Probing the angle of birefringence due to long range axion hair from pulsars

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Rotating neutron star or pulsar can be a possible source of pseudo Nambu Goldstone bosons or axions which can mediate long range axionic hair outside of the pulsar. When the electromagnetic radiation is emitted from a pulsar and passes through the long range axion hair, the axion rotates the polarization of the electromagnetic radiation and produces birefringence. We obtain the angle of birefringence due to this long range axionic hair as 0.42° . This value is within the accuracy of measuring the linear polarization angle of pulsar light which is $\leq 1.0^\circ$. Our result continues to hold as long as the range of the axion hair (inverse of axion mass) is greater than the radius of the pulsar, i.e., $m_a < 10^{-11}$ eV and the axion decay constant $f_a \lesssim \mathcal{O}(10^{17}$ GeV).

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I. INTRODUCTION

Axion was first introduced by Peccei and Quinn (PQ) (1977) to solve the strong CP problem [1–4]. The neutron electric dipole moment (nEDM) d_n depends on a parameter $\bar{\theta}$ and from chiral perturbation theory, we can obtain $d_n \sim 10^{-16} \bar{\theta}$ e.cm. $\bar{\theta}$ is related to the QCD θ angle by $\bar{\theta} = \theta + \arg \det M_q$ [5,6], where M_q is the quark mass matrix. However, the current experimental bound on neutron EDM is $d_n \sim 10^{-26}$ e.cm, which implies $\bar{\theta} \lesssim 10^{-10}$ [7]. The smallness of the nEDM parameter is called the strong CP problem. To solve the strong CP problem, Peccei and Quinn postulated that $\bar{\theta}$ is not just a parameter but it is a dynamical field (scaled by $1/f_a$ to make it dimensionless) which is driven to zero by its own classical potential. It is a pseudo Nambu Goldstone boson which arises due to spontaneous breaking of global $U(1)$ symmetry at a scale f_a which is called the axion decay constant and explicitly breaks at Λ_{QCD} due to nonperturbative QCD effects. These are the QCD axions which solve the strong CP problem. The QCD axion mass (m_a) is related with f_a by $m_a = 5.7 \times 10^{-12} \text{ eV} (\frac{10^{18} \text{ GeV}}{f_a})$. So, to obtain f_a smaller than the Planck scale, the mass of the axion is $m_a \gtrsim 10^{-12}$ eV [8].

There are also other types of pseudo Nambu Goldstone bosons which arise due to string compactifications [9]. The detail studies of axions and axionlike particles (ALPs) are discussed in [9–13]. The axions can also be a possible candidate of dark matter and even they can be ultralight [14–17]. The coupling of axions with other particles depend on the Peccei-Quinn symmetry breaking scale f_a . The axion can couple with photons and the coupling of axion with photons is described by the term $g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} / 4$ where $g_{a\gamma\gamma}$ is the axion photon coupling constant which is inversely proportional to f_a . The axion field is denoted by a , $F_{\mu\nu}$ is the electromagnetic field tensor, and $\tilde{F}^{\mu\nu}$ is its dual.

There are no direct observational confirmations of axions in the Universe. However, one can put bound on the axion decay constant and axion mass in $m_a - f_a$ plane from laboratory, astrophysical, and other experiments. There are some experiments which are searching for solar axions of mass sub-eV range and the axion decay constant $f_a \sim 10^7$ GeV [18,19]. If solar axions are there in nature, then it would violate supernova 1987A bound which requires $f_a \gtrsim 10^9$ GeV. Axions can be a possible candidate of hot dark matter if $f_a \lesssim 10^8$ GeV [20–22]. Cold axions can be produced from vacuum realignment mechanism and provide a component of cold dark matter [23,24]. Axions can also be produced from the decay of axion strings [25–27]. Some laboratory experiments also put bounds on the axion parameters [28–35].

