$Y(4626)$ as a P -wave $[cs][\bar{c}\bar{s}]$ tetraquark stateJian-Rong Zhang *Department of Physics, College of Liberal Arts and Sciences, National University of Defense Technology, Changsha 410073, Hunan, People's Republic of China*

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Motivated by the Belle Collaboration's new observation of $Y(4626)$, we investigate the possibility of its configuration as a P -wave $[cs][\bar{c}\bar{s}]$ tetraquark state from QCD sum rules. Eventually, the extracted mass $4.60^{+0.13}_{-0.19}$ GeV for the P -wave cs -scalar-diquark $\bar{c}\bar{s}$ -scalar-antidiquark state agrees well with the experimental data of $Y(4626)$, which could support its interpretation as a P -wave scalar-scalar $[cs][\bar{c}\bar{s}]$ tetraquark state.

DOI: [10.1103/PhysRevD.102.054006](https://doi.org/10.1103/PhysRevD.102.054006)**I. INTRODUCTION**

Very newly, Belle Collaboration reported the first observation of a vector charmoniumlike state $Y(4626)$ decaying to a charmed-antistrange and anticharmed-strange meson pair $D_s^+ D_{s1}^-(2536)^-$ with a significance of 5.9σ [1]. Its mass and width were measured to be $4625.9^{+6.2}_{-6.0} \pm 0.4$ MeV and $49.8^{+13.9}_{-11.5} \pm 4.0$ MeV, respectively. This state is near the $Y(4660)$ observed in the hidden-charm process $e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$ [2,3] and also consistent with the $Y(4630)$ searched in the $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$ [4,5]. Considering their close masses and widths, $Y(4660)$ and $Y(4630)$ were suggested to be the same resonance [6–8], and there have been various theoretical explanations for them, such as a conventional charmonium [9–11], a $f_0(980)\Psi'$ bound state [12–14], a baryonium state [6,15,16], a hadro-charmonium state [17], a tetraquark state [18–24], and so on.

The new observation of $Y(4626)$ by Belle immediately aroused one's great interest [25–31]. With an eye to the multi-quark viewpoint, an assignment of $Y(4626)$ was proposed as a $D_s^* \bar{D}_{s1}(2536)$ molecular state in a quasipotential Bethe-Salpeter equation approach with the one-boson-exchange model [26]. Later, the mass spectrum of $D_s^* \bar{D}_{s1}(2536)$ system was calculated within the framework of Bethe-Salpeter equations [28], and in the end the authors may not think $Y(4626)$ to be a $D_s^* \bar{D}_{s1}(2536)$ bound state, but something else. Otherwise, some authors employed a multi-quark color

flux-tube model with a multibody confinement potential and one-gluon-exchange interaction to make an exhaustive investigation on the diquark-antidiquark state [30], and they concluded that $Y(4626)$ can be well interpreted as a P -wave $[cs][\bar{c}\bar{s}]$ state.

Under the circumstance, it is interesting and significant to study whether $Y(4626)$ could be a candidate of P -wave $[cs][\bar{c}\bar{s}]$ tetraquark state by different means. It is known that one has to face the complicated nonperturbative problem in QCD while handling a hadronic state. Established firmly on the QCD basic theory, the QCD sum rule [32] acts as one authentic way for evaluating nonperturbative effects, which has been successfully applied to plenty of hadronic systems (for reviews see [33–36] and references therein). Therefore, in this work we are devoted to investigating whether $Y(4626)$ could be a P -wave $[cs][\bar{c}\bar{s}]$ tetraquark state with the QCD sum rule method.

This paper is organized as follows. The QCD sum rule for the P -wave tetraquark state is derived in Sec. II, followed by the numerical analysis in Sec. III. The last part is a brief summary and outlook.

