Reanalysis of the newly observed Ω^* state in a hadronic molecule model

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(Received 24 November 2019; accepted 8 October 2020; published 28 October 2020)

After the recent discovery of the new Ω^* state, the ratio of the branching fractions of $\Omega(2012) \rightarrow \bar{K}\pi\Xi$ relative to the $\bar{K}\Xi$ decay channel was investigated by the Belle Collaboration. The measured 11.9% upper limit to this ratio is in sharp tension with the S -wave $\bar{K}\Xi(1530)$ molecule interpretation for $\Omega(2012)$, which indicates the dominant $\bar{K}\pi\Xi$ three-body decay. We try to explore the possibility of the P -wave molecule assignments for $\Omega(2012)$ [where $\Omega(2012)$ has positive parity]. It is found that the latest experimental measurements are compatible with the $1/2^+$ and $3/2^+$ $\bar{K}\Xi(1530)$ molecular pictures, while the $5/2^+$ $\bar{K}\Xi(1530)$ molecule shows larger $\bar{K}\pi\Xi$ three-body decay than $\bar{K}\Xi$ decay as in the case of an S -wave molecule. Thus, the newly observed $\Omega(2012)$ can be interpreted as the $1/2^+$ or $3/2^+$ $\bar{K}\Xi(1530)$ molecule state according to current experiment data.

DOI: [10.1103/PhysRevD.102.074025](https://doi.org/10.1103/PhysRevD.102.074025)

I. INTRODUCTION

Recently, the Belle Collaboration reported on a new Ω^* state in the $\bar{K}\Xi$ invariant mass distribution via $\Upsilon(1S, 2S, 3S)$ decays, with measured mass $M = 2012.4 \pm 0.7$ (stat) ± 0.6 (syst) MeV and decay width $\Gamma = 6.4^{+2.5}_{-2.0}$ (stat) ± 1.6 (syst) MeV [1]. This discovery aroused the bright-eyed interest of theoreticians in understanding the nature of the excited Ω state. On the one hand, before the discovery of $\Omega(2012)$, only two states, the four-star ground state $\Omega(1672)$ and the three-star excited state $\Omega(2250)$, are listed in a review of the Particle Data Group (PDG) [2] for the $S = -3$ baryon spectrum. In addition, two other two-star states with even higher masses are also mentioned in PDG. The ground state $\Omega(1672)$ is well established in the well-known quark model based on $SU(3)$ -flavor symmetry [3,4]. However, our knowledge of the nature of the $\Omega(2250)$ state and other higher Ω^* states is quite scarce. In particular, the almost 600 MeV mass difference between the ground state $\Omega(1672)$ and the first observed excited state $\Omega(2250)$ is surprising since the

negative-parity orbital excitations of many other baryons are approximately 300 MeV above their respective ground states. And various models, such as the quark model [5,6], the Skyrme model [7], and lattice gauge theory [8], predicted masses of the first orbital ($1P$) excitations of $\Omega(1672)$ with $J^P = 1/2^-$ or $3/2^-$ of around 2000 MeV, which is quite close to the observed value. Stimulated by these facts on the Ω baryon spectrum, recent theoretical works interpreted the newly observed $\Omega(2012)$ as the $1P$ orbital excitation of the ground state Ω baryon and investigated its strong decays via the chiral quark model [9], QCD sum rules [10,11], $SU(3)$ flavor symmetry [12], and the 3P_0 model [13]. The measured width ~ 6 MeV will be almost saturated by the two-body $\bar{K}\Xi$ channel if the $\Omega(2012)$ is treated as the spin-parity- $3/2^-$ $1P$ excited state. From the point of view of the compact decuplet baryon interpretation for $\Omega(2012)$, the troubling mass gap in the current $S = -3$ baryon spectrum will disappear naturally and the $\Delta(1700)$ resonance with the quantum number of $J^P = 3/2^-$ can be assigned as the decuplet partner of $\Omega(2012)$, as pointed out in Ref. [12].

