Heavy dark matter and gravitational waves

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We study the heavy dark matter produced around the temperature of the phase transition. Meanwhile domain walls can be formed after the breakdown of a $\mathbb{Z}_3$ discrete symmetry induced by the first-order phase transition. The generated gravitational waves by domain wall decay are found to be able to be probed by the pulsar timing arrays, and the future square kilometer array.

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I. INTRODUCTION

The phase transition in the Standard model (SM) is confirmed to be a crossover [1], and a first-order phase transition (PT) is predicted in many particle physics models that go beyond the SM [2], such as singlet extended SM [3–10], two Higgs doublet models [11–16], the George-Machacek model [17], and the next-to-minimal supersymmetry model [18,19], etc. A general prediction of the first-order PT is the production of gravitational waves (GWs) [20,21], which happens to be one of the scientific searching goals of the space-based interferometers, including LISA [22], Taiji [23], TianQin [24], DECIGO [25], and BBO [26].

Dark matter (DM) is believed to be one crucial ingredient of the Universe, while its nature is still a mystery. The weak scale weakly interacting massive particle (WIMP) DM currently confronts with severe constraints from direct detection DM experiments, such as XENON1T [27], PandaX [28], and LUX [29]. One possibility to alleviate the situation is to consider the DM production through a PT modified thermal freeze-out mechanism as studied in Refs. [30–32], where the dark sector keeps in thermal equilibrium with the SM in the early Universe. Another scenario is to consider that as the dark sector feebly interacts with the SM, the relic density can be generated through the freeze-in mechanism [33], where the DM is called feebly interacting massive particles (FIMP). The typical temperature for FIMP DM production is close to the DM mass, in this case the DM production may be

highly amplified or diminished by the PT effect, see Refs. [34–39]. Thus the significant PT effects involved in the FIMP production processes should be taken into account.

After the PT, the symmetry of the model breaks. One general prediction of the spontaneous breakdown of a discrete symmetry is the formation of the domain walls (DWs) [40]. The collapse of DWs may produce GWs with peak frequency being around nanoHertz [41–44], which can be probed by pulsar timing array experiments, such as the European pulsar timing array [45], the Parkes pulsar timing array [46], the international pulsar timing array [47], and the NANOGrav [48].

In this paper, we study the heavy fermionic DM production within a $\mathbb{Z}_3$ model, which is well motivated for neutrino and DM physics [49–54], and explore the GW signatures coming from the DW decay during the first-order phase transition. This paper is organized as follows: we first study the PT in a $\mathbb{Z}_3$ model in Sec. II, Sec. III is devoted to the DM study, GWs produced from the first-order PT and DW decay are studied in Sec. IV, we then summarize in Sec. V.

II. PHASE TRANSITION MODEL

In this work, we consider the PT model with the scalar potential given by

$$V = \mu_S^2 |S|^2 + \lambda_S |S|^4 + \frac{\mu_3}{2} (S^3 + S^{\dagger 3}), \quad (1)$$

where $S = (v_s + s + iA)/\sqrt{2}$, and v_s is the vacuum expectation value (VEV) of the s field. The cubic term μ_3 breaks the global $U(1)$ symmetry (under transformation $S \rightarrow e^{i\alpha} S$) with a remanent unbroken $\mathbb{Z}_3$ symmetry. Considering the stationary point condition,

$$\left. \frac{dV(s, A)}{ds} \right|_{s=v_s} = 0, \quad (2)$$

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we get $\mu_s^2 = -\lambda_s v_s^2 - \frac{3\sqrt{2}}{4}\mu_3 v_s$. The masses of the new particles (s , A) are then given by

$$m_s^2 = \mu_s^2 + 3\lambda_s v_s^2 + \frac{3\mu_3 v_s}{\sqrt{2}},$$

$$m_A^2 = \mu_s^2 + \lambda_s v_s^2 - \frac{3\mu_3 v_s}{\sqrt{2}}.$$

The cubic interaction coupling μ_3 and the VEV of s relate to the physical masses through

$$v_s = \sqrt{\frac{3m_s^2 + m_A^2}{6\lambda_s}}, \quad \mu_3 = -\frac{4m_A^2 \sqrt{2\lambda_s(3m_s^2 + m_A^2)}}{9\sqrt{6}m_s^2 + 3\sqrt{6}m_A^2}. \quad (3)$$

The finite-temperature effective potential at a one-loop level is given by

$$V_{\text{eff}}(s, T) = V_0(s) + V_{\text{CW}}(s) + V_{\text{ct}}(s) + V_T(s, T). \quad (4)$$

Here, $V_0(s)$ is the tree-level potential in terms of the classical field,

$$V_0(s) = \frac{\mu_s^2}{2}s^2 + \frac{\mu_3}{2\sqrt{2}}s^3 + \frac{\lambda_s}{4}s^4. \quad (5)$$

The one-loop contribution splits up into a ultraviolet-divergent zero-temperature (Coleman-Weinberg) part, V_{CW} , and a finite-temperature part, V_T .