II. THE P -WAVE $[cs][\bar{c}\bar{s}]$ STATE QCD SUM RULE

Generally, it is possible to get a total spin parity 1^- for a P -wave tetraquark with several diquark choices. Meanwhile, one could note that there have been broad discussions on the “good” diquarks or “bad” diquarks in the tetraquark configurations [37]. Thus, one could represent the P -wave $[cs][\bar{c}\bar{s}]$ tetraquark state basically from the following considerations [38]. A good diquark operator in the attractive antitriplet color channel can be written as $\bar{q}_c \gamma_5 q$, and a bad diquark operator can be written as $\bar{q}_c \gamma q$, which represent 0^+ and 1^+ ,

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respectively. Similarly, one can construct 0^- and 1^- operators as $\bar{q}_c q$ and $\bar{q}_c \gamma \gamma_5 q$, respectively. Lattice studies suggest that diquarks are preferably (energetically) formed into spin 0 configurations [39]. In fact, the solid tetraquark candidates tend to be made of good diquarks. As the concrete examples of QCD sum rules, we have explored some tetraquark states with various diquark configurations [40], and the final results favored the scalar diquark-scalar antidiquark structures, which virtually manifest that a solid tetraquark state should be composed of good diquarks.

In this manner, the P -wave $[cs][\bar{c}\bar{s}]$ state could be described as having the flavor content $[cs][\bar{c}\bar{s}]$ with the spin momentum numbers $S_{[cs]} = 0$, $S_{[\bar{c}\bar{s}]} = 0$, and $S_{[cs][\bar{c}\bar{s}]} = 0$, as well as with the orbital momentum number $L_{[cs][\bar{c}\bar{s}]} = 1$. To characterize such a state, one could first construct the current

$$j_\mu = \epsilon_{def} \epsilon_{d'e'f'} (s_d^T C \gamma_5 c_e) D_\mu (\bar{s}_{d'} \gamma_5 C \bar{c}_{e'}^T), \quad (1)$$

for the P -wave scalar-scalar case, and then

$$j_\mu = \epsilon_{def} \epsilon_{d'e'f'} (s_d^T C c_e) D_\mu (\bar{s}_{d'} C \bar{c}_{e'}^T), \quad (2)$$

for the P -wave pseudoscalar-pseudoscalar case. Here the index T means matrix transposition; C denotes the charge conjugation matrix; the covariant derivative D_μ is introduced to generate $L = 1$; and d, e, f, d' , and e' are color indices.

In general, the two-point correlator

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j_\mu(x) j_\nu^\dagger(0)] | 0 \rangle, \quad (3)$$

can be parametrized as

$$\Pi_{\mu\nu}(q^2) = \frac{q_\mu q_\nu}{q^2} \Pi^{(0)}(q^2) + \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi^{(1)}(q^2). \quad (4)$$

Furthermore, the part of correlator proportional to $-g_{\mu\nu}$ is used to obtain the sum rule, which can be evaluated in two different ways: at the hadronic level and at the quark level. Phenomenologically, $\Pi^{(1)}(q^2)$ can be written as

$$\Pi^{(1)}(q^2) = \frac{\lambda^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} \Pi^{(1)}(s)}{s - q^2}, \quad (5)$$

where M_H denotes the hadron's mass. In the operator product expansion (OPE) side, it can be expressed as

$$\Pi^{(1)}(q^2) = \int_{(2m_c + 2m_s)^2}^{\infty} ds \frac{\rho(s)}{s - q^2}, \quad (6)$$

for which the spectral density $\rho(s) = \frac{1}{\pi} \text{Im} \Pi^{(1)}(s)$.

To derive $\rho(s)$, one works at leading order in α_s and includes condensates up to dimension 8. The strange quark is treated as a light one and the diagrams are considered up to the order m_s . Keeping the heavy-quark mass finite, one uses the heavy-quark propagator in momentum space [41]. The correlator's light-quark part is calculated in the coordinate space and Fourier-transformed to the momentum space in D dimension, which is combined with the heavy-quark part and then dimensionally regularized at $D = 4$ [36,42,43]. At length, it is given by $\rho(s) = \rho^{\text{pert}} + \rho^{\langle \bar{s}s \rangle} + \rho^{\langle g^2 G^2 \rangle} + \rho^{\langle g \bar{s} \sigma \cdot G s \rangle} + \rho^{\langle \bar{s}s \rangle^2} + \rho^{\langle g^3 G^3 \rangle} + \rho^{\langle \bar{s}s \rangle \langle g^2 G^2 \rangle} + \rho^{\langle \bar{s}s \rangle \langle g \bar{s} \sigma \cdot G s \rangle}$, concretely with

