Jackiw-Teitelboim gravity and the models of a Hawking-Page transition for 2D black holes

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We propose a top down construction for Jackiw-Teitelboim (JT) gravity using compactification of $D = 5$ gravity theories in the presence of Abelian $[U(1)]$ as well as $SU(2)$ Yang-Mills fields. The background solutions corresponding to the $D = 2$ model have been obtained in the perturbative regime of the theory where the corrections have been estimated over the uncharged JT solutions while treating the gauge couplings as the parameters of the expansion. Our analysis reveals the existence of two classes of solutions, namely, (i) the *interpolating* vacuum solution with AdS_2 in the IR and $Lifshitz_2$ in the UV (which serves as the thermal radiation background for our analysis) and (ii) the charged 2D black hole solution exhibiting $Lifshitz_2$ asymptotics. The analysis on thermal stability reveals the onset of a first-order phase transition at $T \sim T_0$ such that for $T < T_0$ the only possible state is the thermal radiation background without any black hole. On the other hand, as the temperature is gradually increased beyond a certain *crossover* value $T \sim T_2(>T_0)$, an emerging *globally* stable black-hole-dominated phase has been observed which clearly indicates the onset of a Hawking-Page transition in 2D gravity models.

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I. OVERVIEW AND MOTIVATION

In today's literature, the celebrated AdS/CFT correspondence [1,2] is widely accepted to be the most remarkable achievement that took place in modern theoretical physics during past two decades. The underlying mechanism that stands behind this proposal is based on the so-called holographic principle [3], whose primary focus is to understand the strong coupling dynamics in large N quantum field theories using a dual (weakly coupled) stringy or gravitational description living in higher dimensions. Since its discovery, several interesting examples have been discovered as well as tested within the realm of strong/weak conjecture, among which the very recent discovery of an interesting toy model of quantum holography [4,5] is something worthy of praise. This proposal is based on the

original work of Ref. [6] and, therefore, goes under the name of Sachdev-Ye-Kitaev (SYK) models [7,8].

Ever since this proposal has been put forward, there has been growing interest in order to explore the large N near-infrared (IR) dynamics in SYK models using its dual gravitational description(s) that lives in one higher dimension. The collective analyses unveil $(1+1)D$ Jackiw-Teitelboim (JT) dilaton gravity as the dual model [9,10] that has been conjectured. Putting all these pieces together finally leads toward an emergent SYK/ AdS_2 correspondence [11–27]. These analyses have been subsequently extended for charged SYK models and their corresponding dual gravitational counterparts [28–30].

The purpose of the present work is to elaborate an example of dimensional compactification which finally leads toward $(1+1)D$ JT dilaton gravity in the presence of nontrivial matter (gauge) couplings. The parent 5D model [31,32], that we choose to start with, was actually proposed in order to construct electrically charged space-time solutions with anisotropic Lifshitz scaling [33,34]. The Chern-Simons (CS) term in the original 5D model [31] was actually a supergravity-inspired “*FFA*”-type term [31,35]. However, the dimensional reduction, on the other hand, results in some form of the $D = 2$ JT model where both gravity as well as the dilation are found to be nonminimally coupled with the Abelian $[U(1)]$ and the $SU(2)$ Yang-Mills sectors of the theory.

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Below, we enumerate some notable facts about the $D = 2$ model that is constructed in this paper.

- (i) The ground state (vacuum) of the theory has been found to be *interpolating* between a Lifshitz₂ at UV and an AdS₂ in the IR. This we identify as the thermal radiation background for our subsequent analysis in the Euclidean formalism.
- (ii) Thermal excitations have been identified as *charged* black holes with Lifshitz₂ asymptotics. In the Euclidean framework, these black holes serve as the basis for what we call the analog of Hawking-Page (HP) transition [36–41] in 2D gravity models.
- (iii) Considering the Euclidean framework, we perform our analysis of thermal stability in black holes using a canonical ensemble. Our analysis reveals the existence of a certain critical temperature $T \sim T_0 \sim \sqrt{\mu_0}$ such that for $T > T_0$ there exist two possible phases of black holes—one with lower mass and a negative heat capacity and the other with higher mass and a positive heat capacity. As the temperature of the system is reduced below T_0 , the black hole phase with negative heat capacity decays into pure thermal radiation via a *first-order* phase transition at $T = T_0$. On the other hand, as the temperature is increased above T_0 , the lower mass black

hole gradually transits into a globally stable phase of larger mass black hole that is in thermal equilibrium with its surrounding radiation background. During this course of transition, we observe several transition temperatures ($T_0 < T_1 < T_2$) and finally *cross over* to a globally stable phase of black hole for $T > T_2$. We collectively identify all these features as being the analog of HP transitions [36] in the “charged” JT model whose holographic interpretation is yet to be unfolded.

The organization for the rest of the paper is as follows: We construct our $D = 2$ model in Sec. II, where we give a brief account for the dimensional reduction procedure and obtain the corresponding background solutions following a perturbative approach. Adopting to a Euclidean framework in Sec. III, we explore the thermal stability in black holes using a canonical ensemble. Finally, we conclude in Sec. IV.

II. $D = 2$ MODEL

A. A top down construction

We consider a $D = 5$ gravity model minimally coupled to both Abelian (A_M) as well as $SU(2)$ Yang-Mills (YM) fields [A_M^a ($a = 1, 2, 3$)] [31]:

$$S_{5D} = \int d^5x \sqrt{-g} \left(\mathcal{R} - 3\Lambda - \frac{\kappa}{4g_s^2} F_{MN}^a F^{aMN} - \frac{\xi}{4} F_{MN} F^{MN} - \frac{\sigma}{2g_2^2} \frac{\epsilon^{MNPQR}}{\sqrt{-g}} F_{MN}^a F_{PQ}^a A_R \right), \quad (2.1)$$

where $\Lambda (< 0)$ is the cosmological constant.¹ Here κ and ξ are the gauge coupling constants which would be treated as an expansion parameter in the subsequent analysis. The CS piece ($\sim F^a \wedge F^a \wedge A$), on the other hand, is quite ubiquitous to $D = 5$ theories and does not seem to have left with any of its imprints on the reduced $D = 2$ model.

