Study of the decays of S -wave $\bar{D}^*K^*$ hadronic molecules: The scalar $X_0(2900)$ and its spin partners $\tilde{X}_{J(J=1,2)}$

Cheng-Jian Xiao,^{1,*} Dian-Yong Chen,^{2,†} Yu-Bing Dong,^{3,4,5,‡} and Guang-Wei Meng¹

¹*Institute of Applied Physics and Computational Mathematics, Beijing 100088, People's Republic of China*

²*School of Physics, Southeast University, Nanjing 210094, People's Republic of China*

³*Institute of High Energy Physics, Chinese Academy of Sciences,
Beijing 100049, People's Republic of China*

⁴*Theoretical Physics Center for Science Facilities (TPCSF),
CAS, Beijing 100049, People's Republic of China*

⁵*University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China*

(Received 9 October 2020; accepted 15 January 2021; published 8 February 2021)

In this work, we investigate the decays of the fully open-flavor tetraquark state $X_0(2900)$ which was observed by the LHCb collaboration very recently. By using an effective Lagrangian approach, the partial widths of $X_0(2900)$ are estimated, where $X_0(2900)$ is assumed to be an S -wave $\bar{D}^*K^*$ hadronic molecule with $I = 0$. It is found that the estimated results are in agreement with the experimental observations. Moreover, we also predict the decay behaviors of the other unobserved $\tilde{X}_{J(J=1,2)}$, which are the spin partners of the $X_0(2900)$ in the S -wave $\bar{D}^*K^*$ picture. It is pointed out that the $\tilde{X}_1$ state with $I = 0$ is a broad state with the width more than 100 MeV, while another $\tilde{X}_2$ state with $I = 0$ is a narrow state with the width approaching half of that for the $X_0(2900)$. In addition, our results also show that the $\bar{D}^*K$ mode is expected to be the dominant decay mode for both $\tilde{X}_1$ and $\tilde{X}_2$. Searching for those unobserved $\tilde{X}_{J(J=1,2)}$ in the future experiments might be helpful to understand the nature of $X_0(2900)$.

DOI: [10.1103/PhysRevD.103.034004](https://doi.org/10.1103/PhysRevD.103.034004)

I. INTRODUCTION

Until now, the exotic family is no longer thin due to the great efforts from the experimental side. Traces of their existence can be found in the meson sector, the baryon sector as well as the dibaryon sector, namely, the tetraquark states, pentaquark states, and hexaquark states. The $X(3872)$, $D_{s0}^*(2317)$, Z_c , P_c are the typical examples of the remarkable exotic states (more information can be found in the review papers [1–12]). Concerning the constituent quarks, most of the exotic states contain a pair of quark-antiquark, $c\bar{c}$ or $u\bar{u}$ for instance, which makes them hidden flavor. Besides of the hidden-flavor structure, the exotic states can be composed of quarks with fully open flavors. The first fully open-flavor exotic state, as well as the only one before September 2020, was observed in 2016 named $X(5568)$ [13]. It was observed by the D0

collaboration and was expected to be consisting of $\bar{b}sud$, which made it obviously exotic [13]. The $X(5568)$ was interesting and attracted great attention for both experimentalists and theorists [2,5,6,8,9,11,12]. However, the later negative results for the $X(5568)$ from LHCb and CMS collaborations bogged down the interests of the study of fully open-flavor states [14,15].

The situation dramatically changed very recently, since the LHCb collaboration reported their first amplitude analysis of the $B^+ \rightarrow D^+ D^- K^+$ process [16,17] and where they have to introduce one spin-0 $X_0(2900)$ state and another spin-1 $X_1(2900)$ in their analysis in order to describe the data. The resonance parameters of these two new states are

$$\begin{aligned} X_0(2900): M &= 2866 \pm 7 \pm 2 \text{ MeV}, \\ \Gamma &= 57 \pm 12 \pm 4 \text{ MeV}, \end{aligned} \quad (1)$$

and

$$\begin{aligned} X_1(2900): M &= 2904 \pm 5 \pm 1 \text{ MeV}, \\ \Gamma &= 110 \pm 11 \pm 4 \text{ MeV}, \end{aligned} \quad (2)$$

respectively. The P parities are determined to be positive for the spin-0 state, and negative for the spin-1 one based on

*xiaocj@ihep.ac.cn
 †chendy@seu.edu.cn
 ‡dongyb@ihep.ac.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

the D^-K^+ decay channel with the isospins undetermined. Therefore, the $I(J^P)$ quantum numbers of the $X_0(2900)$ and $X_1(2900)$ could be $0/1(0^+)$ and $0/1(1^-)$, respectively.

The observed channel of $X_0(2900)$ and $X_1(2900)$ indicates that the quark components of these two states are $\bar{c}d\bar{s}u$. Such kinds of exotic states have usually been considered as good candidates of tetraquark state, such as hadronic molecular states and compact tetraquark states. In the former case, the four quarks form two hadrons, which are bounded via the strong interaction. In the later case, the quarks form a compact structure. For the particular $\bar{c}d\bar{s}u$ structure, a $D^*\bar{K}^*$ bound state with $I(J^P) = 0(0^+)$ was predicted early in 2010, the mass and width were estimated to be 2848 and 59 MeV, respectively. Moreover, Ref. [18] also predicted a $cs\bar{u}\bar{d}$ state with the mass 2850 MeV. Besides, the charmed partners of the $X(5568)$, whose structures were $su\bar{d}\bar{c}$, were predicted [19,20], however, the mass $M = 2550$ MeV did not fit the present observation.

Stimulated by the observation of the $X_0(2900)$ and $X_1(2900)$, many theoretical analyses of the two resonances have been carried out by employing various approaches [21–40]. References [21] and [24,30] interpreted the $X_0(2900)$ as the compact tetraquark based on the constituent quark model and QCD sum rules, respectively. Moreover, the $X_1(2900)$ was explained as the compact tetraquark state in Refs. [26,30,32,33,35,38]. Applying the chromomagnetic interactions diquark configuration model, the $J^P = 0^+$ resonance was also considered as a radial excited tetraquark, while the $J^P = 1^-$ one was assigned as an orbitally excited tetraquark [23]. However, a calculation based on the extended relativized quark model disfavored the tetraquark interpretation [27].

It should be mentioned that the hadronic molecule assignments were proposed [25,26,28,29,31,33,35,38]. By considering the J^P quantum numbers and mass threshold, the $X_0(2900)$ was explained as the S -wave $\bar{D}^*K^*$ hadronic molecule, while the $X_1(2900)$ was explained as the $\bar{D}_1K$ [29] and P -wave $\bar{D}^*K^*$ hadronic molecules [28]. There were also negative results for the $\bar{D}_1K$ molecule interpretation for the $X_1(2900)$, where it was found that the potential between $\bar{D}_1K$ was too weak to form any bound state [40]. To explore the nature of the $X_0(2900)$ and $X_1(2900)$, the production mechanism was also analyzed [37]. In additions, Ref. [22] considered the triangle singularity to be the origin of $X_0(2900)$ and $X_1(2900)$.

