Gravitational formfactors of the ρ , π , and K mesons in the light-cone QCD sum rules

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By using the quark part of the energy-momentum tensor current, the gravitational formfactors of the ρ , π , and K mesons are calculated within the light-cone sum rules method. In the considered version, the energy-momentum tensor current is not conserved and as a result, for integer spin J , there appear $4J + 2(2J + 1)$ formfactors, which correspond to the conservation (nonconservation) of the energy-momentum tensor current. The average mass radius of ρ , π , and K mesons as well as the gravitational quadrupole moment of the ρ meson are calculated. We also compare our results with the one existing in the literature on these quantities.

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I. INTRODUCTION

During the last years, the energy-momentum tensor (EMT) has become one of the popular research objects for understanding better the structure of hadrons [1,2]. The components of the EMT show how matter couples to the gravitational field. The most fundamental information about a particle is contained in the matrix element of the EMT. The said matrix element is defined in terms of gravitational formfactors (GFFs) (see [2–5]). A useful information about the mechanical properties of hadrons such as mass, spin, total angular momentum, pressure, etc. can be extracted via studying the GFFs. One can obtain some hints about the GFFs from various phenomenological studies of the EMT of particles with different spins. The formalism of GFFs for a spin-1 meson is discussed in [6–8] and for arbitrary spin is generalized in [9]. The model calculations of spin-1 particles are carried out within the AdS/QCD approach [10], the Nambu–Jona-Lazino (NJL) model [11], and the light-cone constituent quark model [12]. The total

GFFs were introduced a long time ago for both spin-0 and spin-1/2 hadrons [1]. The GFFs for spin-1 particles have been discussed in the literature [6,7,10,11,13].

The GFFs of the ρ meson were studied within the light-cone constituent quark model in [12]. It should be emphasized that the study of GFFs for hadrons started with [14] and experimental measurements through deeply virtual Compton scattering [15]. The GFFs were also calculated in chiral perturbation in [16].

The GFFs of the π meson were studied in the chiral limit of a hard-wall AdS/QCD [17], in the NJL model [16], by using the generalized distribution amplitudes [18], in the lattice QCD [19], and in hard electroproduction processes [20]. The GFFs of the K meson were studied in the light-cone quark model [21].

In the present work, we study the GFFs of the ρ , π , and K mesons within the light-cone sum rules (LCSR). By using the results of the GFFs, we also calculate the average mass radius of the said mesons, as well as the gravitational quadrupole moment of the ρ meson (for more details of the definitions of these quantities, see [2]). The LCSR method [22] is a synthesis of the well-known traditional sum rules [23,24] and the theory of hard exclusive processes [25–27]. The LCSR method is powerful in that it not only provides consistently taking into account the hard and soft contributions to exclusive processes but also establishes a bridge to connect the distribution amplitudes (DAs) of hadrons and formfactors. In the LCSR method, the operator product expansion (OPE) is performed over twist rather than

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dimension in traditional sum rules. The main nonperturbative input parameters of the LCSR method are the DAs.

The paper is organized as follows. In Sec. II, we derive the sum rules for the GFFs under study. In Sec. III, we perform the numerical analysis for the sum rules obtained in the previous section. Section IV contains our conclusion.

II. SUM RULES FOR THE ρ , π , AND K MESON GFFS

For the calculation of the GFFs of the mesons under consideration within the light-cone sum rules, we introduce the following correlation function:

$$\Pi_{\mu\nu\lambda}^{V(A)} = i \int d^4x e^{iqx} \langle \mathcal{M}(p) | T \{ T_{\mu\nu}^q(x) J_\lambda^{V(A)}(0) \} | 0 \rangle \quad (1)$$

where $\mathcal{M}$ stands for the meson considered, $J_\lambda^V = \bar{u}\gamma_\lambda d$ is the interpolating current of the ρ meson, $J_\lambda^A = \bar{u}\gamma_\lambda\gamma_5 q$ ($q = d$ or s) is the interpolating current of the π and K mesons, and the $T_{\mu\nu}^q(x)$ is the EMT including only the contribution of quark fields,

$$T_{\mu\nu}^q = \frac{i}{4} \left[\bar{\psi} (\overleftrightarrow{D}_\mu \gamma_\nu + \overleftrightarrow{D}_\nu \gamma_\mu) \psi - g_{\mu\nu} \frac{i}{2} \bar{\psi} (\overleftrightarrow{D} - m_\psi) \psi \right] \quad (2)$$

where $\overleftrightarrow{D}_\mu = \overleftrightarrow{\partial}_\mu \pm igA_\mu^a \frac{\lambda^a}{2}$ and ψ stands for u , d , or s quarks.

In [2], it is obtained that the second term of the EMT can be written as $g_{\mu\nu} \bar{\psi} (\overleftrightarrow{D} - m_\psi) \psi \approx g_{\mu\nu} (1 + \gamma_m) m_\psi \bar{\psi} \psi$ where

γ_m is the anomalous dimension of the mass operator. In the present work, we are working in the chiral limit ($m_\psi \rightarrow 0$), hence the second term of the EMT can be neglected.

We start our consideration by computing the correlation function from the hadronic side. Its representation from the hadronic side is obtained by inserting a complete set of mesons carrying the same quantum numbers as the mesons under consideration and, isolating the contribution of the ground state, we get

$$\Pi_{\mu\nu\lambda}^{V(A)} = \frac{\langle \mathcal{M}(p) | T_{\mu\nu}^q | \mathcal{M}(p') \rangle \langle \mathcal{M}(p') | J_\lambda^{V(A)} | 0 \rangle}{p'^2 - m_\rho^2} + \text{higher states} \quad (3)$$

The matrix elements in Eq. (3) are defined as

$$\langle \rho(p') | J_\lambda^V | 0 \rangle = f_\rho m_\rho \epsilon_\lambda^* \quad (4)$$

for the ρ meson and

$$\langle \mathcal{P}(p') | J_\lambda^A | 0 \rangle = f_\mathcal{P} p'_\lambda \quad (5)$$

for the pion and kaon, where f_ρ is the ρ meson decay constant, m_ρ is its mass, and ϵ_λ is its polarization vector, and $f_\mathcal{P}$ is the decay constant of the pseudoscalar meson $\mathcal{P}$ and p'_λ is its 4-momentum.

The EMT of a spin-1 particle in QCD is defined as (see, for example, [8,11])

$$\begin{aligned} \langle \rho(p) | T_{\mu\nu}^q | \rho(p') \rangle &= 2P_\mu P_\nu \left(-\epsilon'^* \cdot \epsilon A_0(q^2) + \frac{\epsilon'^* \cdot P \epsilon \cdot P}{m_\rho^2} A_1(q^2) \right) \\ &+ 2[P_\mu (\epsilon'_\nu \epsilon \cdot P + \epsilon_\nu \epsilon'^* \cdot P) + P_\nu (\epsilon'_\mu \epsilon \cdot P + \epsilon_\mu \epsilon'^* \cdot P)] J(q^2) \\ &+ \frac{1}{2} (q_\mu q_\nu - g_{\mu\nu} q^2) \left(\epsilon'^* \cdot \epsilon D_0(q^2) + \frac{\epsilon'^* \cdot P \epsilon \cdot P}{m_\rho^2} D_1(q^2) \right) \\ &+ \left[\frac{1}{2} (\epsilon'_\mu \epsilon'_\nu + \epsilon'_\mu \epsilon_\nu + \epsilon'_\nu \epsilon_\mu) q^2 + (\epsilon'_\mu q_\nu + \epsilon'_\nu q_\mu) \epsilon \cdot P - (\epsilon_\mu q_\nu + \epsilon_\nu q_\mu) \epsilon'^* \cdot P - 4g_{\mu\nu} \epsilon'^* \cdot P \epsilon \cdot P \right] E(q^2) \\ &+ \left(\epsilon'_\mu \epsilon'_\nu + \epsilon'_\mu \epsilon_\nu - \frac{1}{2} \epsilon'^* \cdot \epsilon g_{\mu\nu} \right) m_\rho^2 F(q^2) + g_{\mu\nu} (\epsilon'^* \cdot \epsilon m_\rho^2 C_0(q^2) + \epsilon'^* \cdot P \epsilon \cdot P C_1(q^2)) \end{aligned} \quad (6)$$

where $P = \frac{1}{2}(p + p')$ and $q = p' - p$. Here, the superscript q indicates that the considered GFF contains contributions only from the quark part of the EMT.

In Eq. (6), the first six formfactors are individually (separately for quark and gluon fields) energy-momentum conserving and the remaining three are not. Due to the sum of quark and gluon parts, the EMT conservation leads to the constraints $\sum_{a=q,g} F^a(Q^2) = \sum_{a=q,g} C_0^a = \sum_{a=q,g} C_1^a = 0$.

In addition to these constraints, there are the normalization conditions $\sum_{a=q,g} A_0^a(Q^2) = 1$ and $\sum_{a=q,g} J^a(Q^2) = 1$. It should be noted that the formfactor A_0 is related to the energy, J to the spin distribution, and the combinations of D_0 , E , and D_1 to the pressure and shear function.

