Swampland conjectures and slow-roll thawing quintessence

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We examine the swampland conjectures in the context of generic slow-roll thawing quintessence models. Defining $\lambda \equiv |V'(\phi_i)/V(\phi_i)|$ and $K \equiv \sqrt{1 - 4V''(\phi_i)/3V(\phi_i)}$, where ϕ_i is the initial value of ϕ , we find regions of parameter space consistent with both observational data and with the refined de Sitter conjecture, and we show that all such models satisfy the distance conjecture. We quantify the degree of fine-tuning on λ needed to achieve these results.

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Observational evidence [1–7] suggests that approximately 70% of the energy density in the Universe is in the form of a negative-pressure component called dark energy, with the remaining 30% in the form of non-relativistic matter. The dark energy component can be parametrized in terms of its equation of state parameter, w , defined as the ratio of the dark energy pressure to its density:

$$w = p/\rho. \quad (1)$$

Then a cosmological constant, Λ , corresponds to the case $\rho = \text{constant}$ and $w = -1$.

While a model with a cosmological constant and cold dark matter (Λ CDM) is consistent with current observations, there are other models of dark energy that have a dynamical equation of state. The most widely investigated are quintessence models, with a time-dependent scalar field, ϕ , having potential $V(\phi)$ [8–14]. (See Ref. [15] for a review.) In these models, the equation of motion for a scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0, \quad (2)$$

where the Hubble parameter H is given by

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\rho_T/3}. \quad (3)$$

In this equation, a is the scale factor, ρ_T is the total density, and we take $8\pi G = \hbar = c = 1$ throughout. The pressure and density of ϕ are given by

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi), \quad (4)$$

and

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi). \quad (5)$$

In general, one can simply integrate Eq. (2) for any given $V(\phi)$, determine the corresponding $\rho_\phi(a)$, and compare with observational constraints. More recently, however, a number of authors have also considered the consistency of quintessence models with a variety of “swampland conjectures.” The swampland conjectures arise in the context of attempts to derive a quantum theory of gravity within string theory. A variety of these conjectures has been proposed (see, e.g., Refs. [16,17] for some of the earliest work in this area); we will consider two of them in the context of this paper.

The distance conjecture constrains the field excursion $\Delta\phi$ to be small when expressed in Planck units, namely, the following:

Conjecture 1:

$$\Delta\phi \lesssim \mathcal{O}(1). \quad (6)$$

The distance conjecture is longstanding [17] and has a great deal of theoretical support (although see Ref. [18] for mechanisms to evade it).

More recently, Obied *et al.* [19] proposed the condition: Conjecture 2.1:

$$\lambda \equiv |V'/V| \gtrsim \mathcal{O}(1), \quad (7)$$

where V' is the derivative of V with respect to ϕ . This constraint, which is based on the difficulty of constructing a de Sitter vacuum in string theory, is called the de Sitter conjecture.

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The consistency (or lack thereof) of these conjectures with quintessence models has been examined in detail [20–28]. (See also Ref. [29] for multifield quintessence, and Refs. [30–32] for scalar fields beyond quintessence.) In general, it is extremely difficult to reconcile the de Sitter conjecture with standard quintessence models (and it is inconsistent with standard Λ CDM). The problem arises because observations favor w near -1 at moderate redshifts, but w near -1 generally translates into values of λ less than 1. (See Ref. [33] for a mechanism to evade this issue in the context of inflation.)

Subsequently, an alternative de Sitter conjecture, called the refined de Sitter conjecture, was proposed [24,34]. This condition is that the scalar field satisfies *either* Eq. (7) or the following:

Conjecture 2.2:

$$c^2 \equiv -V''/V \gtrsim \mathcal{O}(1). \quad (8)$$

Thus, Eq. (8) requires the potential $V(\phi)$ to be concave, with a lower bound on the curvature.

Equation (8) leads to a natural quintessence model, namely, one in which the scalar field begins initially near a maximum in the potential and then rolls downhill. Such “hilltop” quintessence models have been considered in some detail. In the terminology of Ref. [35], these are “thawing” models, in which the field begins initially with $\dot{\phi} \approx 0$ at some initial value $\phi = \phi_i$. Initially, $w \approx -1$, but the field then evolves with $\dot{\phi}$ increasing with time up to the present, so that w increases with time. Note that in these models, any initial nonzero value of $\dot{\phi}$ is damped by Hubble friction [the second term in Eq. (2)] at early times, so the assumption that $\dot{\phi}_i \approx 0$ is reasonable.