On the other hand, the reported mass of the newly observed $\Omega(2012)$ state lies quite close to the threshold of the $\bar{K}\Xi(1530)$ channel with a binding energy of ~ 15 MeV. The closeness to the thresholds leads naturally to the interpretation of a hadronic molecule composed of $\bar{K}\Xi(1530)$ for $\Omega(2012)$. After the first successful attempt of the hadronic molecule picture on uncovering the composite nature of the deuteron performed by Weinberg [14,15], many exotic states which were

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discovered in recent decades can be described well with the molecule scenarios, such as $D_{s0}^*(2317)$ as a DK molecule, $X(3872)$ as a $D^*\bar{D}$ molecule, and the newly observed series of P_c pentaquarklike states as $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules. Systematic discussions of the hadronic molecule can be found in recent reviews [16–18]. The possibility of $\Omega(2012)$ as the $3/2^--\bar{K}\Xi(1530)$ hadronic molecule is investigated in Refs. [12,19–22]. It is remarkable that the three-body $\bar{K}\pi\Xi$ channel contributes sizable width to $\Omega(2012)$ in the S -wave hadronic molecule scenario, while it has difficulty happening in the compact decuplet baryon assignment. Inspired by this significant difference, the Belle Collaboration tried to distinguish these two interpretations of $\Omega(2012)$ by searching for its three-body decay to $\bar{K}\pi\Xi$ [23]. They did not observe the significant $\Omega(2012)$ signals in the $\bar{K}\pi\Xi$ channel and drew their conclusion with the 90% credibility level upper limits on the ratios of the branching fractions of three-body $\bar{K}\pi\Xi$ to $\bar{K}\Xi$ two-body decay, $\mathcal{B}(\Omega(2012) \rightarrow \bar{K}\pi\Xi)/\mathcal{B}(\Omega(2012) \rightarrow \bar{K}\Xi) < 11.9\%$. It is in sharp tension with the prediction of the S -wave $\bar{K}\Xi(1530)$ molecule assignment for $\Omega(2012)$.

However, it should be noted that the quantum number of $\Omega(2012)$ cannot be determined in experiments at present. Refocusing the processes considered by the Belle Collaboration, the samples of $\Omega(2012)$ were collected in the final states of the Υ decay processes. The parity of $\Omega(2012)$ prefers to be positive if $\Omega(2012)$ is measured via the reaction $\Upsilon \rightarrow \bar{\Omega}\Omega^*$. In that way, the Υ decays in the S -wave, which indicates that more events of $\Omega(2012)$ can be observed in experiments. The positive-parity $\Omega(2012)$ should be assigned as the P -wave $\bar{K}\Xi(1530)$ bound state in the molecular scenario. The existence of the P -wave and even higher partial wave bound states in the hidden charm sector was already suggested with the unitary coupled-channel approaches in Ref. [24]. In this work, we would like to explore the possibility of the P -wave molecule assignments for $\Omega(2012)$. As in our previous work, we investigate the strong decays of $\Omega(2012)$ with the effective Lagrangian approach by treating it as the P -wave molecular states with $J^P = 1/2^+, 3/2^+$, and $5/2^+$, respectively.

This work is organized as follows: In Sec. II, we introduce formalism and some details about the theoretical tools used to calculate the decay modes of exotic hadronic molecular states. In Sec. III, the numerical results and a

discussion are presented. Section IV is devoted to a summary of this work.

II. FORMALISM

In the hadronic molecule picture, the two-body $\bar{K}\Xi$ decay happens via the triangle mechanism where the vector-meson-exchange potential is adopted for the interaction between the $\bar{K}\Xi(1530)$ molecular component and the $\bar{K}\Xi$ final state. And the $\bar{K}\pi\Xi$ three-body decay through the decay of the intermediate $\Xi(1530)$ component happens at tree level. Decay diagrams of the $\Omega(2012)$ molecules are shown in Fig. 1. The partial decay widths of these diagrams can be calculated with the effective Lagrangian approach. As we did previously, the Lorentz covariant L - S scheme proposed in Ref. [25] is used to describe the first vertex in which $\Omega(2012)$ couples to the $\bar{K}\Xi(1530)$ component in the P -wave. The Lagrangians for the different spin parities of $\Omega(2012)$ are presented in the following,

$$\mathcal{L}_{\bar{K}\Xi^*\Omega^*(1/2^+)} = g_{\bar{K}\Xi^*\Omega^*}^{1/2^+} \tilde{g}_{\mu\nu} (\partial^\nu \bar{\Xi}^{*\mu} K - \bar{\Xi}^{*\mu} \partial^\nu K) \Omega_\mu^*, \quad (1)$$

$$\mathcal{L}_{\bar{K}\Xi^*\Omega^*(3/2^+)} = g_{\bar{K}\Xi^*\Omega^*}^{3/2^+} i \epsilon_{\mu\nu\alpha\beta} \tilde{g}^{\alpha\rho} (\partial_\rho \bar{\Xi}^{*\nu} K - \bar{\Xi}^{*\nu} \partial_\rho K) \hat{p}^\beta \Omega_\mu^*, \quad (2)$$