The one-loop Coleman-Weinberg potential is [55,56]

$$V_{\text{CW}}(s) = \sum_i \frac{\eta_i n_i}{64\pi^2} m_i^4(s) \left[\log\left(\frac{m_i^2(s)}{\Lambda^2}\right) - C_i \right], \quad (6)$$

where i runs over the s -dependent mass of all particle species, n_i is the number of degrees of freedom for each species, $\eta_i = +1$ (-1) for scalars (fermions), and $C_i = 3/2$ ($5/6$) for scalars and fermions. Here, we take $\Lambda = v_s$. The counterterms are introduced to prevent the shift of the vacuum driven by the Coleman-Weinberg potential, with

$$V_{\text{ct}}(s) = \frac{\delta\mu_s^2}{2}s^2 + \frac{\delta\mu_3}{2\sqrt{2}}s^3 + \frac{\delta\lambda_s}{4}s^4, \quad (7)$$

where these terms are fixed through following conditions,

$$\left. \frac{d(V_{\text{CW}}(s) + V_{\text{ct}}(s))}{ds} \right|_{s=v_s} = 0,$$

$$\left. \frac{d^2(V_{\text{CW}}(s) + V_{\text{ct}}(s))}{ds^2} \right|_{s=v_s} = 0,$$

$$V_{\text{CW}}(0) - V_{\text{CW}}(v_s) - V_{\text{ct}}(v_s) = 0. \quad (8)$$

The one-loop finite-temperature correction is evaluated to be [56,57]

$$V_T(s, T) = \frac{T^4}{2\pi} \sum_i n_i J_{B,F}\left(\frac{m_i^2(s, T)}{T^2}\right), \quad (9)$$

where the function $J_{B,F}$ is given by

$$J_{B,F}(y) = \pm \int_0^\infty dx x^2 \ln \left[1 \mp \exp\left(-\sqrt{x^2 + y}\right) \right], \quad (10)$$

with the upper (lower) sign corresponding to bosonic (fermionic) contributions. Here, the thermal masses of the scalars are

$$m_s^2(s, T) = m_s^2 + \frac{4\lambda_s}{12}T^2, \quad (11)$$

$$m_A^2(s, T) = m_A^2 + \frac{\lambda_s}{3}T^2. \quad (12)$$

The onset condition of the first-order phase transition is [58–60]:

$$\Gamma \sim T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T} \sim 1, \quad (13)$$

which means that the nucleation temperature (T_n) is obtained when the number of bubbles for bubble nucleation per horizon volume and per horizon time is of order unity. Here,

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2} \left(\frac{ds}{dr} \right)^2 + V_{\text{eff}}(s, T) \right] \quad (14)$$

can be extremized to find the solution of the bounce of configuration of the nucleated bubbles, i.e., the bounce configuration of the field connects the $\mathbb{Z}_3$ broken vacuum (true vacuum) and the $\mathbb{Z}_3$ preserving vacuum (false vacuum), with the boundary conditions

$$\lim_{r \rightarrow \infty} s = 0, \quad \left. \frac{ds}{dr} \right|_{r=0} = 0. \quad (15)$$

There are two crucial parameters for GW calculation: (1) the latent heat of the first-order PT normalized by the radiative energy, i.e., $\alpha = \frac{\Delta\rho}{\rho_R}$ with $\Delta\rho$ being the released latent heat from the phase transition to the energy density of

TABLE I. The benchmark points for achieving the first-order PT.

