Interaction potential between heavy $Q\bar{Q}$ in a color octet configuration in the QGP from a study of hybrid Wilson loops

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We investigate the interaction between a heavy quark-antiquark pair in color octet configuration in a gluon plasma. We calculate nonperturbatively the effective thermal potential for hybrid states with static $Q\bar{Q}$ octet and an adjoint gluonic source. We compare the results for two different gluonic sources, which lead to completely different potentials in the confined phase. We find that they show identical thermal potentials to large distances, and interpret this potential as the interaction potential between the static Q and the $\bar{Q}$ in a color octet configuration. We present results for the potential in gluon plasma for moderately high temperatures $\lesssim 2T_c$. The implications of our result are discussed.

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I. INTRODUCTION

Quarkonia, mesonic bound states of heavy quark and antiquark, provide one of the most important signatures of the formation of quark-gluon plasma (QGP) in relativistic heavy ion collision experiments. It was suggested in the early days of collider studies of QGP [1] that the screening of the color charge inside QGP will lead to dissolution of J/ψ states, which can be observed by modification of the dilepton peak. Various theoretical approaches have been formulated to study the behavior of $Q\bar{Q}$ bound states in the plasma. See [2] for a recent comprehensive review [3]. In particular, an effective thermal potential approach to the problem has been formulated in [4] in perturbation theory, in the hard thermal loop (HTL) approximation. The behavior of such a potential has been examined in the effective field theory language for different distance scales [5]. While the formulation of Ref. [4] is for the theoretical abstraction of an isolated heavy $Q\bar{Q}$ placed in the plasma, the effective potential introduced there remains an important part of a description of heavy quark systems in plasma using the open quantum system language [6,7]. It is possible to evaluate this potential nonperturbatively, from numerical calculations of thermal Wilson loop [8]. Calculations based on Bayesian analysis [8,9] or modeling of the low-frequency peak [10] suffer from large systematics.

A new method, based on splitting of the thermal Wilson loop, has led to a well-controlled extraction of the potential from Wilson loop data [11].

In theoretical descriptions of the phenomenology of quarkonia in plasma, one typically needs the interaction between the Q and the $\bar{Q}$ not only in a color singlet configuration but also in the color octet configuration. This is because the interaction of the $Q\bar{Q}$ with the medium can change its color configuration from singlet to octet and vice versa, and therefore study of evolution of the $Q\bar{Q}$ pair in plasma involves both color configurations. In particular, in the open quantum system approach developed to study quarkonia in medium [6], the singlet and octet potentials are essential ingredients [7]. Also the $Q\bar{Q}$ pair may be in a color octet state at production time, and the time taken for it to go to a color singlet combination may be larger than the formation time of the plasma [12]. However, nonperturbative information about the octet potential is not available in the literature. Part of the problem is the inherent difficulty in defining an octet state in a gauge-invariant setup.

For the singlet, the potential describes the time evolution of the thermal correlator of the nonrelativistic vector current,

$$C_>(t, \vec{r}) \equiv \int d^3x \langle \chi^\dagger(t, \vec{x}) \sigma_k \mathbb{U}^\dagger \psi(t, \vec{x} + \vec{r}) \times \psi^\dagger(0, \vec{x} + \vec{r}) \sigma_k \mathbb{U} \chi(0, \vec{x}) \rangle. \quad (1)$$

Here $\mathbb{U}$ is a suitable gauge connection such that the current is gauge invariant. ψ, χ are two-component, nonrelativistic fields: ψ annihilate a quark and χ create an antiquark. σ_k are the Pauli matrices and the angular brackets denote thermal average. The σ_k do not affect the $\mathcal{O}(M_Q^0)$ potential. If one

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has a system where the sole interaction term is a potential $V(\vec{r})$ between the quark and the antiquark, then it is easy to show that, to leading order in $1/M_Q$, $C_>(t, \vec{r})$ satisfies

$$\left(i\partial_t - \frac{\nabla_{\vec{r}}^2}{M_Q}\right)C_>(t, \vec{r}) = V(\vec{r})C_>(t, \vec{r}). \quad (2)$$

We then define a potential [4] by equating the left-hand side of Eq. (2) to $V(t, \vec{r})C_>(t, \vec{r})$ (staying within leading order of $1/M_Q$), where the interaction effects are summarized in a time-dependent $V(t, \vec{r})$. An effective thermal potential, $V_T(\vec{r})$, can then be defined in the large t limit, if the limit exists: $V_T(\vec{r}) = \lim_{t \rightarrow \infty} V(t, \vec{r})$. In the static limit, modulo renormalization factor, $C_>(t, \vec{r})$ reduces to a Minkowski-time Wilson loop:

$$W_M(t, \vec{r}) = \frac{1}{3} \text{Tr} \mathbb{P} e^{i \int_0^t dt_1 A_0(t_1, \vec{r})} \mathbb{U}(t; \vec{r}, \vec{0}) \mathbb{P} e^{i \int_t^0 dt_2 A_0(t_2, \vec{0})} \times \mathbb{U}(0; \vec{0}, \vec{r}) \quad (3)$$

and Eq. (2) reduces to

$$i\partial_t \log W_M(t, \vec{r}) \xrightarrow{t \rightarrow \infty} V_T(\vec{r}), \quad (4)$$

which defines the effective thermal potential for the singlet channel [4].

Equation (2) describes the time evolution of a thermal correlation function, and not of a $Q\bar{Q}$ wave function. It reduces naturally to Eq. (4) in the static limit, which can lead to a nonperturbative calculation of the potential [8]. The effective thermal potential defined by Eq. (2) is complex in general [4]. It has been argued that the potential Eq. (2) can be re-interpreted in terms of evolution of a $Q\bar{Q}$ wave function [13,14]. In this language, the evolution of the $Q\bar{Q}$ pair is described by a stochastic Hamiltonian, and $V_T^{\text{im}}(\vec{r})$ is related to fluctuation of the stochastic noise. The effective thermal potential remains an essential ingredient in such open quantum system studies of quarkonia in plasma.

For the octet potential, one can proceed in a similar way, starting with a point-split nonrelativistic current

$$J^a(\vec{r}; \vec{x}, \vec{x}_0, t) = \psi^\dagger(\vec{x} + \vec{r}; t) \sigma_k \mathbb{U}(\vec{x} + \vec{r}, \vec{x}_0; t) \times T^a \mathbb{U}(\vec{x}_0, \vec{x}; t) \chi(\vec{x}; t) \quad (5)$$

and looking at the time derivative of the correlator $C_>^{aJ^a}(t, \vec{r})$ a la Eq. (4). The current J^a is gauge dependent and the correlator $C_>^{aJ^a}(t, \vec{r})$ needs to be defined in a fixed gauge. Unfortunately, standard gauge fixed definitions of $C_>^{aJ^a}(t, \vec{r})$ lead to a system which may be very different from what was intended: e.g., in the temporal gauge $C_>^{aJ^a}(t, \vec{r})$ actually describes, in the static limit, a $\bar{Q}Q_{\text{adj}}Q$ system [15]. In the literature one usually employs the

Coulomb gauge; nonperturbatively, $C_>^{aJ^a}(t, \vec{r})$ is not defined in the Coulomb gauge, and a further fixing of temporal gauge along $\vec{x}_0$ gets us back to a $\bar{Q}Q_{\text{adj}}Q$ system [15].

When we talk about $\bar{Q}Q$ in color octet combination in the context of quarkonia, we have in mind a system where $\bar{Q}Q$ is interacting with an adjoint gluonic source. To mimic this system, we could start with a trial current like

$$J_G(\vec{r}; \vec{x}, \vec{x}_0, t) = \bar{\psi}(\vec{x} + \vec{r}; t) \mathbb{U}(\vec{x} + \vec{r}, \vec{x}_0; t) G^a(\vec{x}_0; t) \times T^a \mathbb{U}(\vec{x}_0, \vec{x}; t) \chi(\vec{x}; t), \quad (6)$$

which is a color singlet combination of the color-octet quark-antiquark system and an adjoint gluonic source at a time slice t , and then look at the correlator $C_G(t, \vec{r}) = \langle J_G^\dagger(t) J_G(0) \rangle$. With a judicious choice of G , it is possible to ensure that J_G does not have overlap with a configuration where the quark-antiquark system is in color-singlet state. A color singlet state consisting of $Q\bar{Q}|_o$ and adjoint gluon source is called a hybrid state. At zero temperature, hybrid potentials have been studied in detail in the literature [16]. For us, the important information is that in certain regimes, the hybrid current can be used to define an octet potential [17,18].

