Beauty baryon nonleptonic decays into decuplet baryons and CP -asymmetries based on an $SU(3)$ -flavor analysis

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We consider charmless weak decays of beauty baryons into decuplet baryons and pseudoscalar mesons in a general framework based on the $SU(3)$ -flavor decomposition of the decay amplitudes. The dynamical assumption independent $SU(3)$ analysis accounts for the effects of an arbitrarily broken $SU(3)$ symmetry in these decays. An alternative approach in terms of quark diagrams is also provided and compared with the $SU(3)$ decomposition in the limit of an exact $SU(3)$ -flavor symmetry. Furthermore, the symmetries of the effective Hamiltonian is used to relate or neglect reduced $SU(3)$ amplitudes to derive several sum rule relations between amplitudes and relations between CP asymmetries in these decays and identify those that hold even if $SU(3)$ is broken.

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I. INTRODUCTION

In recent times, bottom baryon decays have come under increased scrutiny chiefly due to the large number of b -baryon production at the LHCb [1]. Among several other interesting rare decays of b baryons, LHCb has analyzed CP asymmetries in bottom baryon charmless decays to two-body and multibody hadronic final states [2–10]. The theoretical foundation laid out to study B meson decays [11–51] is also useful to study bottom and charmed baryon decay amplitude sum rules and CP asymmetry relations for various decay modes. In particular, the general framework of $SU(3)$ analysis in beauty mesons as well as charm meson decays [52–63] into two pseudoscalars (PP), pseudoscalar-vector boson (PV), and two vector mesons (VV) can be easily extended to study bottom and charmed baryon decaying into an octet baryon and a pseudoscalar meson [64–86].

On the other hand, experimental evidences suggest a b baryon to a decuplet baryon transition alongside the pseudoscalar(vector) meson is also possible. In this paper,

we consider such hadronic beauty-baryon decays featuring a decuplet of light baryons and a pseudoscalar meson based on the $SU(3)$ decomposition of the decay amplitudes in terms of $SU(3)$ -reduced amplitudes. An alternate approach based on the flavor flow along quark lines is also presented, which is equivalent to the $SU(3)$ decomposition in the limit of an exact $SU(3)$ -flavor symmetry. In contrast to previous studies, our approach [19,87,88] facilitates an $SU(3)$ decomposition of the decays in terms of $SU(3)$ -reduced amplitudes without any particular set of assumptions about the underlying dynamics.

The formalism of the $SU(3)$ decomposition of the decay amplitudes in terms of the reduced $SU(3)$ amplitudes is presented in Sec. II. The results are summarized in the Appendixes A and B. The flavor flow diagrams are indicated in Appendix C. The relations between the amplitudes for beauty baryon decays into a decuplet of light baryons and pseudoscalar mesons are derived in Sec. III. The effects of $SU(3)$ breaking on account of a s -quark mass are considered in Sec. IV. The corresponding relations between CP asymmetries are derived in Sec. V. We finally conclude in Sec. VI.

II. FORMALISM

In this paper, we study the charmless decay of an antitriplet ($\bar{\mathbf{3}}$) beauty baryon into a decuplet baryon (B) and an octet pseudoscalar meson (P), i.e., $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10})P(\mathbf{8})$. This decay is described by a Hamiltonian with $\Delta Q = 0$ and $\Delta S = -1, 0$ (equivalent to the ΔI_3 and ΔY representation). The possible decays can be divided into two subclasses,

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namely the $\Delta S = 0$ and $\Delta S = -1$ transitions. The allowed final state $SU(3)$ representations (**f**) are **8**, **10**, **27**, **35**. There are twenty physical processes possible for $\Delta S = 0$ and another twenty for $\Delta S = -1$. In Appendix A, each of these decay modes are decomposed in terms of the $SU(3)$ reduced amplitudes that add up to forty. Since the physical η and η' mesons are admixtures of the octet η_8 and singlet η_1 mesons, a study of $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10})P(\mathbf{1})$ is also necessary. Therefore,

one has to take into account four (two each for $\Delta S = -1$ and $\Delta S = 0$) additional independent $SU(3)$ amplitudes. Since no assumption about the particular form of the effective Hamiltonian has been made yet, these forty four reduced amplitudes are all independent of each other, and no amplitude relation exists between the decay modes. The expression of the amplitudes in terms of reduced $SU(3)$ amplitudes is concisely given as

$$\mathcal{A}(i \rightarrow f_b f_m) = (-1)^{I_3 - \frac{Y}{2} - \frac{T}{3}} \sum_{\substack{\{\mathbf{f}, \mathbf{R}\} \\ Y^b + Y^m = Y^f, Y^f - Y^i = Y^H \\ I_3^b + I_3^m = I_3^f, I_3^f - I_3^i = I_3^H}} C_{I_3^b I_3^m I_3^f}^{\mathbf{f}_b \mathbf{f}_m \mathbf{f}} \left(\begin{matrix} \mathbf{f}_b & \mathbf{f}_m & \mathbf{f} \\ (Y^b, I^b, I_3^b) & (Y^m, I^m, I_3^m) & (Y^f, I^f, I_3^f) \end{matrix} \right) \\ \times \left(\begin{matrix} \mathbf{f} & \bar{\mathbf{i}} & \mathbf{R} \\ (Y^f, I^f, I_3^f) & (-Y^i, I^i, -I_3^i) & (Y^H, I^H, I_3^H) \end{matrix} \right) C_{I_3^f I_3^i I_3^H}^{\mathbf{f} \bar{\mathbf{i}} \mathbf{R}} \langle \mathbf{f} \| \mathbf{R}_I \| \mathbf{i} \rangle, \quad (1)$$

where, $C_{A,B,C}^{a,b,c}$ are the $SU(2)$ Clebsch-Gordon coefficients and

$$\left(\begin{matrix} \mathbf{R}_a & \mathbf{R}_b & \mathbf{R}_c \\ (Y^a, I^a, I_3^a) & (Y^b, I^b, I_3^b) & (Y^c, I^c, I_3^c) \end{matrix} \right) \quad (2)$$

are the $SU(3)$ isoscalar coefficients [89–93] obtained by coupling the representations $\mathbf{R}_a \otimes \mathbf{R}_b \rightarrow \mathbf{R}_c$. T is the triality¹ of a $SU(3)$ representation of the initial state conjugate $\bar{\mathbf{i}}$, and the phase factor appearing in front ensures that the correct signs are assigned to the individual initial b -baryon antitriplet. Given a form of effective Hamiltonian ($\mathcal{H}_{\text{eff}}$), it can be $SU(3)$ decomposed,

$$\mathcal{H}_{\text{eff}} = \sum_{\substack{\{\mathbf{Y}, \mathbf{I}, \mathbf{I}_3\} \\ \mathbf{R}}} \mathcal{F}_{\mathbf{R}}^{\{\mathbf{Y}, \mathbf{I}, \mathbf{I}_3\}} \mathbf{R}_I, \quad (3)$$

where $\mathcal{F}_{\mathbf{R}}^{\{\mathbf{Y}, \mathbf{I}, \mathbf{I}_3\}}$ depends on the $SU(3)$ CG coefficients appearing in front of the $SU(3)$ representations ($\mathbf{R}_I$). Moreover, $\mathcal{F}_{\mathbf{R}}^{\{\mathbf{Y}, \mathbf{I}, \mathbf{I}_3\}}$ also contains additional factors entering Eq. (3) in the form of Wilson coefficients and CKM elements. It is also important to note that by knowing the dynamical coefficients for different isospin values in a given $SU(3)$ representation, one can drop the isospin Casimir label (I) and express the Wigner-Eckart reduced matrix element $\langle \mathbf{f} \| \mathbf{R} \| \mathbf{i} \rangle$, in its usual form, independent of the isospin I label. By using completeness of $SU(3)$ CG coefficients up to a phase factor,

¹The triality (T) of a representation (p, q) of $SU(3)$ is given by $(p - q) \bmod 3$. For example, **3** of $SU(3)$ has triality 1.

$$\langle \mathbf{f} \| \mathbf{R}_I \| \mathbf{i} \rangle = \underbrace{\mathcal{F}_{\mathbf{R}}^{\{\mathbf{Y}, \mathbf{I}, \mathbf{I}_3\}}}_{\text{dynamical Coeff. of } \mathcal{H}} \sqrt{\frac{\dim \mathbf{f}}{\dim \mathbf{R}}} \langle \mathbf{f} \| \mathbf{R} \| \mathbf{i} \rangle. \quad (4)$$

The lowest order effective Hamiltonian [94–96] for charmless b -baryon decays consists of $\Delta S = -1$ and $\Delta S = 0$ parts. Each part is composed from the operators $O_1, \dots, O_{10}$. The complete Hamiltonian can be written as

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left[\lambda_u^{(s)} (C_1(O_1^{(u)} - O_1^{(c)}) + C_2(O_2^{(u)} - O_2^{(c)})) \right. \\ - \lambda_t^{(s)} \sum_{i=1,2} C_i O_i^{(c)} - \lambda_t^{(s)} \sum_{i=3}^{10} C_i O_i^{(s)} \\ + \lambda_u^{(d)} (C_1(O_1^{(u)} - O_1^{(c)}) + C_2(O_2^{(u)} - O_2^{(c)})) \\ \left. - \lambda_t^{(d)} \sum_{i=1,2} C_i O_i^{(c)} - \lambda_t^{(d)} \sum_{i=3}^{10} C_i O_i^{(d)} \right], \quad (5)$$

where $V_{ub}V_{us}^* = \lambda_u^s$, $V_{ub}V_{ud}^* = \lambda_u^d$, $V_{tb}V_{ts}^* = \lambda_t^s$, $V_{tb}V_{td}^* = \lambda_t^d$ are the CKM elements and C_i s are the Wilson coefficients. O_1 and O_2 are the “tree” operators,

$$O_1^{(u)} = (\bar{u}_L^i \gamma^\mu b_L^j)(\bar{s}_L^j \gamma_\mu u_L^i) \\ O_1^{(c)} = (\bar{c}_L^i \gamma^\mu b_L^j)(\bar{s}_L^j \gamma_\mu c_L^i) \\ O_2^{(u)} = (\bar{u}_L^i \gamma^\mu b_L^i)(\bar{s}_L^j \gamma_\mu u_L^j) \\ O_2^{(c)} = (\bar{c}_L^i \gamma^\mu b_L^i)(\bar{s}_L^j \gamma_\mu c_L^j), \quad (6)$$

