Polarization effects in the search for a dark vector boson at e^+e^- colliders

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We argue that the search for dark vector boson through $e^+e^- \rightarrow Z_d\gamma$ can determine the Lorentz structure of $Z_d l^+ l^-$ couplings with the detection of leptonic decays $Z_d \rightarrow l^+ l^-$. We assume a general framework that the dark vector boson interacts with ordinary fermions through vector and axial-vector couplings. As a consequence of Ward-Takahashi identity, Z_d is transversely polarized in the limit $m_{Z_d} \ll \sqrt{s}$. On the other hand, the fraction of longitudinal Z_d is non-negligible for m_{Z_d} comparable to $\sqrt{s}$. Such polarization effects can be analyzed through angular distributions of final-state particles in Z_d decays. Taking $l^\pm \equiv \mu^\pm$, we study the correlation between Z_d angle relative to e^- beam direction in e^+e^- center of momentum frame and μ^- angle relative to the boost direction of Z_d in Z_d rest frame. This correlation is shown to be useful for probing the Lorentz structure of $Z_d l^+ l^-$ couplings. We discuss the measurement of such correlation in Belle II detector, taking into account the detector acceptance and energy resolution.

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I. INTRODUCTION

Searching for dark matter (DM) is one of the major endeavors in the present day particle physics community. The efforts in direct and indirect detections as well as productions of DMs in LHC so far have not produced positive results. Recently, there are growing interests to search for DM related phenomena with huge statistics and high precision measurements. These phenomena involve the hidden sector [1–6], which is assumed to interact with Standard Model (SM) particles through certain messengers. A popular proposal for such a messenger particle is the so-called dark photon, which mixes with the $U(1)$ hypercharge field B_μ in SM,

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2}\frac{\varepsilon_\gamma}{\cos\theta_W}B_{\mu\nu}A'^{\mu\nu} - \frac{1}{4}A'_{\mu\nu}A'^{\mu\nu}, \quad (1)$$

where A'_μ is the dark photon field and $A'_{\mu\nu} \equiv \partial_\mu A'_\nu - \partial_\nu A'_\mu$. The above mixing induces electromagnetic couplings, $\mathcal{L}_{\text{em}} = \varepsilon_\gamma e J_{\text{em}}^\mu A'_\mu$, between the dark photon and SM fermions, which generate rich phenomenology [7]. On the other hand, the neutral current couplings between the same set of particles are further suppressed by the factor $m_{A'}^2/m_Z^2$ for $m_{A'} \ll m_Z$ with $m_{A'}$ the dark photon mass. However,

independent neutral current couplings can be generated through mass mixing between the messenger particle and the Z boson [8–10]. In this case, the messenger particle is often referred to as Z' boson. The mass mixing term $\delta m^2 Z'_\mu Z^\mu$ can induce neutral current couplings $\mathcal{L}_{\text{NC}} = (g\varepsilon_Z/\cos\theta_W)J_{\text{NC}}^\mu Z'_\mu$ with $\varepsilon_Z \equiv \delta m^2/m_Z^2$. For a general scenario that both kinetic and mass mixings are present, the interactions between dark boson and SM fermions are given by

$$\mathcal{L}_{\text{int}} = \left(\varepsilon_\gamma e J_{\text{em}}^\mu + \varepsilon_Z \frac{g}{\cos\theta_W} J_{\text{NC}}^\mu \right) Z_{d,\mu}, \quad (2)$$

with Z_d the dark boson, which is the generalization of A' and Z' .¹

We note that the physical masses of Z_d and Z are given by diagonalizing of the following mass matrix:

$$\mathbf{M}_0^2 = \begin{pmatrix} m_{Z^0}^2 & -\varepsilon_Z m_{Z^0}^2 \\ -\varepsilon_Z m_{Z^0}^2 & m_{Z_d}^2 \end{pmatrix} + \begin{pmatrix} 0 & -\varepsilon_\gamma \tan\theta_W m_{Z^0}^2 \\ -\varepsilon_\gamma \tan\theta_W m_{Z^0}^2 & (2\varepsilon_\gamma \varepsilon_Z \tan\theta_W m_{Z^0}^2 + \varepsilon_\gamma^2 \tan^2\theta_W m_{Z^0}^2) \end{pmatrix}, \quad (3)$$

where the second term arises from the diagonalization of kinetic Lagrangian $\mathcal{L}_{\text{gauge}}$. To ensure the positivity of

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¹Strictly speaking, ε_Z in Eq. (2) is $\delta m^2/m_{Z^0}^2$ with m_{Z^0} the Z boson mass before mass matrix diagonalization. Nevertheless, for numerical calculations, we take $\varepsilon_Z \equiv \delta m^2/m_Z^2$ as the correction is of higher orders.

physical masses, one requires the determinant of $\mathbf{M}_0^2$ be positive, i.e., $\det(\mathbf{M}_0^2) = (m_{Z_d}^2 m_{Z_0}^2 - (\delta m^2)^2) > 0$. We note that the second term of the mass matrix does not give additional contribution to the determinant. This condition constrains models that generate masses for Z , Z_d , and the mass mixing between the two. For our interested scenario $\varepsilon_Z \simeq \varepsilon_\gamma$ with the latter in the range $(10^{-3}-10^{-4})$, i.e., current limits from dark photon searches at e^+e^- colliders, $m_{Z_d}^0$ should be greater than $(10-100)$ MeV. In Ref. [9], two Higgs doublets and one scalar Higgs singlet are introduced such that $m_{Z_0}^2 = g^2(v_1^2 + v_2^2)/(4\cos^2\theta_W)$, $m_{Z_d}^0 = g_d^2(v_2^2 + v_d^2)$, and $\delta m^2 = g_d g v_2^2/(2\cos\theta_W)$. Here g_d is the dark boson gauge coupling to the second Higgs doublet and the scalar Higgs singlet, while v_1 (v_2) and v_d are vacuum expectation values of neutral scalar in the first (second) Higgs doublet and that of the scalar Higgs singlet. The above-mentioned lower limit on m_{Z_d} requires $g_d > 4 \times (10^{-5}-10^{-4})$ if $(v_d^2 + v_2^2) \simeq (v_1^2 + v_2^2) \simeq (246 \text{ GeV})^2$.