Since in Eq. (6) we have nine formfactors, we need nine independent Lorentz structures for the determination of these formfactors. We choose the following structures:

$$\begin{aligned}\Pi_{\mu\nu\lambda}^V &= \epsilon'^*_{\lambda} p_{\mu} q_{\nu} \Pi_1^V + p_{\mu} g_{\mu\nu} \epsilon'^*_{\lambda} \cdot q \Pi_2^V + \epsilon'^*_{\mu} p_{\lambda} q_{\nu} \Pi_3^V \\ &+ \epsilon'^*_{\lambda} q_{\mu} q_{\nu} \Pi_4^V + p_{\nu} p_{\lambda} q_{\mu} \epsilon'^*_{\lambda} \cdot q \Pi_5^V + q_{\mu} q_{\nu} p_{\lambda} \epsilon'^*_{\lambda} \cdot q \Pi_6^V \\ &+ \epsilon'^*_{\mu} g_{\mu\nu} \Pi_7^V + p_{\lambda} g_{\mu\nu} \epsilon'^*_{\lambda} \cdot q \Pi_8^V + q_{\mu} g_{\nu\lambda} \epsilon'^*_{\lambda} \cdot q \Pi_9^V \\ &+ \dots\end{aligned}\quad (7)$$

The most general form of the matrix element of the EMT between spinless particles can be written in the following way:

$$\begin{aligned}\langle \mathcal{P}(p) | T_{\mu\nu}^q | \mathcal{P}(p') \rangle &= 2P_{\mu} P_{\nu} F_0^q(q^2) \\ &+ \frac{1}{2} (q_{\mu} q_{\nu} - q^2 g_{\mu\nu}) F_1^q(q^2) \\ &+ 2m_{\mathcal{P}}^2 F_2^q(q^2) g_{\mu\nu}\end{aligned}\quad (8)$$

where P and q are as defined earlier. The first two formfactors are individually EMT-conserving and the last one is nonconserving. The conservation of the EMT leads to the following constraints:

$$\sum_{a=q,g} F_2^a(q^2) = 0 \quad (9)$$

$$\sum_{a=q,g} F_0^a(q^2) = 1 \quad (10)$$

Another constraint follows from the low-energy pion theorem [20,28–30]:

$$\lim_{m_{\pi} \rightarrow 0} \sum_{a=q,g} F_1^a(q^2) = -1 \quad (11)$$

Using Eqs. (5) and (8), from Eq. (3) for the phenomenological part of the correlation function, we get

$$\Pi_{\mu\nu\lambda}^A = P_{\mu} P_{\nu} P_{\lambda} \Pi_1^A + q_{\mu} q_{\nu} P_{\lambda} \Pi_2^A + \dots \quad (12)$$

Now, we calculate the correlation function from the QCD side. Using the explicit forms of the interpolating currents J_{λ}^A and $T_{\mu\nu}^q$ and by using the Wick theorem for the theoretical part of the correlation function, we get

$$\begin{aligned}\Pi_{\mu\nu\lambda}^V &= -\frac{1}{4} \int d^4x e^{iqx} \langle \mathcal{M}(p') | [\bar{u}(x) \gamma_{\mu} \overleftrightarrow{D}_{\nu} S(x) \gamma_{\lambda} q(0) \\ &+ \bar{u}(0) \gamma_{\lambda} \overleftrightarrow{D}_{\mu} S(x) \gamma_{\nu} q(x) + \bar{u}(x) \gamma_{\nu} \overleftrightarrow{D}_{\mu} S(x) \gamma_{\lambda} q(0) \\ &+ \bar{u}(0) \gamma_{\lambda} \overleftrightarrow{D}_{\nu} S(x) \gamma_{\mu} q(x)] | 0 \rangle\end{aligned}\quad (13)$$

and $\Pi_{\mu\nu\lambda}^A$ can be obtained by letting $\gamma_{\lambda} \rightarrow \gamma_{\lambda} \gamma_5$. From this formula, it follows that for the calculation of the correlation function from the QCD side, the expression of the quark propagator in the presence of a background field is needed. This expression was obtained in [22] as

$$\begin{aligned}S_q(x) &= \frac{i \not{x}}{2\pi^2 x^4} - \frac{ig_s}{16\pi^2 x^2} \int_0^1 du [\bar{u} \sigma_{\alpha\beta} \not{x} + \not{x} \sigma_{\alpha\beta} u] G^{\alpha\beta} \\ &- \frac{ie_q}{16\pi^2 x^2} \int_0^1 du [\bar{u} \sigma_{\alpha\beta} \not{x} + \not{x} \sigma_{\alpha\beta}] F^{\alpha\beta}\end{aligned}\quad (14)$$

where $G^{\alpha\beta}$ and $F^{\alpha\beta}$ are the gluon and photon field strength tensors, respectively. Performing relevant calculations for the correlation function from QCD, we get

$$\begin{aligned}\Pi_{\mu\nu\lambda}^{V(A)} &= -\frac{1}{4} \left\{ \frac{i}{2\pi^2} \sum_i \int du \left[\frac{1}{4} \langle \mathcal{M} | \bar{u} \Gamma_i d | 0 \rangle \text{tr} \left\{ \Gamma_i \gamma_{\nu} \left(\frac{\gamma_{\mu}}{x^4} - \frac{4x_{\mu} \not{x}}{x^6} \right) (\gamma_{\lambda} \gamma_5) \right\} + \frac{1}{4} \langle \mathcal{M} | \bar{u} \Gamma_i d | 0 \rangle \right. \right. \\ &\times \text{tr} \left\{ \Gamma_i (\gamma_{\lambda} \gamma_5) \left(\frac{\gamma_{\mu}}{x^4} - \frac{4x_{\mu} \not{x}}{x^6} \right) \gamma_{\nu} \right\} + (\mu \leftrightarrow \nu) \Big] \\ &- \frac{ig_s}{16\pi^2} \sum_i \int du \left[\frac{1}{4} \langle \mathcal{M} | \bar{u} \Gamma_i G^{\alpha\beta} q | 0 \rangle \text{tr} \left\{ \Gamma_i (\gamma_{\lambda} \gamma_5) \left(\left(\frac{\gamma_{\mu}}{x^2} - \frac{2x_{\mu} \not{x}}{x^4} \right) \bar{u} \sigma_{\alpha\beta} \right. \right. \right. \\ &+ u \sigma_{\alpha\beta} \left(\frac{\gamma_{\mu}}{x^2} - \frac{2x_{\mu} \not{x}}{x^4} \right) \gamma_{\nu} \Big\} + \frac{1}{4} \langle \mathcal{M} | \bar{u} \Gamma_i G^{\alpha\beta} q | 0 \rangle \\ &\times \text{tr} \left\{ \Gamma_i \gamma_{\nu} \left(\bar{u} \left(\frac{\gamma_{\mu}}{x^2} - \frac{2x_{\mu} \not{x}}{x^4} \right) \sigma_{\alpha\beta} + u \sigma_{\alpha\beta} \left(\frac{\gamma_{\mu}}{x^2} - \frac{2x_{\mu} \not{x}}{x^4} \right) \right) (\gamma_{\lambda} \gamma_5) \right\} + (\mu \leftrightarrow \nu) \Big] \Big\}.\end{aligned}\quad (15)$$

In these expressions, $\Gamma_i = \{1, \gamma_5, \gamma_{\mu}, i\gamma_{\mu} \gamma_5, \frac{1}{\sqrt{2}} \sigma_{\mu\nu}\}$ is the full set of Dirac matrices. The matrix elements $\langle \mathcal{M} | \bar{u} \Gamma_i d | 0 \rangle$ and $\langle \mathcal{M} | \bar{u} \Gamma_i G_{\alpha\beta} d | 0 \rangle$ are expressed in terms of the meson distribution amplitudes (DAs) of different twists. These DAs are the main nonperturbative parameters of the light-cone sum rules. The relevant matrix elements are (see [31–36])

$$\begin{aligned}\langle \rho(p) | \bar{u}(x) \gamma_{\mu} d(0) | 0 \rangle &= f_{\rho} m_{\rho} \left\{ \frac{\epsilon'^* \cdot x}{p \cdot x} p_{\mu} \int_0^1 du e^{i\bar{u} p x} \left[\phi_{\parallel}(u) + \frac{m_{\rho}^2 x^2}{16} A_{\parallel}(u) \right] + \left(\epsilon'^*_{\mu} - p_{\mu} \frac{\epsilon'^* \cdot x}{p \cdot x} \int_0^1 du e^{i\bar{u} p x} g_{\perp}^v(u) \right) \right. \\ &- \left. \frac{1}{2} x_{\mu} \frac{\epsilon'^* \cdot x}{(p \cdot x)^2} m_{\rho}^2 \int_0^1 du e^{i\bar{u} p x} [g_3(u) + \phi_{\parallel}(u) - 2g_{\perp}^v(u)] \right\}\end{aligned}\quad (16)$$

$$\langle \rho(p) | \bar{u}(x) i \gamma_\mu \gamma_5 d(0) | 0 \rangle = -\frac{i}{4} \epsilon_{\mu\nu\alpha\beta} \epsilon'^{\nu} p^\alpha x^\beta f_\rho m_\rho \int_0^1 du e^{i\bar{u}p \cdot x} g_\perp^a(u) \quad (17)$$

$$\langle \rho(p) | \bar{u}(x) G_{\alpha\beta} \gamma_\mu d(0) | 0 \rangle = \frac{f_\rho m_\rho}{i g_s} p_\mu (\epsilon'_\alpha{}^* p_\beta - \epsilon'_\beta{}^* p_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_1 + \bar{u}\alpha_3)p \cdot x} \mathcal{V}(\alpha_i) \quad (18)$$

$$\langle \rho(p) | \bar{u}(x) \tilde{G}_{\alpha\beta} i \gamma_\mu \gamma_5 d(0) | 0 \rangle = \frac{i f_\rho m_\rho}{g_s} p_\mu (\epsilon'_\alpha{}^* p_\beta - \epsilon'_\beta{}^* p_\alpha) \int \mathcal{D}\alpha_i e^{i(\alpha_1 + \bar{u}\alpha_3)p \cdot x} \mathcal{A}(\alpha_i) \quad (19)$$

and

$$\langle \mathcal{P}(p) | \bar{u}(x) \gamma_5 d(0) | 0 \rangle = -i \mu_{\mathcal{P}} \int_0^1 du e^{i\bar{u}p \cdot x} \phi_{\mathcal{P}}(u) \quad (20)$$