$O_3, \dots, O_6$ are the “gluonic penguin” operators,

$$\begin{aligned}
O_3^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} (\bar{q}_L^j \gamma_\mu q_L^i) \\
O_4^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} (\bar{q}_L^j \gamma_\mu q_L^i) \\
O_5^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} (\bar{q}_R^j \gamma_\mu q_R^i) \\
O_6^{(s)} &= (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} (\bar{q}_R^j \gamma_\mu q_R^i), \quad (7)
\end{aligned}$$

and finally $O_7, \dots, O_{10}$ are the four “electroweak penguin” (EWP) operators,

$$\begin{aligned}
O_7^{(s)} &= \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} e_q (\bar{q}_R^j \gamma_\mu q_R^i), \\
O_8^{(s)} &= \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} e_q (\bar{q}_R^j \gamma_\mu q_R^i), \quad (8)
\end{aligned}$$

$$\begin{aligned}
O_9^{(s)} &= \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} e_q (\bar{q}_L^j \gamma_\mu q_L^i), \\
O_{10}^{(s)} &= \frac{3}{2} (\bar{s}_L^i \gamma^\mu b_L^j) \sum_{q=u,d,s} e_q (\bar{q}_L^j \gamma_\mu q_L^i). \quad (9)
\end{aligned}$$

$\mathcal{H}_{\text{eff}}$ is a linear combinations of four quark operators of the form $(\bar{q}_1 b)(\bar{q}_2 q_3)$. These operators transform as $3 \otimes 3 \otimes \bar{3}$ under $SU(3)$ flavor and can be decomposed into sums of irreducible operators corresponding to irreducible $SU(3)$ representations: $\mathbf{15}, \bar{\mathbf{6}}, \mathbf{3}^{(6)}, \mathbf{3}^{(\bar{3})}$ where the superscript index: “6” (“ $\bar{3}$ ”) indicates the origin of $\mathbf{3}$ out of the two possible representations arising from the tensor product of q_1 and q_2 . The $SU(3)$ triplet representation of quarks (q_i)

and its conjugate denoting the antiquarks ($\bar{q}_i$) consist of the flavor states,

$$q_i = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad \bar{q}_i = \begin{pmatrix} \bar{d} \\ -\bar{u} \\ \bar{s} \end{pmatrix}. \quad (10)$$

The unbroken dim-6 effective Hamiltonian contains parts that transform as $\mathbf{3}, \bar{\mathbf{6}}, \mathbf{15}$ under $SU(3)$ flavor. The tree part of the effective Hamiltonian responsible for $b \rightarrow u$ transition has the following $SU(3)$ decomposition:

$$\begin{aligned}
\frac{\sqrt{2}\mathcal{H}_T}{4G_F} &= \left\{ \lambda_u^s \left[\frac{(C_1 + C_2)}{2} \left(-\mathbf{15}_1 - \frac{1}{\sqrt{2}} \mathbf{15}_0 - \frac{1}{\sqrt{2}} \mathbf{3}_0^{(6)} \right) \right. \right. \\
&\quad \left. \left. + \frac{(C_1 - C_2)}{2} (\bar{\mathbf{6}}_1 + \mathbf{3}_0^{(\bar{3})}) \right] \right. \\
&\quad \left. + \lambda_u^d \left[\frac{(C_1 + C_2)}{2} \left(-\frac{2}{\sqrt{3}} \mathbf{15}_{3/2} - \frac{1}{\sqrt{6}} \mathbf{15}_{1/2} - \frac{1}{\sqrt{2}} \mathbf{3}_{1/2}^{(6)} \right) \right. \right. \\
&\quad \left. \left. + \frac{(C_1 - C_2)}{2} (-\bar{\mathbf{6}}_{1/2} + \mathbf{3}_{1/2}^{(\bar{3})}) \right] \right\}. \quad (11)
\end{aligned}$$

The penguin part of the effective Hamiltonian is also $SU(3)$ decomposed below,

$$\begin{aligned}
\frac{\sqrt{2}\mathcal{H}_E}{4G_F} &= \{ -\lambda_t^s [-\sqrt{2}(C_3 + C_4) \mathbf{3}_0^{(6)} + (C_3 - C_4) \mathbf{3}_0^{(\bar{3})}] \\
&\quad -\lambda_t^d [-\sqrt{2}(C_3 + C_4) \mathbf{3}_{1/2}^{(6)} + (C_3 - C_4) \mathbf{3}_{1/2}^{(\bar{3})}] \\
&\quad -\lambda_t^s [-\sqrt{2}(C_5 + C_6) \mathbf{3}_0^{(6)} + (C_5 - C_6) \mathbf{3}_0^{(\bar{3})}] \\
&\quad -\lambda_t^d [-\sqrt{2}(C_5 + C_6) \mathbf{3}_{1/2}^{(6)} + (C_5 - C_6) \mathbf{3}_{1/2}^{(\bar{3})}] \}, \quad (12)
\end{aligned}$$

$$\begin{aligned}
\frac{\sqrt{2}\mathcal{H}_{\text{EWP}}}{4G_F} &= \left\{ -\lambda_t^s \left[\frac{(C_9 + C_{10})}{2} \left(-\frac{3}{2} \mathbf{15}_1 - \frac{3}{2\sqrt{2}} \mathbf{15}_0 + \frac{1}{2\sqrt{2}} \mathbf{3}_0^{(6)} \right) + \frac{(C_9 - C_{10})}{2} \left(\frac{3}{2} \bar{\mathbf{6}}_1 + \frac{1}{2} \mathbf{3}_0^{(\bar{3})} \right) \right] \right. \\
&\quad \left. -\lambda_t^d \left[\frac{(C_9 + C_{10})}{2} \left(-\sqrt{3} \mathbf{15}_{3/2} - \frac{1}{2} \sqrt{\frac{3}{2}} \mathbf{15}_{1/2} + \frac{1}{2\sqrt{2}} \mathbf{3}_{1/2}^{(6)} \right) + \frac{(C_9 - C_{10})}{2} \left(-\frac{3}{2} \bar{\mathbf{6}}_{1/2} + \frac{1}{2} \mathbf{3}_{1/2}^{(\bar{3})} \right) \right] \right\}, \quad (13)
\end{aligned}$$

where we have ignored the contributions from O_7 and O_8 electroweak penguin operators due to the smallness of the Wilson coefficients C_7 and C_8 appearing in front of them. For charmless b decays, $b \rightarrow qc\bar{c}$, $q = d, s$ contributes as an independent $\mathbf{3}$ of $SU(3)$ without affecting the $SU(3)$ structure of Eqs. (11)–(13). With this particular choice of the effective Hamiltonian and allowed final state $SU(3)$ representations, there are five independent $SU(3)$ -reduced matrix elements,

$$\begin{aligned}
&\langle 8 \| \mathbf{3} \| \bar{\mathbf{3}} \rangle, \quad \langle 8 \| \bar{\mathbf{6}} \| \bar{\mathbf{3}} \rangle, \quad \langle 8 \| \mathbf{15} \| \bar{\mathbf{3}} \rangle \\
&\langle 10 \| \mathbf{15} \| \bar{\mathbf{3}} \rangle, \quad \langle 27 \| \mathbf{15} \| \bar{\mathbf{3}} \rangle \quad (14)
\end{aligned}$$

and the $SU(3)$ -reduced amplitudes arising from $\mathcal{H}_T$ are conveniently expressed as follows:

$$\begin{aligned}
c_8 &= -(C_1 + C_2) \langle 8 \| \mathbf{3}^{(6)} \| \bar{\mathbf{3}} \rangle + \sqrt{2}(C_1 - C_2) \langle 8 \| \mathbf{3}^{(\bar{3})} \| \bar{\mathbf{3}} \rangle \\
b_8 &= (C_1 - C_2) \langle 8 \| \bar{\mathbf{6}} \| \bar{\mathbf{3}} \rangle \\
a_8 &= (C_1 + C_2) \langle 8 \| \mathbf{15} \| \bar{\mathbf{3}} \rangle \\
a_{10} &= (C_1 + C_2) \langle 10 \| \mathbf{15} \| \bar{\mathbf{3}} \rangle \\
a_{27} &= (C_1 + C_2) \langle 27 \| \mathbf{15} \| \bar{\mathbf{3}} \rangle. \quad (15)
\end{aligned}$$

The decay amplitudes for all possible $\Delta S = -1$ and $\Delta S = 0$ processes are expressed using Eq. (15) and are given in Tables I and II, respectively.

TABLE I. $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10})P(\mathbf{8})$ decay $\Delta S = -1$ transitions.

$\Delta S = -1$	$SU(3)$ —reduced amplitude	Topological diagrams
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-)$	$\frac{1}{60}(-5\sqrt{2}a_{10} - 3a_{27} - 8a_8 + 4\sqrt{10}b_8)$	$-\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0)$	$\frac{1}{60}(5\sqrt{2}a_{10} + 3a_{27} + 8a_8 - 4\sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-)$	$\frac{1}{60}(-5\sqrt{2}a_{10} + 15a_{27} + 10a_8 + 2\sqrt{15}c_8 - 2\sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}(-E_3 + P_u + T)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0)$	$\frac{1}{30}(-3a_{27} - 3a_8 - \sqrt{15}c_8)$	$-\frac{1}{2\sqrt{6}}(E_2 - E_3 + 2P_u + T)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \eta_8)$	$-\frac{1}{10\sqrt{3}}(3a_{27} - 2a_8 + \sqrt{10}b_8)$	$\frac{1}{6\sqrt{2}}(2E_1 - E_2 - E_3 - T)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+)$	$\frac{1}{60}(5\sqrt{2}a_{10} - 3a_{27} + 2(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8))$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0)$	$\frac{1}{60}(5\sqrt{2}a_{10} + 15a_{27} - 10a_8 - 2\sqrt{15}c_8 + 2\sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}(E_3 - P_u)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^+)$	$\frac{1}{60}(5\sqrt{2}a_{10} - 3a_{27} + 2(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8))$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-)$	$\frac{1}{30}(-5\sqrt{2}a_{10} + 6a_{27} + a_8 + \sqrt{15}c_8 + \sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}(-E_1 - E_3 + P_u + T)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0)$	$\frac{1}{60}(10a_{10} + 9\sqrt{2}a_{27} - \sqrt{2}a_8 - \sqrt{30}c_8 - 2\sqrt{5}b_8)$	$\frac{1}{2\sqrt{3}}(E_1 + E_3 - P_u)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0)$	$\frac{1}{120}(-10a_{10} - 27\sqrt{2}a_{27} - 2(\sqrt{2}a_8 + \sqrt{30}c_8 + 2\sqrt{5}b_8))$	$-\frac{1}{2\sqrt{3}}(E_2 + P_u + T)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \eta_8)$	$\frac{1}{120}(10\sqrt{3}a_{10} + 3\sqrt{6}a_{27} - 2(\sqrt{6}a_8 + 3\sqrt{10}c_8 + 2\sqrt{15}b_8))$	$\frac{1}{6}(2E_1 - E_2 + 2E_3 - 3P_u - T)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+)$	$\frac{1}{60}(5\sqrt{2}a_{10} - 3a_{27} + 2(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8))$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^+)$	$\frac{1}{60}(5\sqrt{6}a_{10} - 3\sqrt{3}a_{27} + 2\sqrt{3}a_8 + 6\sqrt{5}c_8 + 2\sqrt{30}b_8)$	$\frac{1}{\sqrt{2}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^-)$	$\frac{1}{60}(12\sqrt{2}a_{27} - 3\sqrt{2}a_8 + \sqrt{30}c_8 - 2\sqrt{5}b_8)$	$\frac{1}{2\sqrt{3}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0)$	$\frac{1}{30}(3a_{27} + 3a_8 - \sqrt{15}c_8 + \sqrt{10}b_8)$	$-\frac{1}{\sqrt{6}}P_u$
$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \pi^-)$	$\frac{1}{30}(-12a_{27} + 3a_8 - \sqrt{15}c_8 + \sqrt{10}b_8)$	$-\frac{1}{\sqrt{6}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \pi^0)$	$\frac{1}{60}(-3\sqrt{2}a_{27} - 3\sqrt{2}a_8 + \sqrt{30}c_8 - 2\sqrt{5}b_8)$	$\frac{1}{2\sqrt{3}}P_u$
$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \eta_8)$	$\frac{1}{60}(3\sqrt{6}a_{27} + 3\sqrt{6}a_8 - 3\sqrt{10}c_8 + 2\sqrt{15}b_8)$	$-\frac{1}{2}P_u$
$\mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^0)$	$\frac{1}{30}(-3\sqrt{3}a_{27} - 3\sqrt{3}a_8 + 3\sqrt{5}c_8 - \sqrt{30}b_8)$	$\frac{1}{\sqrt{2}}P_u$