While quintessence generically produces a time-varying value for w , a successful model must closely mimic Λ CDM in order to be consistent with current observations. Hence, a viable model should yield a present-day value of w close to -1 . This fact has been exploited in a number of papers that explored the evolution of a scalar field subject to the constraint that w must be close to -1 [36–38]. By imposing this constraint, one can reduce an infinite number of models to a finite set of behaviors for $w(a)$ characterized entirely by the values of V'/V and V''/V at the initial value of the scalar field, ϕ_i . Here we will use this methodology to determine the parameter space consistent with current observations and allowed by the distance conjecture and the refined de Sitter conjecture.

Constraints on quintessence models from the refined de Sitter conjecture have been examined previously by Agrawal and Obied [39] and by Raveri *et al.* [23]. The former examined a potential consisting of a constant plus a negative quadratic, while the latter used a potential of the form

$$V(\phi) = B \cos(c\phi). \quad (9)$$

Our own investigation most closely resembles that in Ref. [23], so we will place our own results in the context of that paper. Raveri *et al.* allowed for a variation in B , c , and ϕ_i (the initial value of ϕ), and their results indicate that $c > 1$ is already disfavored by current observational constraints. Of course, it is obvious (and noted in Ref. [23]) that arbitrarily large values of c are possible if ϕ_i is sufficiently small, but such conditions require a high degree of fine-tuning and can be destabilized by quantum fluctuations.

Our approach differs somewhat from that in Ref. [23]. We use the formalism of Refs. [36–38] to examine arbitrary thawing models with w near -1 . While it is possible to map the parameter space of Ref. [23] onto this formalism, our approach corresponds to a different set of priors on these parameters, which will result in a different set of constraints. Further, our approach leads naturally to a useful relation between $\Delta\phi$ in the distance conjecture and V''/V in the refined de Sitter conjecture. Finally, we explore the exact degree of fine-tuning needed in models that satisfy the refined de Sitter conjecture for generic thawing models.

Consider a thawing scalar field with a potential $V(\phi)$ characterized by $\lambda \equiv |V'/V| < 1$. Following Refs. [36–38], we note that such a potential will lead naturally to $1 + w \ll 1$ at present, and the evolution of ϕ and w will follow a well-defined, limited set of trajectories. Note that our assumption violates Eq. (7), so that for the models considered here, the refined swampland conjecture will require Eq. (8) to be satisfied. The evolution of ϕ as a function of t in this case is [37]

$$\phi(t) = \phi_i + \frac{V'(\phi_i)}{V''(\phi_i)} \left(\frac{\sinh(kt)}{kt_\Lambda \sinh(t/t_\Lambda)} - 1 \right), \quad (10)$$

where ϕ_i is the initial value of ϕ , t is the time, t_Λ is the value of t given by

$$t_\Lambda = \frac{2}{\sqrt{3V(\phi_i)}}, \quad (11)$$

and the constant k characterizes the curvature of the potential:

$$k \equiv \sqrt{\frac{3V(\phi_i)}{4} - V''(\phi_i)}. \quad (12)$$

We can use Eq. (10) to find the field excursion $\Delta\phi$ between $t = 0$ and $t = t_0$ (the present time). Taking the background expansion to be approximately Λ CDM, we have

$$t_0 = t_\Lambda \tanh^{-1}(\sqrt{\Omega_{\phi 0}}), \quad (13)$$

and following Refs. [36–38], we define

$$K \equiv kt_\Lambda = \sqrt{1 - \frac{4V''(\phi_i)}{3V(\phi_i)}}. \quad (14)$$

Then Eq. (10) allows us to express the field excursion as

$$|\Delta\phi| = \left| \frac{4\lambda}{3(1-K^2)} \left(\frac{(1-\Omega_{\phi 0})^{\frac{1}{2}(1-K)} [(1+\sqrt{\Omega_{\phi 0}})^K - (1-\sqrt{\Omega_{\phi 0}})^K]}{2K\sqrt{\Omega_{\phi 0}}} - 1 \right) \right|. \quad (15)$$

Note that in the limit where $K \rightarrow 1$ ($V'' \rightarrow 0$), we regain the expression in Ref. [23] for $|\Delta\phi|$ when $V'(\phi) = \text{const.}$ In Fig. 1, we show the relationship between $|\Delta\phi|$, λ , and K given by Eq. (15). (We take $\Omega_{\phi 0} = 0.69$ throughout.)