BMs	λ_s	m_s (GeV)	m_A (GeV)	$v_s(T_n)$ (GeV)	T_n (GeV)	α	β/H_n
BM1	1.52	14991.4	40278.8	14878.4	12235.2	0.005	14310.7
BM2	1.27	7889.7	21655.6	8859.8	6291.1	0.006	6991.3
BM3	0.35	3843.2	9959.3	7686.8	4856.6	0.004	5428.6

the plasma background; (2) the inverse time duration of the phase transition, which is defined as

$$\frac{\beta}{H_n} = T \left. \frac{d(S_3(T)/T)}{dT} \right|_{T=T_n}. \quad (16)$$

We show three benchmarks estimated by analyzing the PT dynamics utilizing COSMOTRANSITION [61] in Table I, which will be used for the study of DM and GWs in the following sections.

III. DARK MATTER

References [62,63] considered the weak scale WIMP DM being the pseudoscalar A and analyzed the GW production at first-order electroweak PT. In this work, we are going to study heavy fermionic DM production at the high scale PT assisted by the pseudoscalar A . Concretely, we consider two Dirac fermions interacting with the $\mathbb{Z}_3$ scalar through the Lagrangian,

$$\mathcal{L} \supset m_{\psi_1} \bar{\psi}_1 \psi_1 + m_{\psi_2} \bar{\psi}_2 \psi_2 + y_s S \psi_1 \bar{\psi}_2 + y_s S^\dagger \bar{\psi}_1 \psi_2. \quad (17)$$

The complex scalar and the two fermionic particles have $\mathbb{Z}_3$ charges: 1, 1 -1. After the $\mathbb{Z}_3$ symmetry breaking, the two Dirac fermions ψ_1 and ψ_2 mixed each other and the lightest one may serve as a DM candidate (denoted as χ); the mass matrix is given by

$$\begin{pmatrix} \frac{1}{2}(-\sqrt{M} + m_{\psi_1} + m_{\psi_2}) & 0 \\ 0 & \frac{1}{2}(\sqrt{M} + m_{\psi_1} + m_{\psi_2}) \end{pmatrix}, \quad (18)$$

where $M = -2m_{\psi_1}m_{\psi_2} + m_{\psi_1}^2 + m_{\psi_2}^2 + 4y_s^2 v_s^2(T)$.

We first consider the case where fermionic DM particles χ can freeze out, while the dark scalars remain in thermal equilibrium with the SM bath. As in Refs. [64–67], we assume that the DM keep in thermal equilibrium with the thermal bath, being massless outside the bubble and massive inside the bubble. As DM particles enter the bubble, their interactions are put abruptly out of equilibrium, which prevents their annihilation and therefore constitute the relics. Considering $m_\chi = x_f T_n \sim 20T_n$, one has the DM mass $m_\chi \sim 10^{-6}$ GeV for the nucleation temperature $T_n \sim 10^{-5}$ GeV. With the nonequilibrium explicitly given by the condition

$$\frac{m_\chi^{\text{in}}}{T_n} \gtrsim 24 - \log \frac{T_n}{\text{TeV}} - \frac{3}{2} \log \frac{m_\chi^{\text{in}}/T_n}{24} + 4 \log y_s, \quad (19)$$

the DM relic density can be calculated as

$$\Omega_{\text{DM}} h^2 \simeq 0.17 \left(\frac{T_n}{\text{TeV}} \right) \left(\frac{m_\chi^{\text{in}}/T_n}{30} \right)^{5/2} \frac{e^{-m_\chi^{\text{in}}/T_n}}{e^{-30}}, \quad (20)$$

where the $m_\chi^{\text{in}} = y_s v_s(T_n)$ with $m_{\psi_1} = m_{\psi_2} = 0$. The observed relic abundance is obtained for m_χ and T_n corresponding to the solid line in Fig. 1. For the PT temperatures shown in the three benchmark points in Fig. 1 (red, orange, yellow stars), we found that the Yukawa couplings are required by the out-of-equilibrium condition [Eq. (19)] to be $y_s = 20.5, 14.4, 12.5$, which are too large and will break the perturbativity.

To address this contradiction, we instead consider a feebly interacting strength of the Yukawa coupling y_s , and the complex scalar S keeps in thermal equilibrium with the SM, such that particle χ becomes a FIMP DM. In the freeze-in mechanism, DM may be produced through the decay channel $s \rightarrow \bar{\psi}\chi$, the scattering channels $s(A)s(A) \rightarrow \bar{\psi}\chi$ via the s channel with the mediator being s , and the t channel of $ss \rightarrow \chi\bar{\chi}$, as shown in the Fig. 2. The annihilation cross sections and decay width for these processes are given in the Appendix. Introducing the dimensionless evolution variable $x \equiv m_\chi/T$, the Boltzmann equation that governs the evolution of DM number density (here, $Y_i \equiv n_i/s$) is given by

FIG. 1. The solid line is calculated by Eq. (20) for $\Omega h^2 = 0.12$. The stars correspond to the benchmark points in Table I.

