$X_0(2866)$ as a $D^*\bar{K}^*$ molecular stateMing-Zhu Liu,¹ Ju-Jun Xie,^{2,3,4,*} and Li-Sheng Geng^{1,5,4,6,†}¹*School of Physics, Beihang University, Beijing 100191, China*²*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*³*School of Nuclear Sciences and Technology, University of Chinese Academy of Sciences, Beijing 101408, China*⁴*School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China*⁵*Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*⁶*Beijing Advanced Innovation Center for Big Data-Based Precision Medicine, School of Medicine and Engineering, Beihang University, Beijing 100191, China*

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Very recently the LHCb Collaboration reported the discovery of two open charm tetraquark states, $X_0(2866)$ and $X_1(2904)$. In the present work, we study the $D^{(*)}$ and $\bar{K}^{(*)}$ interaction in the one-boson exchange model and show that $X_0(2866)$ can be understood as a $D^*\bar{K}^*$ molecule with $I(J^P) = 0(0^+)$, or at least it has a large molecular component. On the other hand, $X_1(2904)$ can not be interpreted as a molecular state. Inspired by the discovery of $X_0(2866)$ and the fact that the $D^*\bar{K}^*$ interaction is strong enough to generate a bound state, we also discuss the likely existence of other open charm molecules. In the meson-meson sector, two molecules near the mass thresholds of DD^* and D^*D^* with $I(J^P) = 0(1^+)$ are obtained, and using the heavy quark flavor symmetry their $\bar{B}\bar{B}^*$ and $\bar{B}^*\bar{B}^*$ counterparts are also predicted. In the meson-baryon sector, seven open charm molecules with $I = 1/2$ near the mass thresholds of $D^{(*)}\Sigma_c^{(*)}$ naturally appear, as dictated by the heavy quark spin symmetry.

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I. INTRODUCTION

Since the Belle Collaboration discovered $X(3872)$ [1], a lot of exotic states that can not easily fit into the conventional quark model have been reported by collaborations all over the world, which not only offers the chance to study the strong interaction, but also challenges existing theories and models. There are different interpretations of these states, such as molecular states, compact multiquark states, traditional hadrons, and kinematics effects. Among them, molecular picture is popular, which was conjectured four decades ago from a direct analogy to the deuteron and the nuclear force that binds it [2,3]. After extensive studies on the production, decay, and mass spectra of these exotic states, some were regarded as robust molecular candidates, such as $X(3872)$, $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ [4–16]. The particular property of these molecular

candidates is that they carry hidden charm number. An intriguing question is then whether open charm molecular candidates exist.

In 2003, the BABAR Collaboration reported a narrow structure in the $D_s^+\pi^0$ invariant mass distribution with a mass of 2.32 GeV and quantum number $J^P = 0^+$ [17], whose mass is much lower than the quark model prediction [18]. To solve this problem, a molecule composed of DK and $D_s\eta$ was proposed by many theoreticians [19–24]. Moreover, its heavy quark spin partner $D_{s1}(2460)$ can also be described as a D^*K molecule [24–26]. Thus $D_{s0}(2317)$ and $D_{s1}(2460)$ can be treated as hadronic molecular candidates with open charm. In the molecular picture their quark contents are $c\bar{s}q\bar{q}$. If the quark contents are changed to $c\bar{s}\bar{q}\bar{q}$, DK will become $D\bar{K}$ at the hadron level. Whether or not such hadronic molecules exist is of great interests both theoretically and experimentally.