TABLE II. $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10})P(\mathbf{8})$ decay $\Delta S = 0$ transitions.

$\Delta S = 0$	$SU(3)$ —reduced amplitude	Topological diagram
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-)$	$\frac{1}{30}(-5\sqrt{2}a_{10} + 6a_{27} + a_8 + \sqrt{15}c_8 + \sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}(-E_1 - E_3 + P_u + T)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0)$	$\frac{1}{120}(10a_{10} - 9\sqrt{2}a_{27} - 4(\sqrt{2}a_8 + \sqrt{30}c_8 + 2\sqrt{5}b_8))$	$\frac{1}{2\sqrt{3}}(E_1 - E_2 + E_3 - 2P_u - T)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \eta_8)$	$\frac{1}{24}(-2\sqrt{3}a_{10} - 3\sqrt{6}a_{27})$	$-\frac{1}{6}(E_1 + E_2 + E_3 + T)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \pi^+)$	$\frac{1}{60}(5\sqrt{6}a_{10} - 3\sqrt{3}a_{27} + 2\sqrt{3}a_8 + 6\sqrt{5}c_8 + 2\sqrt{30}b_8)$	$\frac{1}{\sqrt{2}}(E_2 + P_u)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \bar{K}^0)$	$\frac{1}{60}(10a_{10} + 9\sqrt{2}a_{27} - \sqrt{2}a_8 - \sqrt{30}c_8 - 2\sqrt{5}b_8)$	$\frac{1}{2\sqrt{3}}(E_1 + E_3 - P_u)$
$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^+)$	$\frac{1}{60}(5\sqrt{2}a_{10} - 3a_{27} + 2(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8))$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-)$	$\frac{1}{60}(-5\sqrt{2}a_{10} + 15a_{27} + 10a_8 + 2\sqrt{15}c_8 - 2\sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}(-E_3 + P_u + T)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0)$	$\frac{1}{60}(5\sqrt{2}a_{10} + 15a_{27} - 10a_8 - 2\sqrt{15}c_8 + 2\sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}(E_3 - P_u)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-)$	$\frac{1}{60}(-5\sqrt{2}a_{10} - 3a_{27} - 8a_8 + 4\sqrt{10}b_8)$	$-\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \pi^0)$	$\frac{1}{60}(-12a_{27} + 3a_8 - \sqrt{15}c_8 - 3\sqrt{10}b_8)$	$\frac{1}{2\sqrt{6}}(E_1 - E_2 - P_u - T)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \eta_8)$	$\frac{1}{60}(-5\sqrt{3}a_8 - 3\sqrt{5}c_8 + \sqrt{30}b_8)$	$-\frac{1}{6\sqrt{2}}(E_1 + E_2 - 2E_3 + 3P_u + T)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+)$	$\frac{1}{60}(5\sqrt{2}a_{10} - 3a_{27} + 2(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8))$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^0)$	$\frac{1}{60}(5\sqrt{2}a_{10} + 3a_{27} + 8a_8 - 4\sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}E_1$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^+)$	$\frac{1}{60}(5\sqrt{2}a_{10} - 3a_{27} + 2(a_8 + \sqrt{15}c_8 + \sqrt{10}b_8))$	$\frac{1}{\sqrt{6}}(E_2 + P_u)$
$\mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^-)$	$\frac{1}{30}(12a_{27} - 3a_8 + \sqrt{15}c_8 - \sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \rightarrow \Delta'^- \bar{K}^0)$	$\frac{1}{30}(3\sqrt{3}a_{27} + 3\sqrt{3}a_8 - 3\sqrt{5}c_8 + \sqrt{30}b_8)$	$-\frac{1}{\sqrt{2}}P_u$
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \pi^-)$	$\frac{1}{60}(-12\sqrt{2}a_{27} + 3\sqrt{2}a_8 - \sqrt{30}c_8 + 2\sqrt{5}b_8)$	$-\frac{1}{2\sqrt{3}}(P_u + T)$
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \pi^0)$	$\frac{1}{60}(-3\sqrt{2}a_{27} - 3\sqrt{2}a_8 + \sqrt{30}c_8 - 2\sqrt{5}b_8)$	$\frac{1}{2\sqrt{3}}P_u$
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \eta_8)$	$\frac{1}{60}(3\sqrt{6}a_{27} + 3\sqrt{6}a_8 - 3\sqrt{10}c_8 + 2\sqrt{15}b_8)$	$-\frac{1}{2}P_u$
$\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^0)$	$\frac{1}{30}(-3a_{27} - 3a_8 + \sqrt{15}c_8 - \sqrt{10}b_8)$	$\frac{1}{\sqrt{6}}P_u$

An alternate description of decay amplitudes is obtained in terms of topological quark diagrams. The symmetry properties of the final state decuplet baryons allow five possible diagrams starting from a flavor antitriplet b -baryon, whose light quarks are in flavor antisymmetric states. Those five independent topologies consist of three W exchanges (E_1 , E_2 , E_3), one tree (T), and a penguinlike (P_q) (q being the flavor of the quark going in the loop) amplitude, where the marked quarks are antisymmetrized in the initial state baryon. The sixth diagram, marked as S , contributes exclusively to the decay modes containing the singlet η_1 meson. The mapping between the topological amplitudes and the $SU(3)$ -reduced amplitudes is given below,

$$\begin{aligned} c_8 &= -\frac{1}{8}(-\sqrt{10}E_1 + 3\sqrt{10}E_2 - 2\sqrt{10}E_3 \\ &\quad + 8\sqrt{10}P_u + 2\sqrt{10}T) \\ b_8 &= \frac{1}{4}(\sqrt{15}E_2 - \sqrt{15}E_1) \\ a_8 &= \frac{1}{24}(5\sqrt{6}E_1 + 5\sqrt{6}E_2 - 10\sqrt{6}E_3 + 2\sqrt{6}T) \\ a_{10} &= \frac{1}{3}(2\sqrt{3}E_1 + 2\sqrt{3}E_2 + 2\sqrt{3}E_3 - \sqrt{3}T) \\ a_{27} &= \sqrt{\frac{2}{3}}T. \end{aligned} \quad (16)$$

The topologies E_3 and T are sometimes ignored in accordance with the Korner-Pati-Woo theorem [97–108] based on the “ $\Delta I = 1/2$ ” rule, which suggests that the quark pair produced in weak interaction ending up in the same baryon must be antisymmetric in flavor space. However, the accuracy of such a statement is dependent on the modeling of the baryon-baryon transition and the exact $SU(3)$ flavor symmetry. The details of the arguments made to ignore E_3 and T are reproduced in Appendix D for completeness. We derive several interesting tests [Eqs. (D14)–(D16) of Appendix D] to verify if the contribution from the topological amplitudes E_3 and T are indeed suppressed. We have chosen to include the E_3 and T diagrams throughout the rest of this paper for generality.

The two approaches are equivalent and imply that individual topological amplitudes cannot be expressed in terms of a single $SU(3)$ -reduced amplitude and vice versa in these two basis. An exception to this observation is the T diagram which is expressible entirely in terms of a single $SU(3)$ -reduced amplitude a_{27} as is evident from Eq. (16). In context of the $\mathbf{10} \otimes \mathbf{1}$ transition, involving the singlet meson, it is worth mentioning that the only $SU(3)$ -reduced amplitude, given by $\langle 10 || \mathbf{15} || \bar{\mathbf{3}} \rangle$ needs to be counted independent of the five $SU(3)$ -reduced amplitude contributing to the $\mathbf{10} \otimes \mathbf{8}$ processes. The flavor-flow diagram S is expressed in terms of that single $SU(3)$ -reduced amplitude. It is also important to note that the topology, P_u , originates

purely from the tree operators, i.e., O_1 and O_2 as emphasized in [23–27], even though it is denoted by P_u and referred to as a penguin topology. Moreover, the W -exchange topologies being a four-quark treelike structure also contribute to the tree amplitudes.