FIG. 2. Processes relevant for the freeze-in mechanism of the DM.

$$\begin{aligned} \frac{dY_\chi}{dx} = & \frac{m_s^2 g_s}{2\pi^2} \frac{m_\chi}{H(x)s(x)x^2} \Gamma(s \rightarrow \chi\bar{\psi}) K_1\left(\frac{xm_s}{m_\chi}\right) \\ & + \frac{g_{B_1}^2}{32\pi^4} \frac{m_\chi}{H(x)s(x)x^2} \\ & \times \int_{4m_{B_1}}^\infty d\hat{s} (\hat{s} - 4m_{B_1}^2) \sigma(B_1\bar{B}_1 \rightarrow \chi B_2) \sqrt{\hat{s}} K_1\left(\frac{x\sqrt{\hat{s}}}{m_\chi}\right), \end{aligned} \quad (21)$$

where $H(x)$ denotes the Hubble rate, $\sqrt{\hat{s}}$ is the center-of-mass energy, $g_s(g_{B_1})$ is the number of degrees of freedom of $s(B_1)$, and K_1 is a modified Bessel function of a second kind. As in Refs. [34,35,37–39], to capture the PT effects, we calculate the DM number density by replacing $m_{s,A} \rightarrow m_{s,A}(T)$ with the thermal masses $m_{s,A}(T)$ being given by Eq. (11).

To illustrate the DM production processes at the first-order PT, we calculate the DM relic density evolution in Fig. 3 by taking input parameters from the BM₁ in Table I. As shown in the top panel of Fig. 3, there are four (three) channels that can contribute to DM number density when the thermal effects are (not) included, see solid (dashed) lines. Considering thermal effects, the mass $m_s(T)$ decreases with temperature, as indicated by Eq. (11), the decay channel becomes active and dominates the DM production process before the PT ($T > T_n$). As the Universe cools down, bubble nucleates, the $\mathbb{Z}_3$ symmetry breaks spontaneously, and the DWs form. Just below the nucleation temperature ($T = 8.9$ TeV), the DM production from the decay channel stops because of $m_s(T) < m_\psi + m_\chi$, then the scattering channels dominate the DM production. The bottom panel of Fig. 3 shows that, when the thermal effects are included, the DM relic density is mostly produced by the decay channel, and stops increasing when $T < 8.9$ TeV after the PT. The DM relic density produced through the freeze-in mechanism without considering thermal effects is smaller than the case with thermal effects by two orders, as shown by the dashed line.

FIG. 3. Top: the DM number density evolution of different channels. Bottom: the DM relic density Ωh^2 as a function of temperature T . The vertical dotted and dashed line correspond to $m_s(T) = m_\psi + m_\chi$ and T_n .

IV. STOCHASTIC GRAVITATIONAL WAVES

With the accumulation of the DM density, DWs are formed after breakdown of the discrete symmetry induced by the first-order PT, and GWs are also produced at the same stage. In this section, we calculate stochastic GWs from first-order PT and DW decay produced after the spontaneous breaking of the $\mathbb{Z}_3$ symmetry at high scales.

A. GWs from first-order PT

For the GWs produced from the first-order PT, we focus on the dominant source, i.e., sound waves created in the plasma. The energy density spectrum is given by [20]¹

$$\begin{aligned} \Omega_{\text{sw}}^2(f) = & 2.65 \times 10^{-6} (H_* \tau_{\text{sw}}) \left(\frac{\beta}{H}\right)^{-1} v_b \left(\frac{\kappa_\nu \alpha}{1 + \alpha}\right)^2 \\ & \times \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \left(\frac{f}{f_{\text{sw}}}\right)^3 \left(\frac{7}{4 + 3(f/f_{\text{sw}})^2}\right)^{7/2}. \end{aligned} \quad (22)$$

¹Reference [68] includes an additional suppression factor to the GW spectrum as a function of the lifetime of the sound waves, which is not important for our calculation.