The sign change in the axion potential from the coupling to gluons at large densities allows axions to be sourced by the neutron stars or pulsars [36–38]. The coherent

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oscillation of the axion potential in free space produces the axionic density. If the axions have coupling with the nucleons, then pulsars immersed in the axionic potential develop long-range axion hair. The pulsar also emits electromagnetic radiation, since the dipole axis is not aligned along the spin axis. As the dipole axis precesses around the spin axis, there is a synchrotron radiation along the magnetic poles that appears as the pulsed signal in a cone swept out by the radio beam. Rotating neutron star or pulsar can also form binaries with other neutron star, white dwarf, and black hole, and there can be ultralight scalar or pseudoscalar axion radiation [36–39] or vector boson radiation [40] due to orbital period decay. Ultralight vector bosons can also mediate between the Sun and the planets and contribute to the perihelion precession of planets [41].

Strong dipolar magnetic field induces $\mathbf{E} \cdot \mathbf{B}$ density outside the pulsar which can also be a source of pseudoscalar axion and can rotate the polarization vector of the electromagnetic radiation [42]. There can also be galactic axion dark matter background which can rotate the polarization of the pulsar light [43]. The ALP dark matter background also rotates the cosmic microwave background modes which constrain the mass of the axion and axion photon coupling constant [44]. There is also a study where photon can change its polarization when it passes through neutrino gas [45]. Axions can also be probed from superradiance phenomena in a polarimetric measurement for a black hole [46,47]. Constraints on the rotation of the polarization angle can also be obtained from the protoplanetary disk polarimetry [48], the active galactic nuclei [49], and quasar polarization [50].

In this paper, we show that pulsar can be a possible source of axions and it can produce a long range axionic hair outside of the pulsar. When the electromagnetic radiation passes through this long range axion hair, it rotates the polarization of electromagnetic radiation and produces birefringence. The obtained birefringent angle due to axion photon interaction is within the accuracy of measuring the linear polarization angle of pulsar light which is $\leq 1.0^\circ$ [43,48,51]. The obtained birefringent angle strictly continues to hold for axions with inverse mass greater than the radius of pulsar (10 km) which gives $m_a < 10^{-11}$ eV.

The paper is organized as follows. In Sec. II, we describe the long range axion profile for an isolated pulsar. In Sec. III, we explain the photon propagation through the long range axion hair and calculate the birefringent angle. Finally, in Sec. IV, we summarize and explain our result.

We use the units $\hbar = c = 1$ throughout the paper.

II. THE LONG RANGE AXION PROFILE FOR A PULSAR

A. Effective potential of axion in vacuum

The interaction of axion with other standard model particles below the PQ and electroweak breaking scale is governed by the Lagrangian [8],

$$\mathcal{L} = \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{a}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{1}{4} a g_{a\gamma\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a} j_{a,0}^\mu, \quad (1)$$

where the first term denotes the kinetic term of the axion field, the second term denotes the coupling of axion with the gluon field, the third term denotes the axion field couples with the photon field, and the last term denotes the derivative coupling of axion field with the quark field q through an axial vector current $j_{a,0}^\mu = g_q^0 \bar{q} \gamma^\mu \gamma_5 q$. $\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\sigma\delta} G^{\sigma\delta}$ is the dual of gluon field $G_{\mu\nu}$. The axion photon coupling is defined as $g_{a\gamma\gamma}^0 = \frac{\alpha}{2\pi f_a} \frac{E}{N}$, where α is the electromagnetic fine structure constant and E/N is the ratio of the electromagnetic to the color anomaly. Note the coupling of axion with the other standard model fields is inversely proportional to f_a .

We can give a chiral rotation to the quark field $q \rightarrow e^{i\gamma_5 \frac{a}{f_a} Q_a} q$ so that derivative coupling disappears from the quark mass term, where $q = (u, d)$, and $\text{tr} Q_a = 1$. Hence, Eq. (1) becomes

$$\mathcal{L} = \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{1}{4} a g_{a\gamma\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial_\mu a}{2f_a} j_a^\mu - \bar{q}_L M_a q_R + \text{H.c.} \quad (2)$$

The new axion photon coupling constant becomes

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a} \left[\frac{E}{N} - 6 \text{tr}(Q_a Q^2) \right], \quad (3)$$

and the axial current density is

$$j_a^\mu = j_{a,0}^\mu - \bar{q} \gamma^\mu \gamma_5 Q_a q. \quad (4)$$

The quark mass matrix in the new mass basis becomes

$$M_a = e^{i\frac{a}{f_a} Q_a} M_q e^{i\frac{a}{f_a} Q_a}, \quad (5)$$

where $M_q = \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix}$ and $Q = \begin{bmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix}$. m_u and m_d are the masses of the up and down quarks, respectively.