The $D = 2$ theory is obtained using the reduction ansatz [28]

$$ds^2 = \Phi^\alpha d\tilde{s}^2 + \Phi^\beta dx_i^2, \quad A_M^a dx^M = A_\mu^a dx^\mu, \quad A_M dx^M = A_\mu dx^\mu; \quad \alpha, \beta \in \mathbb{R}, \quad (2.2)$$

together with the metric of the reduced space-time,

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu; \quad g_{\mu\nu} = \Phi^\alpha \tilde{g}_{\mu\nu}; \quad \mu, \nu = t, z. \quad (2.3)$$

The $D = 2$ action (modulo a total derivative) could be formally expressed as²

$$S_{2D} = \mathcal{V}_3 \int d^2x \sqrt{-\tilde{g}} (\tilde{\mathcal{R}}\Phi - V(\Phi) - \Phi \mathcal{L}(\tilde{A}_\mu, \tilde{A}_\mu^a)) + S_{\text{GH}} + S_{\text{ct}},$$

$$\mathcal{L}(\tilde{A}_\mu, \tilde{A}_\mu^a) = \frac{\kappa}{4g_s^2} \tilde{F}_{\mu\nu}^a \tilde{F}^{a\mu\nu} + \frac{\xi}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}; \quad V(\Phi) = 3\Lambda\Phi, \quad (2.4)$$

where we set $\alpha = 0$ and $\beta = 2/3$ without any loss of generality. Notice that our model (2.4) is a special case of Ref. [17] with $C = 0$ and $A = -3\Lambda$. Moreover, here $S_{\text{GH}} = -\int dt \sqrt{-\gamma} \mathcal{K} \Phi$ is the standard Gibbons-Hawking (GH) term ($\mathcal{K}$ being the extrinsic curvature associated with the boundary hypersurface [42]) and S_{ct} is the so-called counterterm which cures the divergences of the on-shell action near its asymptotic limits.

¹We set the AdS radius $L = 1$ and $16\pi G = 1$ in the subsequent analysis.

²The methodology of dimensional reduction from $5D$ to $2D$ has been briefly discussed in the Appendix A.

B. Equations of motion

The equations of motion that readily follow from the variation of the parent action (2.4) are listed below:

$$(\nabla_\mu \nabla_\nu - \tilde{g}_{\mu\nu} \square) \Phi + \frac{\xi \Phi}{2} \left(\tilde{F}_{\mu\rho} \tilde{F}_\nu^\rho - \frac{1}{4} \tilde{F}^2 \tilde{g}_{\mu\nu} \right) + \frac{\kappa \Phi}{2g_s^2} \left(\tilde{F}_{\mu\rho}^a \tilde{F}_\nu^{a\rho} - \frac{1}{4} \tilde{F}^{a2} \tilde{g}_{\mu\nu} \right) - \frac{3\Lambda}{2} \Phi \tilde{g}_{\mu\nu} = 0, \quad (2.5a)$$

$$\tilde{\mathcal{R}} - 3\Lambda - \frac{\kappa}{4g_s^2} \tilde{F}_{\mu\nu}^a \tilde{F}^{a\mu\nu} - \frac{\xi}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} = 0, \quad (2.5b)$$

$$\frac{1}{\sqrt{-\tilde{g}}} \partial_\mu (\sqrt{-\tilde{g}} \Phi \tilde{F}^{a\mu\nu}) + \Phi \epsilon^{abc} \tilde{A}_\mu^b \tilde{F}^{c\mu\nu} = 0, \quad (2.5c)$$

$$\partial_\mu (\sqrt{-\tilde{g}} \Phi \tilde{F}^{\mu\nu}) = 0. \quad (2.5d)$$

In order to proceed further, we choose to work with the static metric ansatz

$$ds^2 = e^{2\omega(z)} (-dt^2 + dz^2) \quad (2.6)$$

together with the ansatz for the gauge fields:

$$\tilde{A}_\mu = (\tilde{A}_t(z), 0), \quad (2.7a)$$

$$\tilde{A}_\mu^a = \tilde{A}_t^3(z) \tau^3 dt + \tilde{A}_z^1(z) \tau^1 dz, \quad (2.7b)$$

where $\tau^a = \sigma^a/2i$ are the Pauli matrices of $SU(2)$ YM theory.

Using (2.6), (2.7a), and (2.7b), we finally note down the set of dynamical equations³:

$$2\Phi'' + 6\Lambda\Phi e^{2\omega} + \Phi e^{-2\omega} \left[\xi (\tilde{A}_t')^2 + \frac{\kappa}{g_s^2} ((\chi')^2 + \chi^2 \eta^2) \right] = 0, \quad (2.8a)$$

$$\Phi'' - 2\omega' \Phi' = 0, \quad (2.8b)$$

$$4\omega'' + 6\Lambda e^{2\omega} - e^{-2\omega} \left[\xi (\tilde{A}_t')^2 + \frac{\kappa}{g_s^2} ((\chi')^2 + \chi^2 \eta^2) \right] = 0, \quad (2.8c)$$

$$\Phi \tilde{A}_t'' + \Phi' \tilde{A}_t' - 2\Phi \omega' \tilde{A}_t' = 0, \quad (2.8d)$$

$$(\Phi e^{-2\omega} \chi \eta)' + e^{-2\omega} \Phi \eta \chi' = 0, \quad (2.8e)$$

$$(\Phi e^{-2\omega} \chi')' - e^{-2\omega} \Phi \eta^2 \chi = 0 \quad (2.8f)$$

³In order to simplify our notations, we set $\tilde{A}_t^3 = \chi(z)$ and $\tilde{A}_z^1 = \eta(z)$.

together with the following constraint:

$$\Phi e^{-2\omega} \chi^2 \eta = 0. \quad (2.9)$$

C. Solving the dynamics

We propose the following perturbative method of solving the dynamics⁴ (2.8a)–(2.8f):

$$\mathcal{A}(z) = \mathcal{A}_{(0)}(z) + \xi \mathcal{A}_{(1)}^{ab}(z) + \kappa \mathcal{A}_{(1)}^{na}(z) + \dots, \quad (2.10)$$

which is a perturbation in the gauge coupling parameters over the uncharged background solutions [17]. In the present analysis, we retain ourselves only up to leading order in the perturbative expansion (2.10). The general strategy would be to substitute (2.10) into (2.8a)–(2.8f) and obtain equations of motion at different order in the perturbative expansion.⁵ Here, $\mathcal{A}(z)$ stands for either of the dynamical variables Φ , ω , χ , and η .

Before we proceed further, the following observations are noteworthy to mention:

- (i) Since we are interested in expanding the action (2.4) up to leading order in the gauge coupling, it is sufficient to estimate leading- or zeroth-order solutions for both Abelian and non-Abelian gauge fields.
- (ii) The Abelian sector (2.8d) could be further simplified in terms of other variables,

$$F_{zt} \equiv A_t' = \frac{Q e^{2\omega}}{\Phi}, \quad (2.11)$$

where Q is the corresponding $U(1)$ charge.