$$\begin{aligned} \langle \mathcal{P}(p) | \bar{u}(x) \gamma_\phi \gamma_5 d(0) | 0 \rangle &= -i f_{\mathcal{P}} p_\phi \int_0^1 du e^{i\bar{u}p \cdot x} \left[\phi_{\mathcal{P}}(u) + \frac{m_{\mathcal{P}}^2 x^2}{16} \hat{A}(u) \right] \\ &\quad - \frac{i}{2} f_{\mathcal{P}} m_{\mathcal{P}}^2 \frac{x_\mu}{p \cdot x} \int_0^1 du e^{i\bar{u}p \cdot x} \hat{B}(u) \end{aligned} \quad (21)$$

$$\langle \mathcal{P}(p) | \bar{u}(x) \sigma_{\xi h} \gamma_5 d(0) | 0 \rangle = \frac{i}{6} \mu_{\mathcal{P}} (1 - \tilde{\mu}_{\mathcal{P}}^2) (p_\xi x_h - p_h x_\xi) \int_0^1 du e^{i\bar{u}p \cdot x} \phi_\sigma(u) \quad (22)$$

$$\begin{aligned} \langle \mathcal{P}(p) | \bar{u}(x) \gamma_\mu g_s G_{\alpha\beta}(ux) d(0) | 0 \rangle &= i p_\mu (p_\alpha x_\beta - p_\beta x_\alpha) \frac{1}{p \cdot x} f_{\mathcal{P}} m_{\mathcal{P}}^2 \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)p \cdot x} \mathcal{V}_\parallel(\alpha_i) \\ &\quad + i \left\{ p_\beta \left[g_{\mu\alpha} - \frac{1}{p \cdot x} (p_\mu x_\alpha + p_\alpha x_\mu) \right] \right. \\ &\quad \left. - p_\alpha \left[g_{\mu\beta} - \frac{1}{p \cdot x} (p_\mu x_\beta + p_\beta x_\mu) \right] \right\} \\ &\quad \times f_{\mathcal{P}} m_{\mathcal{P}}^2 \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)q \cdot x} \mathcal{V}_\perp(\alpha_i) \end{aligned} \quad (23)$$

$$\begin{aligned} \langle \mathcal{P}(p) | \bar{u}(x) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(ux) d(0) | 0 \rangle &= p_\mu (p_\alpha x_\beta - p_\beta x_\alpha) \frac{1}{p \cdot x} f_{\mathcal{P}} m_{\mathcal{P}}^2 \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)p \cdot x} \mathcal{A}_\parallel(\alpha_i) \\ &\quad + \left\{ p_\beta \left[g_{\mu\alpha} - \frac{1}{p \cdot x} (p_\mu x_\alpha + p_\alpha x_\mu) \right] \right. \\ &\quad \left. - p_\alpha \left[g_{\mu\beta} - \frac{1}{p \cdot x} (p_\mu x_\beta + p_\beta x_\mu) \right] \right\} \\ &\quad \times f_{\mathcal{P}} m_{\mathcal{P}}^2 \int \mathcal{D}\alpha_i e^{i(\alpha_1 + u\alpha_3)p \cdot x} \mathcal{A}_\perp(\alpha_i) \end{aligned} \quad (24)$$

$$\begin{aligned} \langle \mathcal{P}(p) | \bar{u}(x) \sigma_{\xi h} \gamma_5 g_s G_{\lambda\tau}(ux) d(0) | 0 \rangle &= i \mu_{\mathcal{P}} \left\{ p_\lambda p_\xi \left[g_{h\tau} - \frac{1}{p \cdot x} (p_h x_\tau + p_\tau x_h) \right] - p_\lambda p_h \left[g_{\xi\tau} - \frac{1}{p \cdot x} (p_\xi x_\tau + p_\tau x_\xi) \right] \right. \\ &\quad \left. - p_\tau p_\xi \left[g_{h\lambda} - \frac{1}{p \cdot x} (p_h x_\lambda + p_\lambda x_h) \right] + p_\tau p_h \left[g_{\xi\lambda} - \frac{1}{p \cdot x} (p_\xi x_\lambda + p_\lambda x_\xi) \right] \right\} \\ &\quad \times \int \mathcal{D}\alpha_i e^{i[p + (\alpha_1 + u\alpha_3)q] \cdot x} \mathcal{T}(\alpha_i) \end{aligned} \quad (25)$$

where we have defined

$$\mu_{\mathcal{P}} := \frac{f_{\mathcal{P}} m_{\mathcal{P}}^2}{m_u + m_q}, \quad \tilde{\mu}_{\mathcal{P}} := \frac{m_u + m_q}{m_{\mathcal{P}}} \quad (26)$$

and $\tilde{G}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}G^{\mu\nu}$ is the dual gluon field strength tensor and $\int \mathcal{D}\alpha_i = \int d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. Using the DAs and performing Fourier transformations and Borel transformation with respect to the variables $-p'^2$ on the theoretical part of the correlation function, we get

$$\begin{aligned} \Pi_1^V = & \frac{1}{8}f_\rho m_\rho (\mathcal{I}_1[g_\perp^a(u) + (g_\perp^{a'}(u) + 4g_\perp^v(u))(-1+u), 1] + \mathcal{I}_3[g_\perp^a(u) + g_\perp^{a'}(u)u - 4g_\perp^v(u)u, 1] \\ & + 4m_\rho^2(2\mathcal{I}_1[(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))(-1+u), 2] - 2\mathcal{I}_3[(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))u, 2] \\ & + \mathcal{I}_5[(\alpha_1 + \alpha_3 - \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2] - \mathcal{I}_6[(\alpha_1 + \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2])) \end{aligned} \quad (27)$$

$$\begin{aligned} \Pi_2^V = & \frac{1}{8}f_\rho m_\rho (4(\mathcal{I}_1[\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u) + \phi_\parallel(u) - \phi_\parallel(u)u, 1] + \mathcal{I}_3[-\hat{g}_\perp^v(u) + \hat{\phi}_\parallel(u) + \phi_\parallel(u)u, 1]) \\ & + m_\rho^2(-\mathcal{I}_1[\hat{A}_\parallel(u) + 2A_\parallel(u)(-1+u) + 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))(-1+u), 2] \\ & + \mathcal{I}_3[\hat{A}_\parallel(u) + 2A_\parallel(u)u + 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))u, 2] + 4(\mathcal{I}_5[(\alpha_1 + \alpha_3 - \alpha_3 u)(\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2] \\ & + \mathcal{I}_6[(\alpha_1 + \alpha_3 u)(\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2])) \end{aligned} \quad (28)$$

$$\begin{aligned} \Pi_3^V = & \frac{1}{4}f_\rho m_\rho (2\mathcal{I}_1[\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u), 1] + 2\mathcal{I}_3[-\hat{g}_\perp^v(u) + \hat{\phi}_\parallel(u), 1] \\ & + m_\rho^2(-\mathcal{I}_1[\hat{A}_\parallel(u) - 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))(-1+u), 2] + \mathcal{I}_3[\hat{A}_\parallel(u) - 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))u, 2])) \end{aligned} \quad (29)$$

$$\begin{aligned} \Pi_4^V = & \frac{1}{8}f_\rho m_\rho (-\mathcal{I}_1[g_\perp^{a'}(u) + 4g_\perp^v(u), 1] + \mathcal{I}_3[g_\perp^{a'}(u) - 4g_\perp^v(u), 1] + 4m_\rho^2(-2\mathcal{I}_1[-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u), 2] \\ & - 2\mathcal{I}_3[-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u), 2] + \mathcal{I}_5[-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u), 2] + \mathcal{I}_6[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{A}(\alpha_i)u, 2])) \end{aligned} \quad (30)$$

$$\begin{aligned} \Pi_5^V = & f_\rho m_\rho (2\mathcal{I}_1[(\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u))(-1+u), 2] + 2\mathcal{I}_3[(\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u))u, 2] \\ & + 2m_\rho^2(\mathcal{I}_1[(\hat{A}_\parallel(u) - 2(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))(-1+u))(1-u), 3] \\ & + \mathcal{I}_3[u(-\hat{A}_\parallel(u) + 2(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))u), 3]) - \mathcal{I}_5[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u, 2] \\ & - \mathcal{I}_6[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u, 2])) \end{aligned} \quad (31)$$

$$\begin{aligned} \Pi_6^V = & \frac{1}{4}f_\rho m_\rho (-\mathcal{I}_1[g_\perp^a(u) + 4\hat{g}_\perp^v(u) - 4\hat{\phi}_\parallel(u), 2] - \mathcal{I}_3[g_\perp^a(u) - 4\hat{g}_\perp^v(u) + 4\hat{\phi}_\parallel(u), 2] \\ & + 4m_\rho^2(\mathcal{I}_1[\hat{A}_\parallel(u) - 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))(-1+u), 3] - \mathcal{I}_3[\hat{A}_\parallel(u) - 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))u, 3])) \end{aligned} \quad (32)$$

$$\begin{aligned} \Pi_7^V = & -\frac{1}{8}f_\rho m_\rho (m_\rho^2(\mathcal{I}_1[A_\parallel(u) + 2(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u)), 1] + \mathcal{I}_3[A_\parallel(u) + 2(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u)), 1]) \\ & - 2(\mathcal{I}_2[-g_\perp^v(u) + \phi_\parallel(u)] + \mathcal{I}_4[-g_\perp^v(u) + \phi_\parallel(u)])) \end{aligned} \quad (33)$$

$$\begin{aligned} \Pi_8^V = & \frac{1}{8}f_\rho m_\rho (\mathcal{I}_1[g_\perp^a(u), 1] + \mathcal{I}_3[g_\perp^a(u), 1] + m_\rho^2(-\mathcal{I}_1[\hat{A}_\parallel(u) - 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))(-1+u), 2] \\ & + \mathcal{I}_3[\hat{A}_\parallel(u) - 4(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u))u, 2])) \end{aligned} \quad (34)$$