Such chiral transformation of the quark field helps to move all the nonderivative coupling into the two lightest quarks. Hence, we can integrate out all the other quarks and can work in the two flavor effective theory. M_a contains nonderivative couplings on the axions. So, in the chiral expansion at the leading order, all the nonderivative couplings of axions contain in the pion mass term governed by the Lagrangian

$$\mathcal{L}_\pi \supset 2B_0 \frac{f_\pi^2}{4} \langle U M_a^\dagger + M_a U^\dagger \rangle, \quad (6)$$

where

$$U = e^{\frac{i\Pi}{f_\pi}}, \quad \Pi = \begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{bmatrix}. \quad (7)$$

Here f_π is the pion decay constant, and B_0 is related to the chiral condensate. To derive the effective axion potential to the leading order, we only consider the neutral pion sector. Choosing Q_a proportional to the identity matrix, we can write

$$V(a, \pi^0) = -B_0 f_\pi^2 \left[m_u \cos\left(\frac{\pi^0}{f_a} - \frac{a}{2f_a}\right) + m_d \cos\left(\frac{\pi^0}{f_a} + \frac{a}{2f_a}\right) \right], \quad (8)$$

or

$$V(a, \pi^0) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2f_a}\right)} \times \cos\left(\frac{\pi^0}{f_\pi} - \phi_a\right), \quad (9)$$

where

$$\tan \phi_a = \frac{m_u - m_d}{m_u + m_d} \tan\left(\frac{a}{2f_a}\right), \quad (10)$$

where m_π is the pion mass. On the vacuum, the neutral pion gets a vacuum expectation value and trivially be integrated out. Hence, the effective axion potential becomes

$$V \approx -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2f_a}\right)}. \quad (11)$$

The minimum of the potential is at $\langle a \rangle = 0$, i.e., $\bar{\theta} = 0$ and solves the strong CP problem. The second derivative of the potential gives the axion mass

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}. \quad (12)$$

We want to probe the axions whose mass is lighter than that obtained from Eq. (11) by a factor of $\sqrt{\epsilon}$ and we consider the parameter space $\epsilon \lesssim 0.1$. For these axions, the potential becomes

$$V \approx -\epsilon m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2f_a}\right)}, \quad (13)$$

and consequently in the $m_u = m_d$ limit, the mass of the axion becomes

$$m_a = \frac{m_\pi f_\pi}{2f_a} \sqrt{\epsilon}. \quad (14)$$

B. Effective axion potential at finite density

We are considering the astrophysical objects like pulsars which are made of nonrelativistic matter mostly like neutrons and protons. The axion potential comes from the quark mass term. From Feynman-Hellmann theorem, one can develop the in medium quark condensate. If $H(\lambda)$ denotes a Hermitian operator which depends on a real parameter λ and operates on a normalized eigenvector $|\psi(\lambda)\rangle$ with eigenvalue $E(\lambda)$, then [52]

$$H(\lambda)|\psi(\lambda)\rangle = E(\lambda)|\psi(\lambda)\rangle. \quad (15)$$

Hence, from Feynman-Hellmann theorem, we can write

$$\langle\psi(\lambda)| \frac{d}{d\lambda} H(\lambda) |\psi(\lambda)\rangle = \frac{d}{d\lambda} \langle\psi(\lambda)| H(\lambda) |\psi(\lambda)\rangle. \quad (16)$$

The chiral symmetry is explicitly broken by the quark mass term which is governed by the Hamiltonian

$$\mathcal{H}_{\text{mass}} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \dots, \quad (17)$$

where m_u , m_d , and m_s denote the quark mass terms correspond to the quark fields u , d , and s , respectively. ... denotes the similar contributions from the heavier quarks. We can write Eq. (17) as

$$\mathcal{H}_{\text{mass}} = 2m_q \bar{q}q - \frac{1}{2} \delta m_q (\bar{u}u - \bar{d}d) + m_s \bar{s}s + \dots, \quad (18)$$