In this work, we study the thermal effect on the hybrid Wilson loop and extract information about the thermal modification of the interaction potential between static $Q\bar{Q}$ in color octet configuration in gluon plasma. To our knowledge, this is the first study of the effective thermal potential for a Q and a $\bar{Q}$ in a color octet configuration, though a related quantity, the color octet free energy, has been studied before [19,20]. Preliminary results of this study were presented in [21]. The logic of our calculational method is explained in Sec. II. There we discuss our method of extracting a potential for the hybrid operator state Eq. (6), and what we mean by inference of an octet potential from it. Technical details of the numerical calculation are given in Sec. III. Our results for the potential are given in Sec. IV, and in Sec. V we summarize and discuss the results. Some technical details are relegated to the Appendix: Appendix A discusses the role of smearing in our study, some details relevant for Sec. II can be found in Appendix B, and various systematics of our extraction of the potential can be found in Appendix C.

II. OBSERVABLES AND ANALYSIS

For our study, we use the hybrid current operator Eq. (6) with the chromomagnetic field operator for $G = G^a T^a$: we use the two choices B_z and $B_+ = B_x + iB_y$. Here z is taken to be the separation between the quark and the antiquark. In the static limit, one gets an Wilson loop with insertion of the G field:

$$\begin{aligned}
W_G(\tau; \vec{r}, \vec{x}) &= \frac{1}{3} \text{Tr} \mathbb{P} e^{i \int_0^\tau d\tau_1 A_0(\tau_1, \vec{r})} \mathbb{U}(\tau; \vec{r}, \vec{x}) G^\dagger(\tau; \vec{x}) \\
&\times \mathbb{U}(\tau; \vec{x}, \vec{0}) \mathbb{P} e^{i \int_\tau^0 d\tau_2 A_0(\tau_2, \vec{0})} \mathbb{U}(0; \vec{0}, \vec{x}) \\
&\times G(0; \vec{x}) \mathbb{U}(0; \vec{x}, \vec{r}). \quad (7)
\end{aligned}$$

Going to Minkowski time and taking the long time derivative, similar to Eq. (4), we can define a potential $V^G(\vec{r})$ for the static hybrid states that have an overlap with J_G defined in Eq. (6). The hybrid state created by J_G is a color singlet formed by combination of the adjoint source $G^a(x_0)$ with the color-octet $Q\bar{Q}$ state given by J^a , Eq. (5), which transforms like a color octet with the gauge rotation at x_0 . In this work we have chosen x_0 to be in the middle of

$$\equiv \left(1 + \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \dots \right) \quad (8)$$

Here time direction is shown vertically, the grey circles indicate the magnetic fields, and the empty dots and dashed lines on the Wilson loop indicate color matrix insertion T^a and 00 component of the gluon propagator, respectively. The leading order potential comes from a ladder sum of diagrams like those explicitly shown inside the parentheses in Eq. (8), where the effect of the G insertion is merely a change in color factor due to the color matrix insertions in the Wilson loops. Such factorization will not hold non-perturbatively, where J^G for the B_z operator will give rise to potential for the $L = 0$ state Σ_u^- and the B_+ operator, that for the $L = 1$ state Π_u . Here L refers to angular momentum around the axis joining the quark and antiquark, u denotes CP odd, and $-$ refers to parity for reflection about a plane passing through this axis. The potential for these operators have been studied in the literature [16].

At short distances, we expect the potential for a state like Eq. (6) to give the interaction potential for the color octet $Q\bar{Q}$. This can be formalized in an effective field theory language [17,18] :

$$V_G(r) \sim V_O(r) + \Lambda_G + \mathcal{O}(r^2) \quad (9)$$

where Λ_G is a constant term capturing the effect of the G insertion, and $V_O(r)$ is the potential for the short distance octet $Q\bar{Q}$ state. For the two operator choices here, Λ_G is identical, since there is no difference between the two operators for $r = 0$. So one expects V_G to be same for the two operators in the short distance regime, modulo $\mathcal{O}(r^2)$ effects. This behavior was tested in detail in [18]. While the

Q and $\bar{Q}$, i.e., at $\vec{x} + \vec{r}/2$. For the choice of the operators B_z and B_+ for G , the quantum numbers of these states are such that the Q and the $\bar{Q}$ are in a color octet combination, in a quark model sense: they do not have an overlap with the color singlet $Q\bar{Q}$ state $\psi^\dagger(0, \vec{x} + \vec{r}) \mathbb{U}_\chi(0, \vec{x})$ appearing in Eq. (1).

While the aim of this paper is to calculate the effective thermal potential nonperturbatively, in order to understand the connection of the hybrid Wilson loop W_G to an interaction potential between the heavy Q and $\bar{Q}$ in a color octet combination, it helps to think in terms of perturbation theory. In leading order (LO) of perturbation theory, the effect of the insertion G isolates and one gets the octet potential from the long time behavior of the Wilson loop:

convergence of V_G extracted from the two components of the magnetic field as $r \rightarrow 0$ was verified, the quick onset of the nonperturbative effects make it difficult to identify any G agnostic part of the interaction potential between the $Q\bar{Q}$ in relative color octet configuration.

At finite temperature the situation is expected to be different: in the deconfined phase such nonperturbative effects are expected to be suppressed. In particular, hybrid states like Σ_u^- and Π_u are not expected to survive as bound states. It is worthwhile to study the long distance behavior of operators like W_G , and in particular, examine if the behavior of the Wilson loop is consistent with the description of the state via an interaction potential. If such a potential can be extracted, and the potential has very little dependence on the G field being inserted, then one can interpret the potential as giving information about the interaction between the $Q\bar{Q}$ pair in the color octet configuration.

This is the goal of the current work. We will study the thermal expectation value of an operator like W_G , and investigate whether the time dependence of this quantity can be interpreted in terms of a thermal potential $V^G(\vec{r}; T)$. We will see in Sec. IV that this is indeed the case. Then we will examine the dependence of $V^G(\vec{r}; T)$ on the G field inserted. In this work we have restricted ourselves to the two operators, B_+ and B_z . We will see that, in contrast to the confined phase, the thermal potential for these two operators agree to much larger distances. The most obvious interpretation of such an agreement is that the factorization of the G field implied in Eq. (8) holds in the deconfined phase. We interpret this potential physically, therefore, as

an interaction potential between the Q and the $\bar{Q}$ in a color-octet configuration.

At finite temperatures, the τ extent of the thermal Wilson loop W_G is finite: $0 < \tau < \beta = 1/T$. Just as in the case of the singlet [4], one can define in HTL perturbation theory an effective thermal potential by continuing to Minkowski time and taking a long time derivative *à la* Eq. (4). For completeness, we outline the steps in Appendix B in leading order of HTL perturbation theory. In this order, the factorization implied in Eq. (8) holds and we get, in the Debye screening regime, a complex thermal potential

$$\begin{aligned} V^o(\vec{r}; T) &= V_{\text{re}}^o(\vec{r}; T) - iV_{\text{im}}^o(\vec{r}; T) \\ V_{\text{re}}^o(\vec{r}; T) &= \frac{g^2}{2N_c} \frac{e^{-m_D r}}{4\pi r} - \frac{g^2 C_F}{4\pi} m_D \\ V_{\text{im}}^o(\vec{r}; T) &= \frac{g^2 T}{2\pi} \left[\frac{N_c}{2} \int_0^\infty \frac{dz z}{(z^2 + 1)^2} \right. \\ &\quad \left. - \frac{1}{2N_c} \int_0^\infty \frac{dz z}{(z^2 + 1)^2} \left(1 - \frac{\sin zx}{zx} \right) \right] \quad (10) \end{aligned}$$

where the potential has been defined in the Coulomb gauge [6]. The Debye mass $m_D = \sqrt{\frac{N_c}{3}} gT$ for $SU(N_c)$ gluon plasma in this order of perturbation theory, and $x = m_D r$. It is interesting to compare Eq. (10) to the thermal singlet potential [4]:

$$\begin{aligned} V_T^{\text{re}}(\vec{r}) &= -\frac{g^2 C_F}{4\pi r} e^{-m_D r} - \frac{g^2 C_F}{4\pi} m_D \\ V_T^{\text{im}}(\vec{r}) &= \frac{g^2 C_F T}{2\pi} \int_0^\infty \frac{dz z}{(z^2 + 1)^2} \left(1 - \frac{\sin zx}{zx} \right). \quad (11) \end{aligned}$$

Both $V_{\text{im}}^o(\vec{r}; T)$ and $V_T^{\text{im}}(\vec{r}) \rightarrow TC_F g^2/4\pi$ as $r \rightarrow \infty$. $V_T^{\text{re}}(\vec{r})$ corresponds to the usual physics of Debye screening in medium, such that for sufficiently large screening, the bound states will not form. On the other hand, $V_T^{\text{im}}(\vec{r})$ clearly leads to a broadening of the spectral function peak. It captures the physics of collision with the thermal particles leading to a decoherence of the $Q\bar{Q}$ wave function [6,14].

