}_{gh}^{LL}(K, \bar{K}) + (C_5 + C_7) \mathcal{A}_{gh}^{SP}(K, \bar{K}) \right\}, \tag{B5}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \rightarrow \pi^- \pi^+) = & \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \{a_1 \mathcal{A}_{ab}^{LL}(\pi, \pi) + C_2 \mathcal{A}_{cd}^{LL}(\pi, \pi) + a_2 \mathcal{A}_{ef}^{LL}(\pi, \pi) + C_1 \mathcal{A}_{gh}^{LL}(\pi, \pi)\} \\
& - \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ (a_4 + a_{10}) \mathcal{A}_{ab}^{LL}(\pi, \pi) + (a_6 + a_8) \mathcal{A}_{ab}^{SP}(\pi, \pi) + (C_3 + C_9) \mathcal{A}_{cd}^{LL}(\pi, \pi) \right. \\
& + (C_5 + C_7) \mathcal{A}_{cd}^{SP}(\pi, \pi) + \left(2a_3 + a_4 + 2a_5 + \frac{1}{2} a_7 + \frac{1}{2} a_9 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ef}^{LL}(\pi, \pi) \\
& + \left(a_6 - \frac{1}{2} a_8\right) \mathcal{A}_{ef}^{SP}(\pi, \pi) + \left(C_3 + 2C_4 - \frac{1}{2} C_9 + \frac{1}{2} C_{10}\right) \mathcal{A}_{gh}^{LL}(\pi, \pi) \\
& \left. + \left(2C_6 + \frac{1}{2} C_8\right) \mathcal{A}_{gh}^{LR}(\pi, \pi) + \left(C_5 - \frac{1}{2} C_7\right) \mathcal{A}_{gh}^{SP}(\pi, \pi) \right\}, \tag{B6}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \rightarrow \pi^0 \pi^0) = & -\frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \{a_2 (\mathcal{A}_{ab}^{LL} - \mathcal{A}_{ef}^{LL}) + C_1 (\mathcal{A}_{cd}^{LL} - \mathcal{A}_{gh}^{LL})\} \\
& - \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ \left(a_4 + \frac{3}{2} a_7 - \frac{3}{2} a_9 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ab}^{LL} + \left(a_6 - \frac{1}{2} a_8\right) (\mathcal{A}_{ab}^{SP} + \mathcal{A}_{ef}^{SP}) \right. \\
& + \left(C_3 - \frac{1}{2} C_9 - \frac{3}{2} C_{10}\right) \mathcal{A}_{cd}^{LL} - \frac{3}{2} C_8 \mathcal{A}_{cd}^{LR} + \left(C_5 - \frac{1}{2} C_7\right) (\mathcal{A}_{cd}^{SP} + \mathcal{A}_{gh}^{SP}) \\
& + \left(2a_3 + a_4 + 2a_5 + \frac{1}{2} a_7 + \frac{1}{2} a_9 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ef}^{LL} + \left(2C_6 + \frac{1}{2} C_8\right) \mathcal{A}_{gh}^{LR} \\
& \left. + \left(C_3 + 2C_4 - \frac{1}{2} C_9 + \frac{1}{2} C_{10}\right) \mathcal{A}_{gh}^{LL} \right\}, \tag{B7}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \rightarrow K^- \pi^+) = & \frac{G_F}{\sqrt{2}} V_{ub} V_{us}^* \{a_1 \mathcal{A}_{ab}^{LL}(\pi, \bar{K}) + C_2 \mathcal{A}_{cd}^{LL}(\pi, \bar{K})\} - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ (a_4 + a_{10}) \mathcal{A}_{ab}^{LL}(\pi, \bar{K}) + (a_6 + a_8) \mathcal{A}_{ab}^{SP}(\pi, \bar{K}) \right. \\
& + (C_3 + C_9) \mathcal{A}_{cd}^{LL}(\pi, \bar{K}) + (C_5 + C_7) \mathcal{A}_{cd}^{SP}(\pi, \bar{K}) + \left(a_4 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ef}^{LL}(\bar{K}, \pi) + \left(a_6 - \frac{1}{2} a_8\right) \mathcal{A}_{ef}^{SP}(\bar{K}, \pi) \\