where we define $\bar{q}q = \frac{1}{2}(\bar{u}u + \bar{d}d)$, $m_q = \frac{1}{2}(m_u + m_d)$, and $\delta m_q = m_d - m_u$. Putting $H \rightarrow \int d^3x \mathcal{H}_{\text{QCD}}$ and $\lambda \rightarrow m_q$ in the Feynman-Hellmann theorem, we obtain

$$2\langle\psi(m_q)| \int d^3x \bar{q}q |\psi(m_q)\rangle = \frac{d}{dm_q} \langle\psi(m_q)| \int d^3x \mathcal{H}_{\text{QCD}} |\psi(m_q)\rangle. \quad (19)$$

Multiplying both sides by m_q , we obtain from Eq. (19)

$$2m_q \langle\psi(m_q)| \int d^3x \bar{q}q |\psi(m_q)\rangle = m_q \frac{d}{dm_q} \langle\psi(m_q)| \int d^3x \mathcal{H}_{\text{QCD}} |\psi(m_q)\rangle. \quad (20)$$

Here, we neglect the isospin breaking terms, though it is not necessary.

Consider two cases, $|\psi(m_q)\rangle = |n_N\rangle$ and $|\psi(m_q)\rangle = |0\rangle$, in Eq. (20). $|n_N\rangle$ denotes the ground state with nucleon density n_N and $|0\rangle$ denotes the vacuum state. Taking the differences of these two cases, we obtain

$$2m_q(\langle \bar{q}q \rangle_{n_N} - \langle \bar{q}q \rangle_0) = m_q \frac{d\xi}{dm_q}. \quad (21)$$

The nuclear matter energy density ξ is given by

$$\xi = M_N n_N + \delta\xi, \quad (22)$$

where $\delta\xi$ contributes to the energy density from nucleon kinetic energy and nucleon nucleon interaction. The quark condensate at low density can be expressed by nucleon σ term, σ_N which is defined as

$$\sigma_N = 2m_q \int d^3x (\langle N | \bar{q}q | N \rangle - \langle 0 | \bar{q}q | 0 \rangle). \quad (23)$$

From Eq. (20), we obtain

$$\sigma_N = m_q \frac{dM_N}{dm_q}. \quad (24)$$

Combining Eqs. (21), (22), and (24), we obtain

$$2m_q(\langle \bar{q}q \rangle_{n_N} - \langle \bar{q}q \rangle_0) = \sigma_N n_N + \dots \quad (25)$$

From Gell-Mann-Oakes-Renner relation, we can write

$$2m_q \langle \bar{q}q \rangle_0 = -m_\pi^2 f_\pi^2. \quad (26)$$

In the effective two flavor theory, we define the effective up and down quark mass as

$$m_{u,d}^{\text{eff}} = m_{u,d} \left[1 - \frac{\sum \sigma_N^{u,d} n_N}{m_\pi^2 f_\pi^2} \frac{m_u + m_d}{m_{u,d}} \right]. \quad (27)$$

From Eq. (11), in $m_u = m_d$ limit, the effective axion potential at finite density becomes

$$V = -m_\pi^2 f_\pi^2 \left\{ \left(\epsilon - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} \right) \cos\left(\frac{a}{2f_a}\right) + \mathcal{O}\left(\left(\frac{\sigma_N n_N}{m_\pi^2 f_\pi^2}\right)^2\right) \right\}. \quad (28)$$

C. Axion profile of a pulsar

It has been discussed in [36–38] that if axions have coupling with nucleons, then axions are sourced by pulsars like neutron stars or white dwarfs. In the following, we have discussed how pulsars can be the source of long range axion hair. Inside the pulsar, the nucleon density corrects the quark mass and the axion potential becomes as Eq. (28). Here we take $\sigma_N \sim 59$ MeV from lattice simulation [53]. The square root of the second derivative of the potential at $a = 0$ gives the magnitude of the tachyonic mass of the axion inside the pulsar which is

$$m_T = \frac{m_\pi f_\pi}{2f_a} \sqrt{\frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} - \epsilon}, \quad r < R, \quad (29)$$