The transition induced by QCD penguin operators, given in Eq. (12) is only a $\mathbf{3}$ under $SU(3)$ and the corresponding $SU(3)$ -reduced amplitude is identified with the “penguin” amplitude P_t ,

$$\begin{aligned} P_t &= -\sqrt{2}(C_3 + C_4 + C_5 + C_6)\langle 8 || \mathbf{3}^{(6)} || \bar{\mathbf{3}} \rangle \\ &\quad + (C_3 - C_4 + C_5 - C_6)\langle 8 || \mathbf{3}_0^{(3)} || \bar{\mathbf{3}} \rangle. \end{aligned} \quad (17)$$

A. Relating EWP and tree amplitudes

We begin this section by noting that the $\mathbf{15}$ and $\bar{\mathbf{6}}$ part of the Hamiltonian described in Eqs. (11) and (13) relate the contributions to the decay from the tree and EWP operators [16,22,30], respectively, so as to effectively obey the following relations:

$$\begin{aligned} \mathcal{H}_{15}^{\text{EWP}}(\Delta S) &= -\frac{3}{2} \frac{\lambda_t^{s(d)}(C_9 + C_{10})}{\lambda_u^{s(d)}(C_1 + C_2)} \mathcal{H}_{15}^T(\Delta S) \\ \mathcal{H}_6^{\text{EWP}}(\Delta S) &= -\frac{3}{2} \frac{\lambda_t^{s(d)}(C_9 - C_{10})}{\lambda_u^{s(d)}(C_1 - C_2)} \mathcal{H}_6^T(\Delta S). \end{aligned} \quad (18)$$

These relations are valid independently for both $\Delta S = \{0, -1\}$ decays and remain unaffected by the QCD penguin operators since O_3 and O_4 transforms entirely as a $\mathbf{3}$ under $SU(3)$. *A priori*, the $\mathbf{3}^{(6)}$ and $\mathbf{3}^{(3)}$ operators do not follow a simple relation for arbitrary values of C_1 , C_2 , C_9 , and C_{10} . Since the set of $SU(3)$ -reduced elements remains the same, in analogy to Eq. (15) one can define

$$\begin{aligned} c_8^{\text{EW}} &= (C_9 + C_{10})\langle 8 || \mathbf{3}^{(6)} || \bar{\mathbf{3}} \rangle + \sqrt{2}(C_9 - C_{10})\langle 8 || \mathbf{3}^{(3)} || \bar{\mathbf{3}} \rangle \\ b_8^{\text{EW}} &= -\frac{3}{2}(C_9 - C_{10})\langle 8 || \bar{\mathbf{6}} || \bar{\mathbf{3}} \rangle \\ a_8^{\text{EW}} &= -\frac{3}{2}(C_9 + C_{10})\langle 8 || \mathbf{15} || \bar{\mathbf{3}} \rangle \\ a_{10}^{\text{EW}} &= -\frac{3}{2}(C_9 + C_{10})\langle 10 || \mathbf{15} || \bar{\mathbf{3}} \rangle \\ a_{27}^{\text{EW}} &= -\frac{3}{2}(C_9 + C_{10})\langle 27 || \mathbf{15} || \bar{\mathbf{3}} \rangle. \end{aligned} \quad (19)$$

The EWP reduced amplitudes b_8^{EW} , a_8^{EW} , a_{10}^{EW} , and a_{27}^{EW} are expressed in terms of b_8 , a_8 , a_{10} , and a_{27} defined in Eq. (15),

FIG. 1. Topological diagrams contributing to tree amplitude for antitriplet b -baryon hadronic decays. The first five diagrams contribute to the $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10})P(\mathbf{8})$ processes. The last diagram contributes exclusively to $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10})P(\mathbf{1})$ processes.

$$\begin{aligned} b_8^{\text{EW}} &= -\frac{3C_9 - C_{10}}{2C_1 - C_2} b_8, & a_8^{\text{EW}} &= -\frac{3C_9 + C_{10}}{2C_1 + C_2} a_8, \\ a_{10}^{\text{EW}} &= -\frac{3C_9 + C_{10}}{2C_1 + C_2} a_{10}, & a_{27}^{\text{EW}} &= -\frac{3C_9 + C_{10}}{2C_1 + C_2} a_{27}. \end{aligned} \quad (20)$$

Using numerical values of the Wilson coefficients to leading logarithmic order one obtains

$$\frac{C_9 + C_{10}}{C_1 + C_2} = -1.139\alpha, \quad \frac{C_9 - C_{10}}{C_1 - C_2} = -1.107\alpha. \quad (21)$$

To a good approximation [22], these two ratios of Wilson coefficients can be taken to be a common value [95,96] given by κ ,

$$\kappa = \frac{C_9 + C_{10}}{C_1 + C_2} = \frac{C_9 - C_{10}}{C_1 - C_2} \simeq -1.12\alpha. \quad (22)$$

With this additional assumption, Eq. (20) implies that the $b_8^{\text{EW}}, a_8^{\text{EW}}, a_{10}^{\text{EW}}$, and a_{27}^{EW} are proportional to the tree reduced amplitudes b_8, a_8, a_{10} , and a_{27} ,

$$\begin{aligned} b_8^{\text{EW}} &= -\frac{3}{2}\kappa b_8, & a_8^{\text{EW}} &= -\frac{3}{2}\kappa a_8, \\ a_{10}^{\text{EW}} &= -\frac{3}{2}\kappa a_{10}, & a_{27}^{\text{EW}} &= -\frac{3}{2}\kappa a_{27}. \end{aligned} \quad (23)$$

The equivalence of the $SU(3)$ -reduced amplitudes to topological diagrams allows one to interpret P_i^{EW} as electroweak quark diagrams with one insertion of the electroweak penguin operator,

$$P_{E_i}^{\text{EW}} = -\frac{3}{2}\kappa E_i \quad (24)$$

$$P_T^{\text{EW}} = -\frac{3}{2}\kappa T, \quad (25)$$

where, $i = \{1, 2, 3\}$.

As mentioned earlier, there is no simple relation between the $\mathbf{3}$ part of the EWP Hamiltonian to the tree part. There are, however, decays where the $\mathbf{3}$ part of the Hamiltonian cannot contribute to the formation of final states, which require a pure $\Delta I = 1$ or $\Delta I = 3/2$ transition. For example, the decay $\Lambda_b^0 \rightarrow \Delta^+ K^-$ receives contribution from the $\bar{\mathbf{6}}$ and $\mathbf{15}$ part of the effective Hamiltonian, and in this particular case, the ratio of EWP and tree contributions is given entirely by the simple ratio $-3/2\kappa(\lambda_t^s/\lambda_u^s)$. The $SU(3)$ -reduced amplitudes for the penguin operators are provided in Eq. (B3) and Eq. (B4).

III. AMPLITUDE RELATIONS

The complete decay amplitude is given in terms of tree and penguin $SU(3)$ -reduced amplitudes and the CKM elements,

$$\mathcal{A} = \lambda_u^q \mathcal{A}_{\text{tree}} + \lambda_t^q \mathcal{A}_{\text{penguin}}, \quad (26)$$

where $q = s, d$ denote the $\Delta S = -1, 0$ process. Since the $SU(3)$ operators appear in EW and the tree part of the Hamiltonian in a particular combination, the same amplitude relations between $\Delta S = -1$ and $\Delta S = 0$ processes are satisfied by the EWP part and the tree part. Moreover, the decay products of a spin-1/2 particle decaying to a

spin-3/2 and a spin-0 particle can be in relative angular momentum $l = 1$ or $l = 2$ state. The decay amplitude is further decomposed into P-wave and D-wave parts,

$$\mathcal{A}^P = \lambda_u^q \mathcal{A}_{\text{tree}}^P + \lambda_t^q \mathcal{A}_{\text{penguin}}^P, \quad (27)$$

$$\mathcal{A}^D = \lambda_u^q \mathcal{A}_{\text{tree}}^D + \lambda_t^q \mathcal{A}_{\text{penguin}}^D. \quad (28)$$

Finally, the following decay amplitude relations are obtained for both the P-wave and D-wave parts:

$$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) = \mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-), \quad (29)$$

$$\mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) = \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0), \quad (30)$$

$$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-), \quad (31)$$

$$\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) = \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0), \quad (32)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) &= \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) \\ &= -\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ &= -\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^0), \end{aligned} \quad (33)$$

$$\begin{aligned} \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^-) &= -\frac{1}{\sqrt{2}} \mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \pi^-) \\ &= \frac{1}{\sqrt{2}} \mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^-) \\ &= -\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \pi^-), \end{aligned} \quad (34)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) &= \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^+) \\ &= \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+) = \frac{1}{\sqrt{3}} \mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^+) \\ &= \frac{1}{\sqrt{3}} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^+) \\ &= \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) = \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^+), \end{aligned} \quad (35)$$

$$\begin{aligned} \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0) &= -\sqrt{2} \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \pi^0) \\ &= \sqrt{\frac{2}{3}} \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \eta_8) \\ &= -\frac{1}{\sqrt{3}} \mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^0) \\ &= \frac{1}{\sqrt{3}} \mathcal{A}(\Xi_b^- \rightarrow \Delta'^- \bar{K}^0) \\ &= \sqrt{\frac{2}{3}} \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \eta_8) = -\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^0) \\ &= -\sqrt{2} \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \pi^0). \end{aligned} \quad (36)$$

The $\Delta S = -1$ triangle relations are given below,

$$\begin{aligned} 2\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) \\ + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) = 0, \end{aligned} \quad (37)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ - \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-) = 0, \end{aligned} \quad (38)$$

$$\begin{aligned} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0) - \frac{1}{\sqrt{2}} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) \\ - \frac{1}{\sqrt{2}} \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) = 0, \end{aligned} \quad (39)$$

$$\begin{aligned} \sqrt{2} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) - \frac{1}{\sqrt{2}} \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) \\ - \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0) = 0, \end{aligned} \quad (40)$$

$$\begin{aligned} \sqrt{2} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \eta_8) + \sqrt{\frac{3}{2}} \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) \\ - \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \eta_8) = 0, \end{aligned} \quad (41)$$

$$\begin{aligned} \frac{2}{\sqrt{3}} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0) - \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \eta_8) \\ + \frac{1}{\sqrt{3}} \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0) = 0, \end{aligned} \quad (42)$$

$$\begin{aligned} 2\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ + \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+) = 0, \end{aligned} \quad (43)$$

$$\begin{aligned} 2\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ + \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^+) = 0, \end{aligned} \quad (44)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) - \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ + \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-) = 0, \end{aligned} \quad (45)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) - \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) \\ + \sqrt{2} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0) = 0, \end{aligned} \quad (46)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) + \frac{1}{2} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^+ \pi^-) \\ + \frac{1}{2\sqrt{3}} \mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^+) = 0. \end{aligned} \quad (47)$$

The $\Delta S = 0$ triangle relations are also obtained,

$$\begin{aligned} \sqrt{6} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \pi^+) \\ + \sqrt{3} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = 0, \end{aligned} \quad (48)$$

$$\begin{aligned} \sqrt{3} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \eta_8) - \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) \\ + 2\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) = 0, \end{aligned} \quad (49)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) - \mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-) \\ + \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) = 0, \end{aligned} \quad (50)$$

$$\begin{aligned} \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) - \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ + \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) = 0, \end{aligned} \quad (51)$$

$$\begin{aligned} \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) - \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ - 2\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \pi^0) = 0, \end{aligned} \quad (52)$$

$$\begin{aligned} \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \eta_8) + \sqrt{3}\mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ - 2\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \eta_8) = 0, \end{aligned} \quad (53)$$

$$\begin{aligned} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) + \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) \\ + \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = 0, \end{aligned} \quad (54)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) - \mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-) \\ + \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^0) = 0, \end{aligned} \quad (55)$$

$$\begin{aligned} \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) - \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ - \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^0) = 0, \end{aligned} \quad (56)$$

$$\begin{aligned} \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^+) + \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) \\ + \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = 0, \end{aligned} \quad (57)$$

$$\begin{aligned} \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) \\ + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^+) = 0. \end{aligned} \quad (58)$$