& \left. + \left(C_3 - \frac{1}{2} C_9\right) \mathcal{A}_{gh}^{LL}(\bar{K}, \pi) + \left(C_5 - \frac{1}{2} C_7\right) \mathcal{A}_{gh}^{SP}(\bar{K}, \pi) \right\}, \tag{B8}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \rightarrow \bar{K}^0 \pi^0) = & \frac{G_F}{2} V_{ub} V_{us}^* \{a_2 \mathcal{A}_{ab}^{LL}(\bar{K}, \pi) + C_1 \mathcal{A}_{cd}^{LL}(\bar{K}, \pi)\} + \frac{G_F}{2} V_{tb} V_{ts}^* \left\{ \left(a_4 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ab}^{LL}(\pi, \bar{K}) \right. \\
& + \left(a_6 - \frac{1}{2} a_8\right) \mathcal{A}_{ab}^{SP}(\pi, \bar{K}) + \frac{3}{2} (a_7 - a_9) \mathcal{A}_{ab}^{LL}(\bar{K}, \pi) + \left(a_4 - \frac{1}{2} a_{10}\right) \mathcal{A}_{ef}^{LL}(\bar{K}, \pi) \\
& + \left(a_6 - \frac{1}{2} a_8\right) \mathcal{A}_{ef}^{SP}(\bar{K}, \pi) + \left(C_3 - \frac{1}{2} C_9\right) \mathcal{A}_{cd}^{LL}(\pi, \bar{K}) + \left(C_5 - \frac{1}{2} C_7\right) \mathcal{A}_{cd}^{SP}(\pi, \bar{K}) - \frac{3}{2} C_8 \mathcal{A}_{cd}^{LR}(\bar{K}, \pi) \\
& \left. - \frac{3}{2} C_{10} \mathcal{A}_{cd}^{LL}(\bar{K}, \pi) + \left(C_3 - \frac{1}{2} C_9\right) \mathcal{A}_{gh}^{LL}(\bar{K}, \pi) + \left(C_5 - \frac{1}{2} C_7\right) \mathcal{A}_{gh}^{SP}(\bar{K}, \pi) \right\}, \tag{B9}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \rightarrow K^- K^+) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \{a_2 \mathcal{A}_{ef}^{LL}(K, \bar{K}) + C_1 \mathcal{A}_{gh}^{LL}(K, \bar{K})\} - \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ (a_3 + a_5 + a_7 + a_9) \mathcal{A}_{ef}^{LL}(K, \bar{K}) \right. \\
&\quad + \left(a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 \right) \mathcal{A}_{ef}^{LL}(\bar{K}, K) + (C_4 + C_{10}) \mathcal{A}_{gh}^{LL}(K, \bar{K}) + (C_6 + C_8) \mathcal{A}_{gh}^{LR}(K, \bar{K}) \\
&\quad \left. + \left(C_4 - \frac{1}{2} C_{10} \right) \mathcal{A}_{gh}^{LL}(\bar{K}, K) + \left(C_6 - \frac{1}{2} C_8 \right) \mathcal{A}_{gh}^{LR}(\bar{K}, K) \right\}, \tag{B10}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \rightarrow \bar{K}^0 K^0) &= -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* \left\{ \left(a_4 - \frac{1}{2} a_{10} \right) \mathcal{A}_{ab}^{LL}(\bar{K}, K) + \left(a_6 - \frac{1}{2} a_8 \right) \mathcal{A}_{ab}^{SP}(\bar{K}, K) \right. \\
&\quad + \left(C_3 - \frac{1}{2} C_9 \right) \mathcal{A}_{cd}^{LL}(\bar{K}, K) + \left(C_5 - \frac{1}{2} C_7 \right) \mathcal{A}_{cd}^{SP}(\bar{K}, K) + \left(a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 \right) \mathcal{A}_{ef}^{LL}(\bar{K}, K) \\
&\quad + \left(C_4 - \frac{1}{2} C_{10} \right) \mathcal{A}_{gh}^{LL}(\bar{K}, K) + \left(C_6 - \frac{1}{2} C_8 \right) \mathcal{A}_{gh}^{LR}(\bar{K}, K) + \left(a_6 - \frac{1}{2} a_8 \right) \mathcal{A}_{ef}^{SP}(K, \bar{K}) \\
&\quad + \left(a_3 + a_4 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10} \right) \mathcal{A}_{ef}^{LL}(K, \bar{K}) + \left(C_3 + C_4 - \frac{1}{2} C_9 - \frac{1}{2} C_{10} \right) \mathcal{A}_{gh}^{LL}(K, \bar{K}) \\
&\quad \left. + \left(C_6 - \frac{1}{2} C_8 \right) \mathcal{A}_{gh}^{LR}(K, \bar{K}) + \left(C_5 - \frac{1}{2} C_7 \right) \mathcal{A}_{gh}^{SP}(K, \bar{K}) \right\}. \tag{B11}
\end{aligned}$$