The expressions Eq. (10) and Eq. (11) are only leading order expressions, valid in the HTL limit $T \gg 1/r$. For the distance regime $rT \ll 1$, the thermal correction to the singlet potential has been calculated using effective field theory techniques [5]. At short distances, $V_T^{\text{im}}(\vec{r}) \sim r^2$ [5]. This is indeed the parametric behavior seen in the short distance regime nonperturbatively [11].

Our aim here is to extract the potential for the hybrid operator, $V^G(\vec{r}; T)$, nonperturbatively from the Euclidean Wilson loop Eq. (7), and enquire if the isolation of the G field insertion from an interaction potential for the Q and the $\bar{Q}$ implied in Eq. (8) approximately holds to a larger distance. The expectation for this is based on the fact that in

the deconfined plasma one can expect nonperturbative effects due to string formation to be suppressed. If the G dependence of $V^G(\vec{r}; T)$ is much less than that compared to the confined phase, one can infer $V^G(\vec{r}; T)$ to be giving information about the interaction between the Q and the $\bar{Q}$. In such a case, we will interpret the potential qualitatively as an octet potential. We emphasize that such an inference is understandably qualitative and in no way negates the issues with nonperturbative definition of the octet state discussed in [15] that were outlined in Sec. I.

For the singlet, nonperturbative calculation of $V_T(\vec{r})$ has been carried out, e.g., in [9,10,22], and recently in [11]. The potential, in particular $V_T^{\text{im}}(\vec{r})$, is very different from Eq. (11) in the temperature range $\lesssim 2T_c$. Here for the analysis of W_G we will follow the strategy of [11], which we outline below. See [11] for a more detailed discussion.

At zero temperature, modulo renormalization factors, the Minkowski space Wilson loop has the asymptotic time behavior $W_M \sim e^{-iV(r)t}$ [Eq. (4)], leading to the Euclidean time behavior $W_G \sim e^{-V(r)\tau}$. Going to sufficiently long τ , this behavior results in a plateau in $-\partial_\tau \log W_G$, from which we extract $V(r)$. At finite temperature, one does not see such a plateau. It was pointed out in [11], however, that splitting the Wilson loop in parts “symmetric” and “anti-symmetric” around $\tau = \beta/2$,

$$\begin{aligned} W_T^a(\tau, \vec{r}) &= \sqrt{\frac{W_T(\tau, \vec{r})}{W_T(\beta - \tau, \vec{r})}}, \\ W_T^p(\tau, \vec{r}) &= \sqrt{W_T(\tau, \vec{r}) \times W_T(\beta - \tau, \vec{r})}, \quad (12) \end{aligned}$$

one can extract a plateau structure from $\log W_T^a(\tau, \vec{r}) \approx (\beta/2 - \tau)V_r$ (see also Sec. III). Here $W_T(\tau, \vec{r})$ denotes the thermal expectation value of W_G . This behavior is illustrated in the left panel of Fig. 1, where we show a “local value of the potential,”

$$V_{\text{re}}^{G,\text{eff}}(r, \tau) = \frac{\log W_T^a(\tau, \vec{r})}{(\beta/2 - \tau)} \quad (13)$$

at $1.2 T_c$ from our data set 3. In Appendix C we show also how the plateau structure changes on going to higher temperatures.

The plateau from $W_T^a(\tau, \vec{r})$ gives the real part of the potential, while $W_T^p(\tau, \vec{r})$ contributes to the imaginary part. This can be understood by hypothesizing, motivated by perturbation theory, a spectral decomposition for $P(\tau) = \log W_T^p(\tau, \vec{r})$:

$$\begin{aligned} P(\tau) &= \int_{-\infty}^{\infty} d\omega \sigma(\omega; T) \frac{1}{2} (e^{-\omega\tau} + e^{-\omega(\beta-\tau)}) \\ &\quad + \tau - \text{independent terms} \\ \tau \rightarrow it \Rightarrow i\partial_t P(it) &= \int_{-\infty}^{\infty} d\omega \sigma(\omega; T) \frac{\omega}{2} (e^{-i\omega t} - e^{-\omega\beta} e^{i\omega t}). \quad (14) \end{aligned}$$

FIG. 1. Plot of “local value of the potential”, Eq. (13), from W_T^a and W_T for Set 3, at $1.2 T_c$, at $r/a_s = 4, 8$ & 16 (smearing level = 300) for the $L = 0$ (left) and $L = 1$ (right) operators. For the sake of visibility, the results at $r/a_s = 8$ and 16 have been shifted upwards by addition of 0.03 and 0.06 , respectively.

At large t , the oscillating factors $\exp(\pm i\omega t)$ ensure that only the $\omega \rightarrow 0$ contribution to the integral in Eq. (14) survives. Then $\exp(\beta\omega) \rightarrow 1$ and Eq. (14) shows that $P(it)$ leads to an imaginary potential.

To proceed further, we note that $\exp(-i\omega t) - \exp(i\omega t - \omega\beta) \xrightarrow{t \rightarrow \infty} -2\pi i\omega\delta(\omega)$. Then in order to get a finite potential $-iV_T^{\text{im}}(\vec{r}) = \lim_{t \rightarrow \infty} i\partial_t P(it)$ we need $\sigma(\omega; T) \sim \frac{1}{\omega^2}(1 + \mathcal{O}(\omega))$. Since we expect thermal physics to introduce a distribution function $(1 + n_B(\omega)) \xrightarrow{\omega \rightarrow 0} \frac{T}{\omega}$ in $P(\tau)$ (see Appendix B), we write an expansion in ω , [11]

$$\sigma(\omega; T) = (1 + n_B(\omega)) \left(\frac{c_0}{\omega} + c_1\omega + c_2\omega^3 + \dots \right). \quad (15)$$

This leads to

$$\partial_\tau P(\tau) = \frac{\pi}{\beta} c_0 \cot \frac{\pi\tau}{\beta} + \dots \quad (16)$$

On going to real t and taking the $t \rightarrow \infty$ limit, the leading term leads to the potential $V_T^{\text{im}}(\vec{r}) = \frac{\pi}{\beta} c_0$. The subleading terms in ω , denoted by $\dots$ in Eq. (16), come from the terms with the higher powers of ω in Eq. (15) and do not contribute to the potential. Near $\beta/2$ we will then expect the Wilson loop to behave like

$$W_T(\tau, \vec{r}) = e^{-V_T^{\text{re}}(\vec{r})(\tau - \frac{\beta}{2}) - \frac{\beta}{\pi} V_T^{\text{im}}(\vec{r}) \log \sin(\frac{\pi\tau}{\beta}) - \dots} W_T(\beta/2, \vec{r}) \quad (17)$$

where the higher order terms $\dots$ come from the subleading terms denoted by $\dots$ in Eq. (16).

The form of Eq. (17) is a hypothesis, motivated by perturbation theory and some general arguments that lead to a potential. Strong support of it comes from the data: just

FIG. 2. $P(\tau) = \log W_T^P(\tau, \vec{r})$, where $W_T^P(\tau, \vec{r})$ is the periodic part of the Wilson loop defined in Eq. (12), at $1.2 T_c$ for the data set 3, smearing level 300, shown with the circles. Also shown is the 1σ band of the contribution from the $V_T^{\text{im}}(\vec{r})$ term Eq. (17). The left panel shows results for the $L = 0$ channel and the $L = 1$ channel is shown in the right.

the simple form Eq. (17), without any subleading terms, gives a good description of the Wilson loop data around $\beta/2$. We illustrate this in Fig. 2 for the data at $1.2 T_c$. Here we use Eq. (17) [with only the first term of Eq. (15)] to fit the data for W_G with $G = B_z$. In the figure we show $P(\tau)$ [Eq. (14)], along with the contribution of the $V_r^{\text{im}}(\vec{r})$ term of Eq. (17) to it. The $V_r^{\text{im}}(\vec{r})$ term captures the behavior of the data near $\beta/2$ very well. Figures 1 and 2 give us confidence in the form Eq. (17) for the Wilson loop. The first subleading term of Eq. (15) has a very different behavior near the center than that of the leading term. Its addition does not significantly improve the fit quality (χ^2). While the result obtained for $V_r^{\text{im}}(\vec{r})$ remains consistent with that obtained only with the leading term, the addition of the subleading term can sometimes destabilize the plot if the interval is small, or if the data is not very accurate, as is often the case for W_G . For large distances $rT > 1$ this problem gets more severe. Therefore in this work we stick to just the form in Eq. (17), and choose a suitable interval so that the fit quality is good. Additional plots showing the stability of fitting the Wilson loop with the leading terms, Eq. (17), are shown in Appendix C.