The search for the light vector boson with the reaction $e^+e^- \rightarrow Z_d \gamma$ has been proposed before [11–13]. Particularly, there exist phenomenological studies on dark sectors under the environment of e^+e^- colliders [14–22]. Along this line, the experimental searches for Z_d proceed through the detection of visible and invisible Z_d decays. The visible mode requires a full reconstruction of Z_d peak through measuring the energy and momentum of lepton or light hadron pairs from Z_d decays [23–28], while the invisible mode looks for the peak of missing mass at m_{Z_d} given by $M_{\text{mass}}^2 = (P_{e^-} + P_{e^+} - P_\gamma)^2$ [29]. We note that both phenomenological and experimental studies mentioned above consider only the dark photon scenario, i.e., Z_d interacts with SM fermions only via electromagnetic current. On the other hand, since neutral-current coupling is also possible, it is of great importance to simultaneously detect Z_d and measure the Lorentz structure of its coupling to SM fermions. To determine the relative strengths of vector and axial-vector couplings, such as the ratio $g_{f,A}/g_{f,V}$ in the generic structure $e\bar{e}\bar{f}(g_{f,V}\gamma_\mu + g_{f,A}\gamma_\mu\gamma_5)fZ_d^\mu$, it is necessary to measure the angular distributions of final-state fermions from Z_d decays.²

The dark vector boson Z_d produced by $e^+e^- \rightarrow Z_d \gamma$ is polarized. In fact, Z_d must be in one of the transversely polarized states in the limit $\sqrt{s} \gg m_{Z_d}$. This is a direct consequence of Ward-Takahashi identity [30] to be elaborated in the next session. Furthermore, with the presence of both $g_{f,A}$ and $g_{f,V}$, parity symmetry is broken. Hence, there exists a forward-backward asymmetry for the production of each transversely polarized Z_d state, while the production of longitudinal Z_d is forward-backward symmetric. The magnitude of the above asymmetry is directly

related to the degree of parity violation, characterized by the parameter $\rho \equiv 4g_{f,A}g_{f,V}$ under the normalization $g_{f,V}^2 + g_{f,A}^2 = 1$. For a fixed ρ , the asymmetry reaches to the maximum for $m_{Z_d}/\sqrt{s} \rightarrow 0$. Besides the asymmetry in the production of transversely polarized Z_d state, there is also forward-backward asymmetry for the angular distributions of final-state fermions from Z_d decays, which is also controlled by the same parameter ρ . Hence, the correlation between these two asymmetries can be exploited to probe ρ .

The most sensitive search for Z_d through the visible mode $e^+e^- \rightarrow Z_d \gamma \rightarrow e^+e^-\gamma, \mu^+\mu^-\gamma$ is performed by BABAR [24]. Using 514 fb^{-1} of data, the upper limits on the mixing parameter ε is $10^{-4}-10^{-3}$ for m_{Z_d} between 0.02 and 10.2 GeV. Comparable sensitivity to ε is expected at Belle II [31–34] with 500 fb^{-1} of integrated luminosity. Belle II is an electron-positron collider experiment running at the SuperKEKB accelerator. It is a next-generation B-factory experiment aiming to record a data set of 50 ab^{-1} . In this paper, we focus on the prospect of detecting Z_d and measuring the parity violation parameter in its interaction with SM fermions with $e^+e^- \rightarrow Z_d \gamma$ followed by $Z_d \rightarrow \mu^+\mu^-$ decay at Belle II. Backgrounds to this process are QED process $e^+e^- \rightarrow \mu^+\mu^-\gamma^3$ and the resonant production process $e^+e^- \rightarrow \gamma X$ [$X = J/\psi, \psi(2S), \Upsilon(1S), \Upsilon(2S)$] followed by $X \rightarrow \mu^+\mu^-$. We will not consider the decay mode $Z_d \rightarrow e^+e^-$ in this paper since backgrounds to this mode are more complicated, including $e^+e^- \rightarrow e^+e^-(\gamma)$ and $e^+e^- \rightarrow \gamma\gamma(\gamma)$. Since we are mainly interested in probing the parity violation effect by $Z_d f\bar{f}$ coupling, the study on $Z_d \rightarrow \mu^+\mu^-$ is sufficient to make our point. We note that there are recent interests in the signals for 17 MeV protophobic vector boson [35–37] motivated from anomalies in ^8Be and ^4He nuclear transitions [38,39]. Searching for vector boson in e^+e^- colliders for this particular parameter range has been proposed [40,41]. Although we shall not focus on such a specific scenario, we do notice that the protophobic vector boson interacts with the electron through both vector and axial-vector couplings. However, parity violation effects resulting from the presence of both couplings were not considered in the above analyses. For an earlier candidate of light neutral gauge boson [42], its parity-violating effects to low energy neutral current processes were studied quite some time ago [43]. Among those low energy processes, the search for atomic parity violations [44,45] still attracts high attentions in recent years. For phenomenological discussions on this issue under the current dark boson scenario, see Refs. [9,46].

²Here we choose the normalization $g_{f,V}^2 + g_{f,A}^2 = 1$.

³Here we neglect the Z boson exchange diagrams since their entire contribution to the total cross section is less than 1% from our numerical studies.

This paper is organized as follows. In Sec. II, we present the polarized differential cross section of $e^+e^- \rightarrow Z_d \gamma$ for different Z_d polarizations. For $\sqrt{s} \gg m_{Z_d}$, we show that the production of longitudinal Z_d is suppressed due to Ward-Takahashi identity, i.e., Z_d is transversely polarized in such a limit. In Sec. III, we discuss the method for probing the parity violation parameter ρ in e^+e^- colliders. We present angular distributions of leptons arising from polarized Z_d decays. Combining with angular distributions of Z_d in production process, we construct the double angular distribution for the signal process $e^+e^- \rightarrow \gamma Z_d \rightarrow \gamma l^+ l^-$. It will be shown that this double angular distribution depends on ρ^2 rather than ρ . We bin the signal events according to the sign of $J \equiv \cos \theta \times \cos \theta_d$ where θ is the angle of Z_d with respect to the e^- direction in e^+e^- center of momentum (CM) frame while θ_d is the helicity angle of lepton arising from Z_d decay. The asymmetry $\mathcal{A}_{\text{PN}} \equiv (S(J > 0) - S(J < 0)) / (S(J > 0) + S(J < 0))$ with S the number of signal events will be shown to be proportional to ρ^2 , so that it directly reflects the degree of parity violation. In Sec. IV, event numbers of $e^+e^- \rightarrow \gamma \mu^+ \mu^-$ from signal and background are calculated with specific integrated luminosity in Belle II detector, taking into account the detector acceptance and energy resolutions. We also calculate the asymmetry parameter $\mathcal{A}_{\text{PN}}$ which depends on the detector acceptance. It will be shown that the simultaneous fitting to $J > 0$ and $J < 0$ event bins should improve the significance of dark boson detection from simply counting the