$$\mathcal{L}_{\bar{K}\Xi^*\Omega^*(5/2^+)} = g_{\bar{K}\Xi^*\Omega^*}^{5/2^+} \tilde{g}^{\mu\alpha} (\partial_\alpha \bar{\Xi}^{*\mu} K - \bar{\Xi}^{*\mu} \partial_\alpha K) \Omega_{\mu\nu}^*, \quad (3)$$

with $\tilde{g}^{\mu\nu}$ defined as $(g^{\mu\nu} - p^\mu p^\nu / p^2)$, where p denotes the momentum of the initial Ω^* state and $\hat{p} = p/m_{\Omega^*}$. The effective couplings $g_{\bar{K}\Xi^*\Omega^*}^{1/2^+}$, $g_{\bar{K}\Xi^*\Omega^*}^{3/2^+}$, and $g_{\bar{K}\Xi^*\Omega^*}^{5/2^+}$ are estimated with the compositeness criterion which states the relation between the derivative of the self-energy operator of hadron resonance and its compositeness [14,15]. In our molecule scenario where $\Omega(2012)$ is assumed to be the pure $\bar{K}\Xi(1530)$ molecular state, the compositeness of $\Omega(2012)$ is equal to 1; that is, $\chi \equiv 1 - Z = 1$. In general, the compositeness criterion is used to estimate the effective coupling only for the S -wave state. This is because only the self-energy loop of the S -wave composite state can be evaluated model independently, while for the higher partial wave state the loop integral is definitely divergent; see Ref. [16] for detailed statements. Consequently, additional scale parameters which are usually cutoffs in some regulators need to be introduced to cope with the UV divergence

FIG. 1. Decay mechanisms of the $\Omega(2012)$ molecules. The left diagram stands for the $\bar{K}\Xi$ two-body decay and the right one is the $\bar{K}\pi\Xi$ three-body decay. Ω^* and Ξ^* denote $\Omega(2012)$ and $\Xi(1530)$, respectively.

in that case. In this work, we use the compositeness criterion to estimate the P -wave couplings between $\Omega(2012)$ and the $\bar{K}\Xi(1530)$ channel by including a Gaussian form factor in the evaluation of the self-energy operator of $\Omega(2012)$. The cutoff dependence of these couplings will also be given when we present our numerical results. The left vertices in the decay diagrams are the same as the S -wave case, and we take the same convention as in our previous calculation [20].

Finally, two form factors are also included in the loop integrals of the triangle diagrams. The first one is the Gaussian form factor. As discussed in Ref. [26], there are two different Gaussian formulas used commonly in phenomenological analyses, the four-dimensional Euclidean formula [27–31] and the three-dimensional nonrelativistic formula [16,32,33]. They are defined as

$$f_1(p_E^2/\Lambda_0^2) = \exp(-p_E^2/\Lambda_0^2) \quad (4)$$

and

$$f_2(\mathbf{p}^2/\Lambda_0^2) = \exp(-\mathbf{p}^2/\Lambda_0^2), \quad (5)$$

respectively. p_E is the four-dimensional Euclidean Jacobi momentum defined as $m_{\bar{K}}p_{\Xi^*}/(m_{\bar{K}} + m_{\Xi^*}) - m_{\Xi^*}p_{\bar{K}}/(m_{\bar{K}} + m_{\Xi^*})$, and $\mathbf{p}$ is the spatial part of the momenta of $\bar{K}$ and Ξ^* in the rest frame of the $\Omega(2012)$ state. Comparing these two equations, we can find that the form factor f_1 includes an additional constraint on the energy of molecular components, which demands that the center of mass energy is divided, as the mass distribution of compounding particles inside molecular states usually happens for bound states in quantum mechanics. The difference between these two kinds of Gaussian form factors will be presented in Sec. III.