Very recently the LHCb Collaboration reported the discovery of two new states, $X_0(2866)$ and $X_1(2904)$, in the D^+K^- invariant mass spectrum of the $B^\pm \rightarrow D^+D^-K^\pm$ decay [27,28]. Their corresponding quantum number, mass, and width (in units of MeV) are

$$J^P = 0^+ : m = 2866 \pm 7, \quad \Gamma = 57 \pm 13, \quad (1)$$

$$J^P = 1^- : m = 2904 \pm 5, \quad \Gamma = 110 \pm 12. \quad (2)$$

*Corresponding author.
xiejunjun@impcas.ac.cn

†Corresponding author.
lisheng.geng@buaa.edu.cn

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Their statistical significance is more than 5σ . The quark contents of these two states are $cs\bar{q}\bar{q}$, which are the first open charm tetraquark candidates discovered experimentally so far. In Ref. [29] using two kinds of quark models Karliner and Rosner identified $X_0(2866)$ as a $cs\bar{u}\bar{d}$ tetraquark, while $X_1(2904)$ can not be described as a tetraquark state. Since both states are located close to the mass threshold of $D^*\bar{K}^*$, a natural question is whether they are $D^*\bar{K}^*$ molecules. In this work we will investigate whether they can be interpreted as $D^*\bar{K}^*$ bound states within the one-boson exchange (OBE) model.

This paper is organized as follows. We briefly explain the pertinent ingredients of the OBE model in Sec. II. We show in Sec. III that it is possible that $X_0(2866)$ has a large molecular component, while we cannot generate a 2^+ state. Inspired by such possibilities, we further study a number of related systems, including $D^{(*)}D^{(*)}$, $\bar{B}^{(*)}\bar{B}^{(*)}$, and $D^{(*)}\Sigma_c^{(*)}$, and find some molecular candidates. We conclude in the last section.

II. FORMALISM

We explore whether the $D^{(*)}\bar{K}^{(*)}$ interactions are strong enough to form bound states. The relevant Lagrangians are the same as those of the $D^{(*)}D^{(*)}$ interactions [7]. The Lagrangian describing the interaction between a charmed meson and a light meson can be written as

$$\mathcal{L}_{HH\pi} = -\frac{g}{\sqrt{2}f_\pi} \text{Tr}[H^\dagger \vec{\sigma} \cdot \nabla(\vec{\tau} \cdot \vec{\pi})H], \quad (3)$$

$$\mathcal{L}_{HH\sigma} = g_\sigma \text{Tr}[H^\dagger \sigma H], \quad (4)$$

$$\begin{aligned} \mathcal{L}_{HH\rho} = & g_\rho \text{Tr}[H^\dagger \vec{\tau} \cdot \vec{\rho}^0 H] \\ & - \frac{f_\rho}{4M} \epsilon_{ijk} \text{Tr}[H^\dagger \sigma_k \vec{\tau} \cdot (\partial_i \vec{\rho}_j - \partial_j \vec{\rho}_i)H], \end{aligned} \quad (5)$$

$$\begin{aligned} \mathcal{L}_{HH\omega} = & -g_\omega \text{Tr}[H^\dagger \omega^0 H] \\ & + \frac{f_\omega}{4M} \epsilon_{ijk} \text{Tr}[H^\dagger \sigma_k (\partial_i \omega_j - \partial_j \omega_i)H], \end{aligned} \quad (6)$$

where $H = \frac{1}{\sqrt{2}}(D + \vec{D}^* \cdot \vec{\sigma})$, satisfying the constraint of heavy quark spin symmetry. The coupling of π to $D^{(*)}$ are $g = 0.6$ and $f_\pi = 0.132$ GeV. The σ coupling is determined from the $NN\sigma$ coupling in the nonlinear sigma model ($g_{\sigma NN} = \sqrt{2}M_N/f_\pi \simeq 10.1$) and the quark model relation: $g_\sigma = \frac{1}{3}g_{\sigma NN} \simeq 3.4$. The ρ and ω couplings to $D^{(*)}$ are of both electric-type (g_v^*) and magnetic-type (f_v^*). We obtain $g_v = 2.6$ and $f_v = 4.5$ using a common mass $M = 1.867$ GeV for the normalization [7]. Here SU(2) symmetry is adopted in our calculation, regarding strange and charm quarks as spectators. Thus in our work the Lagrangian describing the interaction between $\bar{K}^{(*)}$ and a light meson are the same as that describing the interaction

between $D^{(*)}$ and a light meson. These interactions can also be obtained in the massive Yang-Mills approach [30] or the hidden gauge approach [31], which turn out to be the same as ours in the heavy quark mass limit.