$$\begin{aligned} \Pi_9^V = & \frac{1}{4}f_\rho m_\rho (2(\mathcal{I}_1[\phi_\parallel(u), 1] + \mathcal{I}_3[\phi_\parallel(u), 1]) + m_\rho^2(\mathcal{I}_1[A_\parallel(u) + 2(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u)), 2] \\ & + \mathcal{I}_3[A_\parallel(u) + 2(-2\hat{g}_\perp^v(u) + \hat{g}_3(u) + \hat{\phi}_\parallel(u)), 2] + 2(\mathcal{I}_5[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u, 2] \\ & + \mathcal{I}_6[\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u, 2])) \end{aligned} \quad (35)$$

and

$$\Pi_1^A = f_{\mathcal{P}}(\mathcal{I}_1[\bar{u}^2 \phi_{\mathcal{P}}(u), 1] + \mathcal{I}_3[u^2 \phi_{\mathcal{P}}(u), 1]) \quad (36)$$

$$\Pi_2^A = \frac{1}{2} f_{\mathcal{P}}(\mathcal{I}_1[u(1+u) \phi_{\mathcal{P}}(u), 1] + \mathcal{I}_3[\bar{u}(1+\bar{u}) \phi_{\mathcal{P}}(u), 1]) \quad (37)$$

where the hat denotes integration, for example, as $\hat{f}(u) = \int_0^u dv f(v)$, and the functions $\mathcal{I}_i[f(u), n]$, $\mathcal{I}_i[f(u)]$, and $\mathcal{I}_k[f(u) \mathcal{F}(\alpha_i), n]$ are defined as

$$\begin{aligned} \mathcal{I}_1[f(u), n] &= (-1)^n \int_0^{u_{10}} du \frac{F_{1n}(u)}{(n-1)!(M^2)^{n-1}} e^{-s_1(u)/M^2} \\ &\quad - \left[\frac{(-1)^{n-1}}{(n-1)!} e^{-s_1(u)/M^2} \sum_{\ell=1}^{n-1} \frac{1}{(M^2)^{n-\ell-1}} \frac{1}{s'_1(u)} \left(\frac{d}{du} \frac{1}{s'_1(u)} \right)^{\ell-1} F_{1n}(u) \right]_{u=u_{10}} \end{aligned} \quad (38)$$

$$\mathcal{I}_2[f(u)] = - \int_0^{u_{20}} du f(u) M^2 e^{-s_2(u)/M^2} \quad (39)$$

$$\begin{aligned} \mathcal{I}_3[f(u), n] &= (-1)^n \int_0^{u_{30}} du \frac{F_{3n}(u)}{(n-1)!(M^2)^{n-1}} e^{-s_3(u)/M^2} \\ &\quad - \left[\frac{(-1)^{n-1}}{(n-1)!} e^{-s_3(u)/M^2} \sum_{\ell=1}^{n-1} \frac{1}{(M^2)^{n-\ell-1}} \frac{1}{s'_3(u)} \left(\frac{d}{du} \frac{1}{s'_3(u)} \right)^{\ell-1} F_{3n}(u) \right]_{u=u_{30}} \end{aligned} \quad (40)$$

$$\mathcal{I}_4[f(u)] = - \int_0^{u_{40}} du f(u) M^2 e^{-s_4(u)/M^2} \quad (41)$$

$$\mathcal{I}_5[f(u) \mathcal{F}(\alpha_i), n] = (-1)^n \int_0^1 du \int_0^{y_0} d\alpha_1 \int_{(y_0-\alpha_1)/\bar{u}}^{1-\alpha_1} d\alpha_3 \frac{f(u) \tilde{\mathcal{F}}_5(\alpha_i)}{(n-1)!(M^2)^{n-1}} e^{-s_5(u)/M^2} \quad (42)$$

$$\mathcal{I}_6[f(u) \mathcal{F}(\alpha_i), n] = (-1)^n \int_0^1 du \int_0^{y_0} d\alpha_1 \int_{(y_0-\alpha_1)/u}^{1-\alpha_1} d\alpha_3 \frac{f(u) \tilde{\mathcal{F}}_6(\alpha_i)}{(n-1)!(M^2)^{n-1}} e^{-s_6(u)/M^2} \quad (43)$$

with

$$s_1(u) = s_2(u) = m_{\mathcal{M}}^2 u - \frac{u}{\bar{u}} q^2 \quad (44)$$

$$s_3(u) = s_4(u) = m_{\mathcal{M}}^2 \bar{u} - \frac{\bar{u}}{u} q^2 \quad (45)$$

$$s_5(u) = s_6(u) = m_{\mathcal{M}}^2 \bar{y} - \frac{\bar{y}}{y} q^2 \quad (46)$$

and

$$F_{1n}(u) = \frac{f(u)}{\bar{u}^n} \quad (47)$$

$$F_{3n}(u) = \frac{f(u)}{u^n} \quad (48)$$

$$\tilde{\mathcal{F}}_i(\alpha_i) = \frac{\mathcal{F}(\alpha_i)}{y_i^n}, \quad i = 5, 6 \quad (49)$$

and

$$y_5 = \alpha_1 + \bar{u} \alpha_3 \quad (50)$$

$$y_6 = \alpha_1 + u \alpha_3 \quad (51)$$

$u_{10} = u_{20}$ is a solution of the equation $m_{\mathcal{M}}^2 u - u q^2 / \bar{u} = s_0$ and $u_{30} = u_{40}$ is a solution of the equation $m_{\mathcal{M}}^2 \bar{u} - \bar{u} q^2 / u = s_0$, and finally, y_0 is a solution of the equation $m_{\mathcal{M}}^2 \bar{y} - \bar{y} q^2 / y = s_0$. Note that in Eqs. (38) and (40), the contributions of the surface terms in the leading-twist amplitudes are taken into account.

Defining $Q^2 = -q^2$, equating the corresponding coefficients of the correlation function from the QCD and hadronic parts (after a Borel transformation over $-p^2$), and solving these equations for the nine formfactors of the ρ meson, we get

$$\begin{aligned}
A_0^q(Q^2) = & \frac{1}{8} e^{m_\rho^2/M^2} (\mathcal{I}_1[g_\perp^a(u) + (g_\perp^{a'}(u) + 4g_\perp^v(u))(-1+u), 1] + \mathcal{I}_3[g_\perp^a(u) + g_\perp^{a'}(u)u - 4g_\perp^v(u)u, 1] \\
& + 4m_\rho^2(2\mathcal{I}_1[(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-1+u), 2] - 2\mathcal{I}_3[(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))u, 2] \\
& + \mathcal{I}_5[(\alpha_1 + \alpha_3 - \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2] - \mathcal{I}_6[(\alpha_1 + \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2]))
\end{aligned} \tag{52}$$

$$\begin{aligned}
A_1^q(Q^2) = & \frac{1}{Q^2} e^{m_\rho^2/M^2} m_\rho^2 (\mathcal{I}_1[g_\perp^a(u) + 4\hat{g}_\perp^v(u) - 4(\hat{\phi}_\parallel(u) + \phi_\parallel(u)(-1+u)) + g_\perp^{a'}(u)(-1+u) + 4g_\perp^v(u)(-1+u), 1] \\
& + \mathcal{I}_3[g_\perp^a(u) - 4\hat{g}_\perp^v(u) + 4\hat{\phi}_\parallel(u) + (g_\perp^{a'}(u) - 4g_\perp^v(u) + 4\phi_\parallel(u))u, 1] \\
& + m_\rho^2(-\mathcal{I}_1[\hat{A}_\parallel(u) + 2A_\parallel(u)(-1+u) + 4(2\hat{g}_\perp^v(u) - g_3(u) - \hat{\phi}_\parallel(u) + 2\hat{g}_\perp^v(u)(-1+u) \\
& - 2\hat{\phi}_\parallel(u)(-1+u))(-1+u), 2] + 8m_\rho^2(\mathcal{I}_1[(\hat{A}_\parallel(u) - 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-1+u))(-1+u)^2, 3] \\
& + \mathcal{I}_3[u^2(-\hat{A}_\parallel(u) + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))u), 3]) \\
& + \mathcal{I}_3[\hat{A}_\parallel(u) + 2u(A_\parallel(u) + 4\hat{g}_\perp^v(u)u - 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u) + 2\hat{\phi}_\parallel(u)u)), 2] \\
& + 4\mathcal{I}_5[(\alpha_1 + \alpha_3 - \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2] - 4\mathcal{I}_6[(\alpha_1 + \alpha_3 u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2]))
\end{aligned} \tag{53}$$