where R is the radius of the pulsar. In the vacuum, the potential of the axion is given by Eq. (13) and the corresponding axion mass is Eq. (14). Inside the pulsar, $\sigma_N \neq 0$ and $m_T \gtrsim m_a$. The sign change of the axion potential at high nucleon density allows axions to be sourced by pulsars. Inside of the pulsar, the axion field is tachyonic and resides on one of the local maxima of the axion potential as shown in Fig. 1(a). Throughout inside of the pulsar, the axion field takes a constant value $a = 4\pi f_a$. Now, we compare the gradient energy f_a^2/r^2 required to move the axion from its unstable solution with the gain in potential energy $m_\pi^2 f_\pi^2 (\epsilon - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2})$ by putting $a = 4\pi f_a$ in the expression of the effective axion potential Eq. (28). When the gain in potential energy is greater than the gradient energy, then the axion field rolls down asymptotically to $a = 0$ at $r \rightarrow \infty$. Hence,

$$m_\pi^2 f_\pi^2 \left(\epsilon - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} \right) > \frac{f_a^2}{r_c^2},$$

or

$$r_c > \frac{1}{m_T}. \quad (30)$$

Axions can be sourced by pulsar if the size of the pulsar is larger than a critical size r_c . A typical pulsar of mass $M = 1.4 M_\odot$ and radius $R = 10$ km can be a source of axions if Eq. (30) is satisfied which gives the upper bound on the axion decay constant, $f_a \lesssim 4.41 \times 10^{17}$ GeV. Nuclei cannot be the source of axions for their very small size and only large sized objects like neutron stars, white dwarfs, and pulsars can be the source of axion field. By matching the inside and outside of axion field solution, we get the long range behavior of the axion field. The axion potential has degenerate vacua, and this degeneracy can be broken by higher dimensional operators at Planck scale or finite density effect [54].

The large nucleon number density of the pulsar affects the effective axion potential significantly. Inside the pulsar, the second derivative of the potential Eq. (28) with respect to the axion field is

$$\frac{\partial^2 V}{\partial a^2} = m_\pi^2 f_\pi^2 \left\{ \left(\epsilon - \frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} \right) \frac{1}{4f_a^2} \cos\left(\frac{a}{2f_a}\right) + \mathcal{O}\left(\left(\frac{\sigma_N n_N}{m_\pi^2 f_\pi^2}\right)^2\right) \right\}. \quad (31)$$

Inside of the pulsar, $\sigma_N \neq 0$ and $\frac{\sigma_N n_N}{m_\pi^2 f_\pi^2} > \epsilon$. Hence, the potential has maxima at $a = 0, \pm 4\pi f_a, \dots$ and minima at $a = \pm 2\pi f_a, \pm 6\pi f_a, \dots$, etc.

FIG. 1. (a) Plot of axion potential V with respect to a . We take $m_\pi^2/m_a^2 = 2$. The black dashed line corresponds to inside the pulsar, and the red solid line corresponds to outside the pulsar. (b) Plot of V as a function of distance (r) from the center of pulsar. There is a discontinuity of V at R due to sign change in V . (c) Plot of a as a function of r . (d) Variation of charge with axion mass.

Outside of the pulsar, $\sigma_N = 0$ and the second derivative of the potential becomes

$$\frac{\partial^2 V}{\partial a^2} = m_\pi^2 f_\pi^2 \left\{ \epsilon \frac{1}{4f_a^2} \cos\left(\frac{a}{2f_a}\right) + \mathcal{O}\left(\left(\frac{\sigma_N n_N}{m_\pi^2 f_\pi^2}\right)^2\right) \right\}. \quad (32)$$

Therefore, outside of the pulsar, the potential attains minima at $a = 0, \pm 4\pi f_a, \dots$ and maxima at $a = \pm 2\pi f_a, \pm 6\pi f_a, \dots$, etc.

Inside the pulsar, the axion field is tachyonic and reside on one of the local maxima of the axion potential. Throughout inside of the pulsar, the dynamical axion field takes a constant value $a = 4\pi f_a$ at the nearest local maximum. Outside of the pulsar, the axion field rolls down to the nearest local minimum and asymptotically reaches $a = 0$ value at infinity.