total event excess. The degree of improvement is closely related to $\mathcal{A}_{\text{PN}}$. In addition, it is possible to measure $\mathcal{A}_{\text{PN}}$ in Belle II detector. We estimate the statistical errors for such measurements under Belle II design integrated luminosity. We summarize and conclude in Sec. V.

II. THE POLARIZED DARK BOSON PRODUCTION CROSS SECTION

A. Ward-Takahashi identity and the polarization of Z_d

Let us write the amplitude for $e^-(p_1) + e^+(p_2) \rightarrow Z_d(k_1) + \gamma(k_2)$ as $\mathcal{M} \equiv \mathcal{M}_\mu \epsilon^\mu(k_1)$ with $\epsilon^\mu(k_1)$ the polarization vector of Z_d . Here $\mathcal{M}_\mu$ contains the photon polarization vector. In the case that Z_d is longitudinal, one has $\epsilon^\mu(k_1) = (|\vec{k}_1|, E_{Z_d} \hat{k}_1) / m_{Z_d}$. In the limit that $\sqrt{s} \gg m_{Z_d}$, i.e., Z_d is ultrarelativistic, one has $\epsilon_{Z_d}^\mu = k_1^\mu / m_{Z_d} + \mathcal{O}(m_{Z_d} / E_{Z_d})$. Hence, $\mathcal{M} = \mathcal{M}_\mu k_1^\mu / m_{Z_d} + \mathcal{O}(m_{Z_d} / E_{Z_d})$. However, $\mathcal{M}_\mu k_1^\mu = 0$ in the limit $m_e \rightarrow 0$ as implied by Ward-Takahashi identity [30]. Therefore, the amplitude for a longitudinal polarized Z_d is of the order $m_{Z_d} / \sqrt{s}$.

B. Explicit demonstration of $\mathcal{M}_\parallel$ suppression

The square of $e^-(p_1) + e^+(p_2) \rightarrow Z_d(k_1) + \gamma(k_2)$ amplitude for a given Z_d polarization can be expressed as follows:

$$\begin{aligned} \frac{|\tilde{\mathcal{M}}|^2}{4} = & 16\pi^2 \alpha^2 \epsilon^2 (g_{f,V}^2 + g_{f,A}^2) \left[\frac{u}{t} + \frac{t}{u} + \frac{2m_{Z_d}^2}{tu} (s - 2(p_1 \cdot \epsilon^*)(p_1 \cdot \epsilon) - 2(p_2 \cdot \epsilon^*)(p_2 \cdot \epsilon)) \right] \\ & - 64i\pi^2 \alpha^2 \epsilon^2 g_{f,V} \cdot g_{f,A} \times \left(\frac{1}{t} \right) \times \epsilon^{\rho\beta\sigma\nu} p_{1,\rho} (p_{2,\sigma} - k_{1,\sigma}) \epsilon_\beta \epsilon_\nu^* \\ & + 64i\pi^2 \alpha^2 \epsilon^2 g_{f,V} \cdot g_{f,A} \times \left(\frac{1}{u} \right) \times \epsilon^{\rho\beta\sigma\nu} p_{2,\rho} (p_{1,\sigma} - k_{1,\sigma}) \epsilon_\beta \epsilon_\nu^* \\ & - 128i\pi^2 \alpha^2 \epsilon^2 g_{f,V} \cdot g_{f,A} \times \left(\frac{1}{tu} \right) \times p_{2,\sigma} p_{1,\lambda} k_{1,\rho} \times \epsilon^{\rho\sigma\lambda\nu} ((p_2 \cdot \epsilon) \epsilon_\nu^* + (p_2 \cdot \epsilon^*) \epsilon_\nu) \\ & + 128i\pi^2 \alpha^2 \epsilon^2 g_{f,V} \cdot g_{f,A} \times \left(\frac{1}{tu} \right) \times p_{2,\sigma} p_{1,\lambda} k_{1,\rho} \times \epsilon^{\rho\sigma\lambda\nu} ((p_1 \cdot \epsilon) \epsilon_\nu^* + (p_1 \cdot \epsilon^*) \epsilon_\nu), \end{aligned} \quad (4)$$

where $\tilde{\mathcal{M}}$ is the amplitude with the polarizations of initial fermions and final-state photon summed, α is the fine-structure constant, m_{Z_d} and ϵ_μ are dark boson mass and polarization vector, respectively, s , t , and u are Mandelstam variables. It is clear that those terms proportional to $g_{f,V} \cdot g_{f,A}$ vanish by summing the Z_d polarization, $\sum_\lambda \epsilon_\mu^\lambda \epsilon_\nu^{*\lambda} = -g_{\mu\nu} + k_{1,\mu} k_{1,\nu} / m_{Z_d}^2$. In the CM frame of colliding electrons and positrons, the momenta of initial and final-state particles are given by

$$\begin{aligned} p_1^\mu &= (E, 0, 0, +E), \\ p_2^\mu &= (E, 0, 0, -E), \\ k_1^\mu &= (E_{Z_d}, \omega \sin \theta, 0, \omega \cos \theta), \\ k_2^\mu &= (\omega, -\omega \sin \theta, 0, -\omega \cos \theta), \end{aligned} \quad (5)$$

where ω is the photon energy, $E_{Z_d} = \sqrt{\omega^2 + m_{Z_d}^2}$, and $s = (p_1 + p_2)^2 = 4E^2$. Using energy and momentum

