Attractor mechanism and nonrenormalization theorem in 6D (1, 0) supergravity

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We compute the macroscopic entropy of the supersymmetric rotating dyonic strings carrying linear momentum in 6D (1, 0) supergravity with curvature squared corrections. Our calculation is based on Sen's entropy function formalism applied to the near-horizon geometry of the string solution taking the form of an extremal Bañados-Teitelboim-Zanelli $\times S^3$. The final entropy formula states that the two independent supersymmetric completions of Riemann tensor squared contribute equally to the entropy. A further S^3 compactification of the 6D theory results in a matter coupled 3D supergravity model in which the quantization condition of the $SU(2)_R$ Chern-Simons level implies the horizon value of the dilaton is not modified by higher derivative interactions beyond supersymmetric curvature squared terms.

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I. INTRODUCTION

The most striking feature hidden in the black hole thermodynamics is the area law of the entropy instead of the usual volume law observed in nearly all local physical systems. Efforts devoted to understanding such an area law of entropy have led to the discovery of the holographic principle of quantum gravity realized concretely in the fruitful framework of AdS/CFT correspondence. One precursor of AdS_3/CFT_2 correspondence is precisely the microstate counting of the Bogomol'nyi-Prasad-Sommerfield (BPS) black holes [1] in string theory. In the infinite charge limit, the logarithm of the corresponding 2D CFT density of states successfully recovers the Bekenstein-Hawking formula. Subsequent studies have extended the agreement between the macroscopic and microscopic entropy to large classes of BPS black holes [2,3] in string and M theory (see [4–7] for nice reviews).

When the charges are large but finite, the matching between the macroscopic and microscopic entropy implies deviations from the area law that are suppressed by inverse powers of the large charges.¹ Astonishingly, for a class of 4D static BPS black holes, it has been demonstrated [9,10]

that the supergravity action supplemented by higher derivative terms does provide the right number of deviations needed to match with its CFT counterpart. Specifically, up to leading-order higher derivative corrections, the macroscopic entropy of the 4D black holes agrees precisely with the CFT result computed in Refs. [3,11]:

$$S_{\text{micro}} = 2\pi \sqrt{\frac{1}{6} \hat{q}_0 (C_{ABC} \hat{p}^A \hat{p}^B \hat{p}^C + c_{2A} \hat{p}^A)},$$

where the coefficients C_{ABC} and c_{2A} encode the geometric data about the internal manifold on which string and M theory is compactified. In particular, c_{2A} labels the coefficients in front of the leading higher derivative terms in the low-energy effective action. Comparison between the BPS black hole entropy and Eq. (1) relies on an implicit map from the conserved charges carried by the black hole to the CFT data $\hat{q}_0$ and $\hat{p}^A$. At leading order, this identification is straightforward. However, when higher derivative interactions are switched on, the relation between the black hole charges and $\hat{q}_0$ and $\hat{p}^A$ becomes less obvious and may be modified by higher derivative interactions as pointed out in Refs. [12,13] for a class of 5D spinning BPS black holes. Supersymmetric higher derivative terms in $D > 4$ introduce another subtlety—the non-gauge-invariant terms like $A_{(1)} \wedge \text{Tr}(R \wedge R)$ in $D = 5$ and $B_{(2)} \wedge \text{Tr}(R \wedge R)$ in $D = 6$ which require a careful treatment regarding their contributions to the macroscopic entropy. In fact, Refs. [12,13] have both computed the macroscopic entropy for the same class of spinning BPS black holes with curvature squared corrections. However, their results match only in the static limit.

¹There can also be logarithmic corrections to the area law. However, they are not present in $\mathcal{N} = 4$ theory [8], which is the relevant context of this paper.

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For a subclass of black holes considered in Refs. [12,13], computing higher derivative corrections to the macroscopic entropy can be performed in a different setup. Specifically, this subclass of BPS black holes corresponds to M-theory branes wrapped around cycles in $K3 \times T^2$, which amount to strings or branes in IIA string theory wrapped on $K3 \times S^1$. In the latter case, the leading α' corrections arise from string one-loop effects [14]. In supersymmetric compactifications of string and M theory, one can first retain only the lower-dimensional supergravity multiplet first and subsequently incorporate matter multiplets. Specific to IIA string theory compactified on $K3$, the 10D string frame action reduces to an effective action of the form $e^{-2\phi}R + R^2 + \dots$, where the R^2 term descends from the R^4 term in $D = 10$. Supersymmetrizing this type of action can be readily performed utilizing the dilaton Weyl multiplet within the framework of off-shell $\mathcal{N} = 2$ supergravity. The resulting supersymmetric R^2 action is relatively simple, independent of the dilaton field. On the other hand, compactifications of 11D supergravity on Calabi-Yau threefolds leads to a 5D supergravity model of the schematic form $R + f(\phi)R^2 + \dots$. In this case, the 5D standard Weyl multiplet is the preferable building block. Rescaling the metric in either case inevitably complicates the curvature squared superinvariants. Therefore, neatness of the higher derivative action selects the suitable off-shell supergravity multiplets. There is the view that, in higher derivative theories, one can always perform field redefinitions under which the structure of the higher derivative terms are not preserved while the physics remains the same. Therefore, the specific form of the higher derivative terms is not crucial. However, this also means that one is free to choose a frame in which the calculation can be done most efficiently. As we will see later, such a convenient frame for studying supersymmetric black holes naturally comes from the off-shell formulation of supergravity, without which the field equations will be much more complicated.