The discussion of the potential requires a low- ω peak in the spectral function decomposition of W_G . In earlier literature, this peak has been hypothesized to have a Lorentzian or Gaussian structure. However, the form in Eq. (17) leads to an asymmetric peak: while for $\omega \approx V_r$ it gives a Breit-Wigner structure, the fall-off from the peak is very different in the large- ω and small- ω side. The limiting behaviors are

$$\begin{aligned} \rho_{\text{low}}(r; \omega) &\approx \sqrt{\frac{2}{\pi}} \frac{V_{\text{im}}}{(V_{\text{re}} - \omega)^2 + V_{\text{im}}^2} \quad |V_{\text{re}} - \omega|, V_{\text{im}} \ll T \\ &\sim (\omega - V_{\text{re}})^{-(1 - \frac{\beta V_{\text{im}}}{\pi})} \quad \omega - V_{\text{re}} \gg T \\ &\sim e^{-\beta(V_{\text{re}} - \omega)} (V_{\text{re}} - \omega)^{-(1 - \frac{\beta V_{\text{im}}}{\pi})} \quad \omega - V_{\text{re}} \ll -T \end{aligned} \quad (18)$$

An asymmetric peak structure has also been suggested in Ref. [23], where the spectral function for thin Wilson loops was discussed from HTL perturbation theory. It agrees with Eq. (18) near the peak, but starts disagreeing away from it. The peak in Eq. (18) takes into account the fact that the nonpotential modes have been sufficiently suppressed by smearing, so that only the potential part, Eq. (17), contributes. This is supported by the data.

III. DETAILS OF THE CALCULATION

As mentioned in the previous section, our primary quantity is the nonperturbatively estimated value of the Wilson loop W_G , Eq. (7), in a gluonic plasma, at moderately high temperatures $\lesssim 2T_c$. Since we need a very fine grid of points for the extraction of the potential from the Wilson loop, we have used a space-time anisotropic discretization, with $\xi = a_s/a_\tau = 3$. We have generated lattices with the anisotropic Wilson action

$$\begin{aligned} S_W &= -3V(\beta_s P_s + \beta_\tau P_t), \\ \text{where } P_s &= \frac{1}{3VN_c} \sum_x \sum_{i < j} \text{ReTr} U_{ij}(x), \\ P_t &= \frac{1}{3VN_c} \sum_x \sum_i \text{ReTr} U_{0i}(x) \end{aligned} \quad (19)$$

are the spatial and temporal plaquette variables, $V = N_s^3 \times N_\tau$, x runs over all space-time points, i runs over all spatial indices, and $U_{mn}(x)$ is the gauge connection around the mn direction plaquette at x . We follow the method of Ref. [24] to get β_s, β_τ ; ξ is estimated from comparison of spatial and temporal Wilson loops. The lattice parameters we use are given in Table I. For each set, short Monte Carlo runs are made at closely spaced N_τ to find the N_τ for deconfinement transition, and then other temperatures are obtained by varying N_τ . The value of N_τ at T_c , determined by the peak of the Polyakov loop susceptibility, is shown in Table I. For the measurements we

TABLE I. List of lattices used in our calculation. For each set, #meas Wilson loop measurements were done for each of #init parallel runs, giving a statistics of #meas $\times$ #init. See text for other details.

Set	β_s	β_τ	$N_\tau(T_c)$	Size	#init	#meas	Smearing
1	2.53	15.95	30	$48^3 \times 20$	47	25	200
2	2.57	16.53	38	$32^3 \times 25$	91	100	150, 200, 250, 300
				$32^3 \times 32$	91	100	150, 200, 250, 300
3	2.60	16.98	45	$40^3 \times 23$	91	90	150, 200, 250, 300, 400
				$40^3 \times 30$	91	90	150, 200, 250, 300, 400
				$40^3 \times 38$	89	90	150, 200
					89	270	250
					89	360	300, 400
				$30^3 \times 60$	91	140	200
				$30^3 \times 72$	91	450	200

ran multiple Monte Carlo chains: first $\#_{\text{init}}$ number of well-decorrelated, thermalized configurations were generated. From each of these starting configurations long Monte Carlo chains were constructed: each chain was further thermalized, and then $\#_{\text{meas}}$ measurements made, two measurements being separated by (100 timelink multilevel [25] hits + 100 update sweeps). Each update step consisted of 1 heatbath + 3 overrelaxation steps, whereas for the multilevel only heatbath steps were used.

We therefore have $\#_{\text{init}} \times \#_{\text{meas}}$ measurements of timelike Wilson loops, i.e., the Euclidean time version of W_M , with magnetic field insertions [Eq. (6)] at the middle of the spatial connection. All averages and errors are done via a bootstrap analysis; for the bootstrap, we have further averaged the $\mathcal{O}(10^3)$ measurements into $\mathcal{O}(10^2)$ blocks ($\mathcal{O}(10)$ for set 1). The magnetic field operators have been implemented using the clover construction. As in the singlet case [11], we do APE smearing [26] of the spatial links to reduce the nonpotential effects. At each APE step, a spatial gauge link is replaced by $\text{Proj}_{SU(3)}(\alpha \times \text{link} + \sum \text{spatial staples})$, where we kept $\alpha = 2.5$. We have looked at data from a number of APE steps (shown in Table I). With higher number of APE steps, the data quality decreases, but the effect of the nonpotential terms also decreases, making it easier to reach a plateau and extract a potential.

The use of spatial gauge link smearing is well-known in potential studies, and detailed nonperturbative studies exist. In our context, it is instructive, however, to understand its effect in the leading order for the Wilson loop. This is discussed in Appendix A. Some illustrations of its effect on the extracted potential can be found in Appendix C.

In this work, we have given all physical quantities in temperature units. Conversion to physical units, if needed, can be done by setting T_c to 280 MeV, which is the value obtained by fixing the string tension: $\sqrt{\sigma} = 0.44$ GeV. The spatial size of the lattices are > 1.8 fm, with the lattice of set 1 being 3.4 fm.

IV. W_G AND THE $Q\bar{Q}$ INTERACTION POTENTIAL

We have argued in Sec. II that the data for the thermal Wilson loop $W_T(\tau, \vec{r})$ near $\beta/2$ can be modelled by Eq. (17). In this section we will present results for the effective thermal potential for the hybrid operator $J_G(t, \vec{r})$, Eq. (6), obtained from fitting the data for W_G in a symmetric interval around $\beta/2$ over which the fits with the simple forms discussed in Sec. II show a plateau. More examples of such plateau are shown in Appendix C. The results presented here are obtained with a bootstrap analysis, where the parameter values within each bootstrap sample are obtained by a χ^2 fit with the full covariance matrix. The statistical error for a given fit range is the (16,84) percentile band of the bootstrap distribution. We present results for the real part of the potential in Sec. IV A, and in Sec. IV B the results for the imaginary part are

FIG. 3. Hybrid potentials below T_c for the $L = 0$ (B_z) and $L = 1$ (B_+) channels. The results are obtained from the $30^3 \times 72$ lattices in Table I. The results for $L = 1$ channel have been shifted slightly along x-axis in the plot, for ease of viewing.

shown. The quoted errors in Sec. IV A and Sec. IV B also include effect of varying the fit range within the plateau region, and spread over smearing levels. Discussion of various systematics related to the results presented in this section have been put in Appendix C.

A. $V_{\text{re}}^G(\vec{r}; T)$

As discussed in Sec. II, the results for $V_{\text{re}}^G(\vec{r}; T)$ are obtained from $W_T^a(\tau, \vec{r})$ with the hybrid current operators. At $T = 0$ the hybrid potential has been studied in detail in the literature. In Fig. 3 we show the potential obtained by us below T_c for the two operator insertions B_z and B_+ . Strictly speaking the lattices here are at $0.63 T_c$ (see Table I); but in the gluonic theory one expects very little thermal effect at this temperature, and we indeed checked that our results are in very good agreement with a recent analysis of $T = 0$ hybrid potential, Ref. [16]. Since this is, in effect, a zero temperature potential, the results shown in Fig. 3 are from a conventional exponential fit. We did check, however, that an analysis along the lines of that prescribed in Sec. II gives results that completely agree with the exponential fit, and, in particular, that the imaginary part of the potential is zero.

From Eq. (9) we would expect that, at short distance, the potentials extracted for the two channels would agree and give the octet potential, modulo an additive constant. As the figure shows, the potentials do seem to agree at very short distances $\lesssim 0.1$ fm, showing the repulsive behavior expected of the octet channel. A detailed analysis of the short distance part has been made in [18]. At larger distances, nonperturbative effects start dominating, and the potentials for the two channels have a different nature. At such distances an interpretation of the hybrid potential as the color octet potential between the $Q\bar{Q}$ is no longer tenable. While both the $L = 0$ and $L = 1$ channels are attractive at long distances, supporting bound states for Σ_u^- and Π_u respectively, quantitatively the long distance

FIG. 4. Comparison of the $V_{\text{re}}^G(\vec{r}; T)$ obtained for the $L = 0$ (B_z) and $L = 1$ (B_+) channels. (Left) $1.2 T_c$, (middle) $1.5 T_c$ and (right) $2.0 T_c$. The points for B_+ have been shifted slightly along x-axis in the plot.

attractive part is very different for the two channels, and also from the long distance part of the singlet.