conservation, we have $E_{Z_d} = E + m_{Z_d}^2/(2\sqrt{s})$ and $\omega = E - m_{Z_d}^2/(2\sqrt{s})$. Let us denote the amplitude for each polarization as $\mathcal{M}_+$, $\mathcal{M}_-$, and $\mathcal{M}_\parallel$ for right-handed, left-handed, and longitudinal polarized dark boson final state, respectively. We have

$$\begin{aligned} |\bar{\mathcal{M}}_+|^2 &= \frac{8\pi^2\alpha^2\varepsilon^2}{(t-m_e^2)(u-m_e^2)} \\ &\times [(1+\cos^2\theta)(s^2+m_{Z_d}^4) + \rho\cos\theta(s-m_{Z_d}^2)^2], \\ |\bar{\mathcal{M}}_-|^2 &= \frac{8\pi^2\alpha^2\varepsilon^2}{(t-m_e^2)(u-m_e^2)} \\ &\times [(1+\cos^2\theta)(s^2+m_{Z_d}^4) - \rho\cos\theta(s-m_{Z_d}^2)^2], \\ |\bar{\mathcal{M}}_\parallel|^2 &= \frac{8\pi^2\alpha^2\varepsilon^2}{(t-m_e^2)(u-m_e^2)} (4m_{Z_d}^2s\sin^2\theta), \end{aligned} \quad (6)$$

where the normalization $g_{f,V}^2 + g_{f,A}^2 = 1$ has been taken and $\rho = 4g_{f,V}g_{f,A}$. The absolute value of ρ essentially describes the degree of parity violation. It is shown that $|\bar{\mathcal{M}}_\parallel|^2$ is suppressed by $m_{Z_d}^2/s$ compared to $|\bar{\mathcal{M}}_\pm|^2$. In Fig. 1, we present the fraction of matrix element square as a function of $\cos\theta$ for each helicity state of Z_d . We take the $V-A$ case with $g_{f,V} = -g_{f,A} = 1/\sqrt{2}$ for illustration. The upper, middle, and lower panels correspond to $m_{Z_d}/\sqrt{s} = 0.1, 0.3$, and 0.8 , respectively. For the first two cases, one can see that the longitudinal fraction is no more than 10%. In addition, $|\bar{\mathcal{M}}_-|^2$ dominates the forward direction ($0 \leq \cos\theta \leq 1$), while $|\bar{\mathcal{M}}_+|^2$ dominates the backward direction. For the third case, the longitudinal fraction is non-negligible and the fractions for helicity $+1$ and -1 states are almost identical due to the suppression of ρ -dependent terms, i.e., the forward-backward asymmetry approaches to zero in the limit $s \rightarrow m_{Z_d}^2$.

The polarized differential cross section is readily calculated with

$$\frac{d\sigma_i}{d\cos\theta} = \frac{1}{32\pi s} \left(1 - \frac{m_{Z_d}^2}{s}\right) |\mathcal{M}_i|^2, \quad (7)$$

with $i = +, -, \text{ and } \parallel$. To check our result, we take $\varepsilon = 1$ and sum over contributions from all polarizations. In the limit $s \gg m_e^2, m_{Z_d}^2$, our result approaches to the differential cross section of $e^+e^- \rightarrow \gamma\gamma$. The results for polarized differential cross section are shown in Fig. 2. In this calculation, we take $\varepsilon = 7 \times 10^{-4}$ for $m_{Z_d}/\sqrt{s} = 0.1, 0.3$, and 0.8 , respectively, for illustrations. This ε value is reachable by Belle II with 500 fb^{-1} luminosity for m_{Z_d} around 1 GeV [31]. Similar to the case of amplitude square, the longitudinal polarized contribution is suppressed for small $m_{Z_d}/\sqrt{s}$. We present in Fig. 3 the fraction of

FIG. 1. The fraction of matrix element square for helicity $+1$, -1 , and longitudinal dark boson final state. We take the $V-A$ case with $g_{f,V} = -g_{f,A} = 1/\sqrt{2}$. The ratio $m_{Z_d}/\sqrt{s}$ is taken to be $0.1, 0.3$, and 0.8 on upper, middle, and lower panels, respectively.

differential cross section in each Z_d polarization. Each fraction is represented by a region of specific color.

III. PROBING PARITY VIOLATION EFFECTS

IN $e^+e^- \rightarrow Z_d\gamma \rightarrow Z_d\mu^+\mu^-$

Now we consider the angular distributions of Z_d decays. Through mixing with SM gauge bosons, Z_d can decay to SM leptons with the polarized differential decay rate

$$\frac{d\Gamma_{l^+l^-}^i}{d\cos\theta_d} = \frac{\beta}{32\pi m_{Z_d}} |\mathcal{M}(Z_d^{(i)} \rightarrow l^+l^-)|^2, \quad (8)$$

with $i = +, -, \text{ and } \parallel$, $\beta = p_l/E_l = \sqrt{1 - 4m_l^2/m_{Z_d}^2}$, and θ_d the angle between l^- direction in the Z_d rest frame and the Z_d direction in e^+e^- CM frame. Thus, we obtain

FIG. 2. The polarized differential cross section for $e^+e^- \rightarrow Z_d \gamma$ with $\varepsilon = 7 \times 10^{-4}$ at Belle II for $m_{Z_d}/\sqrt{s} = 0.1, 0.3$, and 0.8 , respectively.

$$\begin{aligned} \frac{d\Gamma_{l^+l^-}^+}{d\cos\theta_d} &= \frac{\alpha\varepsilon^2\beta}{2m_{Z_d}} [2g_{l,V}^2 m_l^2 + (1 + \cos^2\theta_d) p_l^2 + \rho \cos\theta_d E_l p_l], \\ \frac{d\Gamma_{l^+l^-}^-}{d\cos\theta_d} &= \frac{\alpha\varepsilon^2\beta}{2m_{Z_d}} [2g_{l,V}^2 m_l^2 + (1 + \cos^2\theta_d) p_l^2 - \rho \cos\theta_d E_l p_l], \\ \frac{d\Gamma_{l^+l^-}^{\parallel}}{d\cos\theta_d} &= \frac{\alpha\varepsilon^2\beta}{m_{Z_d}} [g_{l,V}^2 m_l^2 + \sin^2\theta_d p_l^2]. \end{aligned} \quad (9)$$