For an isolated pulsar of constant density, the axion field equation of motion becomes [36,38]

$$\nabla^\mu \nabla_\mu \left(\frac{a}{2f_a} \right) = \begin{cases} -m_T^2 \sin\left(\frac{a}{2f_a}\right) \text{sgn}\left\{\cos\left(\frac{a}{2f_a}\right)\right\} & (r < R), \\ m_a^2 \sin\left(\frac{a}{2f_a}\right) \text{sgn}\left\{\cos\left(\frac{a}{2f_a}\right)\right\} & (r > R). \end{cases} \quad (33)$$

The sgn function takes care of the absolute value of $|\cos(\frac{a}{2f_a})|$ in the potential. Inside the pulsar, the equation of motion of axion field is satisfied by the field value $a = 4\pi f_a$. If a neutron star or pulsar is immersed in a low mass axionic potential, then the solution of the axion field outside of the pulsar falls off with distance like Yukawa interaction. Assuming the spacetime outside the pulsar is Schwarzschild, the axion field equation of motion becomes

$$\left(1 - \frac{2GM}{r}\right) \frac{d^2 a}{dr^2} + \frac{2}{r} \left(1 - \frac{GM}{r}\right) \frac{da}{dr} = m_a^2 a, \quad (34)$$

where M is the mass of the pulsar and G is the universal gravitational constant. Here we take the approximation $\sin(\frac{a}{2f_a}) \sim \frac{a}{2f_a}$ for small value of $\frac{a}{2f_a}$.

At a large distance ($r \gg 2GM$) from the pulsar, Eq. (34) becomes

$$\frac{d^2 a}{dr^2} + \frac{2}{r} \frac{da}{dr} = m_a^2 a \dots \quad (35)$$

Assuming, the solution of the axion field is $a = \chi(r)/r$, Eq. (35) becomes $\chi'' - m_a^2 \chi = 0$, where the prime denotes the derivative of the axion field with respect to r .

This equation yields $\chi = C_1 e^{m_a r} + C_2 e^{-m_a r}$, where C_1 and C_2 are the integration constants. Since $a \rightarrow 0$ as $r \rightarrow \infty$ implies, $C_1 = 0$. Hence, the axion field equation becomes $a \sim q_a e^{-m_a r}/r$, where we substitute C_2 as q_a . For small axion mass ($m_a \ll 1/R$), the axion field behaves as a long range hair with an effective axion charge q_a so that the axion Compton wavelength is larger than the size of the star.

In the Schwarzschild background, the massive long range Yukawa axion field will not satisfy the Klein-Gordon equation (33),

$$\nabla^\mu \nabla_\mu a = m_a^2 a. \quad (36)$$

To solve Eq. (36), we expand the axion field in a perturbative way where the perturbation parameter is GM/R and the leading order term is the long range Yukawa term. Let the axion field be

$$a(r) = a_0(r) + \frac{GM}{R} a_1(r) + \mathcal{O}\left(\left(\frac{GM}{R}\right)^2\right), \quad (37)$$

where $a_0(r) = q_a \frac{e^{-m_a r}}{r}$ is the leading order Yukawa term. Following the same method as [41], the axion field solution becomes

$$a(r) = \frac{q_a e^{-m_a r}}{r} \left[1 + \frac{GM}{r} \{1 - m_a r \ln(m_a r) + m_a r e^{2m_a r} E_i(-2m_a r)\} \right] + \mathcal{O}\left(\left(\frac{GM}{R}\right)^2\right), \quad (38)$$

where E_i is the exponential integral function. From the continuity of axion field at the surface of the pulsar, we obtain the effective axion charge

$$q_a = 4\pi f_a R e^{m_a R} \left[1 + \frac{GM}{R} \{1 - m_a R \ln(m_a R) + m_a R e^{2m_a R} E_i(-2m_a R)\} \right]^{-1} + \mathcal{O}\left(\left(\frac{GM}{R}\right)^{-2}\right). \quad (39)$$

In the limit $GM/R \ll 1$ and $m_a \rightarrow 0$, we obtain $q_a \sim 4\pi f_a R$ [36].