At finite temperatures, the long-distance nonperturbative behavior is suppressed, and one may expect to be able to extract information about the octet potential over longer distances. This is exactly what we found for $V_{\text{re}}^G(\vec{r}; T)$.

In Fig. 4 we compare the results of $V_{\text{re}}^G(\vec{r}; T)$ extracted from the Wilson loop with B_z and B_+ insertions, following Sec. II. Note that the potential can be extracted from the Wilson loop modulo an additive renormalization constant (see Appendix B). For the results in this section, we have fixed the additive renormalization constant by demanding that the $T = 0$ singlet potential at the shortest distance $r = a_s$ matches the lattice discretized Coulomb potential:

$$V_s(r = a_s, T = 0) = -g^2 C_F \int \frac{d^3 k}{8\pi^3} \frac{\cos k_3 a_s}{4 \sum_i \sin^2(k_i a_s/2)} \quad (20)$$

where for the coupling g^2 we have used the “boosted coupling” $g^2(r \sim a) = \frac{6}{\sqrt{\beta_s \beta_t} \sqrt{P_s P_t}}$; P_s, P_t are the plaquette variables defined in Eq. (19). The choice of the coupling and the matching point is somewhat arbitrary, and no detailed study of systematics of the subtraction was done;

so the potentials should be taken to be defined modulo a small, temperature-independent additive constant. We stress that the additive normalization of the singlet at $T = 0$ fixes the renormalization of both $V_{\text{re}}^s(\vec{r}; T)$ and $V_{\text{re}}^G(\vec{r}; T)$ at all temperatures.

The difference between Figs. 4 and 3 is stark: there is no nonperturbative rising part of the potential above T_c , and the potentials for $L = 0$ and $L = 1$ agree very well (within errors) to the distance studied. As per the discussion of Sec. II, we will infer this potential as the interaction potential between the $Q\bar{Q}$ in the color octet configuration.

In Fig. 5 we show $V_{\text{re}}^G(\vec{r}; T)$ at different temperatures below and above T_c . As indicated in the figure, a constant has been subtracted from $V_{\text{re}}^G(\vec{r}; T)$ below T_c for showing it in the same scale. As the figure shows, the potential at $0.75 T_c$ matches very well with that at $0.63 T_c$, supporting our earlier assertion that they both are consistent with the potential at $T = 0$. The figure clearly displays the effect of the deconfinement transition on the potential: the potential above T_c is repulsive at all distances. There is no trace of the long distance nonperturbative attractive part present in the potential below T_c . As we already mentioned, the channel independence of the potential above T_c , shown in Fig. 4, leads us to associate this potential with the interaction potential $V_{\text{re}}^o(\vec{r}; T)$ between the color octet $Q\bar{Q}$.

FIG. 5. Temperature dependence of $V_{\text{re}}^G(\vec{r}; T)$; (left) $L = 0$ and (right) $L = 1$ channels. As indicated in the label, a constant has been subtracted from the potential below T_c for convenience of showing it with the above T_c results. Also the data points for $0.63 T_c$ have been shifted slightly to the left for visual clarity.

FIG. 6. (Left) Comparison of the singlet and hybrid potentials $V_{\text{re}}^G(\vec{r}; T)$ above T_c . For $V_{\text{re}}^G(\vec{r}; T)$, results for both $L = 0$ and $L = 1$ channels are shown (with the $L = 1$ points slightly shifted along x axis for ease of viewing). In the right panel, we show this comparison down to a distance $rT_c \sim 2.4$, for $1.5 T_c$. In this plot the long distance part is from the coarser lattice of set 1.

While this is the first nonperturbative study of $V_{\text{re}}^o(\vec{r}; T)$ from lattice, an observable constructed with Wilson lines, which was interpreted in perturbation theory as free energy of a Coulomb gauge fixed color octet $Q\bar{Q}$ pair in the plasma, has been studied nonperturbatively before [19,20]. For the singlet channel, the free energy is real, and is identical to the potential at $T = 0$. In the plasma, it has been found to be close to the real part of the potential; see [11] for a comparison of the two. The (perturbatively defined) color octet free energy in Coulomb gauge is qualitatively similar to the behavior of $V_{\text{re}}^o(\vec{r}; T)$ shown in Fig. 5. Note that the nonperturbative interpretation of such a Coulomb gauge fixed free energy is far from obvious; see Ref. [27].

Since, as we argued, the hybrid potential above T_c can be interpreted as the interaction potential between heavy $Q\bar{Q}$ in a color octet configuration, it is interesting to compare it with the potential for the $Q\bar{Q}$ in a color singlet combination. In Fig. 6 we display together the octet and singlet potentials above T_c . The octet potential is much flatter than the singlet potential. Also at each temperature, the two potentials approach the same temperature-dependent constant. Physically one expects this; at sufficiently long distance the interaction between the Q and the $\bar{Q}$ is expected to vanish; the remnant constant then may be interpreted as a thermal correction to mass of the quark.

In the right panel of Fig. 6 we check this behavior down to longer distances using the larger, but coarser, set 1 data. Both these behaviors are qualitatively consistent with the expectations from perturbation theory, Eq. (10) and Eq. (11). We also note that the convergence of the singlet and octet potentials happen at shorter distances at higher temperatures. This is also expected, since the difference is $\propto \frac{e^{-m_D r}}{r}$, which is smaller at higher temperatures, where m_D is larger. However, there are some quantitative differences from perturbation theory: in particular, the differences in the asymptotic values at two temperatures is larger than what Eq. (10) predicts.

To further check the conformity with the perturbative behavior, we look at $\delta V_T^{\text{re}}(r) = V_T^{\text{re}}(r+1) - V_T^{\text{re}}(r)$. In leading order perturbation theory, the ratio of this quantity in singlet and octet channels is $-(N_c^2 - 1)$. As Fig. 7 shows, our data is consistent with this.

B. $V_{\text{im}}^G(\vec{r}; T)$

The effective thermal potential is in general complex [4], with the imaginary part of the potential related to damping and decoherence mechanisms [6]. In the case of the singlet channel, where the potential is attractive and leads to spectral function peaks for sufficiently massive quarks, the imaginary part controls the width of the spectral

FIG. 7. Comparison of $\delta V_T^{\text{re}}(r) = V_T^{\text{re}}(r+1) - V_T^{\text{re}}(r)$ for singlet and octet channels, at $1.2 T_c$ (left), $1.5 T_c$ (middle) and $2 T_c$ (right). The points for B_z have been slightly shifted along the horizontal axis.

FIG. 8. $V_{\text{im}}^G(\vec{r}; T)$ extracted from $L = 0$ (B_z) and $L = 1$ (B_+) channels, at $1.2 T_c$ (left) and $1.5 T_c$ (right). The points for B_+ have been slightly shifted horizontally. The singlet channel result at the same temperature, $V_{\text{im}}^s(\vec{r}; T)$, is also shown.

FIG. 9. Temperature dependence of $V_{\text{im}}^G(\vec{r}; T)$, for $L = 0$ (left) and $L = 1$ (right) channels. The points for $1.5 T_c$ have been slightly shifted along the horizontal axis.

function peak. Such an interpretation is not available here, and the imaginary part is to be understood as introducing decoherence in the $Q\bar{Q}$ system during its evolution as octet.

The extraction of the imaginary part from the hybrid operators of the sort used here turns out to be more problematic, and we can only get partial information about them. In particular, we were not able to get reliable results for $V_{\text{im}}^G(\vec{r}; T)$ at $2 T_c$, and show results only for $1.2 T_c$ and $1.5 T_c$.

In Fig. 8 we compare the results for $V_{\text{im}}^G(\vec{r}; T)$ with insertion of the B_z and B_+ operators. Here we see a different behavior from that seen in Fig. 4 for $V_{\text{re}}^G(\vec{r}; T)$: while at short distances, the results for the two insertions agree within statistical error, at longer distances $rT \gtrsim 1$ they do not agree at the 1σ level. The disagreement in Fig. 8 is not large, and we have also noted in Sec. II the issue with the stability of the fit for $rT > 1$, so we cannot make a definitive statement here; but clearly, based on our data, we cannot justify identifying $V_{\text{im}}^G(\vec{r}; T)$ as color octet potential for large distances. For short distances $rT \lesssim 1$ $V_{\text{im}}^G(\vec{r}; T)$ is independent of G (for the two choices of G we have used), and we will infer this short distance potential as a color octet $Q\bar{Q}$ potential.