For the $D\bar{K}$, $D^*\bar{K}$, and $D\bar{K}^*$ systems the exchanged light mesons can be σ , ρ , or ω . While π , σ , ρ , and ω are allowed for the $D^*\bar{K}^*$ system. From the Lagrangians of Eqs. (3)–(6) we can derive the OBE potentials for the $D^{(*)}\bar{K}^{(*)}$ systems as

$$\begin{aligned} V_\pi(\vec{r}) = & \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{g^2}{6f_\pi^2} [-\vec{\sigma}_1 \cdot \vec{\sigma}_2 \delta(\vec{r}) \\ & + \vec{\sigma}_1 \cdot \vec{\sigma}_2 m_\pi^3 W_Y(m_\pi r) \\ & + S_{12}(\vec{r}) m_\pi^3 W_T(m_\pi r)], \end{aligned} \quad (7)$$

$$V_\sigma(\vec{r}) = -g_\sigma^2 m_\sigma W_Y(m_\sigma r), \quad (8)$$

$$\begin{aligned} V_\rho(\vec{r}) = & \vec{\tau}_1 \cdot \vec{\tau}_2 \left[g_\rho^2 m_\rho W_Y(m_\rho r) \right. \\ & + \frac{f_v^2}{4M^2} \left(-\frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \delta(\vec{r}) \right. \\ & + \frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 m_\rho^3 W_Y(m_\rho r) \\ & \left. \left. - \frac{1}{3} S_{12}(\hat{r}) m_\rho^3 W_T(m_\rho r) \right) \right], \end{aligned} \quad (9)$$

$$\begin{aligned} V_\omega(\vec{r}) = & g_\omega^2 m_\omega W_Y(m_\omega r) \\ & + \frac{f_v^2}{4M^2} \left[-\frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \delta(\vec{r}) \right. \\ & + \frac{2}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 m_\omega^3 W_Y(m_\omega r) \\ & \left. - \frac{1}{3} S_{12}(\hat{r}) m_\omega^3 W_T(m_\omega r) \right], \end{aligned} \quad (10)$$

where the dimensionless functions $W_Y(x)$ and $W_T(x)$ are defined as

$$W_Y(x) = \frac{e^{-x}}{4\pi x}, \quad (11)$$

$$W_T(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{4\pi x}. \quad (12)$$

As the charmed mesons involved in our calculation are not pointlike particles, we introduce a form factor to take into account their finite size. Here we use a monopolar form factor (for more details we refer to Refs. [7,14]),

$$F(q, m, \Lambda) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \quad (13)$$

TABLE I. Matrix elements of spin-spin and tensor operators for the partial waves considered in this work.

Molecule	Partial waves	J^P	$\vec{\sigma}_1 \cdot \vec{\sigma}_2$	$S_{12} = 3\vec{\sigma}_1 \cdot \hat{r}\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$D^*\bar{K}^*$	1S_0 - 5D_0	0^+	$\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & -2 \end{pmatrix}$
$D^*\bar{K}^*$	3S_1 - 3D_1	1^+	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$
$D^*\bar{K}^*$	1D_2 - 5S_2 - 5D_2	2^+	$\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -\sqrt{\frac{2}{5}} & \frac{2}{\sqrt{7}} \\ -\sqrt{\frac{2}{5}} & 0 & \sqrt{\frac{14}{5}} \\ \frac{2}{\sqrt{7}} & \sqrt{\frac{14}{5}} & \frac{3}{7} \end{pmatrix}$

where m and q are the mass and four-momentum of the exchanged meson, respectively.