Given $g_{l,V}^2 + g_{l,A}^2 = 1$ and $\rho = 4g_{l,V}g_{l,A}$, we have $g_{l,V}^2 = (1 + \sqrt{1 - \rho^2/4})/2$ for $|g_{l,V}| \geq |g_{l,A}|$, while $g_{l,V}^2 = (1 - \sqrt{1 - \rho^2/4})/2$ for $|g_{l,V}| \leq |g_{l,A}|$. The double differential distribution of final-state leptons is given by

FIG. 3. The fraction of differential cross section in each Z_d polarization as represented by regions of different colors. The range for each color adds up to unity.

$$\frac{d^2P}{dkd\xi} = \frac{1}{\sigma_T \cdot \Gamma_{l^+l^-}} \sum_i \left(\frac{d\sigma^i}{d\cos\theta} \right) \cdot \left(\frac{d\Gamma_{l^+l^-}^i}{d\cos\theta_d} \right), \quad (10)$$

with $\kappa = \cos\theta$, $\xi = \cos\theta_d$, $\Gamma_{l^+l^-}$ the unpolarized $Z_d \rightarrow l^+l^-$ decay width, and σ_T the total Z_d production cross section. We first observe that the double differential distribution $d^2P/dkd\xi$ only depends on ρ^0 and ρ^2 . Second, the ρ^2 -dependent term in the double distribution is given by $\rho^2\beta(1 - m_{Z_d}^2/s)^2 J/(1 - \kappa^2)$ with $J = \kappa\xi$. The sign of this contribution is determined by the sign of J . This contribution vanishes at the dark boson production threshold, $s = m_{Z_d}^2$, or at the threshold for Z_d decaying into the lepton pair, i.e., $\beta = 0$. It also vanishes if either κ or ξ is integrated to the full range.

To analyze the double distribution, we integrate κ from -1 to 0 , i.e., we consider leptonic decays of Z_d produced in the backward direction. The forward-backward asymmetry of l^- in its helicity angle is presented in Fig. 4. In three panels, we present the angular distribution of l^- for $|\rho| = 0, 1$, and 2 , respectively. The case $|\rho| = 0$ implies either $g_{l,A} = 0$ or $g_{l,V} = 0$ with the former corresponding to the dark-photon scenario, while $|\rho| = 2$ corresponds to either $V - A$ or $V + A$ cases. On the upper panel, we separate results into $|g_{l,V}| \geq |g_{l,A}|$ and $|g_{l,V}| \leq |g_{l,A}|$. These two cases coincide in the middle and lower panels with $\beta = 1$, i.e., $m_l \rightarrow 0$. The asymmetries in the upper and lower panels are small either due to a small β or to the suppression

in $(1 - m_{Z_d}^2/s)^2$ with $m_{Z_d}/\sqrt{s} = 0.8$. Significant asymmetry is seen in the middle panel with large $|\rho|$. In general, we may define the following asymmetry parameter:

$$\mathcal{A}_{\text{PN}} \equiv \frac{S(\kappa \cdot \xi > 0) - S(\kappa \cdot \xi < 0)}{S(\kappa \cdot \xi < 0) + S(\kappa \cdot \xi > 0)}, \quad (11)$$

where the subscript PN indicates that $\mathcal{A}_{\text{PN}}$ describes the difference in signal event rate as $\kappa\xi$ reverses its sign. In limits of $\beta \rightarrow 1$ and $m_{Z_d} \ll \sqrt{s}$, we have

$$\mathcal{A}_{\text{PN}} = \frac{3}{4} \left(\frac{\rho^2}{4} \right) \frac{-\ln(1 - \kappa_m^2)}{\ln(\frac{1+\kappa_m}{1-\kappa_m}) - \kappa_m}, \quad (12)$$

where κ_m is the maximum of κ . The minimum of κ is assumed to be $-\kappa_m$. It is found that $\mathcal{A}_{\text{PN}}$ is not very sensitive to κ_m . $\mathcal{A}_{\text{PN}} = 0.64 \times (\rho^2/4)$ for $\kappa_m = 0.95$, and $0.55 \times (\rho^2/4)$ for $\kappa_m = 0.80$. In the next section, we shall calculate $\mathcal{A}_{\text{PN}}$ and its associated uncertainty with acceptance cuts in Belle II detector.

IV. THE PROSPECT OF MEASURING PARITY-VIOLATING DARK BOSON INTERACTIONS IN BELLE II

In this section, we discuss the search for dark vector boson and the possible measurement of parity violation parameter ρ through determining $\mathcal{A}_{\text{PN}}$ in Belle II detector. We shall begin by considering the detector acceptance of Belle II and compare our sensitivity estimation for the dark photon search through $e^+e^- \rightarrow A'\gamma$ with $A' \rightarrow \mu^+\mu^-$ with BABAR result at 514 fb^{-1} and the projected sensitivity of Belle II at 500 fb^{-1} . These comparisons are important for validating our approach. Next, we consider Belle II at the full integrated luminosity 50 ab^{-1} and extend our discussions to the dark boson scenario with a nonvanishing ρ .

A. Sensitivity for the dark photon search at Belle II

To illustrate our points in previous sections, we take the dark photon mass as 0.5 and 2 GeV , respectively, as benchmark values. These two mass values satisfy $m_{Z_d}/\sqrt{s} \ll 1$ so that the dark photons are produced in transversely polarized states. Hence, for the generalization to dark boson scenario in the next subsection, we shall see that the asymmetry parameter $\mathcal{A}_{\text{PN}}$ will be significant.