We include $\tau_{\text{sw}} = \min[\frac{1}{H_*}, \frac{R_*}{U_f}]$ with $H_* R_* = v_b (8\pi)^{1/3} \times (\beta/H)^{-1}$ to consider the duration of the phase transition [69]. The root-mean-square fluid velocity is estimated as [21,70,71]

$$\bar{U}_f^2 \approx \frac{3}{4} \frac{\kappa_\nu \alpha}{1 + \alpha}. \quad (23)$$

The efficiency factor κ_ν describes the latent heat transferred into the kinetic energy [72]. We use bubble wall velocity $v_b = 1$ for this study.²

B. GWs from DW decay

For the DW decay, we consider the gravitational radiation produces in the radiation dominated era. The spectrum of the GWs from the DW decay is estimated to be $\Omega_{\text{GW}}^{\text{dw}} h^2 \propto f^3$ when $f < f_{\text{peak}}$, and $\Omega_{\text{GW}}^{\text{dw}} h^2 \propto f^{-1}$ when $f \geq f_{\text{peak}}$ [44]. The amplitude of GWs at the present time t_0 is [43,44]

$$\Omega_{\text{GW}}^{\text{dw}} h^2 (t_0)_{\text{peak}} \simeq 5.20 \times 10^{-20} \times \tilde{\epsilon}_{\text{gw}} \mathcal{A}^4 \left(\frac{10.75}{g_*} \right)^{1/3} \times \left(\frac{\sigma_{\text{wall}}}{1 \text{ TeV}^3} \right)^4 \left(\frac{1 \text{ MeV}^4}{\Delta V} \right)^2 \quad (24)$$

at the peak frequency, which is given by the Hubble parameter at the decay time [44]:

$$\begin{aligned} f^{\text{dw}}(t_0)_{\text{peak}} &= \frac{a(t_{\text{dec}})}{a(t_0)} H(t_{\text{dec}}) \\ &\simeq 3.99 \times 10^{-9} \text{ Hz} \mathcal{A}^{-1/2} \\ &\times \left(\frac{1 \text{ TeV}^3}{\sigma_{\text{wall}}} \right)^{1/2} \left(\frac{\Delta V}{1 \text{ MeV}^4} \right)^{1/2}, \end{aligned} \quad (25)$$

where the area parameter $\mathcal{A} = 1.2$ for the $\mathbb{Z}_3$ symmetry potential [43]. We take the efficiency parameter $\tilde{\epsilon}_{\text{gw}} = 0.7$ [44] and the degree of freedom $g_* = 10.75$ at the decay time of the domain walls [43]. The bias term ΔV was introduced to make the domain walls decay before they overclose the Universe [77–80], which explicitly breaks the discrete symmetry and may arise from a Planck-suppressed higher dimensional operator [81], a tadpole term [82], or the QCD instanton effect [83,84]. The bias term ΔV here explicitly breaks the $\mathbb{Z}_3$ symmetry and therefore determines the decay time of the DW,

²We note that this velocity is relatively larger than that of the electroweak baryogenesis calculations [4,9,10,73–75]. For the theoretical uncertainties in gravitational wave production from first-order PT, see Ref. [76].

$$t_{\text{dec}} \approx \mathcal{A} \sigma_{\text{wall}} / (\Delta V). \quad (26)$$

To avoid overclosing the Universe, the domain wall decay should fulfill the following relation,

$$\sigma_{\text{wall}} < 2.93 \times 10^4 \text{ TeV}^3 \mathcal{A}^{-1} (0.1 \text{ sec} / t_{\text{dec}}). \quad (27)$$

Further, the lower limit on the magnitude of the bias term can be obtained by requiring the DW decay before the BBN with $t_{\text{dec}} \leq 0.01 \text{ sec}$ [85,86],

$$\Delta V \gtrsim 6.6 \times 10^{-2} \text{ MeV}^4 \mathcal{A} \left(\frac{\sigma_{\text{wall}}}{1 \text{ TeV}^3} \right). \quad (28)$$

Here, the magnitude of the bias term and that of the potential around the core of DWs should satisfy the condition of $\Delta V \ll V$, in order not to affect the PT dynamics. The DW solution [87] and the calculation of the DW tension are as follows. Introducing the phase of the singlet as $S = v_s e^{i\phi}$, we obtain the potential of ϕ ,

$$V = \frac{\mu_S^2}{2} v_s^2 + \frac{\lambda_S}{4} v_s^4 + \frac{\mu_3}{2\sqrt{2}} v_s^3 \cos(3\phi). \quad (29)$$