Using the above form factor, the functions δ , W_Y , and W_T in Eqs. (7)–(10) need to be changed accordingly,

$$\delta(r) \rightarrow m^3 d(x, \lambda), \quad (14)$$

$$W_Y(x) \rightarrow W_Y(x, \lambda), \quad (15)$$

$$W_T(x) \rightarrow W_T(x, \lambda), \quad (16)$$

with $\lambda = \Lambda/m$. The corresponding functions d , W_Y , and W_T read

$$d(x, \lambda) = \frac{(\lambda^2 - 1)^2}{2\lambda} \frac{e^{-\lambda x}}{4\pi}, \quad (17)$$

$$W_Y(x, \lambda) = W_Y(x) - \lambda W_Y(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \frac{e^{-\lambda x}}{4\pi}, \quad (18)$$

$$W_T(x, \lambda) = W_T(x) - \lambda^3 W_T(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \lambda^2 \left(1 + \frac{1}{\lambda x}\right) \frac{e^{-\lambda x}}{4\pi}. \quad (19)$$

In addition, since D -wave interactions may play an important role in forming hadronic molecules, we also consider the D -wave interaction in the $D^*\bar{K}^*$ system, which will produce two kinds of spin operators, spin-spin $\vec{\sigma} \cdot \vec{\sigma}$ and tensor S_{12} . We define the tensor operator as $S_{12} = 3\vec{\sigma}_1 \cdot \hat{r}\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$. The specific matrix elements of spin-spin and tensor operators for the $D^*\bar{K}^*$ system are displayed in Table I.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. $cs\bar{q}\bar{q}$

In Figs. 1 and 2, we show the binding energies of the $I = 0$ $D^{(*)}\bar{K}^{(*)}$ systems as a function of the cutoff. The OBE potentials of $D\bar{K}$, $D^*\bar{K}$, and $D\bar{K}^*$ are the same, and the only difference of these states are the kinetic energy. We present the binding energy dependence of these states on

the cutoff in Fig. 1. The results indicate that a bound state near the mass threshold of $D\bar{K}^*$ is likely to exist, but with a cutoff much larger than the preferred value of about 1 GeV. An even larger cutoff is needed for the $D\bar{K}$ and $D^*\bar{K}$ systems to bind. Therefore we conclude that the existence of molecular states near the $D\bar{K}$ and $D^*\bar{K}$ mass thresholds is unlikely.

For the $D^*\bar{K}^*$ system, there are three spin-parity assignments, $J = 0$, $J = 1$, and $J = 2$, for S -wave interactions (which we focus on in the present work). From the results shown in Fig 2, it seems that two bound states with $J^P = 0^+$ and $J^P = 1^+$ with $I = 0$ are likely. Therefore $X_0(2866)$ could be assigned as a molecule state near the mass threshold $D^*\bar{K}^*$ with $J^P = 0^+$. In addition, our results do not support the existence of a $J^P = 2^+$ state because of the rather large cutoff needed.¹

In our framework, if one considers $X_1(2904)$ as a $D^*\bar{K}^*$ bound state, the interaction between D^* and $\bar{K}^*$ is P -wave, which results a much more repulsive potential. Thus $X_1(2904)$ can not be identified as a $D^*\bar{K}^*$ molecule in this work.

Furthermore, we note that the cutoff needed to make the mass of $X_0(2866)$ to agree with the LHCb data is about 1.2 GeV, slightly larger than the preferred value of 1 GeV. As a result, such a relatively large cutoff may indicate the existence of a sizable compact tetraquark component. In this sense, the conclusion of our present study does not necessarily contradict that of Ref. [29], at least not strongly.

B. $cc\bar{q}\bar{q}$

If we replace the s quark in the $cs\bar{q}\bar{q}$ system with a c quark, the system becomes $cc\bar{q}\bar{q}$ with a charm number $C = 2$. At the hadron level, this system corresponds to $D^{(*)}D^{(*)}$. In our previous work [7], assuming that $X(3872)$ is a 1^{++} bound state, we have already investigated the $\bar{D}^{(*)}D^{(*)}$ system in the OBE model. Two $0(1^+)$ hadronic molecules with $C = 2$ near the DD^* and D^*D^* mass

¹We did not find any $I = 1$ $D^*\bar{K}^*$ bound states with a reasonable cutoff because the $I = 1$ interactions of π and ρ are not only opposite to but also only one third of the $I = 0$ interactions.