IV. $SU(3)$ BREAKING EFFECTS

The $SU(3)$ structure of the unbroken Hamiltonian is modified by this term and to the first order in strange quark

mass; the broken Hamiltonian is made of the following $SU(3)$ representations [19,52,54,56,61,109–111]:

$$\begin{aligned} (3 \oplus \bar{6} \oplus 15) \otimes (1 + \epsilon 8) \\ = (3 \oplus \bar{6} \oplus 15) + \epsilon(3_i \oplus \bar{6}_i \oplus 15_1 \oplus 15_2 \oplus 15_3^1 \\ \oplus 15_3^2 \oplus 15' \oplus 24_2 \oplus 24_3 \oplus 42), \end{aligned}$$

where the subscript $i = 1, 2, 3$ indicates the origin of that representation coming from $3, \bar{6}, 15$ of the unbroken Hamiltonian, respectively, the superscript is for multiplicities of a particular representation, and ϵ is the $SU(3)$ -breaking parameter. $SU(3)$ breaking effects will induce higher $SU(3)$ representations, and some of the amplitude relations will cease to hold as there are now thirteen independent $SU(3)$ -reduced matrix elements. The isospin relation,

$$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) = -\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-), \quad (59)$$

and isospin triangle relations, for $\Delta S = -1$,

$$\begin{aligned} 2\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) \\ + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) = 0, \end{aligned} \quad (60)$$

and for $\Delta S = 0$,

$$\begin{aligned} \sqrt{6}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \pi^+) \\ + \sqrt{3}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = 0, \end{aligned} \quad (61)$$

continue to hold. In addition, arbitrary $SU(3)$ -breaking but isospin conserving effects still forbid $\Delta I = 2$ and $\Delta I = \frac{5}{2}$ transitions, which results in general amplitude sum rules,

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) + 2\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) + \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ - \sqrt{2}\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0) - \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-) - \sqrt{2}\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0) - \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+) \\ + \sqrt{2}\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^-) + \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0) + \mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \pi^-) + \sqrt{2}\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \pi^0) = 0 \end{aligned} \quad (62)$$

$$\begin{aligned} 2\sqrt{3}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) + \sqrt{2}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \pi^+) + \sqrt{6}\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) - \sqrt{6}\mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) - \sqrt{6}\mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-) \\ - 2\sqrt{6}\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \pi^0) - \sqrt{6}\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) - \sqrt{6}\mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) + \sqrt{6}\mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^-) \\ + \sqrt{2}\mathcal{A}(\Xi_b^- \rightarrow \Delta^- \bar{K}^0) + 2\sqrt{3}\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \pi^-) + \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \pi^0) = 0. \end{aligned} \quad (63)$$

V. CP RELATIONS

The general decay amplitude for a spin-1/2 b -baryon (B) to a spin 0 pseudoscalar (P) and a spin-3/2 (D) is given by

$$\mathcal{M} = -iq_\mu \bar{u}_D^\mu (a + b\gamma_5) u_B, \quad (64)$$

where $\bar{u}_D^\mu$ is the Rarita-Schwinger spinor for the spin-3/2 decuplet baryon, q_μ is the momentum of the pseudoscalar meson, and u_B is the spinor for the initial spin-1/2 b baryon. The two coefficients a and b contain the CKM elements as well as the same flavor structure as $\mathcal{A}_{\text{tree}}$ and $\mathcal{A}_{\text{penguin}}$. The total decay rate for an unpolarized b baryon described by the decay amplitude of the form in Eq. (64) is given by

$$\Gamma(\mathcal{B} \rightarrow \mathcal{DP}) = \frac{|\mathbf{p}_D|}{8\pi m_B^2} \left(\frac{2m_B^2}{3m_D^2} 2m_B(E_D + m_D) |\mathbf{p}_D|^2 \right) \times \left(|a|^2 + \frac{|\mathbf{p}_D|^2}{(E_D + m_D)^2} |b|^2 \right), \quad (65)$$

where $|\mathbf{p}_D|$ is the magnitude of the three-momentum of the decuplet baryon in the rest frame of the initial b baryon. The decay products can be in any one of the two possible relative angular momentum states, $l = 1$ and $l = 2$ identified as **P** wave and **D** wave, respectively. The kinematic factors for **P** wave and **D** wave are given by

$$P = \sqrt{\frac{2}{3}} \frac{|\mathbf{p}_D|}{m_D} \sqrt{2m_B^3(E_D + m_D)} \mathcal{A}^P, \quad (66)$$

$$D = \sqrt{\frac{2}{3}} \frac{|\mathbf{p}_D|}{m_D} \sqrt{2m_B^3(E_D - m_D)} \mathcal{A}^D, \quad (67)$$

where $\mathcal{A}^P$ and $\mathcal{A}^D$ are defined in Eq. (27) and Eq. (28). In terms of the **P**-wave and **D**-wave contributions introduced, the total decay rate Γ is

$$\Gamma(\mathcal{B} \rightarrow \mathcal{DP}) = \frac{|\mathbf{p}_D|}{8\pi m_B^2} (|P|^2 + |D|^2) \quad (68)$$

$$= \frac{m_B(E_D + m_D) |\mathbf{p}_D|^3}{6\pi m_D^2} \times \left(|\mathcal{A}^P|^2 + \frac{|\mathbf{p}_D|^2}{(E_D + m_D)^2} |\mathcal{A}^D|^2 \right). \quad (69)$$

A_{CP} is defined [110] subsequently as

$$A_{CP} = \frac{\Gamma(\mathcal{B} \rightarrow \mathcal{DP}) - \Gamma(\bar{\mathcal{B}} \rightarrow \bar{\mathcal{D}}\bar{\mathcal{P}})}{\Gamma(\mathcal{B} \rightarrow \mathcal{DP}) + \Gamma(\bar{\mathcal{B}} \rightarrow \bar{\mathcal{D}}\bar{\mathcal{P}})} = \frac{\Delta_{CP}(\mathcal{B} \rightarrow \mathcal{DP})}{2\tilde{\Gamma}(\mathcal{B} \rightarrow \mathcal{DP})}, \quad (70)$$

where

$$\tilde{\Gamma}(\mathcal{B} \rightarrow \mathcal{DP}) = \frac{1}{2} (\Gamma(\mathcal{B} \rightarrow \mathcal{DP}) + \Gamma(\bar{\mathcal{B}} \rightarrow \bar{\mathcal{D}}\bar{\mathcal{P}})).$$

A_{CP} is the sum of CP violating contribution from the δ_{CP}^P and δ_{CP}^D with the appropriate phase-space factor multiplied,

$$A_{CP}(\mathcal{B} \rightarrow \mathcal{DP}) = \frac{\tau_B}{\mathcal{BR}(\mathcal{B} \rightarrow \mathcal{DP})} \frac{m_B(E_D + m_D) |\mathbf{p}_D|^3}{6\pi m_D^2} \times \left(\delta_{CP}^P + \frac{|\mathbf{p}_D|^2}{(E_D + m_D)^2} \delta_{CP}^D \right), \quad (71)$$

where τ_B is the lifetime of the beauty baryon.

By definition,

$$\delta_{CP}^l(\mathcal{B} \rightarrow \mathcal{DP}) = -4\mathbf{J}\text{Im}[\mathcal{A}_{\text{tree}}^{l*}(\mathcal{B} \rightarrow \mathcal{DP})\mathcal{A}_{\text{penguin}}^l(\mathcal{B} \rightarrow \mathcal{DP})], \quad (72)$$

J being the well-known Jarlskog invariant. Based on the amplitude relations for the tree and penguin parts obtained in Eq. (29), the following δ_{CP}^a relations [112–114] are obtained:

$$\delta^l(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) = -\delta^l(\Xi_b^0 \rightarrow \Delta^+ K^-), \quad (73)$$

$$\delta^l(\Lambda_b^0 \rightarrow \Xi'^0 K^0) = -\delta^l(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0), \quad (74)$$

$$\delta^l(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = -\delta^l(\Xi_b^0 \rightarrow \Sigma'^+ K^-), \quad (75)$$

$$\delta^l(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) = -\delta^l(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0), \quad (76)$$

$$\begin{aligned} \delta^l(\Lambda_b^0 \rightarrow \Delta^+ K^-) &= -\delta^l(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) \\ &= \delta^l(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ &= -\delta^l(\Xi_b^0 \rightarrow \Xi'^0 K^0), \end{aligned} \quad (77)$$

$$\begin{aligned} \delta^l(\Xi_b^- \rightarrow \Sigma'^0 K^-) &= \frac{1}{2} \delta^l(\Xi_b^- \rightarrow \Xi'^0 \pi^-) \\ &= -\frac{1}{2} \delta^l(\Xi_b^- \rightarrow \Delta^0 K^-) \\ &= -\delta^l(\Xi_b^- \rightarrow \Sigma'^0 \pi^-), \end{aligned} \quad (78)$$

$$\begin{aligned} \delta^l(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) &= \delta^l(\Lambda_b^0 \rightarrow \Xi'^- K^+) \\ &= \delta^l(\Xi_b^0 \rightarrow \Xi'^- \pi^+) = \frac{1}{3} \delta^l(\Xi_b^0 \rightarrow \Omega^- K^+) \\ &= -\frac{1}{3} \delta^l(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = -\delta^l(\Lambda_b^0 \rightarrow \Sigma'^- K^+) \\ &= -\delta^l(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) = -\delta^l(\Xi_b^0 \rightarrow \Xi'^- K^+), \end{aligned} \quad (79)$$

$$\begin{aligned} \delta^l(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0) &= 2\delta^l(\Xi_b^- \rightarrow \Xi'^- \pi^0) \\ &= \frac{2}{3} \delta^l(\Xi_b^- \rightarrow \Xi'^- \eta_8) = \frac{1}{3} \delta^l(\Xi_b^- \rightarrow \Omega^- K^0) \\ &= -\frac{1}{3} \delta^l(\Xi_b^- \rightarrow \Delta'^- \bar{K}^0) = -\frac{2}{3} \delta^l(\Xi_b^- \rightarrow \Sigma'^- \eta_8) \\ &= -\delta^l(\Xi_b^- \rightarrow \Xi'^- K^0) = -2\delta^l(\Xi_b^- \rightarrow \Sigma'^- \pi^0), \end{aligned} \quad (80)$$