- (iii) The dilaton equation of motion (2.8b) could be recast as

$$\Phi'(z) = -\frac{e^{2\omega}}{2}, \quad (2.12)$$

whose solution may be expressed as

$$\Phi(z) \simeq -\int \frac{dz}{2} e^{2\omega}. \quad (2.13)$$

1. Interpolating vacuum

In order to find vacuum solutions, the first step would be to note down zeroth-order solutions⁶ for both the dilaton and the metric from (B1a) and (B2a):

⁴Here ab and na stand for respective perturbative corrections (to the uncharged JT solutions [17]) due to Abelian and non-Abelian sectors, respectively, of the $D = 2$ model (2.4).

⁵See Appendix B for details.

⁶See Appendix C for the details of the derivation.

$$e^{2\omega_{(0)}^{\text{vac}}} = -\frac{2}{3\Lambda z^2}, \quad (2.14a)$$

$$\Phi_{(0)}^{\text{vac}} = -\frac{1}{3\Lambda z}. \quad (2.14b)$$

Combining (2.14a) and (2.6), it is by now quite evident that the vacuum solution corresponds to AdS_2 space-time in standard Poincaré coordinates

$$ds^2 \sim z^{-2}(-dt^2 + dz^2), \quad (2.15)$$

where we identify $z \rightarrow 0$ as the asymptotic UV limit of the bulk space-time. On the other hand, $z \rightarrow \infty$ stands for the IR limit [17].

Next, we note down solutions corresponding to (B4a) and (B4b):

$$\chi_{(0)}^{\text{vac}} \simeq \log z, \quad (2.16)$$

$$\eta_{(0)}^{\text{vac}} \simeq \frac{1}{z(1 + \log z)}, \quad (2.17)$$

where we have multiplied (B4b) by $\chi_{(0)}/\eta_{(0)}$ and used (B3a).

On the other hand, using (B2b) and (B2c), the solutions corresponding to $\omega_{(1)}^{ab}$ and $\omega_{(1)}^{na}$ can be found as

$$(\omega_{(1)}^{ab})^{\text{vac}} = 10z^2 + \frac{C}{z} + \frac{Q^2\Lambda}{6}z^2(1 - 3\log z), \quad (2.18a)$$

$$(\omega_{(1)}^{na})^{\text{vac}} = -\frac{3\Lambda}{z}. \quad (2.18b)$$

Finally, the leading-order solutions for dilaton may be obtained from (B1b) and (B1c):

$$(\Phi_{(1)}^{ab})^{\text{vac}} = \frac{-3(C - 20z^3) + 4Q^2\Lambda z^3(1 - \frac{3}{4}\log z)}{9\Lambda z^2}, \quad (2.19a)$$

$$(\Phi_{(1)}^{na})^{\text{vac}} \simeq \frac{1}{z^2}. \quad (2.19b)$$

Combining all these pieces together, we find

$$\begin{aligned} ds_{\text{vac}}^2 &= e^{2\omega^{\text{vac}}}(-dt^2 + dz^2) \\ &\approx e^{2\omega_{(0)}^{\text{vac}}}(1 + 2\xi(\omega_{(1)}^{ab})^{\text{vac}} + 2\kappa(\omega_{(1)}^{na})^{\text{vac}})(-dt^2 + dz^2). \end{aligned} \quad (2.20)$$

Given the above metric structure (2.20), it is customary to explore various asymptotic limits associated to it. For example, near the IR ($z \rightarrow \infty$) region, one finds

$$e_{\text{IR}}^{2\omega^{\text{vac}}} \simeq -\frac{2}{3\Lambda z^2} - \frac{2}{9\Lambda}(60\xi + Q^2\xi\Lambda(1 - 3\log z)) + \mathcal{O}(z^{-3}), \quad (2.21)$$

which clearly reveals an emerging AdS_2 geometry.

On the other hand, the UV ($z \rightarrow 0$) limit of the metric reveals

$$\begin{aligned} e_{\text{UV}}^{2\omega^{\text{vac}}} &\simeq \frac{1}{z^3} \left(-\frac{4\xi C}{3\Lambda} + 4\kappa \right) - \frac{2}{3\Lambda z^2} \\ &\quad - \frac{2}{9\Lambda}(60\xi + Q^2\xi\Lambda(1 - 3\log z)) + \mathcal{O}(z^2), \end{aligned} \quad (2.22)$$

an emerging Lifshitz₂ geometry with dynamical exponent $z_{\text{dyn}} = \frac{3}{2}$. A careful analysis further reveals that, for $\xi = \kappa = 0$, the resulting geometry becomes AdS_2 in both the asymptotic limits. This confirms that gauge fields in the theory are actually responsible for the change in asymptotic (UV) structure as found earlier in Refs. [30,34].

2. 2D black holes

The zeroth-order or uncharged background solutions $\omega_{(0)}$ and $\Phi_{(0)}$ may be expressed as

$$e^{2\omega_{(0)}^{\text{BH}}} = -\frac{8\mu}{3\Lambda \sinh^2(2\sqrt{\mu}z)}, \quad (2.23a)$$

$$\Phi_{(0)}^{\text{BH}} = -\frac{2\sqrt{\mu}}{3\Lambda} \coth(2\sqrt{\mu}z). \quad (2.23b)$$

Below, we note down first-order solutions corresponding to the metric and the dilaton. In order to simplify our analysis, we consider the following change of coordinate:

$$z = \frac{1}{2\sqrt{\mu}} \coth^{-1} \left(\frac{\rho}{\sqrt{\mu}} \right). \quad (2.24)$$

Using (2.24), the solution corresponding to (B2b) can be expressed as

$$\begin{aligned} (\omega_{(1)}^{ab})^{\text{BH}} &\simeq \frac{\rho}{\sqrt{\mu}} + \left[\frac{\rho}{2\sqrt{\mu}} \cdot \log \left(\frac{\sqrt{\mu} + \rho}{\sqrt{\mu} - \rho} \right) - 1 \right] \\ &\quad - \frac{3Q^2\Lambda}{16\mu^{3/2}} \left\{ 2\sqrt{\mu}(1 + \log \rho) \right. \\ &\quad \left. + \rho \log \rho \cdot \log \left(\frac{\sqrt{\mu} - \rho}{\sqrt{\mu} + \rho} \right) \right. \\ &\quad \left. + \rho \left[\text{Li} \left(2, \frac{\rho}{\sqrt{\mu}} \right) - \text{Li} \left(2, \frac{-\rho}{\sqrt{\mu}} \right) \right] \right\}. \end{aligned} \quad (2.25)$$