FIG. 1. Binding energies of $(I = 0)$ $D\bar{K}$, $D^*\bar{K}$, and $D\bar{K}^*$ molecules as a function of the cutoff.

FIG. 2. Binding energies of $D^*\bar{K}^*$ ($I = 0$) molecules as a function of the cutoff.

thresholds are predicted. In Fig 3, we display the dependence of the binding energy of the two systems on the cutoff. It seems to be very likely that two hadronic molecules close to the DD^* and D^*D^* thresholds exist, which should be

FIG. 3. Binding energies of D^*D and D^*D^* molecules as a function of the cutoff.

clarified by future experiments. We note that the doubly charmed tetraquark states have already been extensively investigated in a number of approaches such as the non-relativistic quark model [32], chiral perturbation theory [33], sum rules [34], and lattice QCD [35,36]. After the doubly charmed baryon Ξ_{cc} was discovered by the LHCb Collaboration in 2017, studies of doubly charmed tetraquark states in the diquark picture were updated with the charmed diquark mass extracted from Ξ_{cc} [37–40]. However, it is not yet settled whether the masses of open charm tetraquark states are below or above the threshold of two charmed mesons.

C. $bb\bar{q}\bar{q}$

Using the heavy quark flavor symmetry, one can straightforwardly correlate the $\bar{B}^{(*)}\bar{B}^{(*)}$ system with the $D^{(*)}D^{(*)}$ system. Since the potential in the bottom sector is the same as that in the charm sector, the binding energy is larger than that in the charm sector as shown in Fig. 4. Thus deeper bound states are more likely to exist in the open bottom sector, consistent with previous studies [37–43].

D. $ccqq\bar{q}\bar{q}$

In our previous studies, we have already confirmed that there is a complete multiplet of hadronic molecules constrained by heavy quark spin symmetry in the $\bar{D}^{(*)}\Sigma_c^{(*)}$ system in light of the LHCb pentaquark states [7,14], but these hadronic molecules have a hidden charm number. Here, we further explore whether there exist open charm molecules in the $D^{(*)}\Sigma_c^{(*)}$ system. In terms of G -party [44], the OBE potential of $D^{(*)}\Sigma_c^{(*)}$ can be easily derived from that of $\bar{D}^{(*)}\Sigma_c^{(*)}$. In this system the cutoff should not be treated as a free parameter because the cutoff can be determined by reproducing the binding energy of $P_c(4312)$ [14]. Within the OBE model and with $\Lambda = 1.119$ GeV, one can study the $D^{(*)}\Sigma_c^{(*)}$ system. The results

FIG. 4. Binding energies of $\bar{B}^*B$ and $\bar{B}^*\bar{B}^*$ molecules as a function of the cutoff.

TABLE II. Binding energies of $D^{(*)}\Sigma_c^{(*)}$ molecules with $I = 1/2$. The uncertainties are generated by varying the OBE potentials from their central values by 15%, accounting for potential breaking of heavy quark symmetry.

Molecule	I	J^P	Binding Energy (MeV)	Mass (MeV)
$D\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	$31.7^{+16.6}_{-13.9}$	4289.3
$D\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$32.5^{+16.8}_{-14.1}$	4352.5
$D^*\Sigma_c$	$\frac{1}{2}$	$\frac{1}{2}^-$	$18.4^{+11.9}_{-9.3}$	4444.6
$D^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$57.4^{+24.8}_{-21.9}$	4405.6
$D^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{1}{2}^-$	$19.2^{+12.7}_{-9.8}$	4507.8
$D^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$32.1^{+17.0}_{-14.2}$	4494.9
$D^*\Sigma_c^*$	$\frac{1}{2}$	$\frac{3}{2}^-$	$61.4^{+25.9}_{-23.7}$	4465.6

are displayed in Table II, which show the existence of seven open charm molecules made of $D^{(*)}\Sigma_c^{(*)}$. We further note that the binding energies of the $D^{(*)}\Sigma_c^{(*)}$ states are larger than those of the $\bar{D}^{(*)}\Sigma_c^{(*)}$ states, because the π and ω exchanged potentials change sign in the former case compared to those in the latter case.