for both $l = \mathbf{P}$ and $l = \mathbf{D}$. Since, A_{CP} depends on the masses of the initial and final baryons as well as the final state meson [71,114], some approximation is needed to obtain the A_{CP} relations between various modes. In the U -spin limit [115], by ignoring $\mathbf{p}_D$ and m_D differences between such modes, CP violation relations can be experimentally verified using the relation [50,86,115],

$$\frac{A_{CP}(\mathcal{B}_i \rightarrow \mathcal{D}_j \mathcal{P}_k)}{A_{CP}(\mathcal{B}_l \rightarrow \mathcal{D}_m \mathcal{P}_n)} \simeq -\frac{\tau_{\mathcal{B}_i}}{\tau_{\mathcal{B}_l}} \frac{\mathcal{BR}(\mathcal{B}_l \rightarrow \mathcal{D}_m \mathcal{P}_n)}{\mathcal{BR}(\mathcal{B}_i \rightarrow \mathcal{D}_j \mathcal{P}_k)}, \quad (81)$$

where i, j, k , and l, m, n are indices corresponding to the various hadron belonging to the above mentioned δ_{CP} relations. There is a further simplification in case $i = l$, resulting in

$$\frac{A_{CP}(\mathcal{B}_i \rightarrow \mathcal{D}_j \mathcal{P}_k)}{A_{CP}(\mathcal{B}_i \rightarrow \mathcal{D}_m \mathcal{P}_n)} \simeq -\frac{\mathcal{BR}(\mathcal{B}_i \rightarrow \mathcal{D}_m \mathcal{P}_n)}{\mathcal{BR}(\mathcal{B}_i \rightarrow \mathcal{D}_j \mathcal{P}_k)}, \quad (82)$$

where the uncertainties due to lifetime measurement also cancel out [67]. The decay asymmetry parameter (α) can be measured from an angular distribution study of the final states provided that the subsequent decay of the decuplet baryon is parity violating. The relative strength of the D-wave contribution [72,114] is extracted from α ,

$$\alpha = \frac{2\text{Re}(\mathcal{A}^{P*} \mathcal{A}^D) |\mathbf{p}_D|/E_D + m_D}{|\mathcal{A}^P|^2 + |\mathcal{A}^D|^2 (|\mathbf{p}_D|/E_D + m_D)^2}. \quad (83)$$

By systematically taking into account both partial wave contributions, a reliable prediction for A_{CP} relations is possible.

VI. CONCLUSION

We have explored an hadronic antitriplet ($\bar{\mathbf{3}}$) beauty baryon into a decuplet baryon (B) and an octet pseudoscalar meson (P), i.e., $B_b(\bar{\mathbf{3}}) \rightarrow B(\mathbf{10})P(\mathbf{8})$, based on the $SU(3)$ decomposition of the decay amplitudes in a general framework. This extends our previous analysis [88] of the antitriplet beauty baryon decays into the octet or singlet of a light baryon and a pseudoscalar meson and completes the application of the method to decays

involving any noncharmed baryon. We have shown that in the most general case, the forty distinct decay modes require forty independent reduced $SU(3)$ amplitudes to describe all possible $\Delta S = -1$ and $\Delta S = 0$ processes. The dimension-6 effective Hamiltonian and allowed final state $SU(3)$ representations constrain the number of independent $SU(3)$ -reduced matrix elements to five. An alternative approach in terms of quark diagrams is also provided and compared with the $SU(3)$ decomposition in the limit of an exact $SU(3)$ -flavor symmetry. We explicitly demonstrate a one to one correspondence between the quark diagrams and $SU(3)$ -reduced matrix elements. Both the approaches indicate that there exist several amplitude relations between different decay modes. We explicitly derive those sum rules relations between decay amplitudes as well as relations between CP asymmetries. We further probe the $SU(3)$ -breaking effects in the decay amplitudes at leading order in the $SU(3)$ -breaking parameter and identify those amplitude relations that survive even when the $SU(3)$ flavor symmetry is no longer exact.

APPENDIX A

The $SU(3)$ decomposition of $\Delta S = -1$ processes for a generic Hamiltonian without any dynamical assumptions,

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 \bar{K}^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Omega'^- K^+) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \pi^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Omega'^- \bar{K}^0) \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{15}} & \sqrt{\frac{2}{15}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{6\sqrt{5}} & \frac{1}{6\sqrt{5}} & \frac{1}{2} & \frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{30}} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{21}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\frac{2}{15}} & -\sqrt{\frac{2}{15}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{6\sqrt{5}} & \frac{1}{2} & -\frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{30}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{21}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{\sqrt{30}} & -\frac{1}{\sqrt{30}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{6} & -\frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{30}} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{21}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{2}{3\sqrt{5}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{3} & 0 & -\frac{1}{3\sqrt{2}} & 0 & \sqrt{\frac{1}{10}} & 0 & \frac{1}{\sqrt{5}} & 0 & \frac{1}{2\sqrt{3}} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{2}{3\sqrt{5}} & 0 & 0 \\ -\frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & 0 & 0 & \frac{1}{15} & \frac{1}{15} & 0 & \frac{1}{\sqrt{6}} & 0 & \frac{1}{10} & 0 & \frac{3}{35} & 0 & \frac{1}{\sqrt{7}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & -\frac{1}{6} & \frac{1}{2\sqrt{2}} & -\frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{30}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{21}} & -\frac{1}{2\sqrt{5}} & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{2}{3\sqrt{5}} & 0 & 0 \\ \frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{3\sqrt{5}} & -\frac{1}{3\sqrt{5}} & 0 & -\frac{1}{3\sqrt{2}} & 0 & \frac{1}{\sqrt{30}} & 0 & \sqrt{\frac{1}{35}} & 0 & \frac{1}{2\sqrt{21}} & \frac{1}{2\sqrt{3}} & \frac{\sqrt{2}}{2} & -\frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{5}} & 0 & 0 \\ \frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{3\sqrt{5}} & -\frac{1}{3\sqrt{5}} & 0 & -\frac{1}{3\sqrt{2}} & 0 & \frac{1}{\sqrt{30}} & 0 & \sqrt{\frac{1}{35}} & 0 & \frac{1}{2\sqrt{21}} & -\frac{1}{2\sqrt{3}} & -\frac{\sqrt{2}}{2} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{5}} & 0 & 0 \\ -\frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{6} & 0 & -\frac{1}{3\sqrt{2}} & \sqrt{\frac{1}{15}} & \frac{1}{\sqrt{30}} & -\frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{21}} & -\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{15}} & -\frac{3}{\sqrt{10}} & \frac{1}{6\sqrt{5}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} \\ \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{7}{6\sqrt{10}} & \frac{1}{3\sqrt{10}} & \frac{1}{3\sqrt{2}} & \frac{1}{6} & -\frac{1}{3} & \frac{1}{2\sqrt{15}} & \frac{1}{\sqrt{15}} & \sqrt{\frac{3}{70}} & -\frac{1}{\sqrt{10}} & -\frac{5}{2\sqrt{42}} & \frac{1}{2\sqrt{10}} & -\frac{1}{2\sqrt{30}} & \frac{1}{6\sqrt{5}} & -\frac{6}{\sqrt{10}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{6}} \\ \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{15}} & \frac{1}{4\sqrt{6}} & -\frac{1}{4\sqrt{6}} & \frac{11}{12\sqrt{10}} & -\frac{2\sqrt{3}}{3} & -\frac{1}{3\sqrt{2}} & \frac{1}{12} & \frac{1}{3} & \frac{\sqrt{3}}{4} & \frac{1}{\sqrt{15}} & 0 & -\frac{1}{\sqrt{10}} & -\frac{\sqrt{2}}{4} & \frac{1}{2\sqrt{10}} & -\frac{1}{2\sqrt{30}} & \frac{7}{12\sqrt{5}} & -\frac{7}{12\sqrt{10}} & \frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{6}} \\ \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{30}} & \frac{1}{\sqrt{30}} & 0 & -\frac{1}{4\sqrt{3}} & 0 & \frac{1}{4\sqrt{3}} & 0 & -\frac{3}{\sqrt{70}} & 0 & \frac{3}{4\sqrt{14}} & \frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{10}} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{1}{4} & \frac{3}{4\sqrt{2}} \\ -\frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{3}{4\sqrt{5}} & \frac{1}{2\sqrt{5}} & -\frac{1}{6} & -\frac{1}{2\sqrt{2}} & \frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{30}} & \frac{1}{\sqrt{30}} & \frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{5}} & -\frac{5}{4\sqrt{21}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{15}} & -\frac{7}{12\sqrt{5}} & \frac{7}{12\sqrt{10}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{4\sqrt{3}} \\ -\frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4\sqrt{15}} & -\frac{1}{\sqrt{15}} & 0 & \frac{1}{2\sqrt{6}} & 0 & \frac{1}{2\sqrt{10}} & 0 & -\frac{1}{35} & 0 & \frac{1}{4\sqrt{7}} & -\frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{4} & -\frac{1}{2\sqrt{2}} & \frac{1}{4} \\ \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{6}} & \frac{1}{2\sqrt{6}} & -\frac{7}{6\sqrt{10}} & \frac{1}{3\sqrt{10}} & -\frac{1}{3\sqrt{2}} & \frac{1}{6} & \frac{1}{3} & \frac{1}{2\sqrt{15}} & -\frac{1}{\sqrt{15}} & \sqrt{\frac{3}{70}} & \frac{1}{\sqrt{10}} & -\frac{5}{2\sqrt{42}} & -\frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{30}} & -\frac{1}{6\sqrt{5}} & \frac{6}{\sqrt{10}} & \frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{6}} \\ -\frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{5}} & \frac{1}{2\sqrt{5}} & -\frac{1}{6} & 0 & \frac{1}{3\sqrt{2}} & \sqrt{\frac{1}{15}} & -\frac{1}{\sqrt{30}} & -\frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{21}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{15}} & \frac{1}{3\sqrt{10}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} \\ -\frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & -\frac{1}{4\sqrt{3}} & \frac{1}{4\sqrt{3}} & \frac{3}{4\sqrt{5}} & \frac{1}{2\sqrt{5}} & \frac{1}{6} & -\frac{1}{2\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{30}} & -\frac{1}{\sqrt{30}} & \frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & -\frac{5}{4\sqrt{21}} & \frac{1}{2\sqrt{3}} & -\frac{1}{2\sqrt{15}} & \frac{7}{6\sqrt{10}} & -\frac{7}{12\sqrt{5}} & \frac{1}{2\sqrt{6}} & \frac{1}{4\sqrt{3}} \\ \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{15}} & \frac{1}{4\sqrt{6}} & -\frac{1}{4\sqrt{6}} & \frac{11}{12\sqrt{10}} & -\frac{2\sqrt{3}}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{12} & -\frac{1}{3} & \frac{\sqrt{3}}{4} & -\frac{1}{\sqrt{15}} & 0 & \frac{1}{\sqrt{10}} & -\frac{1}{4} & -\frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{30}} & -\frac{7}{12\sqrt{5}} & \frac{7}{12\sqrt{10}} & -\frac{1}{4\sqrt{3}} & -\frac{1}{4\sqrt{6}} \\ -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{30}} & -\frac{1}{\sqrt{30}} & 0 & \frac{1}{4\sqrt{3}} & 0 & -\frac{1}{4\sqrt{3}} & 0 & \frac{3}{\sqrt{70}} & 0 & -\frac{3}{4\sqrt{14}} & \frac{\sqrt{2}}{2} & -\frac{1}{2\sqrt{10}} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{1}{4} & \frac{3}{4\sqrt{2}} \\ \frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & \frac{1}{4} & -\frac{1}{4} & \frac{1}{4\sqrt{15}} & \frac{1}{\sqrt{15}} & 0 & -\frac{1}{2\sqrt{6}} & 0 & -\frac{1}{2\sqrt{10}} & 0 & \frac{1}{\sqrt{35}} & 0 & -\frac{1}{4\sqrt{7}} & -\frac{\sqrt{2}}{2} & \frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{4} & -\frac{1}{2\sqrt{2}} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \langle 8 || \mathbf{15}_1 || \bar{\mathbf{3}} \rangle \\ \langle 8 || \mathbf{6}_1 || \bar{\mathbf{3}} \rangle \\ \langle 10 || \mathbf{15}_1 || \bar{\mathbf{3}} \rangle \\ \langle 10 || \mathbf{15}'_1 || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{15}_1 || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{24}_1 || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{24}_2 || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_1 || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_2 || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{42}_1 || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{42}_2 || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{48}_1 || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{48}_2 || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{15}'_1 || \bar{\mathbf{3}} \rangle \\ \langle 8 || \mathbf{3}_0 || \bar{\mathbf{3}} \rangle \\ \langle 8 || \mathbf{15}_0 || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{15}_0 || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_0 || \bar{\mathbf{3}} \rangle \\ \langle 10 || \mathbf{15}_0 || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{42}_0 || \bar{\mathbf{3}} \rangle \end{pmatrix} \quad (A1)$$