Then kinetic term of ϕ is

$$\mathcal{L}_{\text{kinetic}}(\phi) = \eta^2 (\partial_\mu \phi) (\partial^\mu \phi), \quad (30)$$

where $\eta^2 = v_s^2/2$. The field equation of

$$\partial_\mu \frac{\partial \mathcal{L}_{\text{kinetic}}}{\partial_\mu (\partial \phi)} + \frac{\partial V}{\partial \phi} = 0 \quad (31)$$

yields

$$\frac{d^2 \phi}{dz^2} - \frac{1}{3B^2} \sin(3\phi) = 0, \quad (32)$$

where we note

$$\frac{1}{B^2} = -\frac{9}{4} \mu_3 v_s^2, \quad \phi = \frac{4}{3} \arctan(e^{\frac{z}{B}}). \quad (33)$$

We then consider a planar DW orthogonal to the z axis [88], i.e., $\phi(z)$, and estimate the DW tension as

$$\begin{aligned} \sigma_{\text{wall}} &= \int dz \rho_{\text{wall}}(z), \\ &= \int \left(\left| \frac{dS}{dz} \right|^2 + V \left(\frac{S(z)}{\sqrt{2}} \right) - V \left(\frac{v_s}{\sqrt{2}} \right) \right) dz. \end{aligned} \quad (34)$$

With the σ_{wall} at hand, the ΔV can be estimated for a peak frequency from Eq. (25).

FIG. 4. Sensitivities of GW detectors on GWs from the first-order PT and the DW decay. Values of DW tensions and bias terms for the three benchmarks are $(\sigma_{\text{wall}}/\text{TeV}^3 = 5.81 \times 10^3, \Delta V/\text{MeV}^4 = 438.0)$ for BM1, $(\sigma_{\text{wall}}/\text{TeV}^3 = 1.66 \times 10^3, \Delta V/\text{MeV}^4 = 125.2)$ for BM2, and $(\sigma_{\text{wall}}/\text{TeV}^3 = 0.16 \times 10^3, \Delta V/\text{MeV}^4 = 12.4)$ for BM3.

C. Numerical results

We explore the GW signal from DW decay after the PT at high scale where the $\mathbb{Z}_3$ symmetry break spontaneously. For the study, we first calculate the bias term ΔV , which is obtained from Eq. (25), with $f^{\text{dw}}(t_0)_{\text{peak}} \approx 10^{-9}$ Hz at the sensitivity frequency of Pulsar Timing Array (PTA) experiments, and then estimate the peak amplitude of the GWs from DW with Eq. (24). Figure 4 depicts that, for the benchmarks under study, the peak frequency of GWs from DW decay is located around nanoHertz and can be probed by future PTA and square kilometer array experiments. The data of NANOGrav 11 yr restrict the phase transition temperature to be $T_n \leq \mathcal{O}(10^4)$ GeV. For the constraint on DW from the NANOGrav 12.5 yr data, we refer to Ref. [42]. Due to large plasma energy at high PT temperatures, amplitude of GWs from the PT for these benchmarks under study (see Table I) is too low to be reached by future space-base interferometers, such as LIGO [89–92], Einstein Telescope [93,94], and Cosmic Explorer [95].

V. CONCLUSION AND DISCUSSIONS

In this work, we study the DM production during the first-order PT and explore GWs from decay of DWs formed after the spontaneously breaking of a $\mathbb{Z}_3$ symmetry that is induced by the PT. Our study shows that the fermionic DM mass can be generated though the first-order PT, we found the so-called filtered DM mechanism cannot be realized during the PT process due to the large Yukawa couplings, which are required to fulfill the decouple condition.

Alternatively in the freeze-in mechanism the heavy fermionic DM [$m_\chi \sim \mathcal{O}(10)$ TeV] can be produced around the temperature of the PT, where the particle mass thresholds and DM production channels are altered via the temperature-dependent masses. The GW density can be a two-peak spectrum, and the PTA experiments with the peak frequency sensitivity around nanoHertz can be used to probe the peak frequency that corresponds to the symmetry breaking scale of the discrete symmetry.