Hence, the axion field solutions inside and outside of the pulsar are

$$a = \frac{q_a e^{-m_a r}}{r} \left[1 + \frac{GM}{r} \{1 - m_a r \ln(m_a r) + m_a r e^{2m_a r} E_i(-2m_a r)\} \right] + \mathcal{O}\left(\left(\frac{GM}{R}\right)^2\right), \quad r > R. \\ = 4\pi f_a, \quad r < R. \quad (40)$$

We assume that the spacetime metric outside the pulsar is Schwarzschild because for a typical pulsar ($M = 1.4M_\odot$, $R = 10$ km), GM/R is not very much small. The plot of axion field and axion potential inside and outside of a pulsar and the variation of axion charge with axion mass is shown in Fig. 1 [38]. In Fig. 2, we obtain the axion field as a function of distance numerically and analytically and for different spacetimes. The red curve denotes the axion field

FIG. 2. Variation of the axion field as a function of distance obtained numerically and analytically and for different spacetimes.

by solving Eq. (40) analytically. The blue curve denotes its numerical solution. The green curve denotes the axion field profile in a flat spacetime for nonzero axion mass. The cyan curve denotes the axion profile for the massless axion in the Schwarzschild background.

The spacetime outside of a pulsar is described by the approximate form of the Kerr metric,

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 \\ - r^2(d\theta^2 + \sin^2\theta d\phi^2) - \frac{4GMj}{r} \sin^2\theta d\phi dt, \quad (41)$$

where j is the angular momentum per unit mass and this approximation is valid as long as $j \ll GM$. For a typical pulsar $j = 10^{-6}$ km, $GM = 1.5$ km, and we can neglect the last term. Hence, the metric outside of the pulsar effectively becomes Schwarzschild [42].

III. PHOTON PROPAGATION IN AN AXIONIC FIELD: BIREFRINGENCE

When a linearly polarized pulsar light passes through a long range axionic hair which is originated from the same

pulsar, then due to their dispersion relations the left and the right circular polarization modes will attain opposite corrections. This effect is called birefringence.

The Lagrangian which describes the interaction between the axion couples with two photons is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}g_{a\gamma\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}. \quad (42)$$

Here $g_{a\gamma\gamma}$ is given as

$$g_{a\gamma\gamma} = \frac{c\alpha_{em}}{2\pi f_a}, \quad (43)$$

where c is a model-dependent parameter of order unity and α_{em} is the electromagnetic fine structure constant. The modified Maxwell's field equations in the presence of axion coupling become [42]

$$\nabla \cdot \mathbf{E} = -g_{a\gamma\gamma}(\nabla a) \cdot \mathbf{B}, \quad (44)$$

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = g_{a\gamma\gamma} \left[(\nabla a) \times \mathbf{E} + \mathbf{B} \frac{\partial a}{\partial t} \right], \quad (45)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (46)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad (47)$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field vectors of the electromagnetic radiation. Since the long range Yukawa type axion field has only r dependence, the second term in the rhs of Eq. (45) is zero. Taking curl of Eq. (45) and using Eqs. (46) and (47), we obtain

$$\nabla_\mu \nabla^\mu \mathbf{B} = -g_{a\gamma\gamma}(\nabla a) \times \frac{\partial \mathbf{B}}{\partial t}. \quad (48)$$

Suppose this magnetic field has a harmonic variation and we can write

$$B(x, t) = \mathcal{B}e^{i\phi(x, t)}, \quad (49)$$

where $\mathcal{B}$ is the magnitude of the magnetic field of electromagnetic radiation and ϕ is the phase. A linearly polarized wave is an equal admixture of right and left circularly polarized waves. So, we can write the transverse magnetic field as

$$\mathcal{B}_\pm = \mathcal{B}_i \pm i\mathcal{B}_j, \quad (50)$$

where $\mathcal{B}_i$ and $\mathcal{B}_j$ are the components of $\mathcal{B}$ along $\hat{e}_i$ and $\hat{e}_j$ orthogonal to $\hat{r}$. In the Fourier space, we can write Eq. (48) as

$$k_\mu k^\mu \mathcal{B} = ig_{a\gamma\gamma}(\nabla a \times \omega \mathcal{B}), \quad (51)$$

where $k^\mu = (\omega, \mathbf{k})$ is the photon four-momentum. Equation (51) decouples for the two circularly polarized modes and we obtain

$$k_\mu k^\mu \mathcal{B}_\pm \mp g_{a\gamma\gamma}(\partial_r a)\omega \mathcal{B}_\pm = 0. \quad (52)$$