Let us begin by taking $m_{Z_d} = 0.5 \text{ GeV}$ with $\rho = 0$. In this case, the branching ratio for $Z_d \rightarrow \mu^+\mu^-$ is about 40% from the measurement of $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ [14,47]. The Belle II calorimeter angular coverage is $12.4^\circ \leq \theta_\gamma^{\text{lab}} \leq 155.1^\circ$ [48], which detects final-state photon in the rapidity range $-1.51 \leq \eta_\gamma^{\text{lab}} \leq 2.22$. Since the boost velocity from the laboratory frame to CM frame is $\beta_{\text{CM}} = (E_{e^-} - E_{e^+})/(E_{e^-} + E_{e^+}) = 3/11$, the photon rapidity in the CM frame is given by

FIG. 4. The angular distribution of l^- from decays of Z_d produced in the backward direction by $e^+e^- \rightarrow Z_d\gamma$ for $|\rho| = 0, 1$, and 2 , respectively. The upper panel is for relativistic Z_d with $m_{Z_d} = 0.1\sqrt{s}$ and nonrelativistic lepton with $\beta = 0.1$. The middle panel corresponds to $(\beta, m_{Z_d}/\sqrt{s}) = (1, 0.1)$, while the lower panel corresponds to $(\beta, m_{Z_d}/\sqrt{s}) = (1, 0.8)$.

$\eta_{\gamma}^{\text{CM}} = \eta_{\gamma}^{\text{lab}} + \ln((1 - \beta_{\text{CM}})/(1 + \beta_{\text{CM}}))/2$. Hence, $-1.79 \leq \eta_{\gamma}^{\text{CM}} \leq 1.94$. Furthermore, the angular coverage of K_L -Muon detector [48] is $25^\circ \leq \theta_{\mu^\pm}^{\text{lab}} \leq 150^\circ$. This leads to the muon rapidity range $-1.60 \leq \eta_{\mu^\pm}^{\text{CM}} \leq 1.23$ in the CM frame. Since the signal mass resolution is between 1.5 and 8 MeV in *BABAR* analysis [24], we take it to be 5 MeV for our sensitivity estimation, i.e., the signal and background events are calculated within a 5 MeV $\mu^+\mu^-$ invariant mass window in the vicinity of assumed Z_d mass.

We note that the rapidity cuts preserve $S(\kappa \cdot \xi > 0) = S(\kappa \cdot \xi < 0)$ for $\rho = 0$. Using CalcHEP [49], we find that the signal $e^+e^- \rightarrow \gamma Z_d$ with $Z_d \rightarrow \mu^+\mu^-$ has the cross section $1.84 \times 10^3 \cdot \epsilon^2$ pb by considering both the rapidity cuts and the 40% $Z_d \rightarrow \mu^+\mu^-$ branching ratio, and the cross section for QED background process $e^+e^- \rightarrow \gamma\mu^+\mu^-$ with the same acceptance cut is 7.76×10^{-2} pb. We note that the above parametrization for signal cross section is valid only for $\epsilon < 0.3$ such that the ϵ -dependent Z_d width is less than 10% of the signal mass resolution. With 500 fb^{-1} of integrated luminosity, the Belle II 90% C.L. sensitivity to ϵ is estimated by the following χ^2 function:

$$\chi^2 = 2 \left(n \ln \left(\frac{n}{w} \right) + w - n \right), \quad (13)$$

where n is the observed event number while w is the expected event number. With $n = S + B = (1.84 \times 10^3 \cdot \epsilon^2 + 7.76 \times 10^{-2}) \text{ pb} \cdot 500 \text{ fb}^{-1}$, $w = B = 7.76 \times 10^{-2} \text{ pb} \cdot 500 \text{ fb}^{-1}$, and $\chi^2 = (1.645)^2$ for 90% C.L. sensitivity,⁴ we obtain $\epsilon = 5.9 \times 10^{-4}$, which is consistent with the sensitivity $\epsilon = 5.6 \times 10^{-4}$ given in Belle II physics book for the visible modes $Z_d \rightarrow e^+e^-, \mu^+\mu^-$ [31]. The latter is also comparable to the constraint from *BABAR* search via visible modes at 514 fb^{-1} [24].

We next take $m_{Z_d} = 2 \text{ GeV}$ with $\rho = 0$. In this case, the branching ratio of $Z_d \rightarrow \mu^+\mu^-$ is about 24% [47]. Hence, the signal cross section is around $1.11 \times 10^3 \cdot \epsilon^2$ pb, while the background cross section is 2.54×10^{-2} pb. Following Eq. (13), we obtain Belle II 90% C.L. sensitivity to ϵ as $\epsilon = 5.8 \times 10^{-4}$, which is also not much different from 6.6×10^{-4} given by Belle II physics book for the visible modes $Z_d \rightarrow e^+e^-, \mu^+\mu^-$ [31].

B. Probing the parity violation effects

1. Enhancement on the detection significance

In the case of nonvanishing ρ , we modify the χ^2 function in Eq. (13) into

⁴In principle, one should also consider the interference between signal and background amplitudes for calculating $S + B$. On the other hand, it can be shown that such contribution scales as ϵ^2 , and for the current case the interference part of the cross section is $\sim -1.8 \cdot \epsilon^2$ pb, which is negligible.

$$\chi^2 = 2 \left(n_a \ln \left(\frac{n_a}{w_a} \right) + w_a - n_a \right) + 2 \left(n_b \ln \left(\frac{n_b}{w_b} \right) + w_b - n_b \right), \quad (14)$$

where n_a (w_a) and n_b (w_b) are observed (expected) event numbers in $\kappa \cdot \xi > 0$ and $\kappa \cdot \xi < 0$ bins, respectively. By considering separate event bins, the dark boson detection significance is expected to be improved. With $n_{a,b} = S_{a,b} + B_{a,b}$ and $w_{a,b} = B_{a,b}$, we can show that

$$\chi^2 = \frac{S^2}{B} (1 + \mathcal{A}_{\text{PN}}^2), \quad (15)$$

with the assumption $S_{a,b} \ll B_{a,b}$ and the identity $B_a = B_b = B/2$. We note that S_a and S_b are the event excess in $\kappa \cdot \xi > 0$ and $\kappa \cdot \xi < 0$ event bins, respectively. It is clear that the detection significance increases from $[S/\sqrt{B}] \cdot \sigma$ to $[S\sqrt{(1 + \mathcal{A}_{\text{PN}}^2)}/\sqrt{B}] \cdot \sigma$ by considering separate event bins.