Whether the $X_0(2900)$ and $X_1(2900)$ are compact tetraquarks, hadronic molecules or due to kinetic effects

is unclear so far. In the hadronic molecule scenario, one may remind that the $X_0(2900)$ was explained as a $\bar{D}^*K^*$ molecule [25,26,28,29,31,33,35,38,41], where it was naturally considered to be bounded via the S -wave interaction between the $\bar{D}^*$ and K^* and carried the quantum numbers $J^P = 0^+$ since the angular momentum of the $X_0(2900)$ is 0 from the LHCb measurements [16,17]. As for the isospin quantum number, there is no strong evidence to distinguish it from the experimental side until now. On the other hand, among the mentioned theoretical works on the hadronic molecule interpretation, Molina *et al.* pointed out that the interaction between the $\bar{D}^*$ and K^* was repulsive for the $I = 1$ case in Refs. [33,41] via the coupled channel unitary approach. Meanwhile, the $I = 1$ interpretation was not supported by the calculations based on the one-boson exchange model either [25,29]. Therefore, we follow the S -wave $\bar{D}^*K^*$ interpretation for the $X_0(2900)$ with isospin $I = 0$ to investigate its decay behaviors via the effective Lagrangian approach. In particular, in the S -wave $\bar{D}^*K^*$ hadronic molecule scenario, another two states with $J = 1$ and $J = 2$ can be associated to the $X_0(2900)$ [25,29,31,33]. These two states with $I = 0$ assignment are the partners of the $X_0(2900)$ in the heavy quark spin symmetry, and that with $I = 1$ assignments are their isospin partners. Hereafter, we would refer $\tilde{X}_1$ and $\tilde{X}_2$ to the $J = 1$ and $J = 2$ states, respectively, where one should note that the $\tilde{X}_1$ is not the observed $X_1(2900)$ in Eq. (2) by the LHCb collaboration [16,17]. Within the same molecule scenario, we also investigate the decay behaviors of $\tilde{X}_1$ and $\tilde{X}_2$.

The present paper is assigned as follows. The effective Lagrangians and decays are given in the next section. Section III shows our numerical results and discussion. The summary is presented in the last section.

II. EFFECTIVE LAGRANGIANS AND DECAYS

The effective Lagrangian approach is applied to estimate the decays of experimental observed $X_0(2900)$ in the present work, where it is considered as the S -wave $\bar{D}^*K^*$ hadronic molecules with the isospin $I(X_0) = 0$. Besides, the decays of the predicted $\tilde{X}_1/\tilde{X}_2$, being the spin partners of the $X_0(2900)$ in the S -wave $\bar{D}^*K^*$ picture, are also investigated, where two possible isospins $I(\tilde{X}_1/\tilde{X}_2) = 0$ and $I(\tilde{X}_1/\tilde{X}_2) = 1$ are adopted for our analyses.

We first construct the effective Lagrangians describing the interaction between the molecular state and its components,

$$\mathcal{L}_{X_0}(x) = g_{X_0\tilde{X}_0}(x) \int dy \Phi(y^2) [D^{*-}\mu(x + w_{K^*\bar{D}^*}y) K_\mu^{*+}(x - w_{\bar{D}^*K^*}y) - \bar{D}^{*0\mu}(x + w_{K^*\bar{D}^*}y) K_\mu^{*0}(x - w_{\bar{D}^*K^*}y)] + \text{H.c.}, \quad (3)$$

$$\mathcal{L}_{\tilde{X}_1}(x) = ig_{\tilde{X}_1} \epsilon_{\mu\nu\alpha\beta} \partial^\mu \tilde{X}_1^\nu(x) \int dy \Phi(y^2) [D^{*- \alpha}(x + w_{K^* \bar{D}^*} y) K^{* + \beta}(x - w_{\bar{D}^* K^*} y) \pm \bar{D}^{*0\alpha}(x + w_{K^* \bar{D}} y) K^{*0\beta}(x - w_{\bar{D}^* K^*} y)] + \text{H.c.}, \quad (4)$$

$$\mathcal{L}_{\tilde{X}_2}(x) = g_{\tilde{X}_2} \tilde{X}_2^{\mu\nu}(x) \int dy \Phi(y^2) [D_\mu^{*-}(x + w_{K^* \bar{D}^*} y) K_\nu^{*+}(x - w_{\bar{D}^* K^*} y) \pm \bar{D}_\mu^{*0}(x + w_{K^* \bar{D}} y) K_\nu^{*0}(x - w_{\bar{D}^* K^*} y)] + \text{H.c.}, \quad (5)$$

where the $\pm$ corresponds to $\tilde{X}_{J(J=1,2)}$ states with $I = 1$ and $I = 0$, respectively, the $w_{AB} = m_A/(m_A + m_B)$, and $\Phi(y^2)$ is the correlation function that carries the distribution information of the components in the hadronic molecule. Within the Fourier transformation, $\Phi(y^2) = \int d^4 p / (2\pi)^4 e^{-ipy} \tilde{\Phi}(-p^2)$. It should be mentioned that the Gaussian form $\tilde{\Phi}(p_E^2) = \exp(-p_E^2/\Lambda^2)$ is widely used to estimate the decays of hadronic molecules, where Λ is the model parameter related to the size of the hadronic molecule and p_E is the momentum in the Euclidean space [42–47].

The coupling constants g_{X_0} and $g_{\tilde{X}_1/\tilde{X}_2}$ can be determined by the compositeness condition [42–44]. For the X_0 state, the compositeness condition reads

$$Z = 1 - \Sigma'_{X_0}(m_{X_0}^2) = 0, \quad (6)$$

where Σ_{X_0} is the X_0 meson mass operator that is presented in Fig. 1(a), and Σ'_{X_0} is its derivative. For the $\tilde{X}_{J(J=1,2)}$ state, the compositeness condition reads

$$Z = 1 - \Pi'_{\tilde{X}_1/\tilde{X}_2}(m_{\tilde{X}_1/\tilde{X}_2}^2) = 0, \quad (7)$$

where $\Pi_{\tilde{X}_{J(J=1,2)}}$ are obtained via the mass operators $\Sigma_{\tilde{X}_1}^{\nu\tau}$ and $\Sigma_{\tilde{X}_2}^{\mu\nu\alpha\beta}$ that are presented in Fig. 1(b),

$$\Sigma_{\tilde{X}_1}^{\nu\tau}(p) = g_\perp^{\nu\tau} \Pi_{\tilde{X}_1} + \dots, \quad (8)$$

$$\Sigma_{\tilde{X}_2}^{\mu\nu\alpha\beta}(p) = \frac{1}{2} (g_\perp^{\mu\alpha} g_\perp^{\nu\beta} + g_\perp^{\nu\alpha} g_\perp^{\mu\beta}) \Pi_{\tilde{X}_2} + \dots, \quad (9)$$

where $g_\perp^{\nu\tau} = g^{\nu\tau} - p^\nu p^\tau / p^2$ and $\Pi'_{\tilde{X}_1/\tilde{X}_2}$ is the derivative of $\Pi_{\tilde{X}_1/\tilde{X}_2}$.