The $SU(3)$ -decomposition of $\Delta S = 0$ processes for a generic Hamiltonian without any dynamical assumptions,

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \pi^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^0 K^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 K^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 \pi^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 K^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^+) \\ \mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Delta'^- K^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \pi^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- K^0) \end{pmatrix} = \begin{pmatrix} -\frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{\sqrt{30}} & \frac{1}{2} & \frac{1}{2\sqrt{3}} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{2}}{6} & -\frac{1}{3\sqrt{30}} & -\frac{\sqrt{2}}{3} & -\frac{\sqrt{3}}{3} & -\frac{\sqrt{2}}{6} & \frac{1}{10} & \frac{\sqrt{6}}{5} & \frac{\sqrt{2}}{5} & \frac{\sqrt{2}}{5} & \frac{1}{2\sqrt{21}} & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{15}} & -\frac{1}{4\sqrt{2}} & -\frac{1}{4\sqrt{6}} & \frac{2}{3\sqrt{15}} & \frac{\sqrt{2}}{12} & \frac{1}{3\sqrt{15}} & \frac{1}{6\sqrt{3}} & \frac{2}{3\sqrt{15}} & \frac{\sqrt{2}}{12} & -\frac{1}{20\sqrt{2}} & \frac{2\sqrt{3}}{5} & -\frac{1}{10\sqrt{7}} & \frac{\sqrt{3}}{5} & -\frac{1}{4\sqrt{42}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{4} & \frac{1}{4\sqrt{2}} & 0 & \frac{\sqrt{2}}{4} & 0 & \frac{1}{2} & 0 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & \frac{\sqrt{2}}{2} & 0 & \frac{5}{4\sqrt{14}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{1}{\sqrt{10}} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & -\frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{12} & -\frac{1}{3\sqrt{10}} & \frac{1}{3\sqrt{2}} & -\frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{12} & -\frac{\sqrt{3}}{20} & \frac{\sqrt{2}}{5} & -\frac{\sqrt{2}}{5} & \frac{1}{5\sqrt{2}} & -\frac{1}{4\sqrt{7}} & 0 & 0 & 0 & 0 \\ \frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{10}} & \frac{1}{2\sqrt{15}} & -\frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{6}} & -\frac{4}{3\sqrt{15}} & -\frac{\sqrt{2}}{6} & -\frac{2}{3\sqrt{15}} & -\frac{1}{3\sqrt{3}} & -\frac{4}{3\sqrt{15}} & -\frac{\sqrt{2}}{6} & \frac{1}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{7}} & 0 & \frac{5}{2\sqrt{42}} & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{5}} & -\frac{1}{2\sqrt{5}} & -\frac{1}{\sqrt{30}} & -\frac{1}{4} & -\frac{1}{4\sqrt{3}} & \frac{4\sqrt{2}}{3} & -\frac{\sqrt{2}}{12} & \frac{2\sqrt{2}}{3} & -\frac{1}{3\sqrt{6}} & \frac{4\sqrt{2}}{3} & -\frac{\sqrt{2}}{12} & \frac{1}{4} & 0 & \frac{1}{\sqrt{14}} & 0 & \frac{5}{4\sqrt{21}} & 0 & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{5}} & \frac{1}{6\sqrt{5}} & \frac{1}{\sqrt{30}} & \frac{1}{6} & -\frac{1}{2\sqrt{3}} & -\frac{1}{3\sqrt{30}} & -\frac{7}{6\sqrt{15}} & \frac{1}{3\sqrt{30}} & \frac{\sqrt{2}}{3} & \frac{1}{6\sqrt{30}} & -\frac{1}{6\sqrt{15}} & \frac{7}{30} & \frac{\sqrt{2}}{10} & -\frac{\sqrt{2}}{5} & -\frac{\sqrt{2}}{5} & -\frac{2}{2\sqrt{21}} & -\frac{2}{3} & \frac{1}{3\sqrt{2}} & -\frac{3}{2\sqrt{10}} & \frac{1}{6\sqrt{2}} \\ \frac{1}{2\sqrt{5}} & -\frac{1}{6\sqrt{5}} & -\frac{1}{\sqrt{30}} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & \frac{1}{3\sqrt{30}} & -\frac{4}{6\sqrt{15}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{6}} & -\frac{1}{6\sqrt{30}} & \frac{2}{3\sqrt{15}} & \frac{1}{15} & \frac{\sqrt{2}}{10} & \frac{3}{5\sqrt{14}} & -\frac{\sqrt{2}}{5} & -\frac{1}{\sqrt{21}} & \frac{2\sqrt{2}}{3} & -\frac{1}{3\sqrt{2}} & -\frac{3}{2\sqrt{10}} & -\frac{1}{6\sqrt{2}} \\ 0 & \frac{2}{3\sqrt{5}} & -\frac{\sqrt{2}}{15} & \frac{1}{6} & -\frac{1}{2\sqrt{3}} & \frac{1}{3\sqrt{30}} & \frac{1}{6\sqrt{15}} & -\frac{7}{3\sqrt{30}} & -\frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{6} & \frac{7}{6\sqrt{15}} & \frac{7}{30} & \frac{\sqrt{2}}{10} & -\frac{\sqrt{2}}{5} & -\frac{\sqrt{2}}{5} & -\frac{1}{2\sqrt{21}} & \frac{\sqrt{2}}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{10}} & \frac{1}{6\sqrt{2}} \\ \frac{1}{4\sqrt{5}} & -\frac{3}{4\sqrt{5}} & \frac{\sqrt{10}}{2} & 0 & 0 & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{6} & -\frac{2\sqrt{2}}{3} & -\frac{1}{3\sqrt{6}} & -\frac{2\sqrt{2}}{3} & -\frac{1}{6\sqrt{15}} & \frac{3}{10} & \frac{\sqrt{2}}{5} & \frac{1}{5\sqrt{14}} & -\frac{\sqrt{2}}{5} & -\frac{\sqrt{2}}{2} & 0 & 0 & \frac{1}{\sqrt{10}} & 0 \\ \frac{\sqrt{2}}{4} & -\frac{1}{4\sqrt{15}} & -\frac{1}{2\sqrt{10}} & 0 & 0 & -\frac{\sqrt{2}}{3} & \frac{1}{6\sqrt{5}} & \frac{\sqrt{2}}{3} & \frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{10}} & -\frac{\sqrt{2}}{6} & \frac{1}{2\sqrt{3}} & 0 & -\frac{\sqrt{2}}{14} & 0 & \frac{1}{2\sqrt{7}} & \frac{\sqrt{2}}{15} & 0 & 0 & \frac{1}{\sqrt{6}} \\ -\frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{6} & -\frac{1}{\sqrt{30}} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & -\frac{\sqrt{2}}{3} & \frac{2}{3\sqrt{15}} & -\frac{1}{3\sqrt{30}} & \frac{1}{3\sqrt{6}} & \frac{11}{6\sqrt{30}} & -\frac{4}{3\sqrt{15}} & \frac{1}{15} & \frac{\sqrt{2}}{10} & \frac{3}{5\sqrt{14}} & -\frac{\sqrt{2}}{5} & -\frac{1}{\sqrt{21}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3\sqrt{2}} & \frac{1}{2\sqrt{10}} & -\frac{1}{6\sqrt{2}} \\ 0 & -\frac{2}{3\sqrt{5}} & \frac{\sqrt{2}}{15} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & -\frac{1}{3\sqrt{30}} & -\frac{1}{6\sqrt{15}} & -\frac{4\sqrt{2}}{3} & -\frac{1}{3\sqrt{6}} & \frac{\sqrt{2}}{3} & \frac{\sqrt{2}}{6} & \frac{1}{6} & 0 & -\frac{1}{\sqrt{14}} & 0 & \frac{1}{2\sqrt{21}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3\sqrt{2}} & 0 & \frac{1}{3\sqrt{2}} \\ -\frac{1}{2\sqrt{5}} & \frac{\sqrt{2}}{6} & -\frac{1}{\sqrt{30}} & -\frac{1}{6} & \frac{1}{2\sqrt{3}} & \frac{\sqrt{2}}{3} & -\frac{1}{6\sqrt{15}} & \frac{2\sqrt{2}}{3} & -\frac{1}{3\sqrt{6}} & -\frac{7}{3\sqrt{30}} & \frac{\sqrt{2}}{6} & \frac{1}{6} & 0 & -\frac{1}{\sqrt{14}} & 0 & \frac{1}{2\sqrt{21}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3\sqrt{2}} & 0 & \frac{1}{3\sqrt{2}} \\ -\frac{1}{2\sqrt{5}} & \frac{1}{6\sqrt{5}} & \frac{1}{\sqrt{30}} & -\frac{1}{12} & \frac{1}{4\sqrt{3}} & -\frac{1}{3\sqrt{30}} & -\frac{23}{12\sqrt{15}} & \frac{1}{3\sqrt{30}} & \frac{\sqrt{2}}{3} & \frac{1}{6\sqrt{30}} & \frac{7}{12\sqrt{15}} & \frac{11}{60} & -\frac{\sqrt{2}}{10} & \frac{\sqrt{2}}{5} & \frac{\sqrt{2}}{5} & -\frac{5}{4\sqrt{21}} & \frac{\sqrt{2}}{3} & \frac{1}{3\sqrt{2}} & \frac{3}{2\sqrt{10}} & \frac{1}{6\sqrt{2}} \\ \frac{\sqrt{2}}{5} & -\frac{1}{2\sqrt{15}} & -\frac{1}{\sqrt{10}} & \frac{1}{4\sqrt{3}} & -\frac{1}{4} & \frac{1}{3\sqrt{10}} & -\frac{7}{12\sqrt{5}} & -\frac{1}{3\sqrt{10}} & \frac{1}{3\sqrt{2}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{12\sqrt{5}} & \frac{7}{20\sqrt{3}} & -\frac{1}{10\sqrt{2}} & -\frac{\sqrt{2}}{5} & \frac{1}{5\sqrt{2}} & -\frac{1}{4\sqrt{7}} & -\frac{\sqrt{2}}{15} & -\frac{1}{\sqrt{6}} & \frac{\sqrt{10}}{2} & -\frac{1}{2\sqrt{6}} \\ \frac{1}{2\sqrt{10}} & -\frac{1}{6\sqrt{10}} & -\frac{1}{2\sqrt{15}} & -\frac{1}{6\sqrt{2}} & \frac{1}{2\sqrt{6}} & \frac{1}{\sqrt{15}} & \frac{\sqrt{10}}{2} & -\frac{1}{\sqrt{15}} & 0 & -\frac{1}{2\sqrt{15}} & -\frac{\sqrt{10}}{2} & \frac{11}{30\sqrt{2}} & -\frac{\sqrt{3}}{10} & \frac{2}{5\sqrt{7}} & \frac{\sqrt{3}}{5} & -\frac{5}{2\sqrt{42}} & -\frac{1}{3\sqrt{5}} & \frac{1}{3} & -\frac{1}{2\sqrt{5}} & \frac{1}{6} \\ -\frac{1}{2\sqrt{10}} & \frac{1}{6\sqrt{10}} & \frac{1}{2\sqrt{15}} & \frac{1}{6\sqrt{2}} & -\frac{1}{2\sqrt{6}} & -\frac{1}{\sqrt{15}} & \frac{1}{\sqrt{30}} & \frac{1}{\sqrt{15}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2\sqrt{15}} & \frac{1}{\sqrt{30}} & \frac{4\sqrt{2}}{15} & -\frac{\sqrt{3}}{10} & -\frac{1}{10\sqrt{7}} & \frac{\sqrt{3}}{5} & -\frac{\sqrt{2}}{21} & \frac{1}{3\sqrt{5}} & -\frac{1}{3} & -\frac{1}{2\sqrt{5}} & -\frac{1}{6} \\ \frac{\sqrt{10}}{2} & -\frac{1}{2\sqrt{30}} & -\frac{1}{2\sqrt{5}} & 0 & 0 & -\frac{2}{3\sqrt{5}} & -\frac{1}{6\sqrt{10}} & \frac{2}{3\sqrt{5}} & -\frac{1}{6} & \frac{1}{3\sqrt{5}} & \frac{\sqrt{2}}{6} & -\frac{1}{2\sqrt{6}} & 0 & \frac{\sqrt{2}}{2} & 0 & -\frac{1}{2\sqrt{14}} & -\frac{1}{\sqrt{15}} & 0 & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{2\sqrt{5}} & \frac{1}{6\sqrt{5}} & \frac{1}{\sqrt{30}} & \frac{1}{6} & -\frac{1}{2\sqrt{3}} & \frac{2\sqrt{2}}{3} & \frac{1}{6\sqrt{15}} & -\frac{2\sqrt{2}}{3} & \frac{1}{3\sqrt{6}} & -\frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{6} & -\frac{1}{6} & 0 & \frac{1}{\sqrt{14}} & 0 & -\frac{1}{2\sqrt{21}} & \frac{\sqrt{2}}{3} & -\frac{\sqrt{2}}{3} & 0 & \frac{\sqrt{2}}{3} \end{pmatrix} \begin{pmatrix} \langle 8 || \mathbf{3}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 8 || \mathbf{15}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 8 || \bar{\mathbf{6}}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 10 || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 10 || \mathbf{15}'_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{15}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{24}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{24}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{42}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{42}_{5/2} || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{48}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{48}_{5/2} || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{15}'_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 8 || \mathbf{15}_{3/2} || \bar{\mathbf{3}} \rangle \\ \langle 10 || \mathbf{15}_{1/2} || \bar{\mathbf{3}} \rangle \\ \langle 27 || \mathbf{42}_{5/2} || \bar{\mathbf{3}} \rangle \\ \langle 35 || \mathbf{42}_{1/2} || \bar{\mathbf{3}} \rangle \end{pmatrix}. \quad (\text{A2})$$