$$\begin{aligned}
C_0^q(Q^2) = & \frac{1}{32m_\rho^2} e^{m_\rho^2/M^2} (2(\mathcal{I}_1[A_\parallel(u)m_\rho^2 - 8g_\perp^v(u)m_\rho^2 + 8\hat{g}_\perp^v(u)m_\rho^2 + 4\hat{g}_\perp^v(u)m_\rho^2 - 2g_3(u)m_\rho^2 - 2\hat{\phi}_\parallel(u)m_\rho^2 \\
& - 8\hat{\phi}_\parallel(u)m_\rho^2 + g_\perp^a(u)(2m_\rho^2 - Q^2) - 4g_\perp^v(u)Q^2 - 2\hat{g}_\perp^v(u)Q^2 + 2\hat{\phi}_\parallel(u)Q^2 - 2\phi_\parallel(u)(4m_\rho^2 - Q^2)(-1+u) \\
& + 8g_\perp^v(u)m_\rho^2u - 4g_\perp^v(u)Q^2u + g_\perp^{a'}(u)(2m_\rho^2(-1+u) - Q^2(1+u)), 1] + \mathcal{I}_2[g_\perp^{a'}(u) + 2g_\perp^v(u) - 2\phi_\parallel(u)] \\
& + \mathcal{I}_3[A_\parallel(u)m_\rho^2 + g_\perp^a(u)(2m_\rho^2 - Q^2) + g_\perp^{a'}(u)(2Q^2 + 2m_\rho^2u - Q^2u) - 2(-2\hat{g}_\perp^v(u)m_\rho^2 + g_3(u)m_\rho^2 + \hat{\phi}_\parallel(u)m_\rho^2 \\
& - 4\hat{\phi}_\parallel(u)m_\rho^2 + \hat{g}_\perp^v(u)(4m_\rho^2 - Q^2) + \hat{\phi}_\parallel(u)Q^2 - 4m_\rho^2\phi_\parallel(u)u + \phi_\parallel(u)Q^2u + g_\perp^v(u)(-2Q^2(-2+u) + 4m_\rho^2u)), 1] \\
& + \mathcal{I}_4[-g_\perp^{a'}(u) + 2g_\perp^v(u) - 2\phi_\parallel(u)]) + m_\rho^2(-\mathcal{I}_1[4\hat{A}_\parallel(u)m_\rho^2 - \hat{A}_\parallel(u)Q^2 - 64\hat{g}_\perp^v(u)Q^2 + 32g_3(u)Q^2 + 32\hat{\phi}_\parallel(u)Q^2 \\
& + 8A_\parallel(u)m_\rho^2(-1+u) + 32\hat{g}_\perp^v(u)m_\rho^2(-1+u) - 16g_3(u)m_\rho^2(-1+u) - 16\hat{\phi}_\parallel(u)m_\rho^2(-1+u) \\
& - 2A_\parallel(u)Q^2(-1+u) - 24\hat{g}_\perp^v(u)Q^2(-1+u) + 12g_3(u)Q^2(-1+u) + 12\hat{\phi}_\parallel(u)Q^2(-1+u) \\
& + 32\hat{g}_\perp^v(u)m_\rho^2(-1+u)^2 - 32\hat{\phi}_\parallel(u)m_\rho^2(-1+u)^2 + 16\hat{g}_\perp^v(u)Q^2(-1+u)^2 - 16\hat{\phi}_\parallel(u)Q^2(-1+u)^2, 2] \\
& + 16m_\rho^2(-\mathcal{I}_1[(1-u)(\hat{A}_\parallel(u)(2m_\rho^2 + Q^2)(-1+u) - 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(2m_\rho^2(-1+u)^2 \\
& + Q^2(-1+(-2+u)u))), 3] \\
& + \mathcal{I}_3[u(-\hat{A}_\parallel(u)(2m_\rho^2 + Q^2)u + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-2Q^2 + (2m_\rho^2 + Q^2)u^2)), 3]) \\
& + \mathcal{I}_3[\hat{A}_\parallel(u)(4m_\rho^2 - Q^2) + A_\parallel(u)(8m_\rho^2u - 2Q^2u) + 4(4\hat{g}_\perp^v(u)(2m_\rho^2 + Q^2)u^2 - 4\hat{\phi}_\parallel(u)(2m_\rho^2 + Q^2)u^2 \\
& + (-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-4m_\rho^2u + Q^2(-8+3u))), 2] \\
& + 4\mathcal{I}_5[-\mathcal{V}(\alpha_i)(4Q^2 + (\alpha_1 + \alpha_3 - \alpha_3 u)(4m_\rho^2 + Q^2 - 6Q^2u)) + \mathcal{A}(\alpha_i)(-4Q^2 + 8Q^2u - \alpha_1(Q^2 + m_\rho^2(4-8u) \\
& + 4Q^2u) + \alpha_3(-1+u)(Q^2 + m_\rho^2(4-8u) + 4Q^2u)), 2] - 4\mathcal{I}_6[\mathcal{A}(\alpha_i)(-4Q^2 + \alpha_1Q^2(5-4u) + 8Q^2u \\
& + \alpha_3Q^2(5-4u)u + \alpha_1m_\rho^2(-4+8u) + \alpha_3m_\rho^2u(-4+8u)) - \mathcal{V}(\alpha_i)(4Q^2 \\
& + (\alpha_1 + \alpha_3 u)(4m_\rho^2 + Q^2(-5+6u))), 2]))
\end{aligned} \tag{54}$$

$$\begin{aligned}
C_1^q(Q^2) = & -\frac{1}{2Q^2} e^{m_\rho^2/M^2} (\mathcal{I}_1[g_\perp^a(u)(2m_\rho^2 - Q^2) + g_\perp^{a'}(u)(4m_\rho^2(-1+u) - Q^2(1+u)) + 4(m_\rho^2(4\hat{g}_\perp^v(u) + 2\hat{g}_\perp^v(u) \\
& - g_3(u) - \hat{\phi}_\parallel(u) - 4\hat{\phi}_\parallel(u) - 4\phi_\parallel(u)(-1+u)) + g_\perp^v(u)(4m_\rho^2(-1+u) - Q^2(1+u))), 1] + \mathcal{I}_2[g_\perp^{a'}(u)] \\
& + \mathcal{I}_3[g_\perp^a(u)(2m_\rho^2 - Q^2) + 4g_\perp^v(u)(Q^2(-2+u) - 4m_\rho^2u) - 4m_\rho^2(4\hat{g}_\perp^v(u) - 2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u) - 4\hat{\phi}_\parallel(u) \\
& - 4\phi_\parallel(u)u) + g_\perp^{a'}(u)(2Q^2 + 4m_\rho^2u - Q^2u), 1] - \mathcal{I}_4[g_\perp^{a'}(u)] + 2m_\rho^2(\mathcal{I}_1[-\hat{A}_\parallel(u)m_\rho^2 - 2g_\perp^a(u)Q^2 \\
& - 4A_\parallel(u)m_\rho^2(-1+u) + 4(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(m_\rho^2(-1+u) - Q^2(1+u)) - 4\hat{g}_\perp^v(u)(4m_\rho^2(-1+u)^2 \\
& + Q^2(3 + (-2+u)u)) + 4\hat{\phi}_\parallel(u)(4m_\rho^2(-1+u)^2 + Q^2(3 + (-2+u)u)), 2] + 4m_\rho^2(\mathcal{I}_1[-2(-2\hat{g}_\perp^v(u) + g_3(u) \\
& + \hat{\phi}_\parallel(u))(4m_\rho^2 + Q^2)(-1+u)^3 + \hat{A}_\parallel(u)(4m_\rho^2(-1+u)^2 + Q^2(3 + (-2+u)u)), 3] - \mathcal{I}_3[-2(-2\hat{g}_\perp^v(u) \\
& + g_3(u) + \hat{\phi}_\parallel(u))(4m_\rho^2 + Q^2)u^3 + \hat{A}_\parallel(u)(2Q^2 + (4m_\rho^2 + Q^2)u^2), 3]) + \mathcal{I}_3[\hat{A}_\parallel(u)m_\rho^2 - 2(g_\perp^a(u) - 4\hat{g}_\perp^v(u))Q^2 \\
& + 4A_\parallel(u)m_\rho^2u + 4\hat{g}_\perp^v(u)(4m_\rho^2 + Q^2)u^2 + 4(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(Q^2(-2+u) - m_\rho^2u) - 4\hat{\phi}_\parallel(u)(2Q^2 \\
& + (4m_\rho^2 + Q^2)u^2), 2] + 4\mathcal{I}_5[-\mathcal{V}(\alpha_i)(Q^2 + (\alpha_1 + \alpha_3 - \alpha_3u)(2m_\rho^2 - Q^2u)) - \mathcal{A}(\alpha_i)(Q^2 - 2Q^2u \\
& + \alpha_1(m_\rho^2(2 - 4u) + Q^2u) + \alpha_3(-1+u)(-Q^2u + m_\rho^2(-2+4u))), 2] - 4\mathcal{I}_6[-\mathcal{V}(\alpha_i)(Q^2 + (2m_\rho^2 \\
& + Q^2(-1+u))(\alpha_1 + \alpha_3u)) + \mathcal{A}(\alpha_i)(-Q^2 + 2Q^2u + \alpha_1(Q^2 - Q^2u + m_\rho^2(-2+4u)) + \alpha_3u(Q^2 - Q^2u \\
& + m_\rho^2(-2+4u))), 2]))
\end{aligned} \tag{55}$$

$$\begin{aligned}
D_0^q(Q^2) = & \frac{1}{8} e^{m_\rho^2/M^2} (\mathcal{I}_1[g_\perp^a(u) + (g_\perp^{a'}(u) + 4g_\perp^v(u))(1+u), 1] + \mathcal{I}_3[g_\perp^a(u) - (-g_\perp^{a'}(u) + 4g_\perp^v(u))(-2+u), 1] \\
& + 4m_\rho^2(2\mathcal{I}_1[(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(1+u), 2] - 2\mathcal{I}_3[(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-2+u), 2] \\
& + \mathcal{I}_5[(-2 + \alpha_1 + \alpha_3 - \alpha_3u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2] \\
& - \mathcal{I}_6[(-2 + \alpha_1 + \alpha_3u)(-\mathcal{V}(\alpha_i) + \mathcal{A}(\alpha_i)(-1+2u)), 2]))
\end{aligned} \tag{56}$$