In this work, we shall revisit the BPS black hole entropy in the IIA setup for reasons below. It is recalled that the $K3$ compactification of IIA string preserves 16 supercharges. The supersymmetric curvature squared action consistent with the same amount of supersymmetry was obtained only recently in 6D off-shell supergravity (and its dimensional reduction) based on the dilaton Weyl multiplet [15,16], where the invariance under eight supercharges is manifest. A further S^1 reduction of the 6D model gives rise to the aforementioned 5D model arising from IIA string on $K3 \times S^1$. In practice, we shall uplift the 5D black hole solutions along the S^1 direction to six dimensions and readdress the issue of the macroscopic entropy there. We emphasize that it is totally equivalent to calculate the entropy of black holes in the 5D model based on the dilaton Weyl multiplet, as circle reduction preserves conserved quantities carried by the solution. We prefer the 6D model for computational simplicity, as the 6D supergravity

multiplet consists of fewer fields. In the 6D setup, we will also explore whether different supersymmetric completions of the Riemann tensor squared give the same contribution to the black hole entropy, by virtue of the fact that there is a unique off-shell formulation of 6D (1, 0) supergravity allowing for well-defined two- and higher derivative superinvariants [15–17] (all curvature squared superinvariants based on the 5D dilaton Weyl multiplet were previously obtained in Refs. [18–20]).² This is advantageous to the 5D setup based on the standard Weyl multiplet [12,13], where only one supersymmetric completion of Riemann tensor squared preserving eight supercharges [21] instead of 16 supercharges is known explicitly. Up to date, there exists no rigorous proof that the other independent supersymmetric completion of the Riemann tensor squared based on standard Weyl multiplet, if one exists, contributes equally to the BPS black hole entropy.

In the next section, we introduce the 6D (1, 0) supergravity action equipped with curvature squared superinvariants. For simplicity, we restrict our discussions to NS-NS (Neveu-Schwarz) sector fields that are captured by the dilaton Weyl multiplet. Upon circle reduction, the 6D model reduces to the 5D ungauged STU model extended by curvature squared terms. Without higher derivative corrections, black hole solutions in the 5D STU model are well studied [22,23]. Among them, we single out the three charge rotating black holes obtained in Ref. [22] and lift them to 6D rotating dyonic strings with linear momentum. The BPS limit of the 6D strings requires equal angular momenta. As a consequence, the solution exhibits an enhanced $U(2) \times \mathbb{R}^2$ isometry, based on which the general ansatz underlying the BPS string solution and compatible with the 6D off-shell supersymmetry can be derived. Following a procedure proposed in Ref. [24], one can then apply the ansatz to construct rotating BPS dyonic string solutions with higher derivative corrections. The entropy of the string solution is determined from its near-horizon geometry in the form of an extremal $BTZ \times S^3$. In Sec. III, by extremizing the entropy function defined for the near-horizon geometry of BPS string solutions in R^2 extended 6D (1, 0) supergravity, we are able to express the entropy of the string solution in terms of the near-horizon charges q_i which are conjugate variables of the parameters appearing in the near-horizon profile of the string. However, in order to compare with existing results in the literature, it is necessary to identify the near-horizon charges q_i in terms of conserved charges measured at infinity. In our case, these are the charges characterizing the number of fundamental strings or wrapped NS5 branes, angular momentum of the string, and the linear momentum excitation. The main task

²There exists also a standard Weyl multiplet in $D = 6$. However, the two-derivative supergravity action based on the standard Weyl multiplet is pathological for the lack of a saddle point [17].

of Sec. IV is to find the relation between near-horizon charges and global charges by solving the asymptotically flat half BPS string solutions and zooming into its near-horizon region. Combining the results from Secs. III and IV, we obtain the following interesting results: (i) the size of the extremal BTZ horizon does receive corrections, although previous work [15,16] has verified that the AdS_3 and S^3 radii are not modified by the curvature squared superinvariants; (ii) our entropy formula recovers precisely the result obtained in Ref. [13] specialized to the K3 compactification; (iii) the two independent supersymmetric completions of the Riemann tensor squared contribute equally to the BPS string entropy. We conclude in Sec. V with discussions emphasizing the nonrenormalization of the horizon value of the dilaton beyond the leading α' correction.

II. THE MODEL

In this section, we set up the 6D supergravity model describing the low-energy dynamics of IIA string on K3 with leading α' corrections that are generated by string one-loop effects. In the low-energy effective action, the massless bosonic fields consist of a metric, an antisymmetric two-form, 24 vectors, and 81 scalars. In the framework of 6D (1, 0) off-shell supersymmetry, the metric, antisymmetric two-form, and one scalar denoted as $\{g_{\mu\nu}, B_{\mu\nu}, L\}$ belong to the bosonic sector of the dilaton Weyl multiplet. Henceforth, we will call L the dilaton field which is related to the string coupling via $g_s^2 = 1/\langle L \rangle_{\text{vev}}$. The black hole solutions to be considered later are fully captured by this field content. We thus write out terms in the effective action composed by the dilaton Weyl multiplet. Vectors and hypermultiplet scalars can be consistently truncated out from the field equations. The leading term in the action takes the form

$$\mathcal{L}_{\text{EH}} = \sqrt{-g}L \left(R + L^{-2} \partial_\mu L \partial^\mu L - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right). \quad (2.1)$$

As explained in Ref. [16], the string frame action given above can be coupled to curvature squared superinvariants without modifying the supersymmetry transformation rule. There are three independent curvature squared superinvariants corresponding to the supersymmetrization of the pure Riemann tensor squared [25]

$$\mathcal{L}_{\text{Riem}^2} = \sqrt{-g} \left[R_{\mu\nu\alpha\beta}(\omega_-) R^{\mu\nu\alpha\beta}(\omega_-) - \frac{1}{4} \epsilon^{\mu\nu\rho\sigma\lambda\tau} B_{\mu\nu} R_{\rho\sigma}{}^\alpha{}_\beta(\omega_-) R_{\lambda\tau}{}^\beta{}_\alpha(\omega_-) \right], \quad (2.2)$$

the supersymmetric completion of the Gauss-Bonnet combination

$$\begin{aligned} \mathcal{L}_{\text{GB}} = \sqrt{-g} & \left[R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2 + \frac{1}{6} R H^2 \right. \\ & - R^{\mu\nu} H_{\mu\nu}^2 + \frac{1}{2} R_{\mu\nu\rho\sigma} H^{\mu\nu\lambda} H^{\rho\sigma}{}_\lambda + \frac{5}{24} H^4 + \frac{1}{144} (H^2)^2 \\ & \left. - \frac{1}{8} (H_{\mu\nu}^2)^2 - \frac{1}{4} \epsilon^{\mu\nu\rho\sigma\lambda\tau} B_{\mu\nu} R_{\rho\sigma}{}^\alpha{}_\beta(\omega_+) R_{\lambda\tau}{}^\beta{}_\alpha(\omega_+) \right], \end{aligned} \quad (2.3)$$