Hence, the dispersion relation for the two circularly polarized modes due to different phase velocities propagating radially from the poles of the pulsar is [42]

$$\omega^2 \left(1 - \frac{2GM}{r} \right)^{-1} - k_r^2 \left(1 - \frac{2GM}{r} \right) = \pm g_{a\gamma\gamma}(\partial_r a)\omega. \quad (53)$$

Hence, we can write the propagation vector from Eq. (53) as

$$k_r = \omega \left(1 - \frac{2GM}{r} \right)^{-1} \mp \frac{g_{a\gamma\gamma}}{2}(\partial_r a). \quad (54)$$

So, the phase shift between the left and the right circularly polarized modes is

$$\Delta\phi = \int_R^\infty (k_r^+ - k_r^-) dr. \quad (55)$$

Using Eq. (54), we can rewrite the phase shift as

$$\Delta\phi = g_{a\gamma\gamma}[a(\infty) - a(R)]. \quad (56)$$

The axion field has a long range behavior outside of the pulsar. Using Eqs. (40), (43), and (56), we can write the phase difference as

$$\Delta\phi = -\frac{c\alpha_{em}q_a e^{-m_a R}}{2\pi f_a R} \left[1 + \frac{GM}{R} \{ 1 - m_a R \ln(m_a R) + m_a R e^{2m_a R} E_i(-2m_a R) \} \right], \quad (57)$$

where the axion field goes to zero at infinity. The positive sign of phase shift implies anticlockwise rotation, and the negative sign of phase shift implies clockwise rotation by looking down the light path.

The observed birefringent angle, which is the angle of rotation of the linear polarization, $(\Delta\theta)$ is half the phase shift $\Delta\phi$ between the two circular polarization modes. Using Eqs. (39), and (57), we obtain

$$\Delta\theta = -c\alpha_{em}, \quad (58)$$

where c is a model-dependent parameter of $\mathcal{O}(1)$ (the exact value of c depends on the ratio of the electromagnetic and the color factor anomaly E/N , which depends on what model we are choosing) and $\alpha_{em} = 1/137$. Hence, we obtain the birefringent angle

$$\Delta\theta = 7.299 \times 10^{-3} \text{ radian} = 0.42^\circ. \quad (59)$$

FIG. 3. The brown shaded region is excluded by SN1987A, the blue shaded region is excluded by direct measurement of the Earth, and the yellow shaded region is excluded by direct measurement of Sun. The gray shaded region is excluded by black hole superradiance measurements [55,56]. The green dotted line denotes the reduced Planck scale. The black continuous line denotes the constraints from big bang nucleosynthesis (BBN) if axion is the dark matter [57]. The violet dotted line corresponds to QCD axions. Our result of the birefringent angle can probe the red shaded region.

Any systematic deviation ($\leq 1.0^\circ$ [43,48,51]) in the linear polarization angle can be due to the long range axion hair. External magnetic field can also give rise such type of rotation of the polarization vector which is called the Faraday effect. The main difference between the optical rotation by long range axionic hair and Faraday effect is that for Faraday effect the birefringent angle is proportional to λ^2 where λ is the wavelength of the electromagnetic wave and for our case, i.e., the optical rotation by axion field, the birefringent angle is independent of λ . The existing constraints on axion parameters and the region in which our result is valid are shown in Fig. 3.

IV. DISCUSSIONS

We have discussed if axions being sourced by pulsar and if it mediates a long range axionic hair outside, then the axion hair can rotate the polarization of the electromagnetic radiation of the pulsar. Here we do not need any external magnetic field for the optical rotation. The birefringent angle that we have first derived is independent of the angular frequency of rotation, radius of the pulsar, mass of the axion, and the axion photon coupling constant. Our result is true for axions of mass $m_a < 10^{-11}$ eV and $f_a \lesssim \mathcal{O}(10^{17})$ GeV. We obtain the birefringent angle as 0.42° if the pulsar has a long range axionic hair. The derived birefringent angle due to axion photon interaction may contribute to the systematic deviation of measuring the linear polarization angle of pulsar light which is $\leq 1.0^\circ$.

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