$$\begin{aligned}
D_1^q(Q^2) = & -\frac{1}{Q^4} e^{m_\rho^2/M^2} m_\rho^2 (-\mathcal{I}_1[g_\perp^a(u)(8m_\rho^2 - Q^2) + g_\perp^{a'}(u)(8m_\rho^2(-1+u) - Q^2(1+u)) + 4((8m_\rho^2 - Q^2)(\hat{g}_\perp^v(u) \\
& - \hat{\phi}_\parallel(u) + \phi_\parallel(u) - \phi_\parallel(u)u) + g_\perp^v(u)(8m_\rho^2(-1+u) - Q^2(1+u))), 1] - \mathcal{I}_3[g_\perp^a(u)(8m_\rho^2 - Q^2) \\
& + 4g_\perp^v(u)(Q^2(-2+u) - 8m_\rho^2u) - 4(8m_\rho^2 - Q^2)(\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u) - \phi_\parallel(u)u) + g_\perp^{a'}(u)(2Q^2 + 8m_\rho^2u - Q^2u), 1] \\
& + m_\rho^2(\mathcal{I}_1[8\hat{A}_\parallel(u)m_\rho^2 + 4g_\perp^a(u)Q^2 - \hat{A}_\parallel(u)Q^2 + 16\hat{g}_\perp^v(u)Q^2 - 32\hat{g}_\perp^v(u)Q^2 + 16g_3(u)Q^2 + 16\hat{\phi}_\parallel(u)Q^2 \\
& - 16\hat{\phi}_\parallel(u)Q^2 + 16A_\parallel(u)m_\rho^2(-1+u) + 64\hat{g}_\perp^v(u)m_\rho^2(-1+u) - 32g_3(u)m_\rho^2(-1+u) - 32\hat{\phi}_\parallel(u)m_\rho^2(-1+u) \\
& - 2A_\parallel(u)Q^2(-1+u) - 8\hat{g}_\perp^v(u)Q^2(-1+u) + 4g_3(u)Q^2(-1+u) + 4\hat{\phi}_\parallel(u)Q^2(-1+u) \\
& + 64\hat{g}_\perp^v(u)m_\rho^2(-1+u)^2 - 64\hat{\phi}_\parallel(u)m_\rho^2(-1+u)^2 + 24\hat{g}_\perp^v(u)Q^2(-1+u)^2 - 24\hat{\phi}_\parallel(u)Q^2(-1+u)^2), 2] \\
& + \mathcal{I}_3[-8\hat{A}_\parallel(u)m_\rho^2 + 4g_\perp^a(u)Q^2 + \hat{A}_\parallel(u)Q^2 - 16\hat{g}_\perp^v(u)Q^2 - 32\hat{g}_\perp^v(u)Q^2 + 16g_3(u)Q^2 + 16\hat{\phi}_\parallel(u)Q^2 \\
& - 64\hat{g}_\perp^v(u)m_\rho^2u + 32g_3(u)m_\rho^2u + 8\hat{g}_\perp^v(u)Q^2u - 4g_3(u)Q^2u + 2A_\parallel(u)(-8m_\rho^2 + Q^2)u - 64\hat{g}_\perp^v(u)m_\rho^2u^2 \\
& + 64\hat{\phi}_\parallel(u)m_\rho^2u^2 - 24\hat{g}_\perp^v(u)Q^2u^2 + 24\hat{\phi}_\parallel(u)Q^2u^2 + 4\hat{\phi}_\parallel(u)(4Q^2 + 8m_\rho^2u - Q^2u), 2] + 8m_\rho^2(\mathcal{I}_1[2(-2\hat{g}_\perp^v(u) \\
& + g_3(u) + \hat{\phi}_\parallel(u))(-1+u)(8m_\rho^2(-1+u)^2 + Q^2(-1+3(-2+u)u)) - \hat{A}_\parallel(u)(8m_\rho^2(-1+u)^2 \\
& + Q^2(5+3(-2+u)u)), 3] + \mathcal{I}_3[-2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))u(-4Q^2 + (8m_\rho^2 + 3Q^2)u^2) + \hat{A}_\parallel(u)(2Q^2 \\
& + (8m_\rho^2 + 3Q^2)u^2), 3]) - 4\mathcal{I}_5[\mathcal{A}(\alpha_i)(-2Q^2 - (\alpha_1 + \alpha_3)(8m_\rho^2 + 3Q^2) + 8(2\alpha_1 + 3\alpha_3)m_\rho^2u \\
& + (4 - 2\alpha_1 + \alpha_3)Q^2u + 2\alpha_3(-8m_\rho^2 + Q^2)u^2) - \mathcal{V}(\alpha_i)(2Q^2 + (8m_\rho^2 + Q^2(3 - 8u))(\alpha_1 + \alpha_3 - \alpha_3u)), 2] \\
& + 4\mathcal{I}_6[\mathcal{A}(\alpha_i)(\alpha_1Q^2(5 - 2u) + 8\alpha_1m_\rho^2(-1+2u) + 8\alpha_3m_\rho^2u(-1+2u) + Q^2(-2 + (4 + \alpha_3(5 - 2u))u)) \\
& - \mathcal{V}(\alpha_i)(2Q^2 + (\alpha_1 + \alpha_3u)(8m_\rho^2 + Q^2(-5+8u))), 2]))
\end{aligned} \tag{57}$$

$$\begin{aligned}
E^q(Q^2) = & \frac{1}{8Q^2} e^{m_\rho^2/M^2} (4(\mathcal{I}_1[g_\perp^a(u)m_\rho^2 - 4\hat{\phi}_\parallel(u)m_\rho^2 + \hat{g}_\perp^v(u)(4m_\rho^2 - Q^2) + \hat{\phi}_\parallel(u)Q^2 - g_\perp^{a'}(u)m_\rho^2(1-u) \\
& - 4g_\perp^v(u)m_\rho^2(1-u) + 4m_\rho^2\phi_\parallel(u)(1-u) - \phi_\parallel(u)Q^2(1-u), 1] + \mathcal{I}_3[g_\perp^a(u)m_\rho^2 + g_\perp^{a'}(u)m_\rho^2u - 4g_\perp^v(u)m_\rho^2u \\
& - (4m_\rho^2 - Q^2)(\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u) - \phi_\parallel(u)u), 1]) + m_\rho^2(-\mathcal{I}_1[\hat{A}_\parallel(u)(4m_\rho^2 - Q^2) + 2(1-u)(A_\parallel(u)(-4m_\rho^2 + Q^2) \\
& + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(4m_\rho^2 + Q^2) + 8(\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u))(2m_\rho^2 + Q^2)(1-u)), 2] \\
& + \mathcal{I}_3[\hat{A}_\parallel(u)(4m_\rho^2 - Q^2) + A_\parallel(u)(8m_\rho^2u - 2Q^2u) + 4u(-(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(4m_\rho^2 + Q^2) \\
& + 4\hat{g}_\perp^v(u)(2m_\rho^2 + Q^2)u - 4\hat{\phi}_\parallel(u)(2m_\rho^2 + Q^2)u), 2] + 16m_\rho^2(\mathcal{I}_1[(1-u)(-4(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))Q^2 \\
& + \hat{A}_\parallel(u)(2m_\rho^2 + Q^2)(1-u) + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(2m_\rho^2 + Q^2)(1-u)^2), 3] \\
& + \mathcal{I}_3[u(-\hat{A}_\parallel(u)(2m_\rho^2 + Q^2)u + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-2Q^2 + (2m_\rho^2 + Q^2)u^2)), 3]) \\
& - 4(\mathcal{I}_5[(\alpha_1 + \alpha_3(1-u))(\mathcal{V}(\alpha_i)(4m_\rho^2u - 3Q^2u + 4m_\rho^2(1-u) + 3Q^2(1-u)) + \mathcal{A}(\alpha_i)(-4m_\rho^2u + 3Q^2u \\
& + 4m_\rho^2(1-u) + 3Q^2(1-u))), 2] + \mathcal{I}_6[(\alpha_1 + \alpha_3u)(-\mathcal{V}(\alpha_i)(4m_\rho^2u + 3Q^2u + 4m_\rho^2(1-u) - 3Q^2(1-u)) \\
& + \mathcal{A}(\alpha_i)(4m_\rho^2u + 3Q^2u - 4m_\rho^2(1-u) + 3Q^2(1-u))), 2]))
\end{aligned} \tag{58}$$

$$\begin{aligned}
F^q(Q^2) = & \frac{1}{16m_\rho^2} e^{m_\rho^2/M^2} (2(\mathcal{I}_1[A_\parallel(u)m_\rho^2 + 2(g_\perp^a(u)m_\rho^2 - 2\hat{g}_\perp^v(u)m_\rho^2 + g_3(u)m_\rho^2 + \hat{\phi}_\parallel(u)m_\rho^2 - 4\hat{\phi}_\parallel(u)m_\rho^2 \\
& + \hat{g}_\perp^v(u)(4m_\rho^2 - Q^2) + \hat{\phi}_\parallel(u)Q^2 - g_\perp^{a'}(u)m_\rho^2(1-u) - 4g_\perp^v(u)m_\rho^2(1-u) + 4m_\rho^2\phi_\parallel(u)(1-u) \\
& - \phi_\parallel(u)Q^2(1-u)), 1] + 2\mathcal{I}_2[g_\perp^v(u) - \phi_\parallel(u)] + \mathcal{I}_3[A_\parallel(u)m_\rho^2 + 2(g_\perp^a(u)m_\rho^2 - 4\hat{g}_\perp^v(u)m_\rho^2 - 2\hat{g}_\perp^v(u)m_\rho^2 \\
& + g_3(u)m_\rho^2 + \hat{\phi}_\parallel(u)m_\rho^2 + 4\hat{\phi}_\parallel(u)m_\rho^2 + \hat{g}_\perp^v(u)Q^2 - \hat{\phi}_\parallel(u)Q^2 + g_\perp^{a'}(u)m_\rho^2u - 4g_\perp^v(u)m_\rho^2u + 4m_\rho^2\phi_\parallel(u)u \\
& - \phi_\parallel(u)Q^2u), 1] + 2\mathcal{I}_4[g_\perp^v(u) - \phi_\parallel(u)] + m_\rho^2(-\mathcal{I}_1[\hat{A}_\parallel(u)(4m_\rho^2 - Q^2) + 2(1-u)(A_\parallel(u)(-4m_\rho^2 + Q^2) \\
& + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(4m_\rho^2 + Q^2) + 8(\hat{g}_\perp^v(u) - \hat{\phi}_\parallel(u))(2m_\rho^2 + Q^2)(1-u)), 2] \\
& + \mathcal{I}_3[\hat{A}_\parallel(u)(4m_\rho^2 - Q^2) + A_\parallel(u)(8m_\rho^2u - 2Q^2u) + 4u(-(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(4m_\rho^2 + Q^2) \\
& + 4\hat{g}_\perp^v(u)(2m_\rho^2 + Q^2)u - 4\hat{\phi}_\parallel(u)(2m_\rho^2 + Q^2)u), 2] + 16m_\rho^2(\mathcal{I}_1[(1-u)(-4(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))Q^2 \\
& + \hat{A}_\parallel(u)(2m_\rho^2 + Q^2)(1-u) + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(2m_\rho^2 + Q^2)(1-u)^2), 3] \\
& + \mathcal{I}_3[u(-\hat{A}_\parallel(u)(2m_\rho^2 + Q^2)u + 2(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-2Q^2 + (2m_\rho^2 + Q^2)u^2)), 3]) \\
& - 4(\mathcal{I}_5[(\alpha_1 + \alpha_3(1-u))(\mathcal{V}(\alpha_i)(4m_\rho^2u - 3Q^2u + 4m_\rho^2(1-u) + 3Q^2(1-u)) + \mathcal{A}(\alpha_i)(-4m_\rho^2u + 3Q^2u \\
& + 4m_\rho^2(1-u) + 3Q^2(1-u))), 2] + \mathcal{I}_6[(\alpha_1 + \alpha_3u)(-\mathcal{V}(\alpha_i)(4m_\rho^2u + 3Q^2u + 4m_\rho^2(1-u) - 3Q^2(1-u)) \\
& + \mathcal{A}(\alpha_i)(4m_\rho^2u + 3Q^2u - 4m_\rho^2(1-u) + 3Q^2(1-u))), 2]))
\end{aligned} \tag{59}$$