$$\begin{aligned} \rho^{\text{pert}} &= -\frac{1}{3 \cdot 5 \cdot 2^{11} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1 - \alpha - \beta) \kappa [r - 5m_c m_s (\alpha + \beta)] r^4, \\ \rho^{\langle \bar{s}s \rangle} &= \frac{\langle \bar{s}s \rangle}{3 \cdot 2^6 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \{ [(2 - \alpha - \beta)m_c + (1 - \alpha - \beta)m_s] r - 3(\alpha - \alpha^2 + \beta - \beta^2)m_s m_c^2 \} r^2 \right. \\ &\quad \left. - m_s \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha(1 - \alpha)} [m_c^2 - \alpha(1 - \alpha)s]^3 \right\}, \\ \rho^{\langle g^2 G^2 \rangle} &= -\frac{m_c \langle g^2 G^2 \rangle}{3^2 \cdot 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1 - \alpha - \beta) (\alpha^3 + \beta^3) \kappa r [(m_c - 3m_s)r - 2m_s m_c^2 (\alpha + \beta)], \\ \rho^{\langle g \bar{s} \sigma \cdot G s \rangle} &= \frac{\langle g \bar{s} \sigma \cdot G s \rangle}{3 \cdot 2^8 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} r \{ -3m_c (\alpha + \beta - 4\alpha\beta) r + m_s \alpha \beta [12m_c^2 - 7(\alpha + \beta)m_c^2 - 5\alpha\beta s] \} \right. \\ &\quad \left. + \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha [m_c^2 - \alpha(1 - \alpha)s] \left\{ \frac{3m_c}{\alpha(1 - \alpha)} [m_c^2 - \alpha(1 - \alpha)s] + 2m_s [5\alpha(1 - \alpha)s - 9m_c^2] \right\} \right\}, \end{aligned}$$

$$\begin{aligned}
\rho^{\langle\bar{s}s\rangle^2} &= \frac{m_c Q \langle\bar{s}s\rangle^2}{3 \cdot 2^4 \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \{-2m_c[m_c^2 - \alpha(1-\alpha)s] + m_s[m_c^2 - 2\alpha(1-\alpha)s]\}, \\
\rho^{\langle g^3 G^3 \rangle} &= -\frac{\langle g^3 G^3 \rangle}{3^2 \cdot 2^{14} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1-\alpha-\beta) \kappa \{[(\alpha^3 + \beta^3)r + 4(\alpha^4 + \beta^4)m_c^2 \\
&\quad - 2m_c m_s (2\alpha^2 + 3\alpha\beta + 2\beta^2)(3\alpha^2 - 4\alpha\beta + 3\beta^2)]r - 4m_s m_c^3 (\alpha + \beta)(\alpha^4 + \beta^4)\}, \\
\rho^{\langle\bar{s}s\rangle\langle g^2 G^2 \rangle} &= \frac{m_c \langle\bar{s}s\rangle \langle g^2 G^2 \rangle}{3^2 \cdot 2^8 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \{(2-\alpha-\beta)(\alpha^3 + \beta^3)m_c^2 - 3[\alpha^2(\beta-1) + \beta^2(\alpha-1)]r \right. \\
&\quad \left. + [2(\alpha^2 + \beta^2)^2 - (\alpha + \beta)^3 - \alpha\beta(\alpha - \beta)^2 - (\alpha^3 + \beta^3)]m_s m_c\} - m_s m_c \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \frac{3\alpha^2 - 3\alpha + 1}{\alpha(1-\alpha)} \right\}, \\
\rho^{\langle\bar{s}s\rangle\langle g\bar{s}\sigma \cdot Gs \rangle} &= \frac{m_c(m_c - m_s) \langle\bar{s}s\rangle \langle g\bar{s}\sigma \cdot Gs \rangle}{3 \cdot 2^5 \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha (6\alpha^2 - 6\alpha + 1),
\end{aligned}$$