2. Measurement of $\mathcal{A}_{\text{PN}}$

Following Eq. (11), we have $\mathcal{A}_{\text{PN}} = (S_a - S_b)/(S_a + S_b)$ with the statistical uncertainties of both numerator and denominator being $\sqrt{B}$. Hence, the uncertainty of $\mathcal{A}_{\text{PN}}$ is given by $\sigma_{\mathcal{A}_{\text{PN}}} = \sqrt{1 + \mathcal{A}_{\text{PN}}^2} (\sqrt{B}/S)$. We note that the asymmetry parameter $\mathcal{A}_{\text{PN}}$ as defined by Eq. (11) is actually independent of the integrated luminosity. It can be calculated by the scattering cross section with appropriate kinematic cuts imposed.

3. Numerical results

Let us assume the scenario that one achieves a 5 standard deviation detection of dark boson at the designed integrated luminosity 50 ab^{-1} based upon counting the overall event excess, i.e., $S = 5\sqrt{B}$ according to Eq. (13). On the other hand, the event excess S_a and S_b for each event bin depends on the asymmetry parameter $\mathcal{A}_{\text{PN}}$. It is to be noted that we shall fix the total event excess $S \equiv S_a + S_b$ regardless of the ρ value. Hence, $\sigma(e^+e^- \rightarrow Z_d \gamma) \cdot \text{Br}(Z_d \rightarrow \mu^+\mu^-)$ is fixed. Since $\sigma(e^+e^- \rightarrow Z_d \gamma) \propto \epsilon^2$ while $\text{Br}(Z_d \rightarrow \mu^+\mu^-)$ depends on ρ , the ϵ value extracted from $S = 5\sqrt{B}$ at 50 ab^{-1} also depends on ρ .

To proceed with our numerical analysis, we first calculate $\mathcal{A}_{\text{PN}}$ and its uncertainty as a function of the ratio $m_{Z_d}/\sqrt{s}$ for a given ρ . In Fig. 5, we present results for $\rho = 0.5, 1.0, 1.5$, and 2.0 , respectively. The results for other ρ 's can be easily inferred since $\mathcal{A}_{\text{PN}} \propto \rho^2$. It is seen that $\mathcal{A}_{\text{PN}}$ is consistent with zero for $\rho = 0.5$ and $\rho = 1$, since the 1σ error bars in these cases reach $\mathcal{A}_{\text{PN}} < 0$ regime. On the other hand, for $\rho = 2$, $\mathcal{A}_{\text{PN}}$ are nonvanishing at more than 2σ for $m_{Z_d}/\sqrt{s} \leq 0.3$. With $\mathcal{A}_{\text{PN}}$ determined, the enhancement on detection significance according to Eq. (15) can be calculated. Let us consider two special scenarios for ρ . The

FIG. 5. $\mathcal{A}_{\text{PN}}$ and its uncertainty as functions of $m_{Z_d}/\sqrt{s}$ for $S/\sqrt{B} = 5$. We present results for $\rho = 0.5, 1.0, 1.5$, and 2.0 , respectively.

first scenario is setting $\varepsilon_\gamma = \varepsilon_Z$ in Eq. (2). Recasting the coupling of Z_d to leptons into the standard form $e\bar{l}(g_{l,V}\gamma_\mu + g_{l,A}\gamma_\mu\gamma_5)lZ_d^\mu$, we have $\varepsilon = 1.18\varepsilon_\gamma$, $g_{l,V} = -0.87$, and $g_{l,A} = -0.5$, which leads to $\rho = 1.74$. The second scenario is the extreme asymmetry case, such as $V - A$ interaction between Z_d and SM fermions. This case is achieved with $\varepsilon_\gamma = \varepsilon_Z \tan \theta_W$ such that $\varepsilon = 0.83\varepsilon_Z$ and $g_{l,A} = -g_{l,V} = 1/\sqrt{2}$, which leads to $\rho = -2$. In this case, the dark boson interacts with the up-type (down-type) quark the same way as it interacts with neutrino (charged lepton). The values of $\mathcal{A}_{\text{PN}}$ and the associated uncertainties in these two scenarios are summarized in Table I for $m_{Z_d} = 0.5$ and 2 GeV. The improved detection significance with Eq. (15) is also presented there. We note that, for $|\rho| = 1.74$, the significance for a nonvanishing $\mathcal{A}_{\text{PN}}$ already reaches 2σ .

It is of interest to estimate S for a 5σ detection at 50 ab^{-1} . Given the background cross sections 7.76×10^{-2} and

$2.54 \times 10^{-2} \text{ pb}$ for $m_{Z_d} = 0.5$ and 2 GeV, respectively, we have $B = 3.88 \times 10^6$ and 1.27×10^6 , respectively. For $S = 5\sqrt{B}$ assumed here, $S = 9850$ and 5634 for $m_{Z_d} = 0.5$ and 2 GeV, respectively. The way S is split into S_a and S_b depends on $\mathcal{A}_{\text{PN}}$. We present values of S_a and S_b for benchmark values of m_{Z_d} and ρ in Table I. Finally, in the last two rows of Table I, we also present $\text{Br}(Z_d \rightarrow \mu^+\mu^-)$ and the corresponding ε extracted from $S = 5\sqrt{B}$ at 50 ab^{-1} .

Before closing this section, we comment on $Z_d \rightarrow \mu^+\mu^-$ branching ratio for a nonvanishing ρ . It is clear that $\text{Br}(Z_d \rightarrow \mu^+\mu^-)$ in the dark boson scenario differs from that of the dark photon case, in particular the decay $Z_d \rightarrow \nu\bar{\nu}$ is possible for the former case. From Eq. (2), one obtains Z_d couplings to leptons and quarks. For $|\rho| = 1.74$ and $m_{Z_d} = 0.5$ GeV, we have $\text{Br}(Z_d \rightarrow e^+e^-) = 22\%$, $\text{Br}(Z_d \rightarrow \mu^+\mu^-) = 21\%$, $\sum_l \text{Br}(Z_d \rightarrow \nu_l\bar{\nu}_l) = 33\%$, and

TABLE I. Presented are the asymmetry parameter $\mathcal{A}_{\text{PN}}$, the improved detection significance with two event bins, the signal event numbers, and extracted ε values assuming 5σ detection significance at 50 ab^{-1} based upon overall event excess, i.e., $S = 5\sqrt{B}$, for benchmark m_{Z_d} and ρ values. The $Z_d \rightarrow \mu^+\mu^-$ branching ratios are also presented.