IV. SUMMARY AND CONCLUSION

The $X_0(2866)$ and $X_1(2904)$ states recently discovered by the LHCb Collaboration are the first open charm tetraquark candidates, which are near the $D^*\bar{K}^*$ mass threshold. Within the one-boson exchange model we systematically investigated the $D^{(*)}\bar{K}^{(*)}$ system and showed that $X_0(2866)$ can be identified as a $D^*\bar{K}^*$ hadronic molecule with $I(J^P) = 0(0^+)$. On the other hand, the molecular interpretation for $X_1(2904)$ is not favored according to our study. As a by-product, we also predicted other likely hadronic molecules with open charm number $C = 2$. In the meson-meson system, we obtained two molecules near the mass thresholds of DD^* and D^*D^*

as well as two molecules near the mass thresholds of $\bar{B}\bar{B}^*$ and $\bar{B}^*\bar{B}^*$ with $I(J^P) = 0(1^+)$. In the meson-baryon system, seven molecules near the mass threshold of $D^{(*)}\Sigma_c^{(*)}$ induced by heavy quark spin symmetry were obtained.

We note that a study of the strong decays of $X_0(2866)$ also supports the molecular interpretation [45], to the extent that both studies seem to point to the fact that compact tetraquark components play a non-negligible role. In addition to more studies on $X_0(2866)$ and $X_1(2904)$, experimental searches for the predicted partner states, either in the hadronic molecular picture or in the compact tetraquark picture, will be crucial to fully disclose the nature of these states.

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Note added in proof:—We noted that, prior to the experiment, three $I = 0$ $D^*\bar{K}^*$ bound states with spin-parity 0^+ , 1^+ , and 2^+ were predicted in the coupled channel unitary approach based on the hidden-gauge formalism [46]. A 0^+ state with a mass of 2848 MeV and a width of 23–59 MeV, close to those of $X_0(2866)$, was predicted, but the approach also predicted a slightly more bound 1^+ state with a mass of 2839 MeV (which does not decay in $D\bar{K}$) and a much more bound 2^+ state with a mass of 2733 MeV.

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- [1] S. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262001 (2003).
 - [2] M. Voloshin and L. Okun, *JETP Lett.* **23**, 333 (1976).
 - [3] A. De Rujula, H. Georgi, and S. Glashow, *Phys. Rev. Lett.* **38**, 317 (1977).
 - [4] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, *Phys. Rep.* **639**, 1 (2016).
 - [5] Z.-F. Sun, J. He, X. Liu, Z.-G. Luo, and S.-L. Zhu, *Phys. Rev. D* **84**, 054002 (2011).
 - [6] X. Liu, Z.-G. Luo, Y.-R. Liu, and S.-L. Zhu, *Eur. Phys. J. C* **61**, 411 (2009).
 - [7] M.-Z. Liu, T.-W. Wu, M. Pavon Valderrama, J.-J. Xie, and L.-S. Geng, *Phys. Rev. D* **99**, 094018 (2019).
 - [8] R. Chen, Z.-F. Sun, X. Liu, and S.-L. Zhu, *Phys. Rev. D* **100**, 011502 (2019).
 - [9] M.-Z. Liu, Y.-W. Pan, F.-Z. Peng, M. Sánchez Sánchez, L.-S. Geng, A. Hosaka, and M. Pavon Valderrama, *Phys. Rev. Lett.* **122**, 242001 (2019).
 - [10] C. Xiao, J. Nieves, and E. Oset, *Phys. Rev. D* **100**, 014021 (2019).
 - [11] C.-J. Xiao, Y. Huang, Y.-B. Dong, L.-S. Geng, and D.-Y. Chen, *Phys. Rev. D* **100**, 014022 (2019).