for the P -wave scalar-scalar case, and with

$$\begin{aligned}
\rho^{\text{pert}} &= -\frac{1}{3 \cdot 5 \cdot 2^{11} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1-\alpha-\beta) \kappa [r + 5m_c m_s (\alpha + \beta)] r^4, \\
\rho^{\langle\bar{s}s\rangle} &= \frac{\langle\bar{s}s\rangle}{3 \cdot 2^6 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \{[-(2-\alpha-\beta)m_c + (1-\alpha-\beta)m_s]r \right. \\
&\quad \left. - 3(\alpha - \alpha^2 + \beta - \beta^2)m_s m_c^2\} r^2 - m_s \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha(1-\alpha)} [m_c^2 - \alpha(1-\alpha)s]^3 \right\}, \\
\rho^{\langle g^2 G^2 \rangle} &= -\frac{m_c \langle g^2 G^2 \rangle}{3^2 \cdot 2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1-\alpha-\beta)(\alpha^3 + \beta^3) \kappa r [(m_c + 3m_s)r + 2m_s m_c^2 (\alpha + \beta)], \\
\rho^{\langle g\bar{s}\sigma \cdot Gs \rangle} &= \frac{\langle g\bar{s}\sigma \cdot Gs \rangle}{3 \cdot 2^8 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} r \{3m_c(\alpha + \beta - 4\alpha\beta)r + m_s \alpha \beta [12m_c^2 - 7(\alpha + \beta)m_c^2 - 5\alpha\beta s]\} \right. \\
&\quad \left. + \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha [m_c^2 - \alpha(1-\alpha)s] \left\{ -\frac{3m_c}{\alpha(1-\alpha)} [m_c^2 - \alpha(1-\alpha)s] + 2m_s [5\alpha(1-\alpha)s - 9m_c^2] \right\} \right\}, \\
\rho^{\langle\bar{s}s\rangle^2} &= \frac{m_c Q \langle\bar{s}s\rangle^2}{3 \cdot 2^4 \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \{-2m_c[m_c^2 - \alpha(1-\alpha)s] - m_s[m_c^2 - 2\alpha(1-\alpha)s]\}, \\
\rho^{\langle g^3 G^3 \rangle} &= -\frac{\langle g^3 G^3 \rangle}{3^2 \cdot 2^{14} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^4} (1-\alpha-\beta) \kappa \{[(\alpha^3 + \beta^3)r + 4(\alpha^4 + \beta^4)m_c^2 \\
&\quad + 2m_c m_s (2\alpha^2 + 3\alpha\beta + 2\beta^2)(3\alpha^2 - 4\alpha\beta + 3\beta^2)]r + 4m_s m_c^3 (\alpha + \beta)(\alpha^4 + \beta^4)\}, \\
\rho^{\langle\bar{s}s\rangle\langle g^2 G^2 \rangle} &= \frac{m_c \langle\bar{s}s\rangle \langle g^2 G^2 \rangle}{3^2 \cdot 2^8 \pi^4} \left\{ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \{-(2-\alpha-\beta)(\alpha^3 + \beta^3)m_c^2 + 3[\alpha^2(\beta-1) + \beta^2(\alpha-1)]r \right. \\
&\quad \left. + [2(\alpha^2 + \beta^2)^2 - (\alpha + \beta)^3 - \alpha\beta(\alpha - \beta)^2 - (\alpha^3 + \beta^3)]m_s m_c\} - m_s m_c \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \frac{3\alpha^2 - 3\alpha + 1}{\alpha(1-\alpha)} \right\}, \\
\rho^{\langle\bar{s}s\rangle\langle g\bar{s}\sigma \cdot Gs \rangle} &= \frac{m_c(m_c + m_s) \langle\bar{s}s\rangle \langle g\bar{s}\sigma \cdot Gs \rangle}{3 \cdot 2^5 \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha (6\alpha^2 - 6\alpha + 1),
\end{aligned}$$

for the P -wave pseudoscalar-pseudoscalar case. It is defined as $r = (\alpha + \beta)m_c^2 - \alpha\beta s$ and $\kappa = 1 + \alpha - 2\alpha^2 + \beta + 2\alpha\beta - 2\beta^2$. The integration limits are $\alpha_{\min} = (1 - \sqrt{1 - 4m_c^2/s})/2$, $\alpha_{\max} = (1 + \sqrt{1 - 4m_c^2/s})/2$, and $\beta_{\min} = \alpha m_c^2/(s\alpha - m_c^2)$. For the four-quark condensate $\langle\bar{s}s\rangle^2$, a general factorization

$\langle\bar{s}s\bar{s}s\rangle = Q\langle\bar{s}s\rangle^2$ [34,44] has been used, where Q is a constant that may be equal to 1 or 2.