The explicit expressions of the mass operators Σ_{X_0} , $\Sigma_{\tilde{X}_1}^{\nu\tau}$ and $\Sigma_{\tilde{X}_2}^{\mu\nu\alpha\beta}$ are

$$\Sigma_{X_0}(p) = g_{X_0}^2 \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}^2(q - w_{\bar{D}^* K^*} p) \frac{-g_{\mu\nu} + q_\mu q_\nu / m_{\bar{D}^*}^2 - g^{\mu\nu} + (p - q)^\mu (p - q)^\nu / m_{K^*}^2}{q^2 - m_{\bar{D}^*}^2 (p - q)^2 - m_{K^*}^2}, \quad (10)$$

$$\Sigma_{\tilde{X}_1}^{\nu\tau}(p) = g_{\tilde{X}_1}^2 \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}^2(q - w_{\bar{D}^* K^*} p) \epsilon_{\mu\nu\alpha\beta} \epsilon_{\eta\tau\rho\sigma} (-ip^\mu) (ip^\eta) \frac{-g^{\alpha\rho} + q^\alpha q^\rho / m_{\bar{D}^*}^2 - g^{\beta\sigma} + (p - q)^\beta (p - q)^\sigma / m_{K^*}^2}{q^2 - m_{\bar{D}^*}^2 (p - q)^2 - m_{K^*}^2}, \quad (11)$$

$$\Sigma_{\tilde{X}_2}^{\mu\nu\alpha\beta}(p) = g_{\tilde{X}_2}^2 \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}^2(q - w_{\bar{D}^* K^*} p) \frac{-g^{\mu\alpha} + q^\mu q^\alpha / m_{\bar{D}^*}^2 - g^{\nu\beta} + (p - q)^\nu (p - q)^\beta / m_{K^*}^2}{q^2 - m_{\bar{D}^*}^2 (p - q)^2 - m_{K^*}^2}. \quad (12)$$

Considering the two-body decays, the X_0 can decay to $\bar{D}K$, the $\tilde{X}_1$ can decay to $\bar{D}^*K$ and $\bar{D}K^*$, and $\tilde{X}_2$ can decay to $\bar{D}K$, $\bar{D}^*K$ and $\bar{D}K^*$. These transitions occur via the

FIG. 1. Mass operators of the X_0 [diagram (a)], and the $\tilde{X}_1/\tilde{X}_2$ [diagram (b)].

triangle diagrams (presented in Fig. 2), where the hadronic molecule and the final state are connected through the $\bar{D}^*$ and K^* by exchanging proper hadrons. Here, the exchanged hadrons can be either pseudoscalar meson and vector meson, including

$$\begin{aligned} P: & \pi, \eta, \eta', \\ V: & \rho, \omega. \end{aligned} \quad (13)$$

As we can see from the Fig. 2, the effective Lagrangians describing the interaction between the charmed (strange) mesons and exchanged hadrons are essential [48,49],

FIG. 2. Diagrams contributing to the processes $X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow D^{(*)-}K^{(*)+}$. Diagram (a) corresponds to the transition from X_0 to $\bar{D}^-K^-$. Diagrams (b) and (c) corresponds to the transition from $\tilde{X}_1$ to $\bar{D}^{*-}K^+$ and $\bar{D}^-K^{*+}$. Diagrams (a)–(c) corresponds to the transitions from $\tilde{X}_2$ to $\bar{D}^-K^+$, $\bar{D}^{*-}K^+$ and $\bar{D}^-K^{*+}$. The P and V stand for the exchanged pseudoscalar mesons and vector mesons, respectively, including the π^0 , η , η' , ρ^0 and ω . Besides, three additional diagrams with the intermediate $\bar{D}^{*0}K^{*0}$ are not presented here, which also contribute to the process $X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow D^{(*)-}K^{(*)+}$ and are considered in the calculation.

$$\mathcal{L}_{D^*DP} = ig_{D^*DP}(D^{*\mu}\partial_\mu P\bar{D} - D\partial_\mu P\bar{D}^{*\mu}), \quad (14)$$

$$\mathcal{L}_{D^*D^*P} = -g_{D^*D^*P}\epsilon^{\mu\nu\alpha\beta}\partial_\mu D_\nu^*P\partial_\alpha\bar{D}_\beta^*, \quad (15)$$

$$\mathcal{L}_{D^*DV} = -g_{D^*DV}\epsilon_{\mu\nu\alpha\beta}D\partial^\mu V^\nu\partial^\alpha\bar{D}^{*\beta} + \text{H.c.}, \quad (16)$$

$$\begin{aligned} \mathcal{L}_{D^*D^*V} = & ig_{D^*D^*V}[D_\mu^*(\partial^\mu\bar{D}_\nu^*V^\nu - \partial^\mu V^\nu\bar{D}_\nu^*) \\ & + (D_\nu^*\partial^\mu V^\nu - \partial^\mu D_\nu^*V^\nu)\bar{D}_\mu^* \\ & + (\partial_\mu D^{*\nu}V^\mu\bar{D}_\nu^* - D^{*\nu}V^\mu\partial_\mu\bar{D}_\nu^*)], \end{aligned} \quad (17)$$

$$\mathcal{L}_{K^*KP} = -ig_{K^*KP}(\bar{K}\partial^\mu P - \partial^\mu\bar{K}P)K_\mu^* + \text{H.c.}, \quad (18)$$

$$\mathcal{L}_{K^*K^*P} = -g_{K^*K^*P}\epsilon^{\mu\nu\alpha\beta}\partial_\alpha\bar{K}_\beta^*P\partial_\mu K_\nu^*, \quad (19)$$

$$\mathcal{L}_{K^*KV} = -g_{K^*KV}\epsilon^{\eta\tau\rho\sigma}\partial_\rho\bar{K}_\sigma^*\partial_\eta V_\tau K + \text{H.c.}, \quad (20)$$

$$\begin{aligned} \mathcal{L}_{K^*K^*V} = & -ig_{K^*K^*V}[(\partial^\mu\bar{K}_\nu^*V^\nu - \bar{K}_\nu^*\partial^\mu V^\nu)K_\mu^* \\ & + \bar{K}_\mu^*(\partial^\mu V^\nu K_\nu^* - V^\nu\partial^\mu K_\nu^*) \\ & + (\bar{K}_\nu^*V^\mu\partial_\mu K^{*\nu} - \partial_\mu\bar{K}_\nu^*V^\mu K^{*\nu})], \end{aligned} \quad (21)$$

where the doublets $D^{(*)}$ and $\bar{D}^{(*)}$ are

$$D^{(*)} = (D^{(*)0}, D^{(*)+}), \quad \bar{D}^{(*)} = \begin{pmatrix} \bar{D}^{(*)0} \\ D^{(*)-} \end{pmatrix}. \quad (22)$$

The P stands for π , η and η' , where

$$\pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}, \quad (23)$$

and the vector meson V can be ρ , ω , where

$$\rho = \begin{pmatrix} \rho^0 & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho^0 \end{pmatrix}. \quad (24)$$

In our numerical calculations, we simple employ the coupling constants $g_{D^*D\pi} = 12.2$, which is estimated via

the experimentally measured decay width of process $D^* \rightarrow D\pi$ [50]. With the help of the HQSS [49],

$$g_{D^*D^*\pi} = \frac{m_D + m_{D^*}}{2} g_{D^*D\pi}. \quad (25)$$

Applying the vector-meson dominance (VMD) method to the process $D^* \rightarrow D\gamma$, one can obtain $g_{D^*D\rho} = 2.82 \text{ GeV}^{-1}$ [49]. The $g_{D^*D^*\rho} = 2.52$ is determined by the same VMD method [48,49]. In addition, the coupling constants $g_{K^*K\pi} = 3.12$ are determined via the experimentally measured decay width of the process $K^* \rightarrow K\pi$ [51]. Moreover, $g_{K^*K\pi}$, $g_{K^*K^*\pi}$, $g_{K^*K\rho}$, and $g_{K^*K^*\rho}$ can be related via a gauge coupling g ,

$$g_{K^*K\pi} = \frac{1}{4}g, \quad g_{K^*K^*\pi} = \frac{1}{4}\frac{g^2 N_c}{16\pi^2 F_\pi}, \quad (26)$$

$$g_{K^*K\rho} = \frac{1}{4}\frac{g^2 N_c}{16\pi^2 F_\pi}, \quad g_{K^*K^*\rho} = \frac{1}{4}g, \quad (27)$$

where $N_c = 3$ is the number of the quark color, $F_\pi = 132 \text{ MeV}$ is the decay constant of the pion. Other coupling constants concerning the $\eta^{(\prime)}$ and ω can be obtained via the $SU(3)$ symmetry.