$$\begin{aligned}
J^q(Q^2) = & \frac{1}{8} e^{m_\rho^2/M^2} (4(\mathcal{I}_1[-\hat{g}_\perp^v(u) + \hat{\phi}_\parallel(u) + \phi_\parallel(u)(-1+u), 1] - \mathcal{I}_3[-\hat{g}_\perp^v(u) + \hat{\phi}_\parallel(u) + \phi_\parallel(u)u, 1]) + m_\rho^2(\mathcal{I}_1[\hat{A}_\parallel(u) \\
& + 2A_\parallel(u)(-1+u) + 4(-2\hat{g}_\perp^v(u) + g_3(u) + \hat{\phi}_\parallel(u))(-1+u), 2] - \mathcal{I}_3[\hat{A}_\parallel(u) + 2A_\parallel(u)u + 4(-2\hat{g}_\perp^v(u) + g_3(u) \\
& + \hat{\phi}_\parallel(u))u, 2] - 4(\mathcal{I}_5[(\alpha_1 + \alpha_3 - \alpha_3u)(\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) - 2\mathcal{V}(\alpha_i)u), 2] + \mathcal{I}_6[(\alpha_1 + \alpha_3u)(\mathcal{A}(\alpha_i) + \mathcal{V}(\alpha_i) \\
& - 2\mathcal{V}(\alpha_i)u), 2]))
\end{aligned} \tag{60}$$

Meanwhile, for the pion and kaon, we obtain

$$F_0^q(Q^2) = -\frac{e^{m_\rho^2/M^2}}{2} (\mathcal{I}_1[\bar{u}^2\phi_P(u), 1] + \mathcal{I}_3[u^2\phi_P(u), 1]) \tag{61}$$

$$F_1^q(Q^2) = -\frac{e^{m_p^2/M^2}}{2} (\mathcal{I}_1[u(1+u)\phi_P(u), 1] + \mathcal{I}_3[\bar{u}(1+\bar{u})\phi_P(u), 1]). \quad (62)$$

III. NUMERICAL ANALYSIS

In this section, we numerically analyze the light-cone sum rules for the GFFs of the ρ , π , and K mesons by using Package X [37]. In the sum rules, we took the mass and decay constant of the mesons under consideration to be $m_\rho = 0.77$ GeV, $m_\pi = 135$ MeV, and $m_K = 497$ MeV and $f_\rho = 0.20$ GeV, $f_\pi = 131$ MeV, and $f_K = 160$ MeV, respectively. Another set of essential input parameters are the meson DAs of different twists. The relevant DAs are given as follows [31–36]:

$$\phi_{\parallel}(u) = 6u\bar{u} \left[1 + 3a_1^{\parallel}\xi + a_2^{\parallel}\frac{3}{2}(5\xi^2 - 1) \right] \quad (63)$$

$$g_3(u) = 1 + \left(-1 - \frac{2}{7}a_2^{\parallel} + \frac{40}{3}\zeta_3^A - \frac{20}{3}\zeta_4^A \right) C_2^{1/2}(\xi) + \left[-\frac{27}{28}a_2^{\parallel} + \frac{5}{4}\zeta_3^A - \frac{15}{16}\zeta_3^A(\omega_3^A + 3\omega_3^V) \right] C_4^{1/2}(\xi) \quad (64)$$

$$g_1^a(u) = 6u\bar{u} \left\{ 1 + a_1^{\parallel}\xi + \left[\frac{1}{4}a_2^{\parallel} + \frac{5}{3}\zeta_3^A \left(1 - \frac{3}{16}\omega_3^A \right) + \frac{35}{4}\zeta_3^V \right] (5\xi^2 - 1) \right\} \quad (65)$$

$$g_1^v(u) = \frac{3}{4}(1 + \xi^2) + a_1^{\parallel}\frac{3}{2}\xi^3 + \left(\frac{3}{7}a_2^{\parallel} + 5\zeta_3^A \right) (3\xi^2 - 1) + \left(\frac{9}{112}a_2^{\parallel} + \frac{105}{16}\zeta_3^V - \frac{15}{64}\zeta_3^A\omega_3^A \right) \times (3 - 30\xi^2 + 35\xi^4) \quad (66)$$

$$A_{\parallel}(u) = 24u^2\bar{u}^2 \quad (67)$$

$$\mathcal{V}(\alpha_i) = 5040(\alpha_1 - \alpha_2)\alpha_1\alpha_2\alpha_3^2 \quad (68)$$

$$\mathcal{A}(\alpha_i) = 360\alpha_1\alpha_2\alpha_3^2 \left[1 + \omega_3^A \frac{1}{2}(7\alpha_3 - 3) \right] \quad (69)$$

for the ρ meson and

$$\phi_P(u) = 6u\bar{u} [1 + a_1^P C_1(\xi) + a_2^P C_2^{3/2}(\xi)] \quad (70)$$

$$\phi_P(u) = 1 + \left(30\eta_3 - \frac{5}{2\mu_P^2} \right) C_2^{1/2}(\xi) + \left(-3\eta_3 w_3^P - \frac{27}{20\mu_P^2} - \frac{81}{10\mu_P^2} a_2^P \right) C_4^{1/2}(\xi) \quad (71)$$

$$\phi_\sigma(u) = 6u\bar{u} \left[1 + \left(5\eta_3 - \frac{1}{2}\eta_3 w_3^P - \frac{7}{20}\mu_P^2 - \frac{3}{5}\mu_P^2 a_2^P \right) C_2^{3/2}(\xi) \right] \quad (72)$$

$$\mathcal{T}(\alpha_i) = 360\eta_3\alpha_1\alpha_2\alpha_3^2 \left[1 + w_3^P \frac{1}{2}(7\alpha_3 - 3) \right] \quad (73)$$

for the pion and kaon. The $C_n^k(x)$ are the Gegenbauer polynomials, $\bar{u} = 1 - u$, and $\xi = 2u - 1$. The values of the parameters inside the DAs at the renormalization scale of $\mu = 1$ GeV are $a_1^{\parallel} = 0$, $a_2^{\parallel} = 0.18 \pm 0.10$, $\zeta_3^A = 0.032 \pm 0.010$, $\zeta_4 = 0.15 \pm 0.10$, $\omega_3^A = -2.1 \pm 1.0$, $\omega_3^V = 3.8 \pm 1.8$, and $\zeta_3^V = 0.013$ for ρ and $a_1^\pi = 0$, $a_2^\pi = 0.44$, $a_1^K = 0.06 \pm 0.03$, $a_2^K = 0.25 \pm 0.15$, $\eta_3 = 0.015$, $w_3^\pi = -3$, and $w_3^K = -1.2 \pm 0.7$ for the pion and kaon. The uncertainties in these parameters in general are estimated to be 30% to 50%.

It should be noted that the ρ meson DAs in the QCD sum rules using the background field method up to the next-to-leading order accuracy is calculated in [38].

From the sum rules for the GFFs, we see that besides the input parameters, they contain two auxiliary parameters, namely the Borel mass parameter, M^2 , and the continuum threshold, s_0 . Apparently, we should find the working region these parameters where measurable GFFs should be independent of them. The lowest bound of the Borel mass parameter is obtained by requiring that the contributions from the highest-twist terms should be considerably smaller (say by 10%) than the contributions from the leading-twist one. The upper bound of M^2 is determined by demanding that the continuum contribution should be less than 30% compared to the total result. The continuum threshold s_0 is obtained by requiring the two-point sum rules reproduce a 10% accuracy of the mass of the corresponding mesons. Taking into account these conditions, we find the working region of M^2 to be

$$\rho: 1.0 \text{ GeV}^2 < M^2 < 2.0 \text{ GeV}^2 \quad (74)$$

$$\pi: 0.6 \text{ GeV}^2 < M^2 < 1.4 \text{ GeV}^2 \quad (75)$$

$$K: 1.0 \text{ GeV}^2 < M^2 < 1.5 \text{ GeV}^2 \quad (76)$$

and the continuum threshold to be

$$\rho: s_0 = 1.4 \pm 0.1 \text{ GeV}^2 \quad (77)$$

$$\pi: s_0 = 0.70 \pm 0.05 \text{ GeV}^2 \quad (78)$$

$$K: s_0 = 1.05 \pm 0.05 \text{ GeV}^2 \quad (79)$$

FIG. 1. The Q^2 dependence of the GFF $A_0^q(Q^2)$ of the ρ meson at $M^2 = 1.0 \text{ GeV}^2$ and $s_0 = 1.4 \text{ GeV}^2$. The results of Sun and Dong [12] in the light-cone constituent quark model, Abidin *et al.* [10] from the AdS/QCD approach, and Freese *et al.* [11] in the NJL model are also shown.