After equating the two expressions (5) and (6), assuming quark-hadron duality, and making a Borel transform, the sum rule can be given by

FIG. 1. The OPE convergence for the P -wave scalar-scalar case is shown by comparing the relative contributions of perturbative, two-quark condensate $\langle \bar{s}s \rangle$, two-gluon condensate $\langle g^2 G^2 \rangle$, mixed condensate $\langle g \bar{s} \sigma \cdot G s \rangle$, four-quark condensate $\langle \bar{s}s \rangle^2$, three-gluon condensate $\langle g^3 G^3 \rangle$, $\langle \bar{s}s \rangle \langle g^2 G^2 \rangle$, and $\langle \bar{s}s \rangle \langle g \bar{s} \sigma \cdot G s \rangle$ from sum rule (7) for $\sqrt{s_0} = 5.2$ GeV.

$$\lambda^2 e^{-M_H^2/M^2} = \int_{(2m_c+2m_s)^2}^{s_0} ds p e^{-s/M^2}. \quad (7)$$

Eliminating the hadronic coupling constant λ , one could yield

$$M_H^2 = \int_{(2m_c+2m_s)^2}^{s_0} ds p e^{-s/M^2} / \int_{(2m_c+2m_s)^2}^{s_0} ds p e^{-s/M^2}. \quad (8)$$

III. NUMERICAL ANALYSIS

Performing the numerical analysis of sum rule (8), the s -quark and the running charm quark masses are chosen as updated values [45]: $m_s = 93_{-5}^{+11}$ MeV and $m_c = 1.27 \pm 0.02$ GeV, respectively. Besides, other input parameters are taken as [32,36]: $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$, $m_0^2 = 0.8 \pm 0.1 \text{ GeV}^2$, $\langle \bar{s}s \rangle = m_0^2 \langle \bar{q}q \rangle$, $\langle g \bar{s} \sigma \cdot G s \rangle = m_0^2 \langle \bar{s}s \rangle$, $\langle g^2 G^2 \rangle = 0.88 \pm 0.25 \text{ GeV}^4$, and $\langle g^3 G^3 \rangle = 0.58 \pm 0.18 \text{ GeV}^6$.

Complying with the standard criterion of sum rule analysis, both the OPE convergence and the pole dominance would be considered to find appropriate work windows for the threshold parameter $\sqrt{s_0}$ and the Borel parameter M^2 : the lower bound of M^2 is gained by analyzing the OPE convergence, and the upper one is obtained by viewing that the pole contribution should be larger than QCD continuum contribution. At the same time, the threshold $\sqrt{s_0}$ characterizes the beginning of continuum state and is empirically about 400–600 MeV above the extracted M_H .

Taking the analysis of P -wave scalar-scalar case as an example, the input parameters would be first kept at their central values. To obtain the lower bound of M^2 , the OPE

FIG. 2. The phenomenological contribution in sum rule (7) for $\sqrt{s_0} = 5.2$ GeV for the P -wave scalar-scalar case. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of M^2 and the dashed line is the relative continuum contribution.