The shorthands are

$$a_i = \begin{cases} C_i + \frac{1}{N_c} C_{i+1}, & \text{for odd } i; \\ C_i + \frac{1}{N_c} C_{i-1}, & \text{for even } i, \end{cases} \tag{B12}$$

$$C_m \mathcal{A}_{ij}^k(M_1, M_2) = i\mathcal{K} \frac{\pi C_F}{N_c^2} m_B^4 f_B f_{M_1} f_{M_2} \{ \mathcal{A}_i^k(C_m, M_1, M_2) + \mathcal{A}_j^k(C_m, M_1, M_2) \}, \tag{B13}$$

where the factor $\mathcal{K} = N_c$ for factorizable amplitudes with $ij = ab$ and ef , and $\mathcal{K} = 1$ for nonfactorizable amplitudes with $ij = cd$ and gh . The expressions of amplitude building block $\mathcal{A}_i^k(C_m, M_1, M_2)$ are given in Appendix C.

APPENDIX C: AMPLITUDE BUILDING BLOCKS

There should be a Sudakov factor corresponding to each WF with the pQCD approach. For the sake of convenience in writing, the shorthands such as $\phi_{B1} = \phi_{B1}(x_1, b_1) e^{-S_B}$, $\phi_{B2} = \phi_{B2}(x_1, b_1) e^{-S_B}$, $\phi_M^a = \phi_M^a(x_2) e^{-S_M}$, $\phi_{M'}^a = \phi_{M'}^a(x_3) e^{-S_{M'}}$, $\phi_M^{p,t} = r_M \phi_M^{p,t}(x_2) e^{-S_M}$, $\phi_{M'}^{p,t} = r_{M'} \phi_{M'}^{p,t}(x_3) e^{-S_{M'}}$ and $\mathcal{A}_i^j = \mathcal{A}_i^j(C_m, M, M')$ will be used in this section. As to the amplitude building block $\mathcal{A}_i^j$, the subscript i corresponds to the indices of Fig. 7, and the superscript j refers to the three possible Dirac structures $\Gamma_1 \otimes \Gamma_2$ of the operator $(\bar{q}_1 q_2)_{\Gamma_1} (\bar{q}_3 q_4)_{\Gamma_2}$, namely $j = LL$ for $(V - A) \otimes (V - A)$, $j = LR$ for $(V - A) \otimes (V + A)$ and $j = SP$ for $-2(S - P) \otimes (S + P)$. The expressions of $\mathcal{A}_i^j$ are written as follows.