On the other hand, substituting (B2c) in (B1c) and using (2.24), we find

$$(\omega_{(1)}^{na})^{\text{BH}} \simeq \frac{\sqrt{\mu}\rho + (\mu - \rho^2) \tanh^{-1}(\frac{\rho}{\sqrt{\mu}})}{2(\mu - 1)\mu^{3/2}}. \quad (2.26)$$

Finally, we note down zeroth-order solutions for gauge fields that readily follow from (B4a) and (B4b):

$$\chi_{(0)}^{\text{BH}} \simeq 1 - \frac{1}{2\sqrt{\mu}} \log\left(\frac{\rho}{\sqrt{\mu}}\right), \quad (2.27a)$$

$$\eta_{(0)}^{\text{BH}} \simeq \frac{(\rho^2 - \mu)}{4\rho\mu^{3/2}} \left(1 - \frac{1}{\sqrt{2\mu}} \log\left(\frac{\rho}{\sqrt{\mu}}\right)\right)^{-2}. \quad (2.27b)$$

Collecting all the pieces together, we note down the black hole metric as

$$ds_{\text{BH}}^2 \simeq -\frac{8}{3\Lambda}(\rho^2 - \mu)(1 + 2\xi(\omega_{(1)}^{ab})^{\text{bh}} + 2\kappa(\omega_{(1)}^{na})^{\text{bh}}) \left(-dt^2 + \frac{d\rho^2}{4(\rho^2 - \mu)^2}\right). \quad (2.28)$$

The black hole solution (2.28) exhibits a horizon at $\rho = \sqrt{\mu}$. On the other hand, substituting $\rho \sim \frac{1}{\delta}$ and thereby taking $\delta \rightarrow 0$ limit reveals $ds_{\text{BH}}^2|_{\rho \rightarrow \infty} \sim \frac{1}{\delta^2}$. This suggests that the UV asymptotic of the space-time approaches Lifshitz₂ geometry with dynamical critical exponent $z_{\text{dyn}} = \frac{3}{2}$.

The Hawking temperature of the black hole (2.28) can be found as

$$T_H = \frac{\sqrt{\mu}}{\pi}. \quad (2.29)$$

Finally, we note down the Wald entropy [43] of the black hole⁷:

$$\begin{aligned} S_W &= 4\pi\Phi|_{\rho=\sqrt{\mu}} \\ &= -\frac{1}{96\Lambda\sqrt{\mu}(|\mu - 1|)} [(|\mu - 1|)\{-33Q^2\Lambda\xi \\ &\quad + 64\mu(1 + 0.89\xi) - 15Q^2\xi(\log \mu)\} - 240\kappa], \end{aligned} \quad (2.30)$$

which clearly shows a discontinuity at $\mu = \mu_0 = 1$. As we probe into details on thermal stability, we identify the above discontinuity as a signature of a first-order transition from an unstable black hole phase to pure radiation.

In order to compare the 2D black hole entropy (2.30) with the black hole entropy in 5D, we uplift the 2D solution into 5D⁸

$$ds_{5D}^2 = ds_{2D}^2 + \Phi^{2/3} dx_i^2, \quad i = 1, 2, 3. \quad (2.31)$$

In a manner similar to the 2D theory, the Wald entropy corresponding to the 5D black hole can be written as [44,45]

$$\begin{aligned} \hat{S}_W &= -2\pi \int_{\Sigma} \left(\frac{\delta \mathcal{L}_{5D}}{\delta \mathcal{R}_{abcd}} \right) \hat{e}_{ab} \hat{e}_{cd} \bar{e} \\ &= -2\pi \int_{\rho=\sqrt{\mu}, t=\text{const}} \left(\frac{\delta \mathcal{L}_{5D}}{\delta \mathcal{R}_{\rho t \rho t}} \right) \hat{e}_{\rho t} \hat{e}_{\rho t} \Phi \prod_{i=1}^3 dx_i \\ &= \mathcal{V}_3 4\pi\Phi|_{\rho=\sqrt{\mu}}, \end{aligned} \quad (2.32)$$

where Σ is the codimension-2 spacelike bifurcation surface with binormal $\hat{e}_{ab}$ which is symmetric in $a \leftrightarrow b$ and normalized as $\hat{e}^{ab} \hat{e}_{ab} = -2$, $\bar{e}$ is the induced volume on Σ , and $\mathcal{R}_{abcd}$ is the Riemann curvature tensor in the theory. Also, the bifurcation surface is at the horizon $\rho = \sqrt{\mu}$, $t = \text{const}$. Notice that, in deriving the final form of the Wald entropy in (2.32), we have used the fact that the Lagrangian density $\mathcal{L}_{5D}$ corresponding to the 5D theory (2.1) contains a term linear in $\mathcal{R}$ only. In addition, we have used the form of the metric given in the ansatz (2.2) explicitly. Hence, from (2.30) and (2.32), we observe that the Wald entropies [per unit volume of the transverse space ($\mathcal{V}_3$)] for the two theories indeed match.

III. THERMAL STABILITY AND HP TRANSITION

We now move on to the Euclidean formalism [46] and explore thermal stability in black holes. To start with, the full Euclidean *on-shell* action is schematically expressed as

$$-S_{2D}^{\text{os}} = S_{\text{grav}}^{\text{os}} + S_{\text{GH}}^{\text{os}} + S_{\text{ct}}^{\text{os}}, \quad (3.1)$$

where each of the above entities on the rhs of (3.1) are estimated using background solutions found in the previous section. This is accompanied by an analytic continuation of the Lorentzian time, $t \rightarrow i\tau$ with a periodicity β . In the present analysis, there exist two distinct periodicities corresponding to the thermal radiation bath (β_{TH}) and the black hole phase (β_{BH}), respectively. The black hole periodicity β_{BH} is uniquely fixed by (2.29). On the other hand, the former (β_{TH}) could be fixed by demanding that the ratio

$$\frac{\beta_{\text{TH}}}{\beta_{\text{BH}}} \approx 2 + \mathcal{O}\left(\frac{1}{\rho_c}\right) \quad (3.2)$$

becomes independent of the radial cutoff $\rho = \rho_c (\rightarrow \infty)$ at large distances [46].