convergence is shown in Fig. 1 by comparing the relative contributions of different condensates from sum rule (7) for $\sqrt{s_0} = 5.2$ GeV. Numerically, the relative perturbative contribution begins to play a dominant role in the OPE side at $M^2 = 3.0 \text{ GeV}^2$, which is increasing with the Borel parameter M^2 . Thereby, the perturbative part could dominate in OPE comparing with other higher dimensional condensate contributions while taking $M^2 \geq 3.0 \text{ GeV}^2$. On the other hand, the upper bound of M^2 is gained by considering the pole dominance phenomenologically. In Fig. 2, the comparison between pole and continuum contributions from sum rule (7) is shown for $\sqrt{s_0} = 5.2$ GeV. The relative pole contribution is approximate to 50% at $M^2 = 3.5 \text{ GeV}^2$ and descending with M^2 . Hence, the pole contribution dominance could be satisfied when $M^2 \leq 3.5 \text{ GeV}^2$. Consequently, the Borel window of M^2 is fixed on $3.0\text{--}3.5 \text{ GeV}^2$ for $\sqrt{s_0} = 5.2$ GeV. In the similar analysis, the proper range of M^2 is gained as $3.0\text{--}3.4 \text{ GeV}^2$ for $\sqrt{s_0} = 5.1$ GeV, and $3.0\text{--}3.7 \text{ GeV}^2$ for $\sqrt{s_0} = 5.3$ GeV. In the chosen work windows, it is expected that the two sides of QCD sum rules have a good overlap and information on the resonance can be safely extracted. The mass M_H of P -wave scalar-scalar $[c\bar{s}][\bar{c}s]$ state is shown as a function of M^2 in Fig. 3, and it is computed to be $4.60 \pm 0.10 \text{ GeV}$ in the work windows. Next, varying the quark masses and condensates, one could arrive at $4.60 \pm 0.10_{-0.04}^{+0.03} \text{ GeV}$ (the first error resulted from the uncertainty due to variation of s_0 and M^2 , and the second error rooting in the variation of QCD parameters) or briefly $4.60_{-0.14}^{+0.13} \text{ GeV}$. At last, taking into account the variation of four-quark condensate factorization factor ρ from 1 to 2, one could get the final mass value

FIG. 3. The dependence on M^2 for the mass M_H of P -wave scalar-scalar $[cs][\bar{c}\bar{s}]$ from sum rule (8) is shown. The ranges of M^2 are 3.0–3.4 GeV^2 for $\sqrt{s_0} = 5.1$ GeV , 3.0–3.5 GeV^2 for $\sqrt{s_0} = 5.2$ GeV , and 3.0–3.7 GeV^2 for $\sqrt{s_0} = 5.3$ GeV , respectively.

$4.60^{+0.13}_{-0.19}$ GeV for the P -wave scalar-scalar case, which is in good agreement with the experimental data of $Y(4626)$ and could support its P -wave scalar-scalar $[cs][\bar{c}\bar{s}]$ tetraquark explanation.

For the P -wave pseudoscalar-pseudoscalar case, Fig. 4 shows the relative contributions of different condensates from sum rule (7) for $\sqrt{s_0} = 5.2$ GeV . Note that the two-quark condensate $\langle\bar{s}s\rangle$ plays an important role on OPE here,

FIG. 4. The OPE convergence for the P -wave pseudoscalar-pseudoscalar case is shown by comparing the relative contributions of perturbative, two-quark condensate $\langle\bar{s}s\rangle$, two-gluon condensate $\langle g^2 G^2 \rangle$, mixed condensate $\langle g\bar{s}\sigma \cdot Gs \rangle$, four-quark condensate $\langle\bar{s}s\rangle^2$, three-gluon condensate $\langle g^3 G^3 \rangle$, $\langle\bar{s}s\rangle\langle g^2 G^2 \rangle$, and $\langle\bar{s}s\rangle\langle g\bar{s}\sigma \cdot Gs \rangle$ from sum rule (7) for $\sqrt{s_0} = 5.2$ GeV .

FIG. 5. The phenomenological contribution in sum rule (7) for $\sqrt{s_0} = 5.2$ GeV for the P -wave pseudoscalar-pseudoscalar case. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of M^2 and the dashed line is the relative continuum contribution.

and, thus, the lower bound of M^2 has to be taken a very high value to meet the convergence condition, whereas, its phenomenological contribution in sum rule (7) is shown in Fig. 5 for $\sqrt{s_0} = 5.2$ GeV , and the Borel parameter should be taken $M^2 \leq 2.5$ GeV^2 to fulfill the pole contribution dominance. For this case, one could note that it is difficult to find reasonable work windows satisfying both good OPE convergence and pole dominance, and graphically the mass M_H 's dependence on M^2 in Fig. 6 is rather unstable. Accordingly, it is not advisable to continue extracting a mass value. In a sideward way, it is consistent with the

FIG. 6. The dependence on M^2 for the mass M_H of P -wave pseudoscalar-pseudoscalar tetraquark state from sum rule (8) is shown.

statement that diquarks are preferably formed into 0^+ good diquarks.