In terms of the effective Lagrangians already given above, we can write out the Feynman amplitudes of the diagrams in Fig. 2. As for the process $X_0 \rightarrow D^-K^+$ [Fig. 2(a)], we have

$$\begin{aligned} \mathcal{M}_{X_0 \rightarrow D^-K^+}^P = & \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{X_0} \right] \\ & \times [ig_{D^*DP}(iq_\mu)][-ig_{K^*KP}(-iq^\nu - ip_4^\nu)] \\ & \times \frac{-g^{\phi\mu} + p_1^\phi p_1^\mu/m_1^2 - g_{\phi\nu} + p_{2\phi} p_{2\nu}/m_2^2}{p_1^2 - m_1^2} \frac{p_2^2 - m_2^2}{p_2^2 - m_2^2} \\ & \times \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1), \end{aligned} \quad (28)$$

$$\begin{aligned}
\mathcal{M}_{X_0 \rightarrow D^- K^+}^V = & \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12} p)^2] \left[\frac{1}{\sqrt{2}} g_{X_0} \right] \\
& \times [g_{D^* D V} \epsilon_{\mu\nu\alpha\beta} (i q^\mu) (-i p_1^\alpha)] [-g_{K^* K V} \\
& \times \epsilon_{\eta\tau\rho\sigma} (-i q^\eta) (-i p_2^\rho)] \frac{-g^{\phi\beta} + p_1^\phi p_1^\beta / m_1^2}{p_1^2 - m_1^2} \\
& \times \frac{-g_\phi^\sigma + p_{2\phi} p_2^\sigma / m_2^2 - g^{\tau\nu} + q^\tau q^\nu / m_q^2}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \\
& \times \mathcal{F}^2(m_q, \Lambda_1). \tag{29}
\end{aligned}$$

The $\mathcal{M}_{X_0 \rightarrow D^- K^+}^P$ and $\mathcal{M}_{X_0 \rightarrow D^- K^+}^V$ are the Feynman amplitudes for the transition from X_0 to $D^- K^+$ with the exchanging pseudoscalar mesons (π^0, η, η') and vector mesons (ρ^0, ω), respectively. The mass of the exchanged meson is m_q . Here, a phenomenological form factor is introduced to represent the off-shell effect of the coupling constant, and we select a dipole form factor [52],

$$\mathcal{F}^2(m_q, \Lambda_1) = \left(\frac{m_q^2 - \Lambda_1^2}{q^2 - \Lambda_1^2} \right)^2, \tag{30}$$

where q is the momentum of the exchange meson.

Another two diagrams with the $\bar{D}^{*0} K^{*0}$ intermediate states, in which the exchanged states are π^- and ρ^- , also contribute to the process $X_0 \rightarrow D^- K^+$; we can obtain the corresponding Feynman amplitude via the isospin symmetry,

$$\mathcal{A}_{X_0 \rightarrow D^- K^+}^{\pi^-} = -2\mathcal{M}_{X_0 \rightarrow D^- K^+}^{\pi^0}, \tag{31}$$

$$\mathcal{A}_{X_0 \rightarrow D^- K^+}^{\rho^-} = -2\mathcal{M}_{X_0 \rightarrow D^- K^+}^{\rho^0}. \tag{32}$$

Similarly, we can write out the Feynman amplitudes for the processes $\tilde{X}_1 \rightarrow D^{*-} K^+$, $\tilde{X}_1 \rightarrow D^- K^*$, $\tilde{X}_2 \rightarrow D^- K$, $\tilde{X}_2 \rightarrow D^{*-} K^+$, and $\tilde{X}_2 \rightarrow D^- K^{*+}$. The detailed expressions are presented in Appendix A.

Now, the total contributions of the processes $X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow D^{(*)-} K^{(*)+}$ are

$$\mathcal{M}^{\text{tot}} = \mathcal{M}^{\pi^0} + \mathcal{M}^\eta + \mathcal{M}^{\eta'} + \mathcal{M}^{\rho^0} + \mathcal{M}^\omega \pm (\mathcal{A}^{\pi^-} + \mathcal{A}^{\rho^-}), \tag{33}$$

where the $\pm$ correspond to $I=0$ and $I=1$ cases, respectively. Finally, we can derive the partial decay widths of the processes $X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow D^{(*)-} K^{(*)+}$,

$$\Gamma(X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow D^{(*)-} K^{(*)+}) = \frac{1}{2J+1} \frac{1}{8\pi M^2} |\vec{p}| |\mathcal{M}^{\text{tot}}|^2, \tag{34}$$

where J and M are the angular momentum and mass of the initial state, respectively, $|\vec{p}|$ is the three-momentum of the final state in the rest frame of the initial state, and the

overline represents the sum of the polarization for the initial and final states.

In terms of the isospin symmetry, the partial decay width of the $\bar{D}^{(*)0} K^{*0}$ is the same as the $D^{(*)-} K^{(*)+}$ mode. Therefore,

$$\begin{aligned}
\Gamma(X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow \bar{D}^{(*)} K^{(*)}) \\
= \Gamma(X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow D^{(*)-} K^{(*)+}) \\
+ \Gamma(X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow \bar{D}^{(*)0} K^{*0}) \\
= 2\Gamma(X_0/\tilde{X}_1/\tilde{X}_2 \rightarrow D^{(*)-} K^{(*)+}). \tag{35}
\end{aligned}$$

III. NUMERICAL RESULTS AND DISCUSSION

As we have mentioned in Sec. I, until now, there is no strong evidence to distinguish the isospin quantum number of the X_0 from the experimental side, while the $I=0$ interpretation was preferred by the theoretical investigations on the spectrum [25,29,33,41]. Here, the estimations of the decays for the X_0 are based on the $I=0$ interpretation. Before estimating the partial decay width, the coupling constants g_{X_0} should be determined. In Fig. 3, the numerical results of the g_{X_0} , as well as the $g_{\tilde{X}_1/\tilde{X}_2}$ are presented via the compositeness condition. Within the determined g_{X_0} , the numerical results of partial decay width for $X_0 \rightarrow \bar{D} K$ process depending on the parameters Λ and Λ_1 are presented in Fig. 4, where the solid lines represent the experimentally measured total decay width of the X_0 . In particular, taking into account systematic and statistic errors, the lower, central and upper limiting values of the total decay width for X_0 are 44.4, 57.0 and 69.6 MeV, respectively. The parameters Λ and Λ_1 are two phenomenological parameters in our model that cannot be determined by the first principle. In the present calculation, we chose the Λ and Λ_1 to be around 1 GeV, where they vary from 0.5 to 1.5 GeV.

Besides the counter plot in Fig. 4, we also present the partial decay width of the $X_0 \rightarrow \bar{D} K$ process depending on

FIG. 3. Coupling constants of g_{X_0} and $g_{\tilde{X}_1/\tilde{X}_2}$.

FIG. 4. The partial decay width of $X_0 \rightarrow \bar{D}K$ with the variation of the parameters Λ and Λ_1 . The solid lines correspond to the experimentally measured decay width of X_0 , where 44.4, 57.0 and 69.6 MeV are the lower limit, central and upper limit values, respectively [17].

the parameter Λ_1 with the typical value 1.0 GeV for Λ in Fig. 5, where the individual contributions from the π , ρ , η , η' and ω mesons are also presented. It can be found that the π -exchange almost contributes the total contribution to the decays of X_0 . In Appendix B, the analog results for another two typical parameter values $\Lambda = 0.5$ GeV and $\Lambda = 1.5$ GeV are also presented.