Using (3.2), the *effective* free energy of the configuration could be obtained using the background subtraction method [46]:

⁷Here by entropy we mean entropy per unit volume of the transverse space, namely, $S_W/\mathcal{V}_3$.

FIG. 1. Behavior of effective free energy ($\Delta\mathcal{F}$) and energy ($\mathcal{E} \sim \mathcal{M}_{\text{BH}}$) of the configuration with temperature $T(\sim\sqrt{\mu})$.

$$\begin{aligned}\Delta\mathcal{F} &= \mathcal{F}_{\text{BH}} - \mathcal{F}_{\text{TH}} \\ &= \frac{1}{18\Lambda(\mu-1)} [(\mu-1)\{-7Q^2\Lambda\xi \\ &\quad + 16\xi(-30 + 0.22\mu)\} + 120\Lambda\kappa].\end{aligned}\quad (3.3)$$

Notice that this free energy is quite unique to our original choices of parameters α and β in (2.2) and, therefore, to the JT gravity model proposed in (2.4). In other words, the background subtraction method used in this paper is also quite pertinent to the choices of parameters α and β .

Finally, using a canonical ensemble ($\mathcal{Z} \sim e^{-\Delta\mathcal{S}_{2D}^{\text{os}}}$), the energy of the black hole configuration may be estimated as

$$\begin{aligned}\mathcal{M}_{\text{BH}} \equiv \langle\mathcal{E}\rangle &= \frac{1}{450\Lambda(\mu-1)^2} (3000\kappa\Lambda(3\mu-1) \\ &\quad - \xi(\mu-1)^2(12000 + 175Q^2\Lambda + 88\mu)).\end{aligned}\quad (3.4)$$

Figure 1 displays the behavior of both the free energy ($\Delta\mathcal{F}$) and the energy ($\sim$ mass) of the (black hole) configuration with temperature ($T \sim \sqrt{\mu}$). In all the subsequent plots, we set $Q = 0.01$, $\xi = 0.009$, and $\kappa = 0.009$. Below, we enumerate the key observations regarding Figs. 1 and 2.

- (i) To start with, we notice that the thermal radiation collapses to form an *unstable* black hole in the region $T_0 < T < T_1$ which can either completely decay into pure radiation without a black hole (for $T < T_0$) or transit to a larger mass black hole for $T > T_0$. Clearly, in this phase, the smaller mass black hole has larger free energy than the thermal radiation bath ($\mathcal{F}_{\text{BH}} > \mathcal{F}_{\text{TH}}$) and is also characterized by a negative heat capacity $C_{\text{BH}} \sim \frac{\partial\langle\mathcal{E}\rangle}{\partial\sqrt{\mu}} < 0$ [Fig. 1(b)]. Therefore, we identify $T_0 < T < T_1$ as a radiation-dominated phase with $\Delta\mathcal{F} > 0$ and $T < T_0$ as a pure radiation phase with $\Delta\mathcal{F} \sim \mathcal{F}_{\text{TH}} < 0$

[Fig. 1(a)]. The corresponding entropy⁸ plot (Fig. 2) reveals the onset of a first-order phase transition at $T_0 \sim \sqrt{\mu_0} = 1$. The $\sqrt{\mu} < 1$ branch [Fig. 2(a)] corresponds to entropy of the configuration that ultimately boils down into pure radiation. This part of the phase diagram represents a rapidly evaporating black hole [Fig. 1(b)] placed in a heat bath which eventually results in a thermal ensemble with *unique* entropy ($\mathcal{S}_{\text{TH}}$). The $\sqrt{\mu} > 1$ branch [Fig. 2(b)], on the other hand, depicts a phase with decreasing difference entropy ($\Delta\mathcal{S}$) for $T \gtrsim T_0$ which corresponds to the fact that the entropy of the unstable black hole approaches the entropy of the thermal bath as it gradually shrinks in size [Fig. 1(b)] with the increase in temperature. This, in turn, is related to the negative heat capacity of the black hole as mentioned earlier.

- (ii) As the temperature of the configuration approaches $T \sim T_1 \sim \sqrt{\mu_1}$, the corresponding mass of the black hole reaches a minima [Fig. 1(b)] and thereafter starts increasing slowly for $T \gtrsim T_1$. We identify this as a *locally*⁹ stable phase of a black hole ($\mathcal{S}_{\text{BH}} \sim \mathcal{S}_{\text{TH}}$) with positive heat capacity $C_{\text{BH}} \sim \frac{\partial\langle\mathcal{E}\rangle}{\partial\sqrt{\mu}} > 0$ [Fig. 1(b)]. However, the effective free energy ($\Delta\mathcal{F}$) of the configuration reveals that this

⁸The entropy ($\mathcal{S}$) that we measure should be understood in two ways. For the $\sqrt{\mu} < 1$ branch, one is left with pure radiation; therefore, we identify the corresponding entropy simply as the entropy of the thermal bath ($\mathcal{S}_{\text{TH}}$). On the other hand, for the $\sqrt{\mu} > 1$ branch, we introduce the notion of *effective* (or difference) entropy, $\Delta\mathcal{S} \sim -\frac{\Delta\mathcal{F}}{\Delta\sqrt{\mu}} \sim |\mathcal{S}_{\text{BH}} - \mathcal{S}_{\text{TH}}|$, which is basically the difference in entropy between two distinct configurations, namely, the black hole and the heat bath surrounding it [see (3.3)]. However, in both cases, the change in total entropy must be positive following the second law of thermodynamics.

⁹By local we mean that this phase is existing only in a small temperature window $T_1 < T < T_2$.

FIG. 2. Behavior of the entropy (S) as a function of temperature $T(\sim\sqrt{\mu})$.

phase is still dominated by the thermal radiation background with $\mathcal{F}_{\text{TH}} < \mathcal{F}_{\text{BH}}$ [Fig. 1(a)]. The total entropy of the configuration is therefore $\sim \mathcal{S}_{\text{TH}}$ and, hence, the difference $\Delta S \sim 0$ over the range $T_1 < T < T_2$ [Fig. 2(b)].

- (iii) Finally, we reach the point of *crossover* $T = T_2$ beyond which the black hole mass ($\mathcal{M}_{\text{BH}}$) starts becoming ever increasing with the increase in temperature and thereby yields a positive heat capacity ($C_{\text{BH}} > 0$) [Fig. 1(b)]. A careful analysis further reveals that this is the point of inflection where the difference entropy vanishes ($\Delta S = 0$) and thereafter starts increasing [Fig. 2(b)] as one approaches the region $T > T_2$. This corresponds to a *globally*¹⁰ stable phase of black hole with $\mathcal{F}_{\text{BH}} < \mathcal{F}_{\text{TH}}$ [Fig. 1(a)]. Needless to say, for $T > T_2$, the entropy content of the configuration is mostly dominated due to the presence of the larger mass black hole which largely contributes to both in the increase in total entropy as well as the difference entropy ($\Delta S \gtrsim 0$) of the system.