IV. SUMMARY AND OUTLOOK

Stimulated by the first observation of a vector charmoniumlike state $Y(4626)$ decaying to a $D_s^+ D_{s1}(2536)^-$ pair, we have calculated the mass of P -wave $[cs][\bar{c}\bar{s}]$ tetraquark state in QCD sum rules. For the P -wave scalar-scalar case, the final result $4.60^{+0.13}_{-0.19}$ GeV is well compatible with the experimental data $4625.9^{+6.2}_{-6.0} \pm 0.4$ MeV of $Y(4626)$, which favors the explanation of $Y(4626)$ as a P -wave scalar-scalar $[cs][\bar{c}\bar{s}]$ tetraquark state. It is of difficulty to find proper work windows to achieve a mass value for the P -wave pseudoscalar-pseudoscalar case, in a sideward way, which is coincident with the picture of $Y(4626)$ as a P -wave scalar-scalar $[cs][\bar{c}\bar{s}]$ tetraquark state.

In the present article, we have devoted time to calculating the mass of P -wave $[cs][\bar{c}\bar{s}]$ state from two-point QCD sum rules, which could provide some evidence for the newly

observed $Y(4626)$ as a P -wave scalar-scalar $[cs][\bar{c}\bar{s}]$ tetraquark state. To finalize the inner structure of $Y(4626)$, undoubtedly it needs further experimental observations and continually theoretical studies. For instance, one could take into account studying the width of the state, which is definitely important and could be obtained by employing three-point QCD sum rules, but then one can expect that there are many Feynman diagrams that should be considered, particularly owing to the derivative operator of interpolating current, which could be researched in some subsequent work after having completed enormous calculations. Anyhow, one can expect that future experimental and theoretical efforts may shed more light on the nature of $Y(4626)$.

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- [1] S. Jia *et al.* (Belle Collaboration), *Phys. Rev. D* **100**, 111103 (R) (2019).
 - [2] X. L. Wang *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **99**, 142002 (2007).
 - [3] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **89**, 111103 (2014).
 - [4] G. Pakhlova *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **101**, 172001 (2008).
 - [5] Y. Xie and Z. Q. Liu, [arXiv:2001.09620](#).
 - [6] D. V. Bugg, *J. Phys. G* **36**, 075002 (2009).
 - [7] F. K. Guo, J. Haidenbauer, C. Hanhart, and U. G. Meissner, *Phys. Rev. D* **82**, 094008 (2010).
 - [8] G. Cotugno, R. Faccini, A. D. Polosa, and C. Sabelli, *Phys. Rev. Lett.* **104**, 132005 (2010).
 - [9] B. Q. Li and K. T. Chao, *Phys. Rev. D* **79**, 094004 (2009).
 - [10] G. J. Ding, J. J. Zhu, and M. L. Yan, *Phys. Rev. D* **77**, 014033 (2008).
 - [11] A. M. Badalian, B. L. G. Bakker, and I. V. Danilkin, *Phys. At. Nucl.* **72**, 638 (2009).
 - [12] F. K. Guo, C. Hanhart, and U. G. Meißner, *Phys. Lett. B* **665**, 26 (2008).
 - [13] Z. G. Wang and X. H. Zhang, *Commun. Theor. Phys.* **54**, 323 (2010).
 - [14] R. M. Albuquerque, M. Nielsen, and R. Rodrigues da Silva, *Phys. Rev. D* **84**, 116004 (2011).
 - [15] C. F. Qiao, *J. Phys. G* **35**, 075008 (2008).
 - [16] N. Lee, Z. G. Luo, X. L. Chen, and S. L. Zhu, *Phys. Rev. D* **84**, 014031 (2011).
 - [17] S. Dubynskiy and M. B. Voloshin, *Phys. Lett. B* **666**, 344 (2008).
 - [18] D. Ebert, R. N. Faustov, and V. O. Galkin, *Eur. Phys. J. C* **58**, 399 (2008).
 - [19] X. W. Liu, H. W. Ke, X. Liu, and X. Q. Li, *Eur. Phys. J. C* **76**, 549 (2016).
 - [20] R. M. Albuquerque and M. Nielsen, *Nucl. Phys. A* **815**, 53 (2009).
 - [21] J. R. Zhang and M. Q. Huang, *Phys. Rev. D* **83**, 036005 (2011).
 - [22] W. Chen and S. L. Zhu, *Phys. Rev. D* **83**, 034010 (2011).
 - [23] Z. G. Wang, *Eur. Phys. J. C* **74**, 2874 (2014); **78**, 518 (2018); **79**, 184 (2019).
 - [24] H. Sundu, S. S. Agaev, and K. Azizi, *Phys. Rev. D* **98**, 054021 (2018).
 - [25] J. Z. Wang, R. Q. Qian, X. Liu, and T. Matsuki, *Phys. Rev. D* **101**, 034001 (2020).
 - [26] J. He, Y. Liu, J. T. Zhu, and D. Y. Chen, *Eur. Phys. J. C* **80**, 246 (2020).
 - [27] X. K. Dong, Y. H. Lin, and B. S. Zou, *Phys. Rev. D* **101**, 076003 (2020).
 - [28] H. W. Ke, X. H. Liu, and X. Q. Li, [arXiv:2004.03167](#).
 - [29] Y. Tan and J. L. Ping, *Phys. Rev. D* **101**, 054010 (2020).
 - [30] C. R. Deng, H. Chen, and J. L. Ping, *Phys. Rev. D* **101**, 054039 (2020).
 - [31] J. F. Giron and R. F. Lebed, *Phys. Rev. D* **101**, 074032 (2020).
 - [32] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys. B* **147**, 385 (1979); **147**, 448 (1979); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Fortschr. Phys.* **32**, 585 (1984).