The second form factor is chosen to be a multipolar formula, as shown in Eq. (6). It is introduced to suppress the off-shell contributions of the exchanged mesons in our triangle diagrams,

$$f_3(q^2) = \frac{\Lambda_1^4}{(m^2 - q^2)^2 + \Lambda_1^4}, \quad (6)$$

where m and q is the mass and momentum of the exchanged particle. The cutoffs Λ_0 and Λ_1 are the free parameters in our calculation, and we vary both of them in the range 0.6–1.4 GeV to scrutinize how the decay behaviors undergo changes as the cutoffs are varied. A specific set of values for Λ_0 and Λ_1 is chosen to give the decay patterns of $\Omega(2012)$ molecules by fitting to the measured total widths.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will present our numerical results on the strong decays of $\Omega(2012)$ in the P -wave $\bar{K}\Xi^*$ molecule

scenarios. First, a short discussion of the estimation of the P -wave effective couplings $g_{\Omega^*\bar{K}\Xi^*}$ will be given. After that, we will show the numerical decay patterns of these P -wave $\bar{K}\Xi^*$ molecules and also the parameter dependence of our results. Following that is a comparison with the latest experimental data and our conclusion. At the end, we try to estimate the partial widths of the three-body $\bar{K}\pi\Xi$ decays which are generated from the rescattering of the $\bar{K}\Xi$ channel for $\Omega(2012)$ states with various quantum numbers.

A. Couplings

As introduced previously, the effective couplings between $\Omega(2012)$ and the $\bar{K}\Xi^*$ channel are estimated with the compositeness condition. We include the form factor f_1 [Eq. (4)] in the calculations of the self-energy operators for the P -wave $\Omega(2012)$ molecules to get rid of the UV divergence. And, for consistently, the same form factor is also included in the estimation of the S -wave coupling. The dependence of these effective couplings for various quantum numbers on cutoff Λ_0 is presented in Fig. 2. It does not escape our attention that the P -wave effective couplings $g_{\Omega^*\bar{K}\Xi^*}^{1/2^+}$, $g_{\Omega^*\bar{K}\Xi^*}^{3/2^+}$, and $g_{\Omega^*\bar{K}\Xi^*}^{5/2^+}$ are more sensitive to the cutoff Λ_0 than the S -wave coupling $g_{\Omega^*\bar{K}\Xi^*}^{3/2^-}$, as expected. And all the couplings decrease when the cutoff gets larger. $5/2^+$ $\Omega(2012)$ has the largest coupling with the $\bar{K}\Xi^*$ channel, $1/2^+$ and $3/2^+$ are next, and $3/2^-$ is smallest. In particular, $g_{\Omega^*\bar{K}\Xi^*}^{1/2^+}$ is quite similar to $g_{\Omega^*\bar{K}\Xi^*}^{3/2^+}$. In spite of some existence of model dependence, the uncertainty of the determination of P -wave effective couplings from the compositeness condition is still under control. As shown in Fig. 2, the largest magnitude of the coupling decrease is less than half of its value in the entire range of Λ_0 . It is our opinion that the P -wave effective couplings obtained with

FIG. 2. The cutoff dependence of the effective coupling constants $g_{\Omega^*\bar{K}\Xi^*}$ for different quantum numbers of Ω^* . The black-dotted, red-square, blue-diamond, and orange-triangular points denote the cases of the $J^P = 3/2^-$ S -wave, $1/2^+$ P -wave, $3/2^+$ P -wave, and $5/2^+$ P -wave Ω^* molecules, respectively.

TABLE I. Partial widths of $\Omega(2012)$ for $\bar{K}\Xi(1530)$ molecules with various quantum numbers. The cutoffs are fixed at $\Lambda_0 = 1.0$ GeV and $\Lambda_1 = 0.8$ GeV. The form factor set (f_1, f_3) is used in the calculation. All of the decay widths are in units of MeV.

Mode	Widths (MeV) with (f_1, f_3)			
	$\Omega(2012) [\bar{K}\Xi(1530)]$			
	S-wave		P-wave	
	$\frac{3}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$
$\bar{K}\Xi$	0.05	6.6	4.4	0.005
$\bar{K}\pi\Xi$ tree level	1.9	0.2	0.2	0.3
$\bar{K}\pi\Xi$ one loop	8.4×10^{-3}	0.02	5.7×10^{-4}	1.0×10^{-4}
Total	1.95	6.8	4.6	0.305
Ratio	38	0.03	0.045	60

the compositeness condition are available for the estimation of the decay widths of the molecular states, especially to estimate the relative ratios of branch fractions among various decay channels.