IV. SUMMARY AND FINAL REMARKS

We conclude with a brief summary of the main outcome in this analysis. We propose a possible formulation of $D = 2$ Jackiw-Teitelboim (JT) gravity using dimensional reduction of the parent $D = 5$ gravity model [31] in the presence of Abelian as well $SU(2)$ Yang-Mills fields. To our surprise, we notice an early (thermal) instability associated to black hole microstates using a canonical ensemble for the $D = 2$ model. We identify that these early black hole states might either decay into pure radiation or switch over to bigger black hole microstates with higher

entropy and lesser free energy. We summarize all these features collectively as being the analog of the Hawking-Page transition [36] in 2D gravity models. This observation also raises a natural question regarding the holographic interpretation of our analysis in terms of (dual) quantum mechanical DOF living in one dimension. We leave this as an exciting direction to be explored in the near future.

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APPENDIX A: DIMENSIONAL REDUCTION FROM 5D TO 2D

In order to analyze the dimensional reduction of the 5D theory (2.1) to 2D, in the following we rewrite the ansatz given in (2.2):

$$ds^2 = \Phi^\alpha d\tilde{s}^2 + \Phi^\beta dx_i^2, \quad A_M^a dx^M = A_\mu^a dx^\mu, \quad A_M dx^M = A_\mu dx^\mu; \quad \alpha, \beta \in \mathbb{R}. \quad (\text{A1})$$

With this choice of the metric, we note that

$$\sqrt{-g} = \sqrt{-\tilde{g}} \Phi^{\alpha+3\beta/2}, \quad (\text{A2})$$

where $\tilde{g}$ is the determinant of the two-dimensional metric.

It is then easy to check that the first two terms in the action (2.1) can be written as

$$\sqrt{-g}(\mathcal{R} - 3\Lambda) = \sqrt{-\tilde{g}} \Phi^{3\beta/2} (\tilde{\mathcal{R}} - 3\Lambda \Phi^\alpha + \Phi^\alpha g^{ij} \mathcal{R}_{ij}). \quad (\text{A3})$$

¹⁰By global we mean that this phase persists for all temperatures $T > T_2$.

In the next step, we rewrite the third term within the curly braces in (A3) as a total derivative term given by

$$\Phi^{\alpha+3\beta/2} g^{ij} \mathcal{R}_{ij} \sim \nabla_\mu [g^{\mu\nu} g^{\lambda\sigma} g_{\lambda\sigma} (\partial_\nu \Phi^\beta) \Phi^{\alpha+\beta/2}], \quad (\text{A4})$$

which therefore does not contribute to the bulk equations of motion (2.5a)–(2.5d). Notice that the JT gravity model (2.4) can be obtained once we set the parameters $\alpha = 0$ and $\beta = 2/3$. In other words, these choices of parameters are quite pertinent to the JT gravity model discussed in this paper. The remaining terms in (2.4) involving gauge fields can be obtained in a similar manner.

APPENDIX B: EQUATIONS OF MOTION AT DIFFERENT ORDERS IN THE PERTURBATION SERIES

In the following, we note down the equations of motion up to leading order in perturbative expansion. For example, substituting (2.10) into (2.8), we obtain

$$\mathcal{O}(0): 2\Phi''_{(0)} + 6\Lambda\Phi_{(0)}e^{2\omega_{(0)}} = 0, \quad (\text{B1a})$$

$$\begin{aligned} \mathcal{O}(\xi): 2(\Phi_{(1)}^{ab})'' + 6\Lambda e^{2\omega_{(0)}}(2\omega_{(1)}^{ab}\Phi_{(0)} + \Phi_{(1)}^{ab}) \\ + \frac{Q^2}{\Phi_{(0)}} e^{2\omega_{(0)}} = 0, \end{aligned} \quad (\text{B1b})$$

$$\begin{aligned} \mathcal{O}(\kappa): 2(\Phi_{(1)}^{na})'' + 6\Lambda e^{2\omega_{(0)}}(2\omega_{(1)}^{na}\Phi_{(0)} + \Phi_{(1)}^{na}) \\ + \frac{1}{g_s^2} \Phi_{(0)} e^{-2\omega_{(0)}} ((\chi'_{(0)})^2 + \chi_{(0)}^2 \eta_{(0)}^2) = 0. \end{aligned} \quad (\text{B1c})$$

Similarly, (2.8c) could be arranged up to leading order in the perturbative expansion as

$$\mathcal{O}(0): 4\omega''_{(0)} + 6\Lambda e^{2\omega_{(0)}} = 0, \quad (\text{B2a})$$

$$\mathcal{O}(\xi): 4(\omega_{(1)}^{ab})'' + 12\Lambda\omega_{(1)}^{ab}e^{2\omega_{(0)}} - \frac{Q^2}{\Phi_{(0)}} e^{2\omega_{(0)}} = 0, \quad (\text{B2b})$$

$$\begin{aligned} \mathcal{O}(\kappa): 4(\omega_{(1)}^{na})'' + 12\Lambda\omega_{(1)}^{na}e^{2\omega_{(0)}} - \frac{e^{-2\omega_{(0)}}}{g_s^2} \\ + ((\chi'_{(0)})^2 + \chi_{(0)}^2 \eta_{(0)}^2) = 0. \end{aligned} \quad (\text{B2c})$$

Substituting (2.10) into the constraint equation (2.9), we obtain

$$\mathcal{O}(0): \Phi_{(0)} e^{-2\omega_{(0)}} \chi_{(0)}^2 \eta_{(0)} = 0, \quad (\text{B3a})$$

$$\begin{aligned} \mathcal{O}(\xi): \Phi_{(0)} (\chi_{(0)} \eta_{(1)}^{ab} + 2\eta_{(0)} \chi_{(1)}^{ab}) \\ + \chi_{(0)} \eta_{(0)} (\Phi_{(1)}^{ab} - 2\Phi_{(0)} \omega_{(1)}^{ab}) = 0, \quad e^{-2\omega_{(0)}} \chi_{(0)}^2 = 0, \end{aligned} \quad (\text{B3b})$$

$$\begin{aligned} \mathcal{O}(\kappa): \Phi_{(0)} (\chi_{(0)} \eta_{(1)}^{na} + 2\eta_{(0)} \chi_{(1)}^{na}) \\ + \chi_{(0)} \eta_{(0)} (\Phi_{(1)}^{na} - 2\Phi_{(0)} \omega_{(1)}^{na}) = 0, \quad e^{-2\omega_{(0)}} \chi_{(0)}^2 = 0. \end{aligned} \quad (\text{B3c})$$