$ \rho $	0.00		1.74		2.00	
m_{Z_d}/GeV	0.5	2.0	0.5	2.0	0.5	2.0
$\mathcal{A}_{\text{PN}}$	(0.0 ± 0.2)	(0.0 ± 0.2)	(0.43 ± 0.22)	(0.44 ± 0.22)	(0.58 ± 0.23)	(0.60 ± 0.23)
Det. Sig. (Eq. (15))	5.0σ	5.0σ	5.4σ	5.5σ	5.8σ	5.8σ
$S(\kappa \cdot \xi > 0)$	4925	2817	7040	4053	7780	4507
$S(\kappa \cdot \xi < 0)$	4925	2817	2810	1581	2070	1127
$\text{Br}(Z_d \rightarrow \mu^+\mu^-)$	40%	24%	21%	7.5%	17%	6.7%
$\varepsilon \cdot 10^4$	3.3	3.2	4.6	5.7	5.1	6.1

$\text{Br}(Z_d \rightarrow \text{hadrons}) = 24\%$. For $m_{Z_d} = 2$ GeV with the same $|\rho|$, the hadronic branching ratio increases to about 74% due to the opening of $Z_d \rightarrow s\bar{s}$ channel. Furthermore, $\sum_l \text{Br}(Z_d \rightarrow \nu_l \bar{\nu}_l) = 11\%$ and $\text{Br}(Z_d \rightarrow e^+ e^-) \simeq \text{Br}(Z_d \rightarrow \mu^+ \mu^-) = 7.5\%$. For $|\rho| = 2$, $\text{Br}(Z_d \rightarrow \mu^+ \mu^-) = 17\%$ and 6.7% for $m_{Z_d} = 0.5$ and 2 GeV, respectively. We remark that our estimation of $\text{Br}(Z_d \rightarrow \text{hadrons})$ depends on the Z_d mass. For $m_{Z_d} = 2$ GeV, we use the quark level process $Z_d \rightarrow q\bar{q}$ to calculate the hadronic decay width. For $m_{Z_d} = 0.5$ GeV, we consider only vector current contribution that can be inferred from the data as mentioned earlier. The axial-vector current is essentially not contributing since it cannot lead to two-pion final state due to parity conservation of strong interaction, nor its coupling to three-pion final state is noticeable at 0.5 GeV from the study of axial-vector spectral function of hadronic τ decays [50].

V. SUMMARY AND CONCLUSION

In this paper, we have pointed out that the dark boson produced by $e^+ e^- \rightarrow Z_d \gamma$ is transversely polarized in the limit $m_{Z_d} \ll \sqrt{s}$. This is a direct consequence of Ward-Takahashi identity. We also demonstrated this property by explicit calculations. The suppressed production of longitudinally polarized dark boson state is shown in Fig. 1 for the $V - A$ limit, i.e., $g_{l,V} = -g_{l,A} = 1/\sqrt{2}$. For $m_{Z_d} \ll \sqrt{s}$, the negative-helicity dark boson dominates the forward region ($\cos\theta > 0$), while the positive-helicity one dominates the backward region ($\cos\theta < 0$). As m_{Z_d} approaches to $\sqrt{s}$, the production of longitudinally polarized dark boson becomes noticeable. Furthermore, the angular distributions of negative- and positive-helicity dark bosons become indistinguishable.

Since we aim for determining the parity violation parameter ρ , we analyze $\mu^-(\mu^+)$ angular distributions from polarized Z_d decays. The double distribution of final state muons $d^2P/d\kappa d\xi$ ($\kappa = \cos\theta$, $\xi = \cos\theta_d$), defined in Eq. (10), was shown to be sensitive to ρ . Explicitly, we

found that $d^2P/d\kappa d\xi = Q_0 + Q_2\rho^2$ with Q_0 an even function of both κ and ξ and Q_2 an odd function of these variables. This implies that the signal event number in the kinematic range $\kappa \cdot \xi > 0$ differs from that with $\kappa \cdot \xi < 0$, which motivates our definition of asymmetry parameter $\mathcal{A}_{\text{PN}}$ proportional to ρ^2 . Besides depending on ρ^2 , $\mathcal{A}_{\text{PN}}$ also depends on the detector acceptance. We calculate numbers of signature and background events for two benchmark masses $m_{Z_d} = 0.5$ and 2 GeV in Belle II detector. The resulting 90% C.L. sensitivity to ε at 500 fb^{-1} integrated luminosity is found to be consistent with that in Belle II physics book for the dark photon scenario.

In the general scenario with nonvanishing ρ , we have seen that the detection significance of dark bosons increases by separately considering events with different signs of $\kappa \cdot \xi$ rather than just counting the overall event excess. The increased χ^2 value is proportional to $\mathcal{A}_{\text{PN}}^2$, as seen from Eq. (15). The numerical values of $\mathcal{A}_{\text{PN}}$ are calculated as a function of $m_{Z_d}/\sqrt{s}$ for $\rho = 0.5, 1.0, 1.5$, and 2.0 , respectively.

In conclusion, we have shown that the detection of dark boson decays into muon pairs in $e^+ e^-$ colliders can probe the parity-violating couplings between the dark boson and SM fermions. Assuming a 5σ event excess in the search for $e^+ e^- \rightarrow \gamma Z_d$ with $Z_d \rightarrow \mu^+ \mu^-$ at Belle II, we have seen that the simultaneous fitting to event numbers in positive and negative $\kappa \cdot \xi$ bins should improve the detection significance to 5.4σ and 5.8σ for input true models with $|\rho| = 1.74$ and $|\rho| = 2.0$, respectively. We have also seen that the significance for a nonvanishing $\mathcal{A}_{\text{PN}}$ can reach 2σ for $\varepsilon_\gamma = \varepsilon_Z$ ($\rho = 1.74$) with $m_{Z_d}/\sqrt{s} \ll 1$. The significance for general values of $m_{Z_d}/\sqrt{s}$ and ρ can be inferred from Fig. 5 with suitable rescaling of the latter parameter.

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