In Fig. 9 we show the temperature dependence of $V_{\text{im}}^G(\vec{r}; T)$. Below T_c $V_{\text{im}}^G(\vec{r}; T)$ is consistent with zero. Above T_c it is very different from zero. In particular, the most striking behavior of $V_{\text{im}}^G(\vec{r}; T)$ is that at $r \rightarrow 0$ it approaches a finite value. It is interesting to see that this behavior is qualitatively consistent with the behavior suggested in Eq. (10), though of course, Eq. (10) is based on HTL perturbation theory and is not valid in short distances.

This behavior is very different from that of the singlet channel, where $V_{\text{im}}^s(\vec{r}; T)(r \rightarrow 0) \rightarrow 0$ [11]. In Fig. 8 we have also shown the imaginary part of the singlet potential. We find that $V_{\text{im}}^s(\vec{r}; T)$ vanishes at $r \rightarrow 0$, but rises $\sim r^2$ at small r [11]. The slope of $V_{\text{im}}^o(\vec{r}; T)$ at small r is smaller than that of $V_{\text{im}}^s(\vec{r}; T)$. Within the accuracy of our results, $V_{\text{im}}^o(\vec{r}; T)$ for $rT < 1$ is consistent with a constant function. We do not find evidence of $V_{\text{im}}^o(\vec{r}; T)$ decreasing with r , as suggested by Eq. (10).

V. DISCUSSION

One way to study the behavior of a heavy $Q\bar{Q}$ pair in quark-gluon plasma is through the introduction of an

effective thermal potential [4,5,14]. The field theoretic definition of such a potential has been given, and it has been calculated in perturbation theory using effective field theory techniques [4,5]. While the real part of the potential connects to the physics of Debye screening, the imaginary part is related to the physics of Landau damping and decoherence of the wave function of the $Q\bar{Q}$ state [14].

Nonperturbative information about the potential of the $Q\bar{Q}$ in a color singlet state is available in the literature [9–11]. The real part of the potential shows the expected Debye screening, but at a quantitative level, differs from the leading order perturbative potential even at $2 T_c$. The deviation from perturbation theory is even stronger in the imaginary part of the potential.

For the phenomenology of quarkonia in quark-gluon plasma, one needs information about the interaction between the heavy Q and $\bar{Q}$ when they are both in a color singlet and a color octet combination. The color singlet $Q\bar{Q}$ can interact with the thermal gluons to go to a color octet combination, and vice versa. In contrast to the singlet potential, very little is known nonperturbatively about the in-medium interaction of $Q\bar{Q}$ in a color octet configuration. One reason for this is the difficulty in nonperturbatively defining a color octet potential [15].

In this paper we investigate the potential for states created by a hybrid operator like J^G , Eq. (6), which is a color singlet formed of J^a [Eq. (5)], a $Q\bar{Q}$ state transforming like color octet at point x_0 , with the gluonic operator $G_a(x_0)$ that transforms like an adjoint representation. G^a is chosen such that the state does not have overlap with the $Q\bar{Q}$ singlet states like the quarkonia states. Our methods for extraction of the thermal potential from the hybrid Wilson loop is similar to the method employed in Ref. [11] for the singlet potential, and is outlined in Sec. II. When we define the potential for such a state, in perturbation theory, G^a factorizes and one gets the potential for the color octet $Q\bar{Q}$ pair. Such a separation does not hold nonperturbatively below T_c , except at very short distances; see Fig. 3.

In the deconfined phase, however, the situation turns out to be very different. In Fig. 4 we show $V_{\text{re}}^G(\vec{r}; T)$ obtained from the states of Fig. 3. Within our calculational accuracy, they agree at all distances explored by us. We explored the agreement to $rT_c \approx 1.5$ at temperatures $\leq 2T_c$, and at $1.5 T_c$ explored to $rT_c \approx 2.4$ with the coarse lattice of Set 1 (Table I). We infer this agreement as isolation of the G^a from the potential, and interpret the hybrid potential as the potential between $Q\bar{Q}$ in the color octet combination. We emphasize that this is an interpretation of the hybrid potential in terms of color octet potential, based on the lack of sensitivity of the potential on G^a .

Our results for the color octet potential are summarized in Fig. 5. The potential is found to be screened above T_c , and does not have any attractive part. This indicates that there will not be any bound states of the heavy $Q\bar{Q}$ in the

plasma in the color octet configuration. At long distances, the potential agrees with the color singlet potential (Fig. 6). Within the accuracy of our data, the data is also consistent with the leading order scaling behavior

$$\frac{\delta V_{\text{re}}^o(\vec{r}; T)}{\delta V_{\text{re}}^s(\vec{r}; T)} = -\frac{C_A/2 - C_F}{C_F};$$

this is demonstrated in Fig. 7.

The thermal effective potential is known to have an imaginary part [4]. The effective potential obtained from the hybrid state also has an imaginary component. The extraction of this part is more difficult, and our results for $V_{\text{im}}^G(\vec{r}; T)$ have large errors. In particular, the results for $rT > 1$ have large systematic uncertainties. The imaginary part of the extracted potential is more difficult to interpret in terms of a color octet potential. For the two hybrid operators we looked at, we found agreement in $V_{\text{im}}^G(\vec{r}; T)$ only up to distances $rT \sim 1$; see Fig. 8. The details of the gluonic operator may become important at larger distances, though we cannot make a definitive statement about this because of the large uncertainties in the data, and also the problems with extraction of the imaginary part, as mentioned in the text. Anyway, with our data we can only meaningfully identify the extracted potential as octet potential for distances $rT \lesssim 1$.

The most striking difference between the imaginary parts of the singlet and the octet potential, shown in Fig. 8, is the behavior at short distances. $V_{\text{im}}^s(\vec{r}; T)$ approaches zero at short distances. On the other hand, $V_{\text{im}}^o(\vec{r}; T)$ acquires a nonzero value even at $r \rightarrow 0$ on crossing T_c . This is consistent with the behavior predicted in perturbation theory, and is also in line with physical intuition [6]. The singlet $Q\bar{Q}$ at very short distances will look like a colorless object to the medium particles, which will not be able to resolve its structure. On the other hand, the medium particles will interact strongly with the color octet $Q\bar{Q}$, leading to damping. The r dependence of the imaginary part is much milder than that of the singlet in the region $rT \lesssim 1$; within the (limited) accuracy of our calculation, $V_{\text{im}}^o(\vec{r}; T)$ is consistent with a constant in this region.

This work provides, to our knowledge, the first non-perturbative information about the interaction between Q and $\bar{Q}$ in a color octet configuration in gluon plasma. It shows that the interaction potential is screened, and does not support any bound state in this channel. Many features of the leading order perturbative potential, Eq. (10), are in qualitative agreement with the nonperturbative results obtained here.

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APPENDIX A: APE SMEARING

As discussed in Sec. III, the extraction of the potential from thin link Wilson loops is difficult, and we do APE smearing [26] of the spatial gauge links. This consists of replacing the spatial gauge links

$$U_i(\vec{x}, \tau) \rightarrow \text{Proj}_{SU(3)} \left[\alpha U_i(\vec{x}, \tau) + \sum_{\substack{1 \leq j \leq 3 \\ j \neq i}} \{ U_j(\vec{x}, \tau) U_i(\vec{x} + a_s \hat{j}, \tau) U_j^\dagger(\vec{x} + a_s \hat{i}, \tau) \right. \\ \left. + U_j^\dagger(\vec{x} - a_s \hat{j}, \tau) U_i(\vec{x} - a_s \hat{j}, \tau) U_j(\vec{x} - a_s \hat{j} + a_s \hat{i}, \tau) \} \right] \quad (\text{A1})$$

iteratively. While the quality of the signal for the Wilson loop deteriorates with the number of smearing steps, the effect of nonpotential terms also decrease, making extraction of potential easier. For this work, we have taken $\alpha = 2.5$, and have done up to 400 steps of APE smearing.

It is instructive to see the effect of APE smearing on the leading order expressions for the Wilson loops. Following [28] we write the effect of smearing on the gauge fields, A_μ , where $V(x, x + a\mu) = e^{iaA_\mu(x)}$. To linear order,

$$A_i^N(Q) = \{f^N(\hat{q}) P_{ij}^T(\hat{q}) + P_{ij}^L(\hat{q})\} A_j(Q) \quad (\text{A2})$$

where

$$f(\hat{q}) = \left(1 - \frac{c}{4} \hat{q}^2\right) \sim e^{-\frac{c}{4} \hat{q}^2}, \quad \hat{q}^2 = \sum_{i=1}^3 \hat{q}_i^2, \quad \hat{q}_i = 2 \sin q_i a_s / 2,$$

$c = \frac{4}{4+\alpha}$, and the projection operators are defined above. For small $q_i a_s$, $\hat{q}_i \rightarrow q_i a_s$ and the projection operators become $P_{ij}^{T,L}(q)$.