B. Partial decay widths of the $\Omega(2012)$ molecules

With the couplings $g_{\Omega^* \bar{K}\Xi^*}$ obtained, the partial widths of the Ω^* molecules can be calculated straightforwardly. The results with the cutoff $\Lambda_0 = 1.0$ GeV and $\Lambda_1 = 0.8$ GeV which are fitted to the measured width of $\Omega(2012)$ are displayed in Table I for the form factor set (f_1, f_3) and Table II for (f_2, f_3) . It is intriguing that the $3/2^-$ S-wave and $5/2^+$ P-wave $\Omega(2012)$ molecules have quite similar decay patterns except for the highly suppressed widths of $5/2^+$ state and the fact that the decay pattern of the $1/2^+$ state is almost the same as that of the $3/2^+$ state. The remarkable difference between these two sets of states is that the dominant decay channel of $1/2^+$ and $3/2^+$ P-wave states is the two-body $\bar{K}\Xi$ channel which is compatible with the experimental observations, while it is the three-body $\bar{K}\pi\Xi$ channel for the $3/2^-$ S-wave and $5/2^+$ P-wave states

TABLE II. Partial widths of $\Omega(2012)$ for $\bar{K}\Xi(1530)$ molecules with various quantum numbers. The cutoffs are fixed at $\Lambda_0 = 1.0$ GeV and $\Lambda_1 = 0.8$ GeV. The form factor set (f_2, f_3) is used in the calculation. All of the decay widths are in units of MeV.

Mode	Widths (MeV) with (f_2, f_3)			
	$\Omega(2012) [\bar{K}\Xi(1530)]$			
	S-wave		P-wave	
	$\frac{3}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$
$\bar{K}\Xi$	0.4	22.7	91.2	0.1
$\bar{K}\pi\Xi$ tree level	1.9	0.2	0.2	0.3
$\bar{K}\pi\Xi$ one loop	0.01	0.01	9.0×10^{-4}	7.0×10^{-4}
Total	2.3	22.9	91.4	0.4
Ratio	4.75	0.009	0.002	3

FIG. 3. Three-body $\bar{K}\pi\Xi$ decay diagrams of $\Omega(2012)$ through the rescattering of $\bar{K}\pi$. Ω^* and Ξ^* denote $\Omega(2012)$ and $\Xi(1530)$, respectively.

which is in sharp tension with the experimental data. It also can be noticed that the form factor f_2 gives a much larger decay width for the $\bar{K}\Xi$ channel than f_1 does. It is similar to the case discussed in Ref. [26]: the exchanged vector mesons (with an energy of about 0.2 GeV) are off the mass shell when the component particles $\bar{K}$ and $\Xi(1530)$ are confined on nearly their mass shells by the form factor f_1 , and its contribution will be suppressed by the form factor f_3 . In addition to the tree-level three-body $\bar{K}\pi\Xi$ decay which happens naturally in the molecule picture, we also try to consider the contribution of the $\bar{K}^*\Xi$ channel by modeling the final state interaction as described in Fig. 3. As shown in the diagram, the $\bar{K}\pi$ interaction happens through the S-channel rescattering with $\bar{K}^*$ as the intermediate resonance. Our results show that whichever form factor set is used, (f_1, f_3) or (f_2, f_3) , the partial widths of one-loop $\bar{K}\pi\Xi$ decays are quite small and can be ignored for both the S-wave and P-wave molecule scenarios.

The cutoff dependence of the partial decay widths of $\Omega(2012)$ states with various quantum numbers are given in Fig. 4 for the form factor set (f_1, f_3) , and Fig. 5 for the (f_2, f_3) case. Also the widths of the P-wave states are more sensitive to both cutoffs Λ_0 and Λ_1 than that of the S-wave state. The partial width of the three-body $\bar{K}\pi\Xi$ channel depends only slightly on the cutoff Λ_0 due to the cutoff dependent coupling constant $g_{\Omega^* \bar{K}\Xi}^{3/2^-}$. The partial widths of the two-body $\bar{K}\Xi$ channel obtained with the form factor set (f_1, f_3) depend a lot on Λ_1 , while they remain almost steady as Λ_0 is varied. In the case where the form factor set (f_2, f_3) is used, however, the two-body $\bar{K}\Xi$ decay widths depend heavily on Λ_0 , and its dependence on Λ_1 is relatively modest but not as slight as the Λ_0 dependence in the case of (f_1, f_3) .