It is not easy to directly compare our results to the experimental data since the partial decay width of the $X_0 \rightarrow \bar{D}K$ process is still unknown from the LHCb measurements. However, it is expected that the branching fraction of the $\bar{D}K$ mode is large since it is the only two-body strong decay mode. Here, we make an approximation that the partial decay width of the $X_0 \rightarrow \bar{D}K$ is the same as the total decay width of X_0 resonance, $\Gamma^{\text{Exp}}(X_0 \rightarrow \bar{D}K) \simeq 44.4\text{--}69.6$ MeV. It can be found in Fig. 4 that the obtained results are in agreement with the experimental measurements in the considered parameter ranges. It could be assigned as a support of the $\bar{D}^*K^*$ hadronic molecule interpretation of the X_0 .

Moreover, with the roughly obtained partial decay width,

$$\Gamma^{\text{Exp}}(X_0 \rightarrow \bar{D}K) \simeq 44.4\text{--}69.6 \text{ MeV}, \quad (36)$$

the model parameters Λ and Λ_1 can be determined and the obtained values of the parameters will be applied to predict the decay behaviors of the unobserved $\tilde{X}_1$ and $\tilde{X}_2$. As we can see from Fig. 4, a series of sets for the values of Λ and Λ_1 can be obtained based on the solid lines: for $\Lambda = 1.0$ GeV, for instance, the corresponding Λ_1 is 1.14–1.33 GeV, where the parameter range for Λ_1 comes from the experimental errors. Besides, the obtained minimal

FIG. 5. Partial decay width of the process $X_0 \rightarrow \bar{D}K$ with the typical parameter value $\Lambda = 1.0$ GeV. Both the total and individual contributions are presented.

value for Λ is 0.74 GeV via $\Gamma^{\text{Exp}}(X_0 \rightarrow \bar{D}K) \simeq 57.0$ MeV. Similarly, we find that the Λ is constrained to be larger than 0.64 and 0.83 GeV, respectively, in terms of $\Gamma^{\text{Exp}}(X_0 \rightarrow \bar{D}K) \simeq 44.4$ MeV and $\Gamma^{\text{Exp}}(X_0 \rightarrow \bar{D}K) \simeq 69.6$ MeV.

The constrained values of the parameter Λ and Λ_1 are applied to predict the decay properties of the $\tilde{X}_1$ and $\tilde{X}_2$, being the spin partners of $X_0(2900)$ in the S -wave $\bar{D}^*K^*$ hadronic molecule scenario. Here, both masses of the $\tilde{X}_1$ and $\tilde{X}_2$ are assigned to be 2866 MeV, which were predicted in Ref. [31]. In Fig. 6, the numerical results of partial decay widths for the $\tilde{X}_1 \rightarrow \bar{D}^*K$ and $\tilde{X}_1 \rightarrow \bar{D}K^*$ processes are presented. For the $I(\tilde{X}_1) = 0$ case, we find that $\Gamma(\tilde{X}_1 \rightarrow \bar{D}^*K)$ varies from 80.4 to 146 MeV within the constrained parameters, which weakly depend to the parameters. The partial decay width for another $\bar{D}K^*$ mode varies from 11.8 to 19 MeV. The numerical results for the $I(\tilde{X}_1) = 1$ case are

FIG. 6. The partial decay widths of the $\tilde{X}_1 \rightarrow \bar{D}^*K$ and $\tilde{X}_1 \rightarrow \bar{D}K^*$ processes with the constrained parameters. The left column corresponds to the results with $I(\tilde{X}_1) = 0$ and the right column is the results with $I(\tilde{X}_1) = 1$. The bands are from the experimental uncertainty.

much smaller compared to those for the $I(\tilde{X}_1) = 0$ case, where the partial decay widths for the $\bar{D}^*K$ and $\bar{D}K^*$ are 12.2–22.5 MeV and 1.94–4.99 MeV, respectively. It could be concluded that for both of the two cases, the $\bar{D}^*K$ mode is the dominant decay mode. Besides, we also find that the $\tilde{X}_1$ with $I = 0$ is a broad state since the corresponding estimated width is more than 100 MeV.

As for another $\tilde{X}_2$ state, the numerical results of its partial decay widths are presented in Fig. 7. One can find that the $\bar{D}^*K$ mode is the dominant decay mode both for the $I(\tilde{X}_2) = 0$ and $I(\tilde{X}_2) = 1$ cases, the corresponding partial decay widths in the constrained parameter range are 8.31–21.4 MeV for the $I(\tilde{X}_2) = 0$ case and 1.17–4.03 MeV for the $I(\tilde{X}_2) = 1$ case. Compared to the $\bar{D}^*K$ mode, the partial decay width for the $\bar{D}K^*$ mode is expected to be, at least, 1 order of magnitude smaller. In particular, the $\Gamma(\tilde{X}_2 \rightarrow \bar{D}K^*)$ is 0.325–0.721 MeV for the $I(\tilde{X}_2) = 0$ case and 0.0423–0.103 MeV for $I(\tilde{X}_2) = 1$ case. Besides the $\bar{D}^*K$ and $\bar{D}K^*$ mode, the $\tilde{X}_2$ can also decay to the $\bar{D}K$, which is the channel that X_0 observed in. The corresponding partial width is 1.65–4.41 MeV for the $I(\tilde{X}_2) = 0$ case, and 0.227–0.787 MeV for the $I(\tilde{X}_2) = 1$ case. Similar to the $\tilde{X}_1$ case, the partial decay widths for the $\tilde{X}_2$ state with $I = 0$ are much larger than that with $I = 1$.

In Table I, we also summarize our predictions for the partial decay widths of $\tilde{X}_1$ and $\tilde{X}_2$ with the typical parameter value $\Lambda = 1.0$ GeV.

Last, we want to emphasize that the predicted decay behaviors of $\tilde{X}_1$ and $\tilde{X}_2$ depend on the cutoff parameters Λ and Λ_1 , the range of which was constrained to be in line with the partial decay width of the $X_0 \rightarrow \bar{D}K$ process. Here, this partial decay width is approximately fixed to be 44.4–69.6 MeV [see Eq. (36)]. The future more precise experiments will help us to reduce the uncertainty

FIG. 7. The partial decay widths of the transitions from $\tilde{X}_2$ to $\bar{D}K$, $\bar{D}^*K$ and $\bar{D}K^*$ with the constrained parameters. The left column corresponds to the results with $I(\tilde{X}_2) = 0$ and the right column is the results with $I(\tilde{X}_2) = 1$. The bands are from the experimental uncertainty.

TABLE I. Predicted partial decay widths for $\tilde{X}_1$ and $\tilde{X}_2$ with the typical parameter value $\Lambda = 1.0$ GeV, the unit is in MeV.

	$\tilde{X}_1$		$\tilde{X}_2$	
	$I = 0$	$I = 1$	$I = 0$	$I = 1$
$\Gamma(\bar{D}K)$	...	...	2.67–3.56	0.392–0.578
$\Gamma(\bar{D}^*K)$	93.1–137	14.6–21.6	13.2–17.2	2.03–2.99
$\Gamma(\bar{D}K^*)$	13.5–18.9	2.62–4.51	0.478–0.565	0.0636–0.0792

of the model parameters, and thus to improve the quality of our predictions given in Figs. 6 and 7 and Table I.