$$\begin{aligned}
\mathcal{A}_a^{LL} &= \int dx_1 dx_2 db_1 db_2 C_i(t_a) \alpha_s(t_a) S_t(x_2) H_{ab}(\alpha_g, \beta_a, b_1, b_2) \{ \phi_{B1} [\phi_M^a (1 + x_2) + (\phi_M^p + \phi_M^t)(\bar{x}_2 - x_2)] \\
&\quad - \phi_{B2} [\phi_M^a - (\phi_M^p + \phi_M^t)x_2] \}, \tag{C1}
\end{aligned}$$

$$\mathcal{A}_a^{LR} = -\mathcal{A}_a^{LL}, \tag{C2}$$

$$\begin{aligned}
\mathcal{A}_a^{SP} &= 2r_{M'} \int dx_1 dx_2 db_1 db_2 C_i(t_a) \alpha_s(t_a) S_t(x_2) H_{ab}(\alpha_g, \beta_a, b_1, b_2) \\
&\quad \times \{ \phi_{B1} [\phi_M^a + \phi_M^p (2 + x_2) - \phi_M^t x_2] - \phi_{B2} [\phi_M^a + \phi_M^p - \phi_M^t] \}, \tag{C3}
\end{aligned}$$

$$\mathcal{A}_b^{LL} = 2 \int dx_1 dx_2 db_1 db_2 C_i(t_b) \alpha_s(t_b) S_t(x_1) H_{ab}(\alpha_g, \beta_b, b_2, b_1) \phi_{B1} \phi_M^p, \quad (C4)$$

$$\mathcal{A}_b^{LR} = -\mathcal{A}_b^{LL}, \quad (C5)$$

$$\mathcal{A}_b^{SP} = \int dx_1 dx_2 db_1 db_2 C_i(t_b) \alpha_s(t_b) S_t(x_1) H_{ab}(\alpha_g, \beta_b, b_2, b_1) 2r_{M'} \{ \phi_{B1} [\phi_M^a x_1 + 2\phi_M^p \bar{x}_1] + 2\phi_{B2} \phi_M^p x_1 \}, \quad (C6)$$

$$\begin{aligned} \mathcal{A}_c^{LL} = & \int dx_1 dx_2 dx_3 db_2 db_3 C_i(t_c) \alpha_s(t_c) S_t(x_2) H_{cd}(\alpha_g, \beta_c, b_2, b_3) \phi_{M'}^a \\ & \times \{ (\phi_{B1} - \phi_{B2}) \phi_M^a (\bar{x}_3 - x_1) - \phi_{B1} (\phi_M^p - \phi_M^t) x_2 \}_{b_1=b_2}, \end{aligned} \quad (C7)$$

$$\begin{aligned} \mathcal{A}_c^{LR} = & \int dx_1 dx_2 dx_3 db_2 db_3 C_i(t_c) \alpha_s(t_c) S_t(x_2) H_{cd}(\alpha_g, \beta_c, b_2, b_3) \phi_{M'}^a \\ & \times \{ (\phi_{B1} - \phi_{B2}) [\phi_M^a (x_1 - \bar{x}_3) + (\phi_M^p + \phi_M^t) x_2] - \phi_{B1} \phi_M^a x_2 \}_{b_1=b_2}, \end{aligned} \quad (C8)$$

$$\begin{aligned} \mathcal{A}_c^{SP} = & \int dx_1 dx_2 dx_3 db_2 db_3 C_i(t_c) \alpha_s(t_c) S_t(x_2) H_{cd}(\alpha_g, \beta_c, b_2, b_3) \\ & \times \{ (\phi_{B1} - \phi_{B2}) (\phi_M^a + \phi_M^p - \phi_M^t) (\phi_{M'}^p + \phi_{M'}^t) (\bar{x}_3 - x_1) + \phi_{B1} x_2 (\phi_M^p + \phi_M^t) (\phi_{M'}^p - \phi_{M'}^t) \}_{b_1=b_2}, \end{aligned} \quad (C9)$$