Finally, from (2.8e) and (2.8f), the zeroth-order equations could be recast as

$$\partial_z (\Phi_{(0)} e^{-2\omega_{(0)}} \chi_{(0)} \eta_{(0)}) + \Phi_{(0)} e^{-2\omega_{(0)}} \eta_{(0)} (\partial_z \chi_{(0)}) = 0, \quad (\text{B4a})$$

$$\partial_z (\Phi_{(0)} e^{-2\omega_{(0)}} (\partial_z \chi_{(0)})) - \Phi_{(0)} e^{-2\omega_{(0)}} \chi_{(0)} \eta_{(0)}^2 = 0. \quad (\text{B4b})$$

APPENDIX C: DERIVATIONS OF ZERO-TH-ORDER SOLUTIONS

In the conformal gauge [17]

$$ds^2 = -e^{2\omega(x^+, x^-)} dx^+ dx^-, \quad x^\pm = (t \pm z), \quad (\text{C1})$$

the zeroth-order equations of motion for the metric and the dilaton can be rewritten as

$$4\partial_+ \partial_- \Phi_{(0)} - 3\Lambda \Phi_{(0)} e^{2\omega_{(0)}} = 0, \quad (\text{C2})$$

$$8\partial_+ \partial_- \omega_{(0)} - 3\Lambda e^{2\omega_{(0)}} = 0, \quad (\text{C3})$$

$$\partial_+ (e^{2\omega_{(0)}} \partial_+ \Phi_{(0)}) = 0, \quad (\text{C4})$$

$$\partial_- (e^{2\omega_{(0)}} \partial_- \Phi_{(0)}) = 0. \quad (\text{C5})$$

Clearly, the solution of (C3) is given by

$$e^{2\omega_{(0)}} = \left(-\frac{8}{3\Lambda}\right) \frac{1}{(x^+ - x^-)^2}. \quad (\text{C6})$$

Now integrating (C4) and (C5), we may write

$$\partial_+ \Phi_{(0)} = \left(-\frac{8}{3\Lambda}\right) \frac{f(x^-)}{(x^+ - x^-)^2}, \quad (\text{C7})$$

$$\partial_- \Phi_{(0)} = \left(-\frac{8}{3\Lambda}\right) \frac{g(x^+)}{(x^+ - x^-)^2}. \quad (\text{C8})$$

Differentiating (C7) with respect to x^- and (C8) with respect to x^+ and substituting them in (C2), we obtain

$$\Phi_{(0)} = \left\{ \frac{\frac{2}{3\Lambda} \frac{\partial_- f(x^-) \cdot (x^+ - x^-) + 2f(x^-)}{(x^+ - x^-)^2}}{\frac{2}{3\Lambda} \frac{\partial_+ g(x^+) \cdot (x^+ - x^-) - 2g(x^+)}{(x^+ - x^-)^2}} \right\}. \quad (\text{C9})$$

After a few easy algebraic steps, the general solution for the dilaton may be written as [17]

$$\Phi_{(0)} = \left(-\frac{2}{3\Lambda}\right) \frac{a + b(x^+ + x^-) + cx^+x^-}{(x^+ - x^-)}, \quad (\text{C10})$$

where a , b , and c are real constants. For the vacuum, we may choose $a = 1$, $b = 0$, and $c = 0$ [17,25].

In the next step, we find the corresponding solutions for the black hole by exploiting the $SL(2, \mathbb{R})$ invariance of the metric (C1). The general solutions can then be written as

$$e^{2\omega_{(0)}} = \left(-\frac{8}{3\Lambda}\right) \frac{\omega^+(x^+)\omega^-(x^-)}{[\omega^+(x^+) - \omega^-(x^-)]^2}, \quad (\text{C11})$$

$$\Phi_{(0)} = \left(-\frac{2}{3\Lambda}\right) \frac{1 - \mu\omega^+(x^+)\omega^-(x^-)}{[\omega^+(x^+) - \omega^-(x^-)]}, \quad (\text{C12})$$

where $\mu(> 0)$ can be considered as the mass of the black hole and $\omega^\pm(x^\pm)$ are *monotonic* functions [17].

Finally, using the conformal transformations

$$\omega^\pm(x^\pm) = \frac{1}{\sqrt{\mu}} \tanh \sqrt{\mu} x^\pm, \quad (\text{C13})$$

(C11) and (C12) can be rewritten, respectively, as

$$e^{2\omega_{(0)}} = \left(-\frac{8}{3\Lambda}\right) \frac{\mu}{\sinh^2(\sqrt{\mu}(x^+ - x^-))}, \quad (\text{C14})$$

$$\Phi_{(0)} = \left(-\frac{2}{3\Lambda}\right) \sqrt{\mu} \coth(\sqrt{\mu}(x^+ - x^-)). \quad (\text{C15})$$