IV. SUMMARY

In the present work, we investigate the decay behaviors of $X_0(2900)$ in the S -wave $\bar{D}^*K^*$ scenario, where the $I(J)^P$ quantum numbers are adopted to be $0(0)^+$ based on the theoretical investigations on the spectrum [25,29,33,41]. With the help of the effective Lagrangian approach, the contributions from the triangle diagrams are estimated. Moreover, in order to represent the off-shell effect of the coupling constants, a phenomenological form factor is considered. The obtained partial decay width for the $X_0 \rightarrow \bar{D}K$ process is in agreement with the experimental data with the model parameters Λ and Λ_1 that are selected to be around 1 GeV. In particular, in the decays of the X_0 , the contribution of the π -exchange approaches the total contribution.

Within the constrained values of model parameters, we further calculate the decay behaviors of the spin partners of the $X_0(2900)$, e.g., the $J^P = 1^+$ and $J^P = 2^+$ S -wave hadronic molecules, which were predicted in theories [25,29,31,33] and referred to be $\tilde{X}_1$ and $\tilde{X}_2$ respectively in the present work. It should be emphasized that the $\tilde{X}_1$ is not the $X_1(2900)$ in Eq. (2) observed by the LHCb collaboration [16,17]. Both the $I = 0$ and $I = 1$ assignments for the $\tilde{X}_{J(J=1,2)}$ are considered in the present calculation, where they are associated with the $X_0(2900)$ via the heavy quark spin symmetry and isospin symmetry. The $\tilde{X}_1$ can decay to $\bar{D}^*K$ and $\bar{D}K^*$, and the $\tilde{X}_2$ can decay to $\bar{D}K$, $\bar{D}^*K$ and $\bar{D}K^*$. In the constrained parameter ranges, the partial decay widths for the $\tilde{X}_1$ state with $I = 0$ are

$$\Gamma(\tilde{X}_1 \rightarrow \bar{D}^*K) = 80.4\text{--}146 \text{ MeV}, \quad (37)$$

$$\Gamma(\tilde{X}_1 \rightarrow \bar{D}K^*) = 11.8\text{--}19.0 \text{ MeV}, \quad (38)$$

and that for the $I = 1$ $\tilde{X}_1$ state are

$$\Gamma(\tilde{X}_1 \rightarrow \bar{D}^*K) = 12.2\text{--}22.5 \text{ MeV}, \quad (39)$$

$$\Gamma(\tilde{X}_1 \rightarrow \bar{D}K^*) = 1.94\text{--}4.99 \text{ MeV}. \quad (40)$$

In addition, the decay behaviors of the $\tilde{X}_2$ are predicted to be, for the $I = 0$ case,

$$\Gamma(\tilde{X}_2 \rightarrow \bar{D}K) = 1.65\text{--}4.41 \text{ MeV}, \quad (41)$$

$$\Gamma(\tilde{X}_2 \rightarrow \bar{D}^*K) = 8.31\text{--}21.4 \text{ MeV}, \quad (42)$$

$$\Gamma(\tilde{X}_2 \rightarrow \bar{D}K^*) = 0.325\text{--}0.721 \text{ MeV}, \quad (43)$$

and for the $I = 1$ case,

$$\Gamma(\tilde{X}_2 \rightarrow \bar{D}K) = 0.227\text{--}0.787 \text{ MeV}, \quad (44)$$

$$\Gamma(\tilde{X}_2 \rightarrow \bar{D}^*K) = 1.17\text{--}4.03 \text{ MeV}, \quad (45)$$

$$\Gamma(\tilde{X}_2 \rightarrow \bar{D}K^*) = 0.0423\text{--}0.100 \text{ MeV}. \quad (46)$$

It could be found that the partial decay widths for the $I = 1$ case are much smaller than that for the $I = 0$ case, for $\bar{D}^*K$ mode of the $\tilde{X}_1$ for instance, the partial decay width for the $I = 0$ case is almost 6 times larger than that for the $I = 1$ case. We also conclude that the $\tilde{X}_1$ state with $I = 0$ is a broad state with the width more than 100 MeV, while others are narrow states. Both for the $\tilde{X}_1$ and $\tilde{X}_2$ state, the $\bar{D}^*K$ mode is the dominant decay mode.

As for the predicted behavior of the $\tilde{X}_1$ and $\tilde{X}_2$ decays, it should be reminded that the results depend on the model parameters Λ and Λ_1 . Those vary in the range that gave results for the $X_0 \rightarrow \bar{D}K$ decay, which are consistent with the present experimental data. The roughly estimated

results will minorly change with the more precise experimental data in the future.

Finally, the observation of the $X_0(2900)$ opened a new area for the fully open multiquark states. The inner structure of the $X_0(2900)$ is still controversial. It is valuable to determine the isospin number of $X_0(2900)$ experimentally. Meanwhile, searching for its spin partners and the flavor partners can also help us to understand the nature of $X_0(2900)$. We hope that more progress can be carried out in the near future.

ACKNOWLEDGMENTS

This project is supported by the National Key R&D Program of China under Grant No. 2017YFA0403200, by the National Natural Science Foundation of China under Grants No. 11947224, No. 11975245 and No. 11775050, by the fund provided to the Sino-German CRC 110 “Symmetries and the Emergence of Structure in QCD” project by the NSFC under Grant No. 12070131001, by the Key Research Program of Frontier Sciences, CAS, Grant No. Y7292610K1, and by National Key Research and Development Program of China under Contracts No. 2020YFA0406300.

APPENDIX A: THE AMPLITUDES OF THE TRANSITION FROM THE $\tilde{X}_1/\tilde{X}_2$ TO $\bar{D}^{(*)}K^{(*)}$

The diagrams contributing to the process $X_{J(J=1,2)} \rightarrow \bar{D}^{(*)}K^{(*)}$ are presented in Fig. 2; we can write out the corresponding Feynman amplitudes. For the $\tilde{X}_1 \rightarrow D^{*-}K^+$ process,

$$\begin{aligned} \mathcal{M}_{\tilde{X}_1 \rightarrow D^{*-}K^+}^P &= \int \frac{d^4q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_1} \epsilon_{\kappa\lambda\gamma\theta} (-ip^\kappa) \epsilon^\lambda(p_1) \right] [g_{D^*D^*P} \epsilon_{\mu\nu\alpha\beta} (ip_3^\mu) (-ip_1^\alpha) \epsilon^\nu(p_3)] \\ &\quad \times [-ig_{K^*KP} (-iq_\phi - ip_{4\phi})] \frac{-g^{\gamma\beta} + p_1^\gamma p_1^\beta/m_1^2 - g^{\theta\phi} + p_2^\theta p_2^\phi/m_2^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1), \end{aligned} \quad (A1)$$

$$\begin{aligned} \mathcal{M}_{\tilde{X}_1 \rightarrow D^{*-}K^+}^V &= \int \frac{d^4q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_1} \epsilon_{\kappa\lambda\gamma\theta} (-ip^\kappa) \epsilon^\lambda(p_1) \right] \{ -ig_{D^*D^*V} [(-ip_1^\tau) g^{\eta\rho} - (iq^\tau) g^{\eta\rho} \\ &\quad + (iq^\eta) g^{\tau\rho} - (ip_3^\eta) g^{\tau\rho} + (ip_3^\rho) g^{\tau\eta} - (ip_1^\rho) g^{\eta\tau}] \epsilon_\tau(p_3) \} [-g_{K^*KV} \epsilon_{\mu\nu\alpha\beta} (-iq^\mu) (-ip_2^\alpha)] \\ &\quad \times \frac{-g_\eta^\gamma + p_1^\gamma p_{1\eta}/m_1^2 - g^{\theta\beta} + p_2^\theta p_2^\beta/m_2^2 - g_\rho^\nu + q_\rho q^\nu/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1). \end{aligned} \quad (A2)$$