Besides the decay widths, the dependence of the relative ratios between the partial decay widths of the $\bar{K}\pi\Xi$ and $\bar{K}\Xi$ channels are also considered; the results are displayed in Fig. 6. Although the decay widths of the $\Omega(2012)$ molecules are cutoff dependent, the characteristic behaviors on the ratios of the branching fractions of the $\bar{K}\pi\Xi$ channel relative to the $\bar{K}\Xi$ channel of the $\bar{K}\Xi^*$ molecules with different spin parities are deserving of special attention. As shown in Fig. 6, the ratios of all the $\Omega(2012)$ molecules do

FIG. 4. Decay widths of the Ω^* molecules with various quantum numbers varying with the cutoffs Λ_0 and Λ_1 obtained with the form factor set of (f_1, f_3) . The left upper, left lower, right upper, and right lower plots are the results of the Ω^* molecule with $J^P = 3/2^-, 3/2^+, 1/2^+$, and $5/2^+$, respectively. The red-solid, green-solid, blue-solid, black-solid, and black-dashed lines denote the Λ_0 dependence of the $\bar{K}\Xi$ channel, the Λ_1 dependence of the $\bar{K}\Xi$ channel, the Λ_0 dependence of the $\bar{K}\pi\Xi$ channel, the Λ_0 dependence of the total width, and the Λ_1 dependence of the total width, respectively. Note that Λ_0 is fixed at 1.0 GeV when Λ_1 is varied and Λ_1 is fixed at 0.8 GeV when Λ_0 is varied.

not change significantly in the entire range of cutoffs relative to the experimental upper limit. The green bands in the plots denote the allowed region of the ratios with experimental data. The spin-parity $1/2^+$ and $3/2^+$ $\bar{K}\Xi(1530)$ molecular assignments for $\Omega(2012)$ are consistent with the experiments when the cutoff is larger than 0.7 GeV, while the $3/2^-$ and $5/2^+$ $\bar{K}\Xi(1530)$ molecular assumptions strongly disagree with the experiments in the range of cutoffs that we considered. Then it can pave another way to understand the nature of the $\Omega(2012)$ state besides the $1P$ orbital excitation of the ground Ω state. Our results suggest that $\Omega(2012)$ might be the P -wave $\bar{K}\Xi(1530)$ molecule state with $J^P = 1/2^+$ or $3/2^+$. Since the decay behaviors of the $1/2^+$ and $3/2^+$ $\bar{K}\Xi(1530)$ molecules are quite similar to each other, it is difficult to distinguish these two possible quantum numbers in the current hadronic molecular framework. Experimental determination of the quantum number for the $\Omega(2012)$ state will be critical to uncovering the mystery of its inner structure in the future.

C. $\bar{K}\pi\Xi$ decays via the rescattering of $\bar{K}\Xi$

We also investigate the three-body $\bar{K}\pi\Xi$ decays of $\Omega(2012)$ by considering $\bar{K}\pi\Xi$ that are generated from the rescattering of $\bar{K}\Xi$. Here two possible rescattering processes are included in our considerations. In the first one, the $\Omega(2012)$ states are suspected to decay into $\bar{K}\Xi$ first. Next the $\bar{K}\Xi$ rescatters to the $\bar{K}\Xi(1530)$ channel, which can be described by the vector meson dominance model. Finally, the three-body $\bar{K}\pi\Xi$ final state is generated through the decay of the intermediate $\Xi(1530)$. In the second process, the intermediate $\bar{K}\Xi$ state rescatters to the $\bar{K}^*\Xi$ channel via a one meson exchange interaction. These two decay mechanisms are shown in Figs. 7 and 8, respectively. Here the quantum numbers of $\Omega(2012)$ are considered to be $1/2^-, 1/2^+, 3/2^-, 3/2^+, 5/2^-,$ and $5/2^+$. We expect that these various quantum numbers of $\Omega(2012)$ can be distinguished using such three-body $\bar{K}\pi\Xi$ decays. Similarly, the partial widths of these diagrams in Figs. 7 and 8 are calculated with the effective Lagrangian

FIG. 5. Decay widths of the Ω^* molecules with various quantum numbers varying with the cutoff Λ_0 and Λ_1 obtained with the form factor set of (f_2, f_3) . The left upper, left lower, right upper, and right lower plots are the results of the Ω^* molecule with $J^P = 3/2^-, 3/2^+, 1/2^+,$ and $5/2^+$, respectively. The red-solid, green-solid, blue-solid, black-solid, and black-dashed lines denote the Λ_0 dependence of the $\bar{K}\Xi$ channel, the Λ_1 dependence of the $\bar{K}\Xi$ channel, the Λ_0 dependence of the $\bar{K}\pi\Xi$ channel, the Λ_0 dependence of the total width, and the Λ_1 dependence of the total width, respectively. Note that Λ_0 is fixed at 1.0 GeV when Λ_1 is varied and Λ_1 is fixed at 0.8 GeV when Λ_0 is varied.