Then the propagator of the smeared fields,

$$G_{ij}^N(Q) = \langle A_i^N(Q) A_j^N(-Q) \rangle \sim f^{2N}(q) \frac{P_{ij}^T(q)}{Q^2 + \Pi_T(\omega_q, \vec{q})} \quad (\text{A3})$$

leads to the spectral function representation

$$G_{ij}^N(q) \equiv \int_{-\infty}^{\infty} \frac{dq_0}{\pi} \frac{\tilde{\rho}_T(q_0, \vec{q})}{q_0 - i\omega_k}, \\ \tilde{\rho}_T(q_0, \vec{q}) \sim f^{2N}(q) \rho_T(q_0, \vec{q}). \quad (\text{A4})$$

This results in a suppression of the nonpotential contribution to Wilson loop, as we discuss in the next section.

APPENDIX B: LO CALCULATION OF POTENTIAL IN HTL

Various strategies in our nonperturbative calculation of the potential has been motivated by insights from perturbation theory and in particular, the expression for the Wilson loop in LO HTL approximation. Here we put together the leading order results for the thin Wilson loop, 3 and 7, in this approximation. This section follows Ref. [4].

We use the Coulomb gauge. Then the gluon propagators are

$$D_{00}(\omega_n, \vec{k}) = \frac{1}{K^2 + \Pi_E(K)} \frac{K^2}{\vec{k}^2}, \quad D_{ij}(\omega_n, \vec{k}) = \frac{1}{K^2 + \Pi_T(K)} \left(\delta_{ij} - \frac{k_i k_j}{\vec{k}^2} \right). \quad (\text{B1})$$

Here K refers to the Euclidean four-momenta $(\omega_n, \vec{k})$. The spectral functions $\rho_E(k_0, \vec{k})$, $\rho_T(k_0, \vec{k})$, introduced through the integral relations

$$\frac{1}{K^2 + \Pi_{T,E}(K)} = \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \frac{\rho_{T,E}(k_0, \vec{k})}{k_0 - i\omega_k}, \quad (\text{B2})$$

provide the connection to Minkowski momenta.

For the singlet channel, the potential in LO will come from diagrams for ordinary Wilson loop similar to the ones shown within parentheses of Eq. (8). They add up to

$$g^2 C_F \int \frac{d^3 k}{8\pi^3} (\cos k_3 r - 1) \left\{ \frac{\tau}{\vec{k}^2 + \Pi_E(0, \vec{k})} + \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \rho_E(k_0, \vec{k}) (1 + n_B(k)) \left(\frac{1}{\vec{k}^2} - \frac{1}{k_0^2} \right) (1 + e^{-\beta k_0} - \mathcal{F}(k_0, \tau)) \right\} \quad (\text{B3})$$

where we define the symmetric and antisymmetric functions

$$\mathcal{F}(k_0, \tau) = e^{-k_0 \tau} + e^{-(\beta - \tau)k_0}, \quad \mathcal{G}(k_0, \tau) = e^{-k_0 \tau} - e^{-(\beta - \tau)k_0}. \quad (\text{B4})$$

The term linear in τ in Eq. (B3) survives in $W_T^a(\tau, \vec{r})$, leading to the potential

$$V_r \equiv g^2 C_F \int \frac{d^3 k}{8\pi^3} \frac{\cos k_3 r - 1}{\vec{k}^2 + \Pi_E(0, \vec{k})} = -\frac{g^2 C_F}{4\pi r} e^{-m_D r} + g^2 C_F \left(-\frac{m_D}{4\pi} + I_a \right) \quad (\text{B5})$$

where the additive divergent term $I_a = \int \frac{d^3 k}{8\pi^3} \frac{1}{k^2} \sim \frac{1}{a}$ in Eq. (B5) results from defining the potential through Wilson loop, forcing $V_r(r \rightarrow 0) \rightarrow 0$. The standard convention of defining potential, used in Eq. (11), sets $I_a \rightarrow 0$, so that we get the familiar Coulomb potential at short distances. This has been done in Sec. IV by fixing the $T = 0$ singlet potential at $r = a_s$ through Eq. (20).

Going to Minkowski time and taking large t , using the relation

$$\lim_{t \rightarrow \infty} i \partial_t \mathcal{F}(k_0, \tau) |_{\tau \rightarrow it} = \lim_{t \rightarrow \infty} k_0 (e^{-ik_0 t} - e^{-\beta k_0} e^{ik_0 t}) \rightarrow -k_0^2 2\pi i \delta(k_0). \quad (\text{B6})$$

we see that the potential picks up contribution from $\rho_E(k_0 \rightarrow 0, k_0 \vec{\rightarrow} 0)$. In leading order of HTL perturbation theory, for $|k_0| \ll |\vec{k}|$, $\rho_T(k_0, \vec{k})$, $\rho_E(k_0, \vec{k})$ in Eq. (B2) behave like [4]

$$\rho_E(k_0, \vec{k}) \approx -\pi m_D^2 \frac{k_0}{2|\vec{k}|(k_0^2 + m_D^2)^2}, \quad \rho_T(k_0, \vec{k}) \approx \pi m_D^2 \frac{\omega}{4|\vec{q}|^5}. \quad (\text{B7})$$

The term with $1/k_0^2$ in the second term of Eq. (B3) then leads to $V_T^{\text{im}}(\vec{r})$ in Eq. (11). The $1/k_3^2$ term does not lead to a potential; Fig. 2 indicates that the contribution of this term is small near $\tau = \beta/2$. As discussed in Sec. A, smearing will suppress $\rho_T(k_0, \vec{k})$.

The diagrams

(B8)

add up to

$$g^2 C_F \int \frac{d^3 k}{8\pi^3} (\cos k_3 r - 1) \left(\frac{1}{k_3^2} - \frac{1}{\vec{k}^2} \right) \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \rho_T(k_0, \vec{k}) (1 + n_B(k_0)) (1 + e^{-\beta k_0} - \mathcal{F}(k_0, \tau)). \quad (\text{B9})$$

Here the gluon lines correspond to transverse gluon propagators. Equation (B9) does not contribute to the potential, as can be seen using Eq. (B6). However, they will contribute to the fit near $\tau \sim \beta/2$. These terms, however, have $\rho_T(k_0, \vec{k})$; as explained in Sec. A, smearing leads to a strong suppression of these terms. When the results for potential stabilize with number of smearing

steps, it indicates that the contribution of these terms have become negligible and we are getting contribution from the potential terms only.

The discussion for the hybrid Wilson loop is similar. The potential contributions in LO come from the diagrams explicitly shown in the right-hand side of Eq. (8), summing up to

$$\langle BB \rangle \times \left\{ 1 + \int \frac{d^3 k}{8\pi^3} \left(g^2 C_F + \frac{g^2}{2N_c} e^{ik_3 r} \right) \left[-\frac{\tau}{\vec{k}^2 + \Pi_E(0, \vec{k})} + \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \rho_E(k_0, \vec{k}) \left(\frac{1}{k_0^2} - \frac{1}{\vec{k}^2} \right) (1 + n_B(k_0)) (1 + e^{-\beta k_0} - \mathcal{F}(k_0, \tau)) \right] \right\} \quad (\text{B10})$$

Renormalizing in the same way as the singlet leads to the potentials Eq. (10). We reiterate that the additive renormalization we have used is fixed by matching of the $T = 0$ singlet potential at $r/a_s = 1$: no separate additive renormalization is used for the octet.

Other diagrams included in $\dots$ in Eq. (8) are

(B11)

and variations: where the gluon lines are at $\tau = 0$ or to the right of B , etc. The sum of their contributions is

$$\langle BB \rangle \times \left\{ \frac{g^2}{2N_c} \int \frac{d^3 k}{8\pi^3} \frac{1 - \cos k_3 r}{k_3^2} - \frac{g^2 N_c}{2} \int \frac{d^3 k}{8\pi^3} \frac{2 - \cos k_3 x - \cos k_3 (r - x)}{k_3^2} \right\} \times \int_{-\infty}^{\infty} \frac{dk_0}{\pi} \rho_T(k_0, \vec{k}) (1 + n_B(k_0)) (1 + e^{-\beta k_0} - \mathcal{F}(k, \tau)). \quad (\text{B12})$$

Using Eq. (B6) we see that they do not contribute to the potential. In the Euclidean time data, smearing suppresses their contribution, due to the $\rho_T(k_0, \vec{k})$ terms.