- [12] Y.-H. Lin and B.-S. Zou, *Phys. Rev. D* **100**, 056005 (2019).
- [13] Y. Yamaguchi, H. García-Tecocoatzí, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi, and M. Takizawa, *Phys. Rev. D* **101**, 091502 (2020).
- [14] M.-Z. Liu, T.-W. Wu, M. Sánchez Sánchez, M. P. Valderrama, L.-S. Geng, and J.-J. Xie, [arXiv:1907.06093](#).
- [15] M. Pavon Valderrama, *Phys. Rev. D* **100**, 094028 (2019).
- [16] M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. A. Oller, and Q. Wang, *Phys. Rev. Lett.* **124**, 072001 (2020).
- [17] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **90**, 242001 (2003).
- [18] S. Godfrey and N. Isgur, *Phys. Rev. D* **32**, 189 (1985).
- [19] T. Barnes, F. Close, and H. Lipkin, *Phys. Rev. D* **68**, 054006 (2003).
- [20] D. Gamermann, E. Oset, D. Strottman, and M. Vicente Vacas, *Phys. Rev. D* **76**, 074016 (2007).
- [21] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, *Phys. Lett. B* **641**, 278 (2006).
- [22] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, *Phys. Rev. D* **76**, 014005 (2007).
- [23] L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, and U.-G. Meißner, *Phys. Rev. D* **87**, 014508 (2013).
- [24] M. Altenbuchinger, L. S. Geng, and W. Weise, *Phys. Rev. D* **89**, 014026 (2014).
- [25] D. Gamermann and E. Oset, *Eur. Phys. J. A* **33**, 119 (2007).
- [26] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, *Phys. Rev. D* **76**, 114008 (2007).
- [27] R. Aaij *et al.* (LHCb Collaboration), [arXiv:2009.00025](#).
- [28] R. Aaij *et al.* (LHCb Collaboration), [arXiv:2009.00026](#).
- [29] M. Karliner and J. L. Rosner, [arXiv:2008.05993](#).
- [30] U. G. Meißner, *Phys. Rep.* **161**, 213 (1988).
- [31] M. Harada and K. Yamawaki, *Phys. Rep.* **381**, 1 (2003).
- [32] S. Zouzou, B. Silvestre-Brac, C. Gignoux, and J. Richard, *Z. Phys. C* **30**, 457 (1986).
- [33] A. V. Manohar and M. B. Wise, *Nucl. Phys.* **B399**, 17 (1993).
- [34] F. S. Navarra, M. Nielsen, and S. H. Lee, *Phys. Lett. B* **649**, 166 (2007).
- [35] Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura, and K. Sasaki, *Phys. Lett. B* **729**, 85 (2014).
- [36] A. Francis, R. J. Hudspith, R. Lewis, and K. Maltman, *Phys. Rev. Lett.* **118**, 142001 (2017).
- [37] M. Karliner and J. L. Rosner, *Phys. Rev. Lett.* **119**, 202001 (2017).
- [38] P. Junnarkar, N. Mathur, and M. Padmanath, *Phys. Rev. D* **99**, 034507 (2019).
- [39] W. Park, S. Noh, and S. H. Lee, *Nucl. Phys.* **A983**, 1 (2019).
- [40] Q.-F. Lü, D.-Y. Chen, and Y.-B. Dong, *Phys. Rev. D* **102**, 034012 (2020).
- [41] B. Wang, Z.-W. Liu, and X. Liu, *Phys. Rev. D* **99**, 036007 (2019).
- [42] L. Leskovec, S. Meinel, M. Pflaumer, and M. Wagner, *Phys. Rev. D* **100**, 014503 (2019).
- [43] M.-T. Yu, Z.-Y. Zhou, D.-Y. Chen, and Z. Xiao, *Phys. Rev. D* **101**, 074027 (2020).
- [44] E. Klempt, F. Bradamante, A. Martin, and J. Richard, *Phys. Rep.* **368**, 119 (2002).
- [45] Y. Huang, J.-X. Lu, J.-J. Xie, and L.-S. Geng, [arXiv:2008.07959](#).
- [46] R. Molina, T. Branz, and E. Oset, *Phys. Rev. D* **82**, 014010 (2010).