approach. The interaction between $\Omega(2012)$ and $\bar{K}\Xi$ is extracted from the experimental width of $\Omega(2012)$ by assuming that it is completely saturated by the $\bar{K}\Xi$ channel with the following effective Lagrangians,

$$\mathcal{L}_{\bar{K}\Xi\Omega^*(1/2^-)} = g_{\bar{K}\Xi\Omega^*}^{1/2^-} \bar{\Xi} K \Omega^*, \quad (7)$$

$$\mathcal{L}_{\bar{K}\Xi\Omega^*(1/2^+)} = g_{\bar{K}\Xi\Omega^*}^{1/2^+} \bar{\Xi} \gamma^5 \tilde{\gamma}^\mu r_\mu K \Omega^*, \quad (8)$$

$$\mathcal{L}_{\bar{K}\Xi\Omega^*(3/2^-)} = g_{\bar{K}\Xi\Omega^*}^{3/2^-} \bar{\Xi} r_\mu K \Omega^*, \quad (9)$$

$$\mathcal{L}_{\bar{K}\Xi\Omega^*(3/2^+)} = g_{\bar{K}\Xi\Omega^*}^{3/2^+} \bar{\Xi} \gamma^5 (\tilde{\gamma}^\alpha g^{\beta\mu} + \tilde{\gamma}^\beta g^{\alpha\mu}) \mathcal{D}_{\alpha\beta} K \Omega^*, \quad (10)$$

$$\mathcal{L}_{\bar{K}\Xi\Omega^*(5/2^-)} = g_{\bar{K}\Xi\Omega^*}^{5/2^-} \bar{\Xi} \mathcal{D}^{\mu\nu} K \Omega_{\mu\nu}^*, \quad (11)$$

$$\begin{aligned} \mathcal{L}_{\bar{K}\Xi\Omega^*(5/2^+)} = & g_{\bar{K}\Xi\Omega^*}^{5/2^+} \bar{\Xi} \gamma^5 (\tilde{\gamma}^\lambda g^{\alpha\mu} g^{\beta\nu} + \tilde{\gamma}^\alpha g^{\lambda\mu} g^{\beta\nu} \\ & + \tilde{\gamma}^\beta g^{\alpha\mu} g^{\lambda\nu}) \mathcal{F}_{\alpha\beta\lambda} K \Omega_{\mu\nu}^* \end{aligned} \quad (12)$$

where

$$\tilde{g}^{\mu\nu} = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{m_{\Omega^*}^2} \right),$$

$$r^\mu = \tilde{g}^{\mu\nu} (k_1 - k_2)_\nu, \quad \tilde{\gamma}^\mu = \tilde{g}^{\mu\nu} \gamma_\nu,$$

$$\mathcal{D}^{\mu\nu} = r^\mu r^\nu - \frac{1}{3} r^\rho r_\rho \tilde{g}^{\mu\nu},$$

$$\mathcal{F}^{\mu\nu\lambda} = r^\mu r^\nu r^\lambda - \frac{1}{5} r^\rho r_\rho (\tilde{g}^{\mu\nu} r^\lambda + \tilde{g}^{\nu\lambda} r^\mu + \tilde{g}^{\lambda\mu} r^\nu),$$

with p, k_1, k_2 the momenta of $\Omega(2012), \bar{K}$, and Ξ , respectively. The form factors f_1 and f_3 are used to cope with the UV divergence and the cutoffs Λ_0 and Λ_1 are taken to have the same values as the calculations in Table I to give a qualitative estimation for the $\bar{K}\pi\Xi$ decays. The extracted coupling constants $g_{\bar{K}\Xi\Omega^*}$ are listed in Table III. The corresponding partial widths are displayed in Table IV. The partial widths of the $\bar{K}\pi\Xi$ channel for all the $\Omega(2012)$

FIG. 6. Ratio of the branching fractions of the three-body $\bar{K}\pi\Xi$ channel relative to the two-body $\bar{K}\Xi$ channel, which varies with the cutoffs Λ_0 and Λ_1 . The upper panels are the results of the Ω^* molecule with $J^P = 1/2^+$ and $3/2^+$, where the left one is obtained with the form factor set of (f_1, f_3) and the right panel is obtained with the form factor set of (f_2, f_3) . The lower panels are for the $J^P = 3/2^-$ and $5/2^+$ cases. The green band in the plot denotes the region allowed by the latest Belle measurements. Note that the Λ_0 is fixed at 1.0 GeV when Λ_1 is varied and the Λ_1 is fixed at 0.8 GeV when Λ_0 is varied.