$$\begin{aligned} \mathcal{A}_d^{LL} = & \int dx_1 dx_2 dx_3 db_2 db_3 C_i(t_d) \alpha_s(t_d) S_t(x_2) H_{cd}(\alpha_g, \beta_d, b_2, b_3) \phi_{M'}^a \\ & \times \{ (\phi_{B1} - \phi_{B2}) [\phi_M^a (x_1 - x_3) + (\phi_M^p + \phi_M^t) x_2] - \phi_{B1} \phi_M^a x_2 \}_{b_1=b_2}, \end{aligned} \quad (C10)$$

$$\begin{aligned} \mathcal{A}_d^{LR} = & \int dx_1 dx_2 dx_3 db_2 db_3 C_i(t_d) \alpha_s(t_d) S_t(x_2) H_{cd}(\alpha_g, \beta_d, b_2, b_3) \phi_{M'}^a \\ & \times \{ (\phi_{B1} - \phi_{B2}) \phi_M^a (x_3 - x_1) - \phi_{B1} (\phi_M^p - \phi_M^t) x_2 \}_{b_1=b_2}, \end{aligned} \quad (C11)$$

$$\begin{aligned} \mathcal{A}_d^{SP} = & \int dx_1 dx_2 dx_3 db_2 db_3 C_i(t_d) \alpha_s(t_d) S_t(x_2) H_{cd}(\alpha_g, \beta_d, b_2, b_3) \{ (\phi_{B1} - \phi_{B2}) (\phi_M^a + \phi_M^p - \phi_M^t) (\phi_{M'}^p - \phi_{M'}^t) (x_1 - x_3) \\ & - \phi_{B1} x_2 (\phi_M^p + \phi_M^t) (\phi_{M'}^p + \phi_{M'}^t) \}_{b_1=b_2}, \end{aligned} \quad (C12)$$

$$\mathcal{A}_e^{LL} = - \int dx_2 dx_3 db_2 db_3 C_i(t_e) \alpha_s(t_e) S_t(\bar{x}_3) H_{ef}(\omega_g, \beta_e, b_2, b_3) \{ \phi_M^a \phi_{M'}^a \bar{x}_3 + 2\phi_M^p [\phi_{M'}^p (1 + \bar{x}_3) + \phi_{M'}^t x_3] \}, \quad (C13)$$

$$\mathcal{A}_e^{LR} = +\mathcal{A}_e^{LL}, \quad (C14)$$

$$\mathcal{A}_e^{SP} = \int dx_2 dx_3 db_2 db_3 C_i(t_e) \alpha_s(t_e) S_t(\bar{x}_3) H_{ef}(\omega_g, \beta_e, b_2, b_3) 2 \{ \phi_M^a \bar{x}_3 (\phi_{M'}^p + \phi_{M'}^t) + 2\phi_M^p \phi_{M'}^a \}, \quad (C15)$$

$$\mathcal{A}_f^{LL} = \int dx_2 dx_3 db_2 db_3 C_i(t_f) \alpha_s(t_f) S_t(x_2) H_{ef}(\omega_g, \beta_f, b_3, b_2) \{ \phi_M^a \phi_{M'}^a x_2 + 2[\phi_M^p (1 + x_2) - \phi_M^t \bar{x}_2] \phi_{M'}^p \}, \quad (C16)$$

$$\mathcal{A}_f^{LR} = +\mathcal{A}_f^{LL}, \quad (C17)$$

$$\mathcal{A}_f^{SP} = \int dx_2 dx_3 db_2 db_3 C_i(t_f) \alpha_s(t_f) S_t(x_2) H_{ef}(\omega_g, \beta_f, b_3, b_2) 2 \{ 2\phi_M^a \phi_{M'}^p + x_2 (\phi_M^p - \phi_M^t) \phi_{M'}^a \}, \quad (C18)$$