and the supersymmetric Ricci scalar squared term whose explicit form can be found in Refs. [16,26]. One can verify that field equations derived from the supersymmetric Ricci scalar squared are proportional to those from the leading-order action (2.1) and, thus, do not modify the solution at the order we are interested in. Moreover, its contribution to the BPS string entropy vanishes [24]. Therefore, the Ricci scalar squared superinvariant will be omitted from later discussions. In the above supersymmetric curvature squared actions, $R_{\mu\nu\rho\sigma}$ is the standard Riemann tensor of the metric, while $R_{\mu\nu}{}^\alpha{}_\beta(\omega_\pm)$ is the curvature defined with respect to the torsionful spin connections $\omega_{\pm\mu}^\alpha{}_\beta$:

$$\begin{aligned} R_{\mu\nu}{}^\alpha{}_\beta(\omega_\pm) &= \partial_\mu \omega_{\pm\nu}^\alpha{}_\beta + \omega_{\pm\mu}^\alpha{}_\gamma \omega_{\pm\nu}^\gamma{}_\beta - (\mu \leftrightarrow \nu), \\ \omega_{\pm\mu}^\alpha{}_\beta &= \omega_\mu^\alpha{}_\beta \pm \frac{1}{2} H_\mu{}^\alpha{}_\beta. \end{aligned} \quad (2.4)$$

The shorthand notations for various contractions of $H_{\mu\nu\rho}$ are defined as

$$\begin{aligned} H^2 &= H_{\mu\nu\rho} H^{\mu\nu\rho}, & H_{\mu\nu}^2 &= H_{\mu\rho\sigma} H_\nu{}^{\rho\sigma}, \\ H^4 &= H_{\mu\nu\sigma} H_{\rho\lambda}{}^\sigma H^{\mu\rho\delta} H^{\nu\lambda}{}_\delta. \end{aligned} \quad (2.5)$$

In summary, we will study the entropy of BPS black holes using the model

$$S_{R+R^2} = \frac{1}{16\pi G_6} \int d^6x \sqrt{-g} \left(\mathcal{L}_{\text{EH}} + \frac{\lambda_1}{16} \mathcal{L}_{\text{Riem}^2} + \frac{\lambda_2}{16} \mathcal{L}_{\text{GB}} \right), \quad (2.6)$$

where the combination with $\lambda_1 = \lambda_2 = \alpha'$ descends from IIA compactified on K3 and enjoys a supersymmetry enhancement. For general values of λ , the effective action (2.6) preserves only eight supercharges. In later discussions, we keep λ general in order to tell apart the contribution to the BPS string entropy from the individual curvature squared superinvariant.

III. MACROSCOPIC ENTROPY FROM EXTREMIZING ENTROPY FUNCTION

The 6D rotating dyonic string [27] that comes from lifting the 5D 3-charge rotating BPS black hole [22] can be put in the form with a manifest $U(2) \times \mathbb{R}^2$ isometry:

$$\begin{aligned}
ds_6^2 &= -a_1^2(r)(dt + \varpi\sigma_3)^2 + a_2^2(r)(dz + A_{(1)})^2 + b(r)^2 dr^2 \\
&\quad + \frac{1}{4}c^2(r)(\sigma_3^2 + d\theta^2 + \sin^2\theta d\phi^2), \\
B_{(2)} &= 2P\omega_2 + d(r)dt \wedge dz + f_1(r)dt \wedge \sigma_3 \\
&\quad + f_2(r)dz \wedge \sigma_3, \\
A_{(1)} &= A_0(r)dt + A_3(r)\sigma_3, \\
L &= L(r),
\end{aligned} \tag{3.1}$$

where in our notation $\sigma_3 = d\psi - \cos\theta d\phi$ and $d\omega_2 = \text{Vol}(S^3)$. The r -dependent functions in (3.1) are given by

$$\begin{aligned}
a_1^2 &= \frac{r^4}{(r^2 + Q_1)(r^2 + Q_2)}, \quad a_2^2 = \frac{r^2 + Q_2}{r^2 + Q_1}, \\
c^2 &= r^2 + P, \quad b^2 = c^2 r^{-2}, \quad \varpi = \frac{\nu}{2r^2}, \\
d &= \frac{r^2}{r^2 + Q_1}, \quad A_0 = \frac{r^2}{r^2 + Q_2}, \quad f_1 = 0, \\
f_2 &= -d\varpi, \quad A_3 = A_0\varpi, \quad L = \frac{r^2}{d(r^2 + P)}.
\end{aligned} \tag{3.2}$$

In the expression above, Q_1 characterizes the number of fundamental string, while Q_2 denotes the linear momentum excitations of the string along the z direction.

The near-horizon limit is attained by enlarging the $r = 0$ region while keeping other parameters fixed. In terms of the new coordinates and parameter defined below,

$$\begin{aligned}
\rho &= r^2, \quad \tau = \frac{2t}{\sqrt{P}Q_1\rho_+}, \quad \tilde{\psi} = \psi + \frac{2\nu}{PQ_1}z, \\
\rho_+^2 &= \frac{Q_2}{Q_1} \left(1 - \frac{\nu^2}{PQ_1Q_2} \right),
\end{aligned} \tag{3.3}$$

the near-horizon geometry can be expressed as an extremal BTZ $\times S^3$:

$$\begin{aligned}
ds_{6,H}^2 &= \frac{P}{4} \left(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2} \right) + \rho_+^2 \left(dz + \frac{\sqrt{P}}{2\rho_+} \rho d\tau \right)^2 \\
&\quad + \frac{P}{4} (\tilde{\sigma}_3^2 + d\theta^2 + \sin^2\theta d\phi^2), \\
B_{(2),H} &= -\frac{P}{4} \cos\theta d\phi \wedge d\tilde{\psi} + \frac{\sqrt{P}\rho_+}{2} \rho d\tau \wedge dz \\
&\quad - \frac{\nu}{2Q_1} dz \wedge d\psi, \quad L_H = \frac{Q_1}{P},
\end{aligned} \tag{3.4}$$

in which $\tilde{\sigma}_3 = d\psi - \cos\theta d\phi$ and evidently the last term in $B_{(2),H}$ is a pure gauge. Locally, the near-horizon geometry preserves eight supercharges [28–30], while the asymptotically flat string solution preserves four supercharges.