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APPENDIX: DECAY WIDTH AND CROSS SECTIONS

The decay width and annihilation cross sections for the processes shown in Fig. 2 are

$$\Gamma(s \rightarrow \chi \bar{\psi}) = \frac{y_s^2 m_s^2 - (m_\chi + m_\psi)^2}{8\pi m_s^3} \times \sqrt{[m_s^2 - (m_\chi + m_\psi)^2][m_s^2 - (m_\chi - m_\psi)^2]}, \quad (\text{A1})$$

$$\begin{aligned} \sigma(\text{AA} \rightarrow s \rightarrow \chi \bar{\psi}) &= \frac{9\mu_3^2 y_s^2 (E_{\text{cm}}^2 - (m_\chi + m_\psi)^2)}{64\pi E_{\text{cm}}^3 \sqrt{E_{\text{cm}}^2 - 4m_A^2 (E_{\text{cm}}^2 - m_s^2)^2}} \\ &\times \sqrt{-2E_{\text{cm}}^2 (m_\chi^2 + m_\psi^2) + (m_\chi^2 - m_\psi^2)^2 + E_{\text{cm}}^4}, \quad (\text{A2}) \end{aligned}$$

$$\begin{aligned} \sigma(\text{ss} \rightarrow s \rightarrow \chi \bar{\psi}) &= \frac{\mu_3^2 y_s^2 (E_{\text{cm}}^2 - (m_\chi + m_\psi)^2)}{32\pi E_{\text{cm}}^3 \sqrt{E_{\text{cm}}^2 - 4m_s^2 (E_{\text{cm}}^2 - m_s^2)^2}} \\ &\times \sqrt{-2E_{\text{cm}}^2 (m_\chi^2 + m_\psi^2) + (m_\chi^2 - m_\psi^2)^2 + E_{\text{cm}}^4}, \quad (\text{A3}) \end{aligned}$$

$$\begin{aligned}
\sigma(ss \rightarrow \psi \rightarrow \chi\bar{\chi}) = & \frac{y_s^4}{8\pi E_{\text{cm}}^2} \times \left(- \left(\left(2\sqrt{E_{\text{cm}}^2 - 4m_\chi^2} (4m_\chi m_\psi (m_\chi - m_s)(m_s + m_\chi) + 3(m_s^2 - m_\chi^2)^2 + 4m_\chi m_\psi^3 + 3m_\psi^4 \right. \right. \right. \\
& + 2m_\psi^2 (E_{\text{cm}}^2 - 3m_s^2 + m_\chi^2) \Big) \Big) / \left(\sqrt{E_{\text{cm}}^4 - 4m_s^2 (m_\psi^2 (E_{\text{cm}}^2 - 2(m_s^2 + m_\chi^2)) + (m_s^2 - m_\chi^2)^2 + m_\psi^4)) \right) \\
& - \left(\left(\log \left(-\sqrt{E_{\text{cm}}^2 - 4m_s^2} \sqrt{E_{\text{cm}}^2 - 4m_\chi^2} - 2(m_s^2 + m_\chi^2 - m_\psi^2) + E_{\text{cm}}^2 \right) \right. \right. \\
& - \log \left(\sqrt{E_{\text{cm}}^2 - 4m_s^2} \sqrt{E_{\text{cm}}^2 - 4m_\chi^2} - 2(m_s^2 + m_\chi^2 - m_\psi^2) + E_{\text{cm}}^2 \right) \\
& - \log \left(-\sqrt{E_{\text{cm}}^2 - 4m_s^2} \sqrt{E_{\text{cm}}^2 - 4m_\chi^2} + 2(m_s^2 + m_\chi^2 - m_\psi^2) - E_{\text{cm}}^2 \right) \\
& + \log \left(\sqrt{E_{\text{cm}}^2 - 4m_s^2} \sqrt{E_{\text{cm}}^2 - 4m_\chi^2} + 2(m_s^2 + m_\chi^2 - m_\psi^2) - E_{\text{cm}}^2 \right) \\
& \times (E_{\text{cm}}^4 + 2(m_s - m_\chi - m_\psi)(m_s + m_\chi + m_\psi)(3m_s^2 + (m_\chi + m_\psi)(5m_\chi - 3m_\psi)) \\
& + E_{\text{cm}}^2 (8m_\psi(m_\chi + m_\psi) - 4m_s^2)) / ((E_{\text{cm}}^4 - 4m_s^2)(E_{\text{cm}}^4 - 2(m_s^2 + m_\chi^2 - m_\psi^2))) \Big) \Big). \quad (\text{A4})
\end{aligned}$$

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