- [33] B. L. Ioffe, in *The Spin Structure of The Nucleon*, edited by B. Frois and V.W. Hughes, and N. de Groot (World Scientific, Singapore, 1997).
- [34] S. Narison, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **17**, 1 (2002).
- [35] P. Colangelo and A. Khodjamirian, in *At the Frontier of Particle Physics: Handbook of QCD*, edited by M. Shifman, Boris Ioffe Festschrift Vol. 3 (World Scientific, Singapore, 2001), pp. 1495–1576.
- [36] M. Nielsen, F. S. Navarra, and S. H. Lee, *Phys. Rep.* **497**, 41 (2010).
- [37] R. L. Jaffe, *Phys. Rev. D* **15**, 267 (1977); R. L. Jaffe and F. E. Low, *Phys. Rev. D* **19**, 2105 (1979); R. L. Jaffe and F. Wilczek, *Phys. Rev. Lett.* **91**, 232003 (2003).
- [38] N. V. Drenska, R. Faccini, and A. D. Polosa, *Phys. Lett. B* **669**, 160 (2008); A. Ali, C. Hambrock, and M. J. Aslam, *Phys. Rev. Lett.* **104**, 162001 (2010).
- [39] C. Alexandrou, Ph. de Forcrand, and B. Lucini, *Phys. Rev. Lett.* **97**, 222002 (2006).
- [40] J. R. Zhang, J. L. Zou, and J. Y. Wu, *Chin. Phys. C* **42**, 043101 (2018); J. R. Zhang, *Phys. Lett. B* **789**, 432 (2019).
- [41] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, *Phys. Rep.* **127**, 1 (1985).
- [42] H. Kim and Y. Oh, *Phys. Rev. D* **72**, 074012 (2005); M. E. Bracco, A. Lozea, R. D. Matheus, F. S. Navarra, and M. Nielsen, *Phys. Lett. B* **624**, 217 (2005); R. D. Matheus, S. Narison, M. Nielsen, and J. M. Richard, *Phys. Rev. D* **75**, 014005 (2007).
- [43] J. R. Zhang, *Eur. Phys. J. C* **79**, 1001 (2019); *Phys. Rev. D* **87**, 076008 (2013); **89**, 096006 (2014); J. R. Zhang and M. Q. Huang, *J. High Energy Phys.* **11** (2010) 057.
- [44] S. Narison, *Phys. Rep.* **84**, 263 (1982); G. Launer, S. Narison, and R. Tarrach, *Z. Phys. C* **26**, 433 (1984); S. Narison, *Phys. Lett. B* **673**, 30 (2009).
- [45] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018); and 2019 update.