In the following, we will adopt the entropy function formalism [6] to compute the macroscopic entropy of the

BPS string solution with higher derivative corrections. The attractor mechanism was initially proposed to compute entropy of extremal black holes in 4D $\mathcal{N} = 2$ supergravities. The procedure summarized here can be readily generalized to AdS₃ by recasting the AdS₃ as U(1) bundled over AdS₂ as we shall do later [see Eq. (3.10)]. Since the near-horizon geometry of a regular extremal black hole is locally AdS₂, it can be parametrized as

$$g_{\mu\nu} dx^\mu dx^\nu = v \left(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2} \right), \quad F_{\rho\tau}^{(i)} = e_i, \quad \phi_i = u_i, \tag{3.5}$$

where “ i ” labels the different vector multiplets. The attractor mechanism states that the near-horizon values $\vec{u}$, $\vec{q}$, and v can be obtained by extremizing the so-called entropy function

$$\mathcal{E}(\vec{u}, \vec{q}, v) = 2\pi(\vec{e} \cdot \vec{q} - f(\vec{u}, \vec{q}, v)) \tag{3.6}$$

at fixed q_i , where $f(\vec{u}, \vec{q}, v)$ is the action evaluated on the geometry (3.5) and q_i are the charges conjugate to the near-horizon variables e_i . In the following, by extremizing the entropy function defined for the near-horizon geometry of BPS string solutions in R^2 extended 6D (1, 0) supergravity, we are able to express the entropy of string solution in terms of q_1 and q_2 . However, in order to compare with existing results in the literature, it is necessary to reexpress the near-horizon charges q_1 and q_2 in terms of charges (Q_i, P, ν) measured at infinity, which we will postpone to the next section.

The near-horizon geometry of the BPS string solution is a direct product of BTZ with S^3 ; we first reduce the 6D theory to $D = 3$. For the purpose of this work, we adopt the minimal ansatz

$$\begin{aligned}
d\hat{s}_6^2 &= ds_3^2 + \frac{P}{4} (\sigma_3^2 + d\theta^2 + \sin^2\theta d\phi^2), \\
\hat{H}_{(3)} &= dB_{(2)} + \frac{P}{4} \sin\theta d\theta \wedge d\phi \wedge d\psi, \quad \hat{L} = L,
\end{aligned} \tag{3.7}$$

which suffices to capture the near-horizon geometry of the BPS string solution even with higher derivative corrections. The fields on the rhs of the equations above are independent of the coordinates on S^3 . Parameter P labels the S^3 radius squared and sets up the physical scale relative to which one can discuss corrections to the leading-order solution, as the 6D model does not have an intrinsic cosmological constant. The dimension reduction of leading two-derivative Lagrangian yields³

³In Ref. [31], it was shown that a more complete consistent S^3 reduction of the two-derivative 6D supergravity coupled to a single chiral tensor multiplet leads to the $\text{SO}(4) \ltimes \mathbb{R}^6$ gauged $\mathcal{N} = 4$ supergravity in $D = 3$. Based on this result, some black string solutions in $D = 6$ were obtained by uplifting solutions in the 3D theory [32].

$$\mathcal{L}_{\text{EH}}^{(3)} = \sqrt{-g}L \left(R - L^{-2} \partial_\mu L \partial^\mu L - \frac{1}{12} |H_{(3)}|^2 + \frac{4}{P} \right),$$

$$H_{(3)} = dB_{(2)}. \quad (3.8)$$

It is straightforward to reduce the supersymmetric curvature squared actions, and the results are

$$\begin{aligned} \mathcal{L}_{\text{Riem}^2}^{(3)} &= \sqrt{-g} \left(4R_{\mu\nu} R^{\mu\nu} - R^2 - \frac{1}{3} H_{(3)} \square H_{(3)} \right. \\ &\quad \left. - \frac{1}{6} R |H_{(3)}|^2 + \frac{1}{48} (|H_{(3)}|^2)^2 \right) + L_{\text{CS}}(\omega_-), \\ \mathcal{L}_{\text{GB}}^{(3)} &= \frac{16}{P} \sqrt{-g} \left(R + \frac{1}{12} |H_{(3)}|^2 \right) + L_{\text{CS}}(\omega_+), \\ L_{\text{CS}}(\omega_-) &= \frac{8}{\sqrt{P}} \left(-\frac{1}{2} \text{tr} \left(\Gamma d\Gamma + \frac{2}{3} \Gamma^3 \right) \right. \\ &\quad \left. - \frac{1}{24} H_{(3)} |H_{(3)}|^2 + \frac{1}{2} H_{(3)} R \right), \\ L_{\text{CS}}(\omega_+) &= \frac{8}{\sqrt{P}} \left(-\frac{1}{2} \text{tr} \left(\Gamma d\Gamma + \frac{2}{3} \Gamma^3 \right) \right. \\ &\quad \left. + \frac{1}{24} H_{(3)} |H_{(3)}|^2 - \frac{1}{2} H_{(3)} R \right), \end{aligned} \quad (3.9)$$

where the parity odd terms are given as three-forms. Next we parameterize the putative metric and two-form

$$\begin{aligned} ds_3^2 &= \frac{\ell^2}{4} \left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} \right) + r_+^2 \left(dz + \frac{\ell \rho}{2r_+} d\tau \right)^2, \\ r_+ &= \frac{\ell}{2e_1}, \quad B_{(2)} = e_2 \rho d\tau \wedge dz, \end{aligned} \quad (3.10)$$

where we choose $\ell > 0$ in our convention. Together with the constant dilaton $L := L_H$, we have four independent variables $\{\ell, e_1, e_2, L_H\}$. Off-shell supersymmetry implies a model-independent relation between e_1 and e_2 :

$$4e_1 e_2 = \ell^2, \quad (3.11)$$

which should be employed after varying the entropy function with respect to $\{\ell, e_1, e_2, L_H\}$. One should also be aware that there is no similar off-shell relation between e_2 and L_H . Although one of the BPS equations relates L to d , e_2 is defined with respect to time τ differing from the original time t by a numerical factor that depends on the detail of the solution.