Now we will solve for w_0 , the present-day value of w , as a function of λ and K . From Ref. [37], we have

$$1 + w(t) = \frac{3}{4} \left(\frac{V'(\phi_i)}{kt_\Lambda V''(\phi_i)} \right)^2 \left(\frac{kt_\Lambda \cosh(kt) \sinh(t/t_\Lambda) - \sinh(kt) \cosh(t/t_\Lambda)}{\sinh^2(t/t_\Lambda)} \right)^2. \quad (16)$$

Taking t equal to the present-day time from Eq. (13), we obtain

$$1 + w_0 = \frac{\lambda^2}{3K^2(1-K^2)^2} \frac{(1-\Omega_{\phi 0})^{1-K}}{\Omega_{\phi 0}} \left[\left(K - \Omega_{\phi 0}^{-\frac{1}{2}} \right) (1 + \sqrt{\Omega_{\phi 0}})^K + \left(K + \Omega_{\phi 0}^{-\frac{1}{2}} \right) (1 - \sqrt{\Omega_{\phi 0}})^K \right]^2. \quad (17)$$

This result is illustrated in Fig. 2. In the limit where $K \rightarrow 1$ ($V'' \rightarrow 0$), we obtain the corresponding expression for $(1 + w_0)/\lambda^2$ for a linear potential [40].

Finally, we can combine Eqs. (15) and (17) to eliminate λ , giving

$$1 + w_0 = \frac{3}{4} \Delta\phi^2 \left[\frac{\left(K - \Omega_{\phi 0}^{-\frac{1}{2}} \right) (1 + \sqrt{\Omega_{\phi 0}})^K + \left(K + \Omega_{\phi 0}^{-\frac{1}{2}} \right) (1 - \sqrt{\Omega_{\phi 0}})^K}{(1 + \sqrt{\Omega_{\phi 0}})^K - (1 - \sqrt{\Omega_{\phi 0}})^K - 2K\sqrt{\frac{\Omega_{\phi 0}}{1-\Omega_{\phi 0}}}(1-\Omega_{\phi 0})^{\frac{K}{2}}} \right]^2. \quad (18)$$

This result is illustrated in Fig. 3.

Now consider the constraints that the refined swampland conjecture [in the form of Eq. (8)] places on these slow-roll thawing models when combined with observational

FIG. 1. For slow-roll thawing quintessence, the field excursion $|\Delta\phi|$ divided by $\lambda \equiv |V'(\phi_i)/V(\phi_i)|$ as a function of $K \equiv \sqrt{1 - 4V''(\phi_i)/3V(\phi_i)}$, where ϕ_i is the initial value of ϕ .

constraints. Observational limits on slow-roll thawing models have been considered by Chiba *et al.* [41], Smer-Berreto and Liddle [42], and Durrive *et al.* [43]. Of these papers, the treatment in Refs. [41,43] is most similar to the discussion here. Both of these papers use the methodology of Refs. [36–38], along with observational data, to provide constraints in the K, w_0 plane. Reference [42] takes a somewhat different approach, deriving observational constraints on the pseudo-Nambu-Goldstone boson (PNGB) potential

$$V(\phi) = M^4[1 + \cos(\phi/f)], \quad (19)$$

and then converting those constraints into limits on K and w_0 . Here we will use the limits derived in Ref. [43], but we will return to the approach in Ref. [42] at the end of the paper.

Reference [43] derives limits on the slow-roll thawing models using the Planck 2015 release [44], type Ia supernova observations from SDSS-II and SNLS [7], and baryon acoustic oscillation measurements [45–47]. The resulting 2σ confidence limits in the plane defined by w_0 and K can be mapped directly onto the models examined here. Using Eq. (18), we can transform these limits into a corresponding

FIG. 2. For slow-roll thawing quintessence, the present-day value of the equation of state parameter, $1 + w_0$, divided by the square of $\lambda \equiv |V'(\phi_i)/V(\phi_i)|$ as a function of $K \equiv \sqrt{1 - 4V''(\phi_i)/3V(\phi_i)}$, where ϕ_i is the initial value of ϕ .

region in the plane defined by $|\Delta\phi|$ and K ; this region is presented in Fig. 4. [Note that we fix $\Omega_{\phi 0} = 0.69$, while Durrive *et al.* marginalize over this quantity. However, their corresponding distribution for $\Omega_{\phi 0}$ (marginalized over w_0) is strongly peaked near our value, and Eqs. (15), (17), and (18) are slowly-varying functions of $\Omega_{\phi 0}$, so our results are relatively insensitive to the assumed value for $\Omega_{\phi 0}$.] If we use the limit $-V''/V > 1$ for the refined de Sitter conjecture, then the region allowed by this conjecture lies to the right of the vertical red dashed line: namely, $K > \sqrt{7/3} = 1.53$. From Fig. 4, it is clear that for the entire region satisfying both the observational constraints and the refined de Sitter conjecture, we have $|\Delta\phi| < 0.3$. Thus, for slow-roll thawing quintessence models consistent

FIG. 3. For slow-roll thawing quintessence, the present-day value of the equation of state parameter, $1 + w_0$, divided by the square of the field excursion, $\Delta\phi$, as a function of $K \equiv \sqrt{1 - 4V''(\phi_i)/3V(\phi_i)}$, where ϕ_i is the initial value of ϕ .