$$\begin{aligned} \mathcal{A}_g^{LL} = & \int dx_1 dx_2 dx_3 db_1 db_2 C_i(t_g) \alpha_s(t_g) H_{gh}(\omega_g, \beta_g, b_1, b_2) \{ \phi_{B2} (\phi_M^p - \phi_M^t) [\phi_{M'}^p (x_1 + \bar{x}_3 + 1) + \phi_{M'}^t (x_1 - x_3)] \\ & - \phi_{B1} [\phi_M^a \phi_{M'}^a x_2 + 2\phi_M^p \phi_{M'}^p (1 + x_2) + 2\phi_M^t \phi_{M'}^t \bar{x}_2 + (\phi_M^p - \phi_M^t) (\phi_{M'}^p + \phi_{M'}^t) (x_1 - x_2 + \bar{x}_3)] \}_{b_2=b_3}, \end{aligned} \quad (C19)$$

$$\begin{aligned} \mathcal{A}_g^{LR} = & - \int dx_1 dx_2 dx_3 db_1 db_2 C_i(t_g) \alpha_s(t_g) H_{gh}(\omega_g, \beta_g, b_1, b_2) \{ (\phi_{B1} - \phi_{B2}) \phi_M^a \phi_{M'}^a (x_1 + \bar{x}_3) + \phi_{B1} [2\phi_M^p \phi_{M'}^p (1 + x_2) \\ & + 2\phi_M^t \phi_{M'}^t \bar{x}_2 + (\phi_M^p + \phi_M^t)(\phi_{M'}^p - \phi_{M'}^t)(x_1 - x_2 + \bar{x}_3)] - \phi_{B2}(\phi_{M'}^p - \phi_{M'}^t)[\phi_M^p(x_1 + \bar{x}_3 + 1) + \phi_M^t(x_1 - x_3)] \} \}_{b_2=b_3}, \end{aligned} \quad (C20)$$

$$\begin{aligned} \mathcal{A}_g^{SP} = & \int dx_1 dx_2 dx_3 db_1 db_2 C_i(t_g) \alpha_s(t_g) H_{gh}(\omega_g, \beta_g, b_1, b_2) \{ \phi_{B1} [\phi_M^a (\phi_{M'}^p - \phi_{M'}^t)(x_1 + \bar{x}_3 - 2) + \phi_{M'}^a (\phi_M^p + \phi_M^t)(2 - x_2)] \\ & - \phi_{B2} [\phi_M^a (\phi_{M'}^p - \phi_{M'}^t)(x_1 - x_3) + \phi_{M'}^a (\phi_M^p + \phi_M^t)] \} \}_{b_2=b_3}, \end{aligned} \quad (C21)$$

$$\begin{aligned} \mathcal{A}_h^{LL} = & \int dx_1 dx_2 dx_3 db_1 db_2 C_i(t_h) \alpha_s(t_h) H_{gh}(\omega_g, \beta_h, b_1, b_2) \{ (\phi_{B1} - \phi_{B2})(\bar{x}_3 - x_1) [\phi_M^a \phi_{M'}^a + (\phi_M^p + \phi_M^t)(\phi_{M'}^p - \phi_{M'}^t)] \\ & + \phi_{B1} x_2 (\phi_M^p - \phi_M^t)(\phi_{M'}^p + \phi_{M'}^t) \} \}_{b_2=b_3}, \end{aligned} \quad (C22)$$

$$\begin{aligned} \mathcal{A}_h^{LR} = & \int dx_1 dx_2 dx_3 db_1 db_2 C_i(t_h) \alpha_s(t_h) H_{gh}(\omega_g, \beta_h, b_1, b_2) \{ (\phi_{B1} - \phi_{B2})(\bar{x}_3 - x_1) (\phi_M^p - \phi_M^t)(\phi_{M'}^p + \phi_{M'}^t) \\ & + \phi_{B1} x_2 [\phi_M^a \phi_{M'}^a + (\phi_M^p + \phi_M^t)(\phi_{M'}^p - \phi_{M'}^t)] \} \}_{b_2=b_3}, \end{aligned} \quad (C23)$$