The entropy function is the Legendre transform of the effective action for the reduced phase space variables $\{\ell, e_1, e_2, L_H\}$. It is constructed as follows. We first parameterize a generic three-dimensional field configuration with a Killing vector ∂_z as

$$\begin{aligned} \hat{g}_{MN} &= g_{mn}(y) dy^m dy^n + \phi(y) (dz + C_m(y) dy^m)^2, \\ \hat{L} &= L(y), \quad \hat{B}_{(2)} = b_m(y) dy^m \wedge dz, \end{aligned} \quad (3.12)$$

where we have used a ‘‘hat’’ to label three-dimensional fields and $m, n = 1, 2$. Note that a term of the form $b(y) dy^1 \wedge dy^2$ in $B_{(2)}$ has been omitted, since it does not contribute to the field strength $H_{(3)}$ which appears in the Lagrangian. In terms of the two-dimensional fields, the action associated with (3.8) and (3.9) can be expressed as

$$S = \int d^2x [\sqrt{-\det g} (\mathcal{L}_0^{(2)} + \mathcal{L}_1^{(2)}) + \mathcal{L}_2^{(2)}], \quad (3.13)$$

where $\sqrt{-\det g} \mathcal{L}_0^{(2)}$ denotes the circle reduction of the two-derivative Lagrangian density (3.8) and $\sqrt{-\det g} \mathcal{L}_1^{(2)}$ is the circle reduction of the diffeomorphism invariant part of the combined higher derivative Lagrangian density $\frac{1}{16} (\lambda_1 \mathcal{L}_{\text{Riem}^2}^{(3)} + \lambda_2 \mathcal{L}_{\text{GB}}^{(3)})$. They are obtained by substituting (3.12) into the corresponding three-dimensional Lagrangian densities. Thus,

$$\begin{aligned} \sqrt{-\det g} [\mathcal{L}_0^{(2)} + \mathcal{L}_1^{(2)}] &= \int dz \sqrt{-\det \hat{g}} [\mathcal{L}_0^{(3)} + \mathcal{L}_1^{(3)}] \\ &= 2\pi \sqrt{-\det \hat{g}} [\mathcal{L}_0^{(3)} + \mathcal{L}_1^{(3)}]. \end{aligned} \quad (3.14)$$

The remaining two-dimensional Lagrangian density $\mathcal{L}_2^{(2)}$ is the dimensional reduction of the pure Lorentz Chern-Simons term in the combined higher derivative Lagrangian above. It is given in Ref. [33] and takes the form

$$\begin{aligned} \mathcal{L}_2^{(2)} &= \frac{K}{2} \pi [R \epsilon^{mn} F_{mn} + \epsilon^{mn} F_{mp} F^{pq} F_{qn}], \\ K &= -\frac{\lambda_1 + \lambda_2}{32\pi \sqrt{P} G_3}, \end{aligned} \quad (3.15)$$

where $F_{mn} = \partial_m C_n - \partial_n C_m$ and $\epsilon^{01} = 1$. Following Ref. [6], we define

$$f(\ell, e_1, e_2, L_H) = \sqrt{-\det g} [\mathcal{L}_0^{(2)} + \mathcal{L}_1^{(2)}] + \mathcal{L}_2^{(2)}, \quad (3.16)$$

evaluated on the ansatz (3.10). Its explicit form is given by

$$\begin{aligned} f(\ell, e_1, e_2, L_H) &= \frac{1}{8G_3} \left[L_H \left(\frac{4e_1 e_2^2}{\ell^3} + \frac{\ell^3}{2e_1 P} - \frac{3\ell}{4e_1} \right) \right. \\ &\quad + \lambda_1 \left(\frac{24e_1^3 e_2^4}{\ell^9} - \frac{3e_1 e_2^2}{\ell^5} + \frac{3}{32e_1 \ell} \right. \\ &\quad \left. + \frac{8e_1^2 e_2^3}{\ell^6 \sqrt{P}} - \frac{3e_2}{2\ell^2 \sqrt{P}} - \frac{1}{4e_1 \sqrt{P}} \right) \\ &\quad + \lambda_2 \left(\frac{3e_2}{2\ell^2 \sqrt{P}} - \frac{8e_1^2 e_2^3}{\ell^6 \sqrt{P}} - \frac{3\ell}{4e_1 P} \right. \\ &\quad \left. \left. - \frac{4e_1 e_2^2}{\ell^3 P} - \frac{1}{4e_1 \sqrt{P}} \right) \right]. \end{aligned} \quad (3.17)$$

Then the entropy function is defined as [6]

$$\mathcal{E} = 2\pi(e_1 q_1 + e_2 q_2 - f(\ell, e_1, e_2, L_H)), \quad (3.18)$$

where q_1 and q_2 are conserved charges conjugate to e_1 and e_2 , respectively. In order to compute the black hole entropy, we must extremize the entropy function for fixed q_1 and q_2 . This is achieved by solving

$$\frac{\partial \mathcal{E}}{\partial \ell} = 0, \quad \frac{\partial \mathcal{E}}{\partial e_1} = 0, \quad \frac{\partial \mathcal{E}}{\partial e_2} = 0, \quad \frac{\partial \mathcal{E}}{\partial L_H} = 0. \quad (3.19)$$

Since we are interested in supersymmetric solutions subject to the extremization equations (3.19), we will impose the BPS equation (3.11) in solving these equations. We emphasize that the extremization equations are derived before imposing the supersymmetry constraint (3.11). We find that the first equation is solved by

$$\ell = \sqrt{P}, \quad (3.20)$$

which together with (3.11) yields the solution to the second equation in (3.19):

$$e_1 = \frac{1}{4} \sqrt{\frac{L_H P + \lambda_1 + \lambda_2}{G_3 \ell q_1}}. \quad (3.21)$$