FIG. 4. Observational constraints on slow-roll thawing quintessence from Ref. [43] mapped onto the $|\Delta\phi|$, K plane using the results of the present paper, where $\Delta\phi$ is the distance traversed by the field up to the present, and $K \equiv \sqrt{1 - 4V''(\phi_i)/3V(\phi_i)}$, with ϕ_i the initial value of ϕ . The area outside of the shaded blue region corresponds to the 2σ excluded region in Ref. [43]. The region to the right of the vertical red line satisfies the refined de Sitter conjecture with $-V''/V > 1$.

with current observations, the distance conjecture becomes redundant; it is satisfied whenever the refined de Sitter conjecture is satisfied, while the converse is not true.

In the same way we can use Eq. (17) to transform the observational limits from Ref. [43] into a corresponding region in the plane defined by λ and K ; this region is presented in Fig. 5. As expected, one can find initial

FIG. 5. Observational constraints on slow-roll thawing quintessence from Ref. [43] mapped onto the λ , K plane using the results of the present paper, where $\lambda \equiv |V'(\phi_i)/V(\phi_i)|$ and $K \equiv \sqrt{1 - 4V''(\phi_i)/3V(\phi_i)}$, with ϕ_i the initial value of ϕ . The area outside of the shaded blue region corresponds to the 2σ excluded region in Ref. [43]. The region to the right of the vertical red line satisfies the refined de Sitter conjecture with $-V''/V > 1$. The region below the black curve was excluded from consideration in Ref. [23] (see text).

conditions consistent with observations for arbitrarily large values of K ; however, as the curvature becomes larger, the value of λ must become increasingly finely tuned to smaller values. This corresponds to placing the field initially closer to a local maximum in the potential. Obviously, these finely tuned initial conditions are less plausible, but the question of what actually defines fine-tuning is, to some extent, arbitrary. However, we can take advantage of the form of the allowed region to make some plausible statements about fine-tuning. For the region

$$\sqrt{7/3} < K < 5, \quad (20)$$

the corresponding value of λ is ~ 0.1 – 1 , which would certainly not be considered fine-tuned. On the other hand, for $K > 6$, we have $\lambda < 0.05$, with λ approaching zero very rapidly as K increases beyond this value. Hence, a plausible region consistent with observations, allowed by the refined de Sitter conjecture, and not particularly fine-tuned, is defined by Eq. (20).

This conclusion differs somewhat from that in Raveri *et al.* [23], who found a strong tension between the refined de Sitter conjecture and potentials of the form given by Eq. (9). However, these two results are not inconsistent. First note that Ref. [23] used the cutoff $c\phi_i < 0.0447$, which eliminates the smallest values of λ . This cutoff corresponds to the region below the black curve in Fig. 5. We have discounted some of this region already as corresponding to fine-tuned values of λ , but this does not account for our large allowed region with $K < 5$. This region is downweighted in Ref. [23] because these authors use a flat prior on the quantity $B/\rho_{\phi 0}$, which disfavors

larger values of c (corresponding to larger K in our discussion). A similar effect can be seen in comparing the observational constraints (without swampland considerations) in Refs. [41,43] to those in Ref. [42]. Both [41,43] find broad observationally allowed regions in the $1 + w_0$, K plane. Reference [42], on the other hand, adopts an approach similar to that of Raveri *et al.*, beginning with a PNGB potential [Eq. (19)] and deriving a corresponding excluded region for K and w_0 . The prior on K in this case is highly nonuniform, resulting in a much smaller allowed region. Indeed, if we were to use the observational constraints of Ref. [42] and repeat our analysis here, our conclusions would be much closer to those of Ref. [23]. Neither approach is intrinsically more correct; the difference simply illustrates that the constraints on quintessence models are very sensitive to the assumed priors.

The results presented here by no means exhaust the various swampland conjectures that have been proposed. For example, Bedroya and Vafa [48] have recently proposed the “trans-Planckian censorship conjecture,” with the corresponding cosmological consequences examined in Ref. [49]. However, the formalism we have utilized here for slow-roll thawing quintessence is particularly well suited for examining the refined de Sitter conjecture in the form of Eq. (8), as this formalism naturally accounts for the behavior of scalar fields near the maximum of potentials with negative curvature.

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