Diagrams that do not satisfy the factorization behavior of Eq. (8) are

(B13)

The left set involves only the symmetric function $\mathcal{F}(k, \tau)$ whereas the right set involves both $\mathcal{F}(k, \tau)$ and $\mathcal{G}(k, \tau)$ of Eq. (B4). The expressions are straightforward, if unilluminating; it is easy to check that they do not contribute to the potential. Also they both involve two or more factors of $\rho_T(k_0, \vec{k})$ and can be suppressed by smearing.

The diagrams Eq. (B12) and Eq. (B13) do not contribute to the potential; however, they contribute in the finite τ Euclidean Wilson loops. For a successful extraction of the potential, we need to identify a plateau where their contributions are negligible. As the structures of these terms demonstrate, smearing lead to their suppression; that is why we get the plateaus demonstrated in Fig. 1 and in Appendix C, from where we can extract the potential.

APPENDIX C: DISCUSSION OF VARIOUS SYSTEMATICS RELATED TO SEC. IV

In this Appendix we present some more details regarding the analysis of the Wilson loop, and the various

systematics that are included in the results quoted in Sec. IV.

First, we present some more details regarding the applicability of our potential extraction method outlined in Sec. II. In particular, we present various plots of the estimate of the effective local potential.

In Fig. 1 we have shown how splitting the Wilson loop into the product of two parts, Eq. (12), makes it possible to extract $V_{\text{re}}^G(\vec{r}; T)$ because of a plateau in the “local potential” $V_{\text{re}}^{G, \text{eff}}(r, \tau)$ [Eq. (13)]. While the plateau is the most prominent in our $1.2 T_c$ set shown there, we can also identify a comfortable plateau at $1.5 T_c$ and $2 T_c$. In Fig. 10 we show $V_{\text{re}}^{G, \text{eff}}(r, \tau)$ at different temperatures for the hybrid Wilson loop with both B_z and B_+ insertion. Similar results are obtained at other distances.

In Fig. 2 we have shown how our proposed structure of the Wilson loop, Eq. (17), describes the “periodic part” of the thermal Wilson loop $W_T^p(\tau, \vec{r})$ [Eq. (12)]. This allows us to extract $V_T^{\text{im}}(\vec{r})$. For a more close-up look at this part, we define a “local value” of $V_T^{\text{im}}(\vec{r})$ from Eq. (16):

FIG. 10. The local potential $V_{re}^{G,eff}(r, \tau)$ for set 3 at different values of N_τ , corresponding to temperatures of $1.2 T_c$ ($N_\tau = 38$), $1.5 T_c$ ($N_\tau = 30$) and $2.0 T_c$ ($N_\tau = 23$), respectively. Left: $L = 0$ channel, at distance $rT_c = 0.67$. Right: $L = 1$ channel, at $rT_c = 1$. Also shown is the 1σ band of our fitted result.

FIG. 11. The local potential $V_{im}^{G,eff}(r, \tau)$ for set 3 at $1.2 T_c$, at different values of r , for the $L = 0$ (left) and $L = 1$ (right) channels. The band shows our fitted estimate of $V_T^{G,eff}(\vec{r})$. As indicated in the figure, the data at $r/a_s = 8$ and 16 have been shifted upwards by a constant for visual clarity.

$$V_{im}^{G,eff}(r, \tau) = \frac{\partial_\tau P(\tau)}{\cot \frac{\pi\tau}{\beta}} \quad (C1)$$

where for ∂_τ we use the finite difference $\frac{1}{2}(P(\tau+1) - P(\tau-1))$, τ being expressed in a_τ units.

In Fig. 11 we show $V_{im}^{G,eff}(r, \tau)$ extracted from our set 3 data at $1.2 T_c$ at some representative distances. $V_{im}^{G,eff}(r, \tau)$ is seen to show a plateau. The plateau quality deteriorates with distance, but is quite reasonable for distances $rT \approx 1$. In Fig. 12 we show $V_{im}^{G,eff}(r, \tau)$ at different temperatures for

FIG. 12. The local potential $V_{im}^{G,eff}(r, \tau)$ for set 3 at different values of N_τ , corresponding to temperatures of $1.2 T_c$ ($N_\tau = 38$), $1.5 T_c$ ($N_\tau = 30$) and $2.0 T_c$ ($N_\tau = 23$), respectively. Left: $L = 0$ channel, at distance $rT_c = 0.67$. Right: $L = 1$ channel, at $rT_c = 0.34$. The 1σ band of our fitted result at the two lower temperatures is also shown.

FIG. 13. Smearing dependence of $V_{\text{re}}^G(\vec{r}; T)$ extracted from the smeared Wilson loops W_G . The top row shows results at $1.2 T_c$ while the bottom row shows results at $1.5 T_c$. The panels to the left are for B_z and those to the right are for B_+ insertions. For ease of viewing, some sets have been slightly shifted along x axis in the plot.

both channels. The quality of the plateau deteriorates with temperature. At $2 T_c$ we do not get a reliable plateau. We do not quote a measurement for $V_T^{\text{im}}(\vec{r})$ at this temperature.

Next we discuss the effect of smearing on our extracted potential, and the size of the discretization error in our results.

1. $V_{\text{re}}^G(\vec{r}; T)$

As discussed in the text, for the spatial gauge connections $\cup$ in Eq. (7) we have used APE smeared links. Smearing also affects the G field. For the singlet potential, it was noticed that the extraction of the potential depends on

FIG. 14. The results for $V_{\text{re}}^G(\vec{r}; T)$ extracted from lattices with different discretization levels. Shown are the results at $1.2 T_c$ (top panels) and $1.5 T_c$ (bottom panels). The left panels correspond to the $L = 0$ channel while the right panels show the $L = 1$ channel results.

FIG. 15. Smearing dependence of the imaginary part of the potential extracted from the hybrid operator inserted Wilson loops. The top row shows results at $1.2 T_c$ while the bottom row shows results at $1.5 T_c$. The panels to the left are for B_z and those to the right are for B_+ insertions. For viewing purpose some sets have been slightly shifted along x axis in the plot.

the smearing level to some extent [11]: one gets a better identified plateau, and the extracted potential seems to change with the smearing level at small levels of smearing, before stabilising at some level. A similar trend is seen in the octet case, except the effect is somewhat enhanced, and

one needs to go to higher levels of smearing before the result becomes insensitive to the smearing level (within our errors). At higher levels of smearing, a better plateau is obtained; at the same time the statistical noise increases. In Fig. 13 we show the potential extracted from Wilson loops

FIG. 16. Comparison of results for $V_{\text{im}}^G(\vec{r}; T)$ at two different lattice spacings. The top row shows results at $1.2 T_c$ while the bottom row shows results at $1.5 T_c$. The panels to the left are for B_z and those to the right are for B_+ insertions.

with different levels of APE smearing, for set 3. We find that the potential saturates only at 300 smearing steps at this cutoff. For the results quoted, we have included a systematic error covering the spread between results from 300 and 400 levels of smearing. For comparison, for the same set, the singlet potential stabilised by 200 smearing steps. For set 2, we find that 200 smearing steps is enough to stabilise the potential, and the systematic error covers data with 200 and 250 smearing steps. For Fig. 13 as well as for other figures shown in cutoff units, we show the unrenormalized data (i.e., the matching to Eq. (20) is not done).

The lattice-discretized results will have discretization errors, which go to zero as one takes the continuum limit. Our lattices are quite fine-grained, so discretization effects are expected to be small. In order to estimate the size of the discretization error, in Fig. 14 we show the results for $V_{\text{re}}^G(\vec{r}; T)$ extracted from two different lattice spacings. Here we added a small overall additive constant to the results at the coarser lattice. We see that the discretization error is much smaller compared to the other uncertainties in our

calculation. We therefore take the results from our finest set, set 3, as indicative of the continuum results. These are the results shown in Sec. IV A.

2. Systematics for $V_{\text{im}}^G(\vec{r}; T)$

For $V_{\text{im}}^G(\vec{r}; T)$ the smearing dependence of the extracted results is shown in Fig. 15. For the B_z operator, the results stabilize quickly: for set 3, already by 200 smearing steps the results seem to have stabilized. For B_+ it is similar, except at $1.2 T_c$ at longer distances. Like for $V_{\text{re}}^G(\vec{r}; T)$, the error bands quoted in Sec. IV B include the spread between smearing levels 300 and 400 for set 3, and that between smearing levels 200 and 250 for set 2.

In comparison to the large errors associated with the extraction of $V_{\text{im}}^G(\vec{r}; T)$, the cutoff effects do not seem to be significant. In Fig. 16 we compare the results for $V_{\text{im}}^G(\vec{r}; T)$ from Set 2 and Set 3. Given the small cutoff dependence, we treat the results from our finest lattices as indicative of continuum results. These are the results shown in Sec. IV B.

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