In turn, the solution for e_1 and the supersymmetry condition (3.11) determines e_2 . The solution to the third equation in (3.19) yields

$$L_H = \frac{4G_3 \ell^3 q_2 + \lambda_2}{P}. \quad (3.22)$$

The last equation in (3.19) is automatically satisfied once $\ell = P$ and the condition (3.11). Finally, substituting the above solutions for ℓ , e_1 , e_2 , and L_H into the entropy function (3.18), we obtain

$$\mathcal{E} = \pi \sqrt{\frac{q_1 (4G_3 \ell^3 q_2 + \lambda_1 + 2\lambda_2)}{G_3 \ell}}. \quad (3.23)$$

The expression above can be recast in a form similar to the Cardy formula

$$\mathcal{E} = 2\pi \sqrt{\frac{c q_1}{6}}, \quad c = \frac{3}{2G_3 \ell} (4G_3 \ell^3 q_2 + \lambda_1 + 2\lambda_2). \quad (3.24)$$

From the dual 2D CFT point of view, the conserved charge q_1 is naturally interpreted as the effective energy level [4] inert under the higher derivative corrections [34]. There-

fore, it can be inferred from the known solution in the two-derivative theory using (3.21). The result is given by

$$q_1 = Q_2 - \frac{\nu^2}{P Q_1}, \quad (3.25)$$

in which the parameters P , Q_1 , Q_2 , and ν are related to conserved charges carried by the rotating dyonic string and are integer valued in string units. The conserved charge q_2 , however, does receive higher derivative corrections generically as it enters the central charge. Thus, its value must be read off from the higher derivative corrected solutions. We will do so in the next section. The simplest way is via computing the horizon value of the dilaton and employing (3.22).

From (3.10) and (3.21), we can solve r_+ in terms of q_1 :

$$r_+^2 = \frac{4\ell^3 G_3 q_1}{L_H P + \lambda_1 + \lambda_2}. \quad (3.26)$$

As shown in the next section, the value of L_H increases when the higher derivative interactions are present. Therefore, the size of the extremal BTZ horizon shrinks due to higher derivative interactions, although the AdS_3 radius remains the same.

IV. THE RENORMALIZED HORIZON VALUE OF DILATON

This section is devoted to compute the value of dilaton curvature squared corrections, using which we can obtain the conserved charge q_2 from (3.22). For this purpose, the ansatz (3.1) for a BPS string solution based on the $U(2) \times \mathbb{R}^2$ isometry is still applicable, since the higher derivative extended field equations will not generate terms violating this symmetry. Supersymmetry requires the undetermined functions in (3.1) to obey certain relations so that the corresponding Killing spinor equations

$$\begin{aligned} 0 &= \left(\partial_\mu + \frac{1}{4} \omega_{\alpha\beta} \gamma^{\alpha\beta} \right) \epsilon^i + \frac{1}{8} H_{\mu\nu\rho} \gamma^{\nu\rho} \epsilon^i, \\ 0 &= \frac{1}{2\sqrt{2}} \gamma^\mu \delta^{ij} \partial_\mu L \epsilon_j - \frac{1}{12\sqrt{2}} L \delta^{ij} \gamma_{\mu\nu\rho} H^{\mu\nu\rho} \epsilon_j \end{aligned} \quad (4.1)$$

admit nontrivial solutions for the symplectic Majorana-Weyl spinor ϵ^i . The indices i and j are raised and lowered by ϵ^{ij} and ϵ_{ij} . We have also set auxiliary fields to 0 consistent with their field equations [24]. For convenience, we introduce the complex Weyl spinor

$$\epsilon = \epsilon_1 + i\epsilon_2 \quad (4.2)$$

and assume the Killing spinor to have the form

$$\epsilon = \Pi(r) \epsilon_0, \quad (4.3)$$

where ϵ_0 is the standard Killing spinor on a round 2-sphere embedded in the 6D spinor obeying the projection conditions

$$\gamma^{012345}\epsilon_0 = -\epsilon_0, \quad \gamma^{01}\epsilon_0 = -\epsilon_0. \quad (4.4)$$

Plugging the ansatz for the bosonic fields and Killing spinor into (4.1), we obtain the sufficient and necessary conditions for the existence of a Killing spinor:

$$0 = A_0(a_2^2(c\dot{\varpi}\dot{A}_0 - c\dot{A}_3 + 2A_3b) - c\dot{\varpi}\dot{d}) + c(a_1^2\dot{\varpi} + A_3\dot{d} - \dot{f}_1) + 2b(f_1 - a_1^2\varpi), \quad (4.5)$$

$$0 = a_1(a_2^2\dot{A}_0 + 2a_2(A_0\dot{a}_2 + \dot{a}_1) + \dot{d}) + A_0a_2(\dot{d} - a_2^2\dot{A}_0), \quad (4.6)$$

$$0 = a_2^2(c(\dot{\varpi}\dot{A}_0 - \dot{A}_3) + 2A_3b) + 2bf_2 - c(\dot{f}_2 + \dot{\varpi}\dot{d}), \quad (4.7)$$

$$0 = -a_2^2\dot{A}_0 + 2a_1\dot{a}_2 + \dot{d}, \quad (4.8)$$

$$0 = b(4A_0f_2\varpi - 4A_3f_2 + c^2 - 4f_1\varpi - P) - c^2\dot{c}, \quad (4.9)$$

$$0 = -a_2^3A_3\dot{A}_0 + a_2(-A_0\dot{f}_2 + 2a_1(\dot{\varpi}\dot{a}_1 + A_3\dot{a}_2) + a_1^2\dot{\varpi} + \dot{f}_1) + a_1a_2^2\dot{A}_3 - a_1\dot{f}_2, \quad (4.10)$$