$$\begin{aligned} \mathcal{A}_h^{SP} = & \int dx_1 dx_2 dx_3 db_1 db_2 C_i(t_h) \alpha_s(t_h) H_{gh}(\omega_g, \beta_h, b_1, b_2) \{ \phi_{B1} [x_2 \phi_{M'}^a (\phi_M^p + \phi_M^t) + (x_1 - \bar{x}_3) \phi_M^a (\phi_{M'}^p - \phi_{M'}^t)] \\ & - \phi_{B2} x_2 \phi_{M'}^a (\phi_M^p + \phi_M^t) \} \}_{b_2=b_3}, \end{aligned} \quad (C24)$$

$$H_{cd}(\alpha, \beta, b_i, b_j) = b_i b_j \{ \theta(b_i - b_j) K_0(b_i \sqrt{\alpha}) I_0(b_j \sqrt{\alpha}) + (b_i \leftrightarrow b_j) K_0(b_j \sqrt{\beta}) \}, \quad (C25)$$

$$H_{ef}(\omega, \beta, b_i, b_j) = -\frac{\pi^2}{4} b_i b_j \{ \theta(b_i - b_j) [J_0(b_i \sqrt{\beta}) + iY_0(b_i \sqrt{\beta})] J_0(b_j \sqrt{\beta}) + (b_i \leftrightarrow b_j) \} \{ J_0(b_i \sqrt{\omega}) + iY_0(b_i \sqrt{\omega}) \}, \quad (C26)$$

$$\begin{aligned} H_{gh}(\omega, \beta, b_i, b_j) = & i \frac{\pi}{2} b_i b_j \{ [J_0(b_i \sqrt{\omega}) + iY_0(b_i \sqrt{\omega})] J_0(b_j \sqrt{\omega}) + (b_i \leftrightarrow b_j) \} \\ & \times \left\{ i \frac{\pi}{2} \theta(\beta) [J_0(b_i \sqrt{\beta}) + iY_0(b_i \sqrt{\beta})] + \theta(-\beta) K_0(b_i \sqrt{-\beta}) \right\}, \end{aligned} \quad (C27)$$

$$\alpha_g = x_1 x_2 m_B^2, \quad (C28)$$

$$\beta_g = (x_2 \bar{x}_3 - \bar{x}_1 x_2 - \bar{x}_3) m_B^2, \quad (C36)$$

$$\omega_g = x_2 \bar{x}_3 m_B^2, \quad (C29)$$

$$\beta_h = x_2 (\bar{x}_3 - x_1) m_B^2, \quad (C37)$$

$$\beta_a = x_2 m_B^2, \quad (C30)$$

$$t_i = \max(1/b_1, 1/b_2, \sqrt{\beta_i}), \quad \text{for } i = a, b; \quad (C38)$$

$$\beta_b = x_1 m_B^2, \quad (C31)$$

$$t_i = \max(1/b_2, 1/b_3, \sqrt{\alpha_g}, \sqrt{|\beta_i|}), \quad \text{for } i = c, d; \quad (C39)$$

$$\beta_c = x_2 (x_1 - \bar{x}_3) m_B^2, \quad (C32)$$

$$t_i = \max(1/b_2, 1/b_3, \sqrt{\beta_i}), \quad \text{for } i = e, f; \quad (C40)$$

$$\beta_d = x_2 (x_1 - x_3) m_B^2, \quad (C33)$$

$$\beta_e = \bar{x}_3 m_B^2, \quad (C34)$$

$$t_i = \max(1/b_1, 1/b_2, \sqrt{\omega_g}, \sqrt{|\beta_i|}), \quad \text{for } i = g, h. \quad (C41)$$

$$\beta_f = x_2 m_B^2, \quad (C35)$$

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