For the $\tilde{X}_1 \rightarrow D^-K^{*+}$ process,

$$\begin{aligned} \mathcal{M}_{\tilde{X}_1 \rightarrow D^-K^{*+}}^P &= \int \frac{d^4q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_1} \epsilon_{\kappa\lambda\gamma\theta} (-ip^\kappa) \epsilon^\lambda(p_1) \right] [ig_{D^*DP} (iq_\mu)] [-g_{K^*K^*P} \epsilon_{\eta\tau\rho\sigma} \\ &\quad \times (-ip_2^\eta) (ip_4^\rho) \epsilon^\sigma(p_4)] \frac{-g^{\gamma\mu} + p_1^\gamma p_1^\mu/m_1^2 - g^{\theta\tau} + p_2^\theta p_2^\tau/m_2^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1), \end{aligned} \quad (A3)$$

$$\begin{aligned} \mathcal{M}_{\tilde{X}_1 \rightarrow D^- K^{*+}}^V &= \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_1} \epsilon_{\kappa\lambda\gamma\theta}(-ip^\kappa) \epsilon^\lambda(p_1) \right] [g_{D^*DV} \epsilon_{\mu\nu\alpha\beta}(iq^\mu)(-ip_1^\alpha)] \{-ig_{K^*K^*V} \\ &\times [(-iq^\rho)g^{\tau\eta} - (-iq^\eta)g^{\rho\tau} + (-ip_2^\tau)g^{\rho\eta} - (-ip_2^\rho)g^{\tau\eta} + (ip_4^\eta)g^{\tau\rho} - (ip_4^\tau)g^{\eta\rho}] \epsilon_\rho(p_4)\} \\ &\times \frac{-g^{\gamma\beta} + p_1^\gamma p_1^\beta/m_1^2 - g_\eta^\beta + p_2^\beta p_{2\eta}/m_2^2 - g_\tau^\beta + q_\tau q^\nu/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1). \end{aligned} \quad (A4)$$

For the $\tilde{X}_2 \rightarrow D^- K^+$ process,

$$\begin{aligned} \mathcal{M}_{\tilde{X}_2 \rightarrow D^- K^+}^P &= \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_2} \epsilon_{\kappa\lambda}(p) \right] [ig_{D^*DP}(-iq_\mu)] [-ig_{K^*KP}(-iq_\nu - ip_{4\nu})] \\ &\times \frac{-g^{\kappa\mu} + p_1^\kappa p_1^\mu/m_1^2 - g_\eta^\mu + p_2^\mu p_{2\eta}/m_2^2 - g_\tau^\mu + q_\tau q^\nu/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1), \end{aligned} \quad (A5)$$

$$\begin{aligned} \mathcal{M}_{\tilde{X}_2 \rightarrow D^- K^+}^V &= \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_2} \epsilon_{\kappa\lambda}(p) \right] [g_{D^*DV} \epsilon_{\mu\nu\alpha\beta}(-iq^\mu)(-ip_1^\alpha)] [-g_{K^*KV} \epsilon_{\eta\tau\rho\sigma}(-iq^\eta)(-ip_2^\rho)] \\ &\times \frac{-g^{\kappa\beta} + p_1^\kappa p_1^\beta/m_1^2 - g^{\lambda\sigma} + p_2^\lambda p_{2\lambda}/m_2^2 - g^{\tau\nu} + q^\tau q^\nu/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1). \end{aligned} \quad (A6)$$

For the $\tilde{X}_2 \rightarrow D^{*-} K^+$ process,

$$\begin{aligned} \mathcal{M}_{\tilde{X}_2 \rightarrow D^{*-} K^+}^P &= \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_2} \epsilon_{\kappa\lambda}(p) \right] [g_{D^*D^*P} \epsilon_{\mu\nu\alpha\beta}(ip_3^\mu)(-ip_1^\alpha) \epsilon^\nu(p_3)] [-ig_{K^*KP}(-iq_\sigma - ip_{4\sigma})] \\ &\times \frac{-g^{\kappa\beta} + p_1^\kappa p_1^\beta/m_1^2 - g^{\lambda\sigma} + p_2^\lambda p_{2\lambda}/m_2^2 - g_\tau^\sigma + q_\tau q^\nu/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1), \end{aligned} \quad (A7)$$

$$\begin{aligned} \mathcal{M}_{\tilde{X}_2 \rightarrow D^{*-} K^+}^V &= \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_2} \epsilon_{\kappa\lambda}(p) \right] \{-ig_{D^*D^*V} [(-ip_1^\tau)g^{\eta\rho} - (iq^\tau)g^{\rho\eta} + (iq^\eta)g^{\tau\rho} - (ip_3^\eta)g^{\tau\rho} \\ &+ (ip_3^\rho)g^{\tau\eta} - (-ip_1^\rho)g^{\eta\tau}] \epsilon_\tau(p_3)\} [-g_{K^*KV} \epsilon_{\mu\nu\alpha\beta}(-iq^\mu)(-ip_2^\alpha)] \\ &\times \frac{-g_\eta^\kappa + p_1^\kappa p_{1\eta}/m_1^2 - g^{\lambda\beta} + p_2^\lambda p_{2\lambda}/m_2^2 - g_\rho^\beta + q^\nu q_\rho/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1). \end{aligned} \quad (A8)$$

For the process $\tilde{X}_2 \rightarrow D^- K^{*+}$,

$$\begin{aligned} \mathcal{M}_{\tilde{X}_2 \rightarrow D^- K^{*+}}^P &= \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_2} \epsilon_{\kappa\lambda}(p) \right] [ig_{D^*DP}(iq_\mu)] [-g_{K^*K^*P} \epsilon_{\eta\tau\rho\sigma}(-ip_2^\eta)(ip_4^\rho) \epsilon^\sigma(p_4)] \\ &\times \frac{-g^{\kappa\mu} + p_1^\kappa p_1^\mu/m_1^2 - g^{\lambda\tau} + p_2^\lambda p_{2\lambda}/m_2^2 - g_\tau^\mu + q_\tau q^\nu/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1), \end{aligned} \quad (A9)$$

$$\begin{aligned} \mathcal{M}_{\tilde{X}_2 \rightarrow D^- K^{*+}}^V &= \int \frac{d^4 q}{(2\pi)^4} \tilde{\Phi}[(p_1 - w_{12}p)^2] \left[\frac{1}{\sqrt{2}} g_{\tilde{X}_2} \epsilon_{\kappa\lambda}(p) \right] [g_{D^*DV} \epsilon_{\mu\nu\alpha\beta}(iq^\mu)(-ip_1^\alpha)] \{-ig_{K^*K^*V} [(-iq^\rho)g^{\tau\eta} \\ &- (-iq^\eta)g^{\rho\tau} + (-ip_2^\tau)g^{\rho\eta} - (-ip_2^\rho)g^{\tau\eta} + (ip_4^\eta)g^{\tau\rho} - (ip_4^\tau)g^{\eta\rho}] \epsilon_\rho(p_4)\} \\ &\times \frac{-g^{\kappa\beta} + p_1^\kappa p_1^\beta/m_1^2 - g_\eta^\beta + p_2^\beta p_{2\eta}/m_2^2 - g_\tau^\beta + q_\tau q^\nu/m_q^2}{p_1^2 - m_1^2} \frac{1}{p_2^2 - m_2^2} \frac{1}{q^2 - m_q^2} \mathcal{F}^2(m_q, \Lambda_1). \end{aligned} \quad (A10)$$

APPENDIX B: PARTIAL DECAY WIDTH OF THE $X_0 \rightarrow \bar{D}K$ WITH TYPICAL PARAMETER VALUES $\Lambda = 0.5$ GeV AND $\Lambda = 1.5$ GeV

Similar to the results in Fig. 5, the partial decay widths of the $X_0 \rightarrow \bar{D}K$ process depending on the parameter Λ_1 with the typical values 0.5 GeV and 1.5 GeV for Λ are presented in Fig. 8.