$$0 = a_2(f_1 - A_0f_2) + a_1(a_2^2A_3 - f_2) + a_1^2a_2\varpi, \quad (4.11)$$

$$0 = \frac{8A_0bf_2\varpi}{c^3} - \frac{\dot{d}}{a_1a_2} - \frac{8A_3bf_2}{c^3} - \frac{8bf_1\varpi}{c^3} - \frac{2Pb}{c^3} + \frac{\dot{L}}{L}, \quad (4.12)$$

$$0 = \Pi(r) - \sqrt{a_1}, \quad (4.13)$$

where dots denote derivatives with respect to r . The third last equation (4.11) is algebraic whose r derivative is preserved on solutions to Eqs. (4.6), (4.8), and (4.10). Equations (4.6) and (4.8) together yield two first integrations

$$A_0 + \frac{a_1}{a_2} = \kappa_1, \quad d + a_1a_2 = \kappa_2, \quad (4.14)$$

in which κ_2 can be set to 0 using the residual gauge symmetry of $B_{(2)}$. We can also choose $\kappa_1 = 0$ by shifting the z coordinate and redefining f_2 .⁴ Subsequently, applying the first equation in (4.14) to (4.10), we find another first integration:

$$a_1a_2A_3 + a_1^2\varpi + f_1 = \kappa_3. \quad (4.15)$$

⁴For the rotating dyonic string solution without linear momentum [24], $\kappa_1 = 1$ is the more common choice.

The equation above together with (4.11) forces $\kappa_3 = 0$. From (4.9) and (4.12), we also find

$$|d|Lc^2 = r^2, \quad (4.16)$$

after choosing the r coordinate so that $c = br$. Using (4.14) and (4.15), we obtain two more first integrations from Eqs. (4.5) and (4.7):

$$f_2 + d\varpi = r^2\kappa_4, \quad A_3 = A_0\varpi + r^2\kappa_5, \quad (4.17)$$

where κ_4 and κ_5 will be set to 0 in order for the solution to obey the asymptotically flat boundary condition. Substituting (4.17) to Eqs. (4.15) and (4.9), we obtain

$$f_1 = 0, \quad c^2 = r^2 + P. \quad (4.18)$$

In summary, the off-shell Killing spinor equations for the asymptotically flat rotating dyonic string ansatz are fully solved provided the following relations are satisfied:

$$A_0 + \frac{a_1}{a_2} = 0, \quad d + a_1a_2 = 0, \quad c^2 = r^2 + P, \quad c = br, \\ |d|Lc^2 = r^2, \quad f_1 = 0, \quad f_2 = -d\varpi, \quad A_3 = A_0\varpi, \quad (4.19)$$

which indicates that the horizon value of L is determined by the near-horizon behavior of d . This is a significant simplification, as the d equation is associated with a conservation law and already admits a first integration.

In terms of the new radial coordinate $\rho = r^2$ and applying the set of relations (4.19), we read off the d equation from the (t, z) component of the $B_{(2)}$ field equation. It turns out to be independent of other undetermined variables in the ansatz and can be integrated twice so that the resulting equation is only second order in ρ derivative. Imposing the asymptotically flat boundary condition, one can set one of the integration constants to 0, and the d equation reads

$$(\rho - (\rho + Q_1)d)d + \frac{(\lambda_2 - \lambda_1)P^2}{2(P + \rho)^2}d^2 \\ - \frac{2\rho\lambda_1 + (\lambda_1 + \lambda_2)P}{P + \rho}\rho dd' + \lambda_1\rho^2d'^2 - \lambda_1\rho^2dd'' = 0, \quad (4.20)$$

where the prime denotes the ρ derivative. The simple form of the d equation has benefited from the precise structure of the off-shell supersymmetric actions. Had we performed a field redefinition, this simplification would be immediately lost. In the context of string compactification, λ_1 and λ_2 are of the same order much smaller than P or Q_1 . The equation above can thus be solved order by order algebraically in the small parameters λ . Here, only the corrections at the first order in λ are meaningful, because higher-order terms will

be modified by R^n ($n > 2$) interactions which are not included in our discussion. To the first order in λ s, the solution of the d equation is given by

$$d(\rho) = \frac{\rho}{\rho + Q_1} \left[1 - \frac{P^2 Q_1^2 (\lambda_1 + \lambda_2) + 2P Q_1^2 (\lambda_1 + \lambda_2) \rho + (Q_1^2 (\lambda_1 + \lambda_2) + (P - Q_1)^2 (\lambda_1 - \lambda_2)) \rho^2}{2(P + \rho)^2 (Q_1 + \rho)^3} \right], \quad (4.21)$$

from which one can see that corrections to the leading-order solution stay small within the entire range of ρ as long as λ are much less than P or Q_1 . In the special case $\lambda_1 = \lambda_2$, the combination of the curvature squared superinvariants is consistent with 16 supercharges, and the solution is independent of P . Meanwhile the conserved charge q_2 (3.22) is not modified by the higher derivative interactions, which is physically more reasonable as the curvature squared terms do not modify the conserved charges carried by the asymptotically flat dyonic string solution. The equal charge case $P = Q_1$ corresponds to a Breckenridge-Myers-Peet-Vafa black hole [2], and the solution also looks quite simple depending only on the sum of λ 's. Near the string horizon at $\rho = 0$,

$$d(\rho) = \frac{\rho}{Q_1} \left(1 - \frac{\lambda_1 + \lambda_2}{2Q_1} \right) + \mathcal{O}(\rho^2). \quad (4.22)$$

Combining this result with the BPS equation relating d and L , we obtain the horizon value of the dilaton with curvature squared corrections:

$$L_H = \frac{Q_1 + \frac{1}{2}(\lambda_1 + \lambda_2)}{P} \Rightarrow q_2 = \frac{1}{4G_3 \ell^3} \left(Q_1 + \frac{1}{2}(\lambda_1 - \lambda_2) \right). \quad (4.23)$$

Substituting the result above to (3.24), we complete the calculation of the macroscopic entropy for the higher derivative corrected rotating BPS dyonic string solution. The final entropy formula takes the form

$$\mathcal{E} = 2\pi \sqrt{\frac{c q_1}{6}}, \quad c = \frac{3}{2G_3 \ell} \left(Q_1 + \frac{3}{2}(\lambda_1 + \lambda_2) \right), \quad q_1 = Q_2 - \frac{\nu^2}{P Q_1}. \quad (4.24)$$