We present our results for the six GFFs of the ρ meson that lead to the conservation of the EMT in Figs. 1–6 and the two GFFs of the π and K meson in Figs. 7–10. In these figures, the shaded regions indicated the uncertainties due to the input parameters and the solid (black) lines represent the central values.

The ρ meson GFFs $A_0(q^2)$ and $J(q^2)$ are related to the mass and charge conservation, hence are subject to the constraint at zero-momentum transfer $A_0(0) = 1$ and $J(0) = 1$ [6,9,10,40]. It is crucial to note that in this work, we took into account only the contribution from the quarks in the EMT.

As we noted that, the correlation function from QCD side can be calculated at sufficiently large negative values

FIG. 2. The same as Fig. 1 but for $A_1^q(Q^2)$.

FIG. 3. The same as Fig. 1 but for $D_0^q(Q^2)$.

FIG. 4. The same as Fig. 1 but for $D_1^q(Q^2)$.

FIG. 5. The same as Fig. 1 but for $E^q(Q^2)$.

FIG. 6. The same as Fig. 1 but for $J^q(Q^2)$.

of Q^2 . The formfactors can be reliably determined at $Q^2 \geq 1 \text{ GeV}^2$ domain. The LCSR method is not applicable for smaller values of Q^2 : $Q^2 < 1 \text{ GeV}^2$. In order to extend the results for the formfactors to the $Q^2 < 1 \text{ GeV}^2$ region, we need some parametrization of the formfactors. The form of the parametrization is chosen in such a way that they should reproduce with high accuracy the LCSR results in the region where the LCSR predictions are reliable. Our numerical analysis shows that the best parametrization of the form

$$F(Q^2) = a \left(1 + \frac{Q^2}{b} \right)^c \quad (80)$$

FIG. 7. The Q^2 dependence of the GFF $F_0^q(Q^2)$ of the π meson at $M^2 = 1.0 \text{ GeV}^2$ $s_0 = 0.7 \text{ GeV}^2$. Our results are compared to the ones in the chiral quark model [39], in the NJL model [11], in the AdS/QCD theory [17], and obtained by using the generalized distribution amplitudes [18].

FIG. 8. The same as Fig. 7 but for $F_1^q(Q^2)$.

where $F(Q^2)$ is some formfactor. For the central values of the parameters, for the formfactors, we obtain the following fitting curves.

$$A_0^q(Q^2) = 0.583 \left(1 + \frac{Q^2}{3.451} \right)^{-0.603} \quad (81)$$

$$A_1^q(Q^2) = 2.247 \left(1 + \frac{Q^2}{1.042} \right)^{-1.329} \quad (82)$$

$$D_0^q(Q^2) = -3.086 \left(1 + \frac{Q^2}{0.160} \right)^{-0.554} \quad (83)$$

$$D_1^q(Q^2) = 21.04 \left(1 + \frac{Q^2}{0.904} \right)^{-1.936} \quad (84)$$

FIG. 9. The Q^2 dependence of the GFF $F_0^q(Q^2)$ of the K meson at $M^2 = 1.25 \text{ GeV}^2$ and $s_0 = 1.05 \text{ GeV}^2$.

FIG. 10. The same as Fig. 9 but for $F_1^q(Q^2)$.

$$E^q(Q^2) = 0.769 \left(1 + \frac{Q^2}{2.603}\right)^{-1.473} \quad (85)$$

$$J^q(Q^2) = 0.413 \left(1 + \frac{Q^2}{2.541}\right)^{-1.351} \quad (86)$$

and

$$F_{0,\pi}^q(Q^2) = 0.876 \left(1 + \frac{Q^2}{2.457}\right)^{-1.704} \quad (87)$$

$$F_{1,\pi}^q(Q^2) = 0.932 \left(1 + \frac{Q^2}{1.951}\right)^{-2.636} \quad (88)$$

$$F_{0,K}^q(Q^2) = 0.770 \left(1 + \frac{Q^2}{3.798}\right)^{-1.879} \quad (89)$$

$$F_{1,K}^q(Q^2) = 0.703 \left(1 + \frac{Q^2}{5.198}\right)^{-3.957} \quad (90)$$

From Figs. 1–6, we deduce the following results: The contributions from the quark sector of the EMT to the GFF for A_0^q seem to make up nearly 60% of the total contributions. For the GFF J , 40% of the total contribution consists of the quark part. The results for both formfactors agree with the expectation that the quark sector in the EMT will add up to almost half of the total contributions to the GFFs. For the rest of the GFFs displayed in Figs. 1–6, we see that the quark contributions seem to be larger than the total obtained in the literature, which would indicate that the gluon contributions should have largely negative contributions.

By looking at Figs. 7 and 9, recalling the fact that we consider only the quark part of the EMT, and in the light of Eq. (10), we conclude that the quark contributions make up nearly 90% (70%) of the formfactor F_0 for the pion (kaon). Examining Figs. 8 and 10 and referring to Eq. (11), one can

see that the gluon contributions to the formfactor F_1 should be large and negative.

As we have already attributed the observation that $A_0^q(0)$ and $J^q(0)$ are not equal to 1 to the fact that we did not include the gluon contributions from the energy-momentum tensor, we would like to also emphasize that the opposite signs of the formfactors A_1^q , D_0^q , and E^q compared to the literature results may be accounted for if one notes that, for instance, in the light-front constituent quark model, the constituent quark masses are large and, in fact, much larger than the chiral limit, and also that there are certain simplifications in the cited approaches. Therefore, we can not only expect differences in the obtained signs of the formfactors but also in the magnitudes.

It should be noted also that only the GFFs A_0 for the ρ meson, F_0 for the pion, and F_0 and F_1 for the kaon are relatively sensitive to the uncertainties in the input parameters, which are encoded in the DAs. For the rest of the GFFs appear to be insensitive to the errors in the input parameters.

At the end of this section, we emphasize that besides mass, there are two other important quantities characterizing the energy distribution, namely the average mass radius and the inertia tensor. The inertia tensor is related to the quadrupole moment of the corresponding meson. Now we present our predictions for the average mass radius, $\langle r^2 \rangle_{\text{mass}}$, of the mesons considered and the quadrupole moment of the ρ meson. The average mass radius $\langle r^2 \rangle_{\text{mass}}$ is obtained in the light-cone frame in [11] as

$$\begin{aligned} \langle r^2 \rangle_{\text{LC}} = & 4 \frac{dA_0(q^2)}{dq^2} \Big|_{q^2=0} \\ & + \frac{1}{3m^2} [2A_0(0) + A_1(0) - 2J(0) + 2E(0)] \end{aligned} \quad (91)$$

for the ρ meson and

$$\langle r^2 \rangle_{\text{LC}} = -4 \frac{dF_0(Q^2)}{dQ^2} \Big|_{Q^2=0} \quad (92)$$

for the pion and kaon. In our work, we obtain

$$\sqrt{\langle r^2 \rangle_\rho} = 0.32 \pm 0.00 \text{ fm} \quad (93)$$

$$\sqrt{\langle r^2 \rangle_\pi} = 0.31 \pm 0.04 \text{ fm} \quad (94)$$

$$\sqrt{\langle r^2 \rangle_K} = 0.24 \pm 0.02 \text{ fm} \quad (95)$$

while it was found to be $\sqrt{\langle r^2 \rangle_\rho} = 0.41 \text{ fm}$ in [12] and

$$\sqrt{\langle r_\pi^2 \rangle} = \begin{cases} 0.27 \text{ fm} \\ 0.29 \text{ fm} \\ 0.34 \text{ fm} \end{cases} \quad (96)$$

[11,13,41]. The empirical value for $\sqrt{\langle r_\pi^2 \rangle}$ extracted from KEKB data is between 0.26 and 0.32 fm [18]. From the presented results for $\sqrt{\langle r_\pi^2 \rangle}$, we find that our result is in good agreement with the predictions of other approaches. We also see that our result on mass radius is smaller than the charge radius which is equal to 0.51 fm for the pion [42].

Finally, the gravitational quadrupole moment for the ρ meson is given in terms of the GFFs as [18]

$$\mathcal{Q}_{\text{mass}} = -\frac{1}{m} \left[-A_0(0) + \frac{1}{2}A_1(0) + 2J(0) - E(0) \right] \quad (97)$$

In our work, we obtain

$$\mathcal{Q}_{\text{mass}} = -\left(0.0512^{+0.0097}_{-0.0073} \right) m_\rho \cdot \text{fm}^2 \quad (98)$$

while it was obtained to be $-0.0322m_\rho \cdot \text{fm}^2$ in [12]. The difference between our result and [18] can attributed to neglecting the gluon part in the EMT.

IV. CONCLUSION

In the present work, we studied the GFFs appearing in the decompositions of the quark part of the energy-momentum tensor for ρ , π , and K mesons within the light-cone QCD sum rules approach. The ρ meson GFFs A_0 and J are related with the mass and spin of the ρ meson and the pion and kaon GFF F_0 is related to the mass. There are following constraints: $A_0(0) = 1$, $J(0) = 1$, and $F_0 = 1$ at zero transfer momentum. We have shown that the quark contributions make up nearly 60%, 40%, and 90% (70%) of the aforementioned GFFs, respectively. The mass radii were obtained to be 0.32 fm, 0.31 fm, and 0.24 fm for the ρ , π , and K , respectively, and our results are in good agreement with the estimations of other approaches such as the AdS/QCD approach [10], NJL model [11], and the light-cone constituent quark model [12]. The small value of the mass radius shows that the energy that makes up the vector meson mass is well concentrated. Our result on the gravitational quadrupole moments approximately 50% larger than the predicted in [12].

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