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- [1] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, *Int. J. Theor. Phys.* **38**, 1113 (1999); *Adv. Theor. Math. Phys.* **2**, 231 (1998).
  - [2] E. Witten, Anti-de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
  - [3] L. Susskind, The World as a hologram, *J. Math. Phys. (N.Y.)* **36**, 6377 (1995).
  - [4] A. Kitaev, Hidden correlations in the Hawking radiation and thermal noise, *Proceedings of the Fundamental Physics Prize Symposium, November 10, Santa Barbara, U.S.A.* (2014).
  - [5] A. Kitaev, A simple model of quantum holography, *Proceedings of the KITP strings seminar and Entanglement program, February 12, April 7, and May 27, Santa Barbara, U.S.A.* (2015).
  - [6] S. Sachdev and J. Ye, Gapless Spin Fluid Ground State in a Random, Quantum Heisenberg Magnet, *Phys. Rev. Lett.* **70**, 3339 (1993).
  - [7] J. Polchinski and V. Rosenhaus, The spectrum in the Sachdev-Ye-Kitaev model, *J. High Energy Phys.* **04** (2016) 001.
  - [8] J. Maldacena and D. Stanford, Remarks on the Sachdev-Ye-Kitaev model, *Phys. Rev. D* **94**, 106002 (2016).
  - [9] C. Teitelboim, Gravitation and Hamiltonian structure in two space-time dimensions, *Phys. Lett. B* **126**, 41 (1983).
  - [10] R. Jackiw, Lower dimensional gravity, *Nucl. Phys.* **B252**, 343 (1985).
  - [11] A. Jevicki, K. Suzuki, and J. Yoon, Bi-local holography in the SYK model, *J. High Energy Phys.* **07** (2016) 007.
  - [12] D. J. Gross and V. Rosenhaus, A generalization of Sachdev-Ye-Kitaev, *J. High Energy Phys.* **02** (2017) 093.
  - [13] D. J. Gross and V. Rosenhaus, The bulk dual of SYK: Cubic couplings, *J. High Energy Phys.* **05** (2017) 092.
  - [14] A. Kitaev and S. J. Suh, The soft mode in the Sachdev-Ye-Kitaev model and its gravity dual, *J. High Energy Phys.* **05** (2018) 183.
  - [15] M. Taylor, Generalized conformal structure, dilaton gravity and SYK, *J. High Energy Phys.* **01** (2018) 010.
  - [16] U. Moitra, S. K. Sake, S. P. Trivedi, and V. Vishal, Jackiw-teitelboim model coupled to conformal matter in the semi-classical limit, *J. High Energy Phys.* **04** (2020) 199.
  - [17] A. Almheiri and J. Polchinski, Models of AdS₂ backreaction and holography, *J. High Energy Phys.* **11** (2015) 014.
  - [18] J. Maldacena, D. Stanford, and Z. Yang, Conformal symmetry and its breaking in two dimensional Nearly Anti-de-Sitter space, *Prog. Theor. Exp. Phys.* **2016**, 12C104 (2016).
  - [19] M. Cvetič and I. Papadimitriou, AdS₂ holographic dictionary, *J. High Energy Phys.* **12** (2016) 008; Erratum, *J. High Energy Phys.* **01** (2017) 120.
  - [20] J. Engelsöy, T. G. Mertens, and H. Verlinde, An investigation of AdS₂ backreaction and holography, *J. High Energy Phys.* **07** (2016) 139.
  - [21] K. Jensen, Chaos in AdS₂ Holography, *Phys. Rev. Lett.* **117**, 111601 (2016).
  - [22] S. R. Das, A. Jevicki, and K. Suzuki, Three dimensional view of the SYK/AdS duality, *J. High Energy Phys.* **09** (2017) 017.
  - [23] S. R. Das, A. Ghosh, A. Jevicki, and K. Suzuki, Three dimensional view of arbitrary q SYK models, *J. High Energy Phys.* **02** (2018) 162.
  - [24] S. R. Das, A. Ghosh, A. Jevicki, and K. Suzuki, Space-time in the SYK model, *J. High Energy Phys.* **07** (2018) 184.
  - [25] H. Kyono, S. Okumura, and K. Yoshida, Deformations of the Almheiri-Polchinski model, *J. High Energy Phys.* **03** (2017) 173.
  - [26] A. Lala and D. Roychowdhury, SYK/AdS duality with Yang-Baxter deformations, *J. High Energy Phys.* **12** (2018) 073.
  - [27] D. Roychowdhury, Holographic derivation of q SYK spectrum with Yang-Baxter shift, *Phys. Lett. B* **797**, 134818 (2019).
  - [28] R. A. Davison, W. Fu, A. Georges, Y. Gu, K. Jensen, and S. Sachdev, Thermoelectric transport in disordered metals without quasiparticles: The Sachdev-Ye-Kitaev models and holography, *Phys. Rev. B* **95**, 155131 (2017).

- [29] A. Gaikwad, L. K. Joshi, G. Mandal, and S. R. Wadia, Holographic dual to charged SYK from 3D Gravity and Chern-Simons, *J. High Energy Phys.* **02** (2020) 033.
- [30] A. Lala and D. Roychowdhury, Models of phase stability in Jackiw-Teitelboim gravity, *Phys. Rev. D* **100**, 124061 (2019).
- [31] Z. Y. Fan and H. Lü, Electrically-charged Lifshitz spacetimes, and hyperscaling violations, *J. High Energy Phys.* **04** (2015) 139.
- [32] Z. Y. Fan and H. Lu, Charged black holes in colored Lifshitz spacetimes, *Phys. Lett. B* **743**, 290 (2015).
- [33] S. Kachru, X. Liu, and M. Mulligan, Gravity duals of Lifshitz-like fixed points, *Phys. Rev. D* **78**, 106005 (2008).
- [34] M. Taylor, Non-relativistic holography, [arXiv:0812.0530](https://arxiv.org/abs/0812.0530).
- [35] L. Romans, Gauged $N = 4$ supergravities in five-dimensions and their magnetovac backgrounds, *Nucl. Phys. B* **267**, 433 (1986).
- [36] S. W. Hawking and D. N. Page, Thermodynamics of black holes in anti-de Sitter space, *Commun. Math. Phys.* **87**, 577 (1983).
- [37] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, Charged AdS black holes and catastrophic holography, *Phys. Rev. D* **60**, 064018 (1999).
- [38] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, Holography, thermodynamics and fluctuations of charged AdS black holes, *Phys. Rev. D* **60**, 104026 (1999).
- [39] S. Carlip and S. Vaidya, Phase transitions and critical behavior for charged black holes, *Classical Quantum Gravity* **20**, 3827 (2003).
- [40] R. G. Cai, S. P. Kim, and B. Wang, Ricci flat black holes and Hawking-Page phase transition in Gauss-Bonnet gravity and dilaton gravity, *Phys. Rev. D* **76**, 024011 (2007).
- [41] Y. S. Myung, Phase transition between non-extremal and extremal Reissner-Nordstrom black holes, *Mod. Phys. Lett. A* **23**, 667 (2008).
- [42] G. Gibbons and S. Hawking, Action integrals and partition functions in quantum gravity, *Phys. Rev. D* **15**, 2752 (1977).
- [43] R. M. Wald, Black hole entropy is the Noether charge, *Phys. Rev. D* **48**, R3427 (1993).
- [44] T. Jacobson, G. Kang, and R. C. Myers, On black hole entropy, *Phys. Rev. D* **49**, 6587 (1994).
- [45] R. Brustein, D. Gorboson, and M. Hadad, Wald's entropy is equal to a quarter of the horizon area in units of the effective gravitational coupling, *Phys. Rev. D* **79**, 044025 (2009).
- [46] E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories, *Adv. Theor. Math. Phys.* **2**, 505 (1998).