The sum of λ 's is related to the total coefficient in front of the $B_{(2)} \wedge \text{Tr}(R \wedge R)$ term or the Riemann tensor squared. It also means each independent supersymmetric completion of the Riemann tensor squared contributes equally to the entropy. In the IIA string embedding, $\lambda_1 + \lambda_2$ is given by $2\alpha'$ [14]. Since entropy is a dimensionless quantity, it does not depend on the choice of units. In particular, we can choose the string units in which the string length $\ell_s = 1$. Also, we recall from Ref. [24] that in the string units

$1/G_3 = 4P^{3/2}$. Therefore, the central charge is equal to $c = 6P(Q_1 + 3)$, indicating that the macroscopic entropy with leading-order higher derivative correction is of the form

$$\mathcal{E} = 2\pi \sqrt{P Q_1 Q_2 - \nu^2} \left(1 + \frac{3}{2Q_1} \right), \quad (4.25)$$

which is consistent with the result obtained in Ref. [13] specialized to the $K3 \times T^2$ compactification. One should also be aware that, in the string units, the parameters P , Q_1 , Q_2 , and ν are all integer valued [22]. The results obtained here and in Ref. [13] agree with the one reported in Ref. [12] only in the static case $\nu = 0$. It should be interesting to connect the macroscopic result to the microscopic one in certain asymptotic limits [35,36].

V. CONCLUSION AND DISCUSSIONS

In this work, we computed the macroscopic entropy of the supersymmetric rotating dyonic strings in 6D (1, 0) supergravity with curvature squared corrections, adopting Sen's entropy function formalism applied to the near-horizon geometry of the string. Upon solving the attractor equations whose solution extremizes the entropy function, we observed that inclusion of the higher derivative interactions does not modify the AdS_3 radius of the BTZ black hole but does shrink the size of the extremal horizon. Expressing the extremized entropy function in terms of the conserved charges carried by the string requires the knowledge of the horizon value of the dilaton which can be extracted only from the complete asymptotically flat string solution. We also showed that, although there are two independent supersymmetric completions of the Riemann tensor squared, the entropy is insensitive to their detailed structures but depends only on the coefficient of the Riemann squared term fixed to be $\alpha'/8$ upon embedding the 6D supergravity model in the $K3$ compactification of IIA string.

Via a circle reduction, the rotating dyonic string becomes the three-charge spinning black hole in the 5D ungauged STU model. Thus, our result can be compared to previous work [12,13] on the macroscopic entropy of supersymmetric black holes in 5D $\mathcal{N} = 2$ supergravity with curvature squared corrections descending from M theory compactified on $K3 \times T^2$. Our entropy formula is in

agreement with the result obtained in Ref. [13] but exhibits a different angular momentum dependence from the result reported in Ref. [12]. In the future, we hope to compute the microscopic entropy of the dual CFT₂ and compare it to the macroscopic result.

String dualities relate IIA string on K3 to heterotic string on T^4 in which the black hole entropy with α' corrections was previously obtained in Refs. [37,38].⁵ A comparison between the entropy of black holes in the heterotic and IIA strings shows that, once α' corrections are taken into account, the black hole entropy is not invariant under the duality transformation.

Locally, the extremal $\text{BTZ} \times S^3$ is equivalent to $\text{AdS}_3 \times S^3$, around which the spectrum of fluctuations has been analyzed in Refs. [15,16]. The fact that the spectrum can be organized into representations of $\text{SU}(1,1|2) \times \text{SU}(1,1) \times \text{SU}(2)$ [15,16] suggests that the S^3 compactification of the 6D model can be described by the matter coupled 3D supergravity model based on the $\text{SU}(1,1|2) \times \text{SU}(1,1)$ algebra. Omitting the curvature squared terms, the action for the massless fields in the supergravity multiplet can be formulated as a Chern-Simons gauge theory. After integrating out auxiliary fields, it takes the form [6,39,40]

$$\begin{aligned} S_{3d} = & \int d^3x \left[k_G \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) \right. \\ & \left. + k_{\text{CS}} \Omega(\Gamma) - (\ell k_G - k_{\text{CS}}) \Omega(A_R) \right], \\ \Omega(\Gamma) = & \frac{1}{2} \text{Tr} \left(\Gamma d\Gamma + \frac{2}{3} \Gamma^3 \right), \\ \Omega(A_R) = & \text{Tr} \left(A_R dA_R + \frac{2}{3} A_R^3 \right), \end{aligned} \quad (5.1)$$

⁵In the recent work [37] revisiting the α' -corrected black hole in heterotic string, the coefficient in front of the supersymmetric black hole should be $\alpha'/8$ instead of α' .

where A_R is the gauge field associated with the $\text{SU}(2)$ inside $\text{SU}(1,1|2)$ and various constants are given by

$$\begin{aligned} k_G = & \frac{PL_H + \alpha'}{16\pi G_3 P} = \frac{\sqrt{P}(Q_1 + 2)}{4\pi}, \\ k_{\text{CS}} = & -\frac{\alpha'}{16\pi G_3 \sqrt{P}} = -\frac{P}{4\pi}. \end{aligned} \quad (5.2)$$

One feature is that the effective Newton's constant depends on the value of dilaton at the string horizon that is affected by the higher derivative interaction via equations of motion. The level of the $\text{SU}(2)_R$ Chern-Simons gauge field is proportional to the central charge appearing in the entropy formula

$$k_R = -\frac{P(Q_1 + 3)}{4\pi}. \quad (5.3)$$

The fact that the quantity above obeys the quantization condition $4\pi k_R = n$, $n = 0, \pm 1, \dots$ [41], implies that the higher derivative corrections to the horizon value of L can be at most quadratic in α' . For instance, α'^2/P^2 may contribute a P -independent integer to $4\pi k_R$. Based on dimension analysis, such a term would come from a supersymmetric R^3 action which is forbidden in both heterotic string and type II string [42]. Therefore, the value of dilation on the string horizon does not receive corrections from higher derivative interactions beyond order α' .

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