states are quite small owing to the triangle loop mechanism. The $3/2^-$ $\Omega(2012)$ has the largest decay width to the $\bar{K}\pi\Xi$ channel at around 5.0×10^{-3} MeV. Note that the value of the $1/2^+$ $\Omega(2012)$ state is several orders of magnitude larger than the $3/2^+$ state. Note that the small $\bar{K}\pi\Xi$ partial

widths obtained in this subsection are irrelevant with the inner structure of $\Omega(2012)$. They can serve as the lower limits of the partial decay widths of the $\bar{K}\pi\Xi$ channel for the $\Omega(2012)$ state, although they cannot help us to discriminate among various quantum numbers of $\Omega(2012)$.

FIG. 7. Three-body $\bar{K}\pi\Xi$ decay diagrams of $\Omega(2012)$ through the rescattering of $\bar{K}\Xi$ with the $\bar{K}\Xi^*$ channel considered. Ω^* and Ξ^* denote $\Omega(2012)$ and $\Xi(1530)$, respectively.

FIG. 8. Three-body $\bar{K}\pi\Xi$ decay diagrams of $\Omega(2012)$ through the rescattering of $\bar{K}\Xi$ with the $\bar{K}\Xi^*$ channel considered. Ω^* and Ξ^* denote $\Omega(2012)$ and $\Xi(1530)$, respectively.

TABLE III. Effective coupling constants $g_{\bar{K}\Xi\Omega^*}$ for $\Omega(2012)$ states with various quantum numbers.

Quantum number	$1/2^-$	$1/2^+$ (GeV $^{-1}$)	$3/2^+$ (GeV $^{-1}$)	$3/2^-$ (GeV $^{-2}$)	$5/2^-$ (GeV $^{-2}$)	$5/2^+$ (GeV $^{-3}$)
$g_{\bar{K}\Xi\Omega^*}$	0.39	0.49	0.85	0.53	1.68	0.71

TABLE IV. Partial widths of three-body $\bar{K}\pi\Xi$ decays for $\Omega(2012)$ states with various quantum numbers. These values are obtained with $\Lambda_0 = 1$ GeV and $\Lambda_0 = 0.8$ GeV. All of the decay widths are in units of MeV.

Quantum number	$1/2^-$	$1/2^+$	$3/2^+$	$3/2^-$	$5/2^-$	$5/2^+$
Rescattering to $\bar{K}\Xi^*$	2.2×10^{-6}	1.2×10^{-4}	9.5×10^{-7}	5.0×10^{-3}	2.1×10^{-5}	1.8×10^{-4}
Rescattering to $\bar{K}^*\Xi$	9.4×10^{-5}	2.6×10^{-4}	4.0×10^{-6}	1.7×10^{-4}	2.2×10^{-6}	2.4×10^{-5}

IV. SUMMARY

The latest observations on the ratio of the branching fractions of $\Omega(2012) \rightarrow \bar{K}\pi\Xi$ relative to the $\bar{K}\Xi$ channel reported by the Belle Collaboration strongly disfavor the S -wave $\bar{K}\Xi(1530)$ molecule interpretation for $\Omega(2012)$. They seem to indicate that $\Omega(2012)$ can be considered the $1P$ orbital excitation of the ground Ω baryon only with $J^P = 3/2^-$. In fact, there is no definite conclusion on the quantum number of the newly observed $\Omega(2012)$ state so far. In this work, we explore the possibility of the $\Omega(2012)$ being a P -wave $\bar{K}\Xi(1530)$ molecular state. Analogous to our previous work, we investigate the strong decays of the P -wave $\Omega(2012)$ molecules with the quantum numbers of $1/2^+$, $3/2^+$, and $5/2^+$ by using the effective Lagrangian approach. It is found that the decay behaviors

of the $1/2^+$ and $3/2^+$ $\Omega(2012)$ molecules are compatible with current experimental data. Then observations suggest that $\Omega(2012)$ might be the P -wave $\bar{K}\Xi(1530)$ molecules with $J^P = 1/2^+$ or $3/2^+$ besides being the $J^P = 3/2^-$ orbital excitation of the ground Ω baryon. The determination of the quantum number of $\Omega(2012)$ would be a landmark experimental feat helping us to understand its nature.

ACKNOWLEDGMENTS

This project is supported by NSFC under Grants No. 11621131001 (CRC110 cofunded by DFG and NSFC), No. 11835015, and No. 11947302, and by the Chinese Academy of Sciences (CAS) under Grant No. XDB34030303.

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