FIG. 8. Partial decay widths of the process $X_0 \rightarrow \bar{D}K$ with the typical parameter values $\Lambda = 0.5$ GeV (left column) and $\Lambda = 1.5$ GeV (right column). Both the total and individual contributions are presented.

-
- [1] H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, *Phys. Rep.* **639**, 1 (2016).
 - [2] A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, *Prog. Theor. Exp. Phys.* **2016**, 062C01 (2016).
 - [3] J. M. Richard, *Few-Body Syst.* **57**, 1185 (2016).
 - [4] H. Clement, *Prog. Part. Nucl. Phys.* **93**, 195 (2017).
 - [5] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, *Prog. Part. Nucl. Phys.* **93**, 143 (2017).
 - [6] A. Esposito, A. Pilloni, and A. Polosa, *Phys. Rep.* **668**, 1 (2017).
 - [7] A. Ali, J. S. Lange, and S. Stone, *Prog. Part. Nucl. Phys.* **97**, 123 (2017).
 - [8] F. K. Guo, C. Hanhart, U. G. Meißner, Q. Wang, Q. Zhao, and B. S. Zou, *Rev. Mod. Phys.* **90**, 015004 (2018).
 - [9] S. L. Olsen, T. Skwarnicki, and D. Zieminska, *Rev. Mod. Phys.* **90**, 015003 (2018).
 - [10] M. Karliner, J. L. Rosner, and T. Skwarnicki, *Annu. Rev. Nucl. Part. Sci.* **68**, 17 (2018).
 - [11] Y. R. Liu, H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, *Prog. Part. Nucl. Phys.* **107**, 237 (2019).
 - [12] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo, and C. Z. Yuan, *Phys. Rep.* **873**, 1 (2020).
 - [13] V. M. Abazov *et al.* (D0 Collaboration), *Phys. Rev. Lett.* **117**, 022003 (2016).
 - [14] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **117**, 152003 (2016).
 - [15] CMS Collaboration, CERN Report No. CMS-PAS-BPH-16-002, 2016.
 - [16] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **125**, 242001 (2020).
 - [17] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **102**, 112003 (2020).
 - [18] J. B. Cheng, S. Y. Li, Y. R. Liu, Y. N. Liu, Z. G. Si, and T. Yao, *Phys. Rev. D* **101**, 114017 (2020).
 - [19] S. S. Agaev, K. Azizi, and H. Sundu, *Phys. Rev. D* **93**, 094006 (2016).
 - [20] W. Chen, H. X. Chen, X. Liu, T. G. Steele, and S. L. Zhu, *Phys. Rev. Lett.* **117**, 022002 (2016).
 - [21] M. Karliner and J. L. Rosner, *Phys. Rev. D* **102**, 094016 (2020).
 - [22] X. H. Liu, M. J. Yan, H. W. Ke, G. Li, and J. J. Xie, *Eur. Phys. J. C* **80**, 1178 (2020).
 - [23] X. G. He, W. Wang, and R. Zhu, *Eur. Phys. J. C* **80**, 1026 (2020).
 - [24] J. R. Zhang, *arXiv:2008.07295*.
 - [25] M. Z. Liu, J. J. Xie, and L. S. Geng, *Phys. Rev. D* **102**, 091502 (2020).
 - [26] H. X. Chen, W. Chen, R. R. Dong, and N. Su, *Chin. Phys. Lett.* **37**, 101201 (2020).
 - [27] Q. F. Lu, D. Y. Chen, and Y. B. Dong, *Phys. Rev. D* **102**, 074021 (2020).
 - [28] Y. Huang, J. X. Lu, J. J. Xie, and L. S. Geng, *Eur. Phys. J. C* **80**, 973 (2020).
 - [29] J. He and D. Y. Chen, *arXiv:2008.07782*.
 - [30] Z. G. Wang, *Int. J. Mod. Phys. A* **35**, 2050187 (2020).
 - [31] M. W. Hu, X. Y. Lao, P. Ling, and Q. Wang, *arXiv:2008.06894*.

- [32] Y. Xue, X. Jin, H. Huang, and J. Ping, [arXiv:2008.09516](#).
- [33] R. Molina and E. Oset, [Phys. Lett. B **811**, 135870 \(2020\)](#).
- [34] T. J. Burns and E. S. Swanson, [Phys. Lett. B **813**, 136057 \(2021\)](#).
- [35] S. S. Agaev, K. Azizi, and H. Sundu, [arXiv:2008.13027](#).
- [36] R. M. Albuquerque, S. Narison, D. Rabetiariivony, and G. Randriamanatrika, [Nucl. Phys. **A1007**, 122113 \(2021\)](#).
- [37] Y. K. Chen, J. J. Han, Q. F. L., J. P. Wang, and F. S. Yu, [arXiv:2009.01182](#).
- [38] H. Mutuk, [arXiv:2009.02492](#).
- [39] T. J. Burns and E. S. Swanson, [Phys. Rev. D **103**, 014004 \(2021\)](#).
- [40] X. K. Dong and B. S. Zou, [arXiv:2009.11619](#).
- [41] R. Molina, T. Branz, and E. Oset, [Phys. Rev. D **82**, 014010 \(2010\)](#).
- [42] S. Weinberg, [Phys. Rev. **130**, 776 \(1963\)](#).
- [43] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y. L. Ma, [Phys. Rev. D **76**, 014005 \(2007\)](#).
- [44] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y. L. Ma, [Phys. Rev. D **76**, 114008 \(2007\)](#).
- [45] Y. Dong, A. Faessler, T. Gutsche, S. Kovalenko, and V. E. Lyubovitskij, [Phys. Rev. D **79**, 094013 \(2009\)](#).
- [46] T. Branz, T. Gutsche, and V. E. Lyubovitskij, [Phys. Rev. D **80**, 054019 \(2009\)](#).
- [47] Y. Huang, C. j. Xiao, Q. F. L., R. Wang, J. He, and L. Geng, [Phys. Rev. D **97**, 094013 \(2018\)](#).
- [48] Z. w. Lin and C. M. Ko, [Phys. Rev. C **62**, 034903 \(2000\)](#).
- [49] Y. s. Oh, T. Song, and S. H. Lee, [Phys. Rev. C **63**, 034901 \(2001\)](#).
- [50] D. Y. Chen, X. Liu, and T. Matsuki, [Phys. Rev. D **91**, 094023 \(2015\)](#).
- [51] D. Y. Chen, X. Liu, and T. Matsuki, [Eur. Phys. J. C **72**, 2008 \(2012\)](#).
- [52] P. Colangelo, F. De Fazio, and T. N. Pham, [Phys. Lett. B **542**, 71 \(2002\)](#).