APPENDIX B

The $\mathcal{T}$ matrix for $\Delta S = 0$ processes assuming the particular form of dim-6 tree Hamiltonian described in Eq. (11),

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \pi^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \pi^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^+) \\ \mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Delta'^- \bar{K}^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \pi^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- K^0) \end{pmatrix} = \begin{pmatrix} -\frac{C_{1,2}^+}{3\sqrt{2}} & \frac{C_{1,2}^+}{5} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ \frac{C_{1,2}^+}{12} & -\frac{3C_{1,2}^+}{20\sqrt{2}} & -\frac{C_{1,2}^+}{15\sqrt{2}} & -\frac{C_{1,2}^-}{3\sqrt{5}} & \frac{C_{1,2}^-}{\sqrt{15}} - \frac{C_{1,2}^+}{\sqrt{30}} \\ -\frac{C_{1,2}^+}{4\sqrt{3}} & -\frac{1}{4}\sqrt{\frac{3}{2}}C_{1,2}^+ & 0 & 0 & 0 \\ \frac{C_{1,2}^+}{2\sqrt{6}} & -\frac{\sqrt{3}C_{1,2}^+}{20} & \frac{C_{1,2}^+}{10\sqrt{3}} & \frac{C_{1,2}^-}{\sqrt{30}} & \frac{C_{1,2}^+}{2\sqrt{5}} - \frac{C_{1,2}^-}{\sqrt{10}} \\ \frac{C_{1,2}^+}{6} & \frac{3C_{1,2}^+}{10\sqrt{2}} & -\frac{C_{1,2}^+}{30\sqrt{2}} & -\frac{C_{1,2}^-}{6\sqrt{5}} & \frac{C_{1,2}^-}{2\sqrt{15}} - \frac{C_{1,2}^+}{2\sqrt{30}} \\ \frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ -\frac{C_{1,2}^+}{6\sqrt{2}} & \frac{C_{1,2}^+}{4} & \frac{C_{1,2}^+}{6} & -\frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ \frac{C_{1,2}^+}{6\sqrt{2}} & \frac{C_{1,2}^+}{4} & -\frac{C_{1,2}^+}{6} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^-}{\sqrt{30}} - \frac{C_{1,2}^+}{2\sqrt{15}} \\ -\frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & -\frac{2C_{1,2}^+}{15} & \frac{1}{3}\sqrt{\frac{2}{5}}C_{1,2}^- & 0 \\ 0 & -\frac{C_{1,2}^+}{5} & \frac{C_{1,2}^+}{20} & -\frac{C_{1,2}^-}{2\sqrt{10}} & \frac{C_{1,2}^-}{2\sqrt{30}} - \frac{C_{1,2}^+}{4\sqrt{15}} \\ 0 & 0 & -\frac{C_{1,2}^+}{4\sqrt{3}} & \frac{C_{1,2}^-}{2\sqrt{30}} & \frac{C_{1,2}^-}{2\sqrt{10}} - \frac{C_{1,2}^+}{4\sqrt{5}} \\ \frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ \frac{C_{1,2}^+}{6\sqrt{2}} & \frac{C_{1,2}^+}{20} & \frac{2C_{1,2}^+}{15} & -\frac{1}{3}\sqrt{\frac{2}{5}}C_{1,2}^- & 0 \\ \frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ 0 & \frac{2C_{1,2}^+}{5} & -\frac{C_{1,2}^+}{10} & -\frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ 0 & \frac{\sqrt{3}C_{1,2}^+}{10} & \frac{\sqrt{3}C_{1,2}^+}{10} & \frac{C_{1,2}^-}{\sqrt{30}} & \frac{C_{1,2}^-}{\sqrt{10}} - \frac{C_{1,2}^+}{2\sqrt{5}} \\ 0 & -\frac{\sqrt{2}C_{1,2}^+}{5} & \frac{C_{1,2}^+}{10\sqrt{2}} & \frac{C_{1,2}^-}{6\sqrt{5}} & \frac{C_{1,2}^-}{2\sqrt{15}} - \frac{C_{1,2}^+}{2\sqrt{30}} \\ 0 & -\frac{C_{1,2}^+}{10\sqrt{2}} & -\frac{C_{1,2}^+}{10\sqrt{2}} & -\frac{C_{1,2}^-}{6\sqrt{5}} & \frac{C_{1,2}^+}{2\sqrt{30}} - \frac{C_{1,2}^-}{2\sqrt{15}} \\ 0 & \frac{1}{10}\sqrt{\frac{3}{2}}C_{1,2}^+ & \frac{1}{10}\sqrt{\frac{3}{2}}C_{1,2}^+ & \frac{C_{1,2}^-}{2\sqrt{15}} & \frac{C_{1,2}^-}{2\sqrt{5}} - \frac{C_{1,2}^+}{2\sqrt{10}} \\ 0 & -\frac{C_{1,2}^+}{10} & -\frac{C_{1,2}^+}{10} & -\frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \end{pmatrix} \begin{pmatrix} \langle 10 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 27 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 8 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 8 \| \bar{\mathbf{6}} \| \bar{3} \rangle \\ \langle 8 \| \mathbf{3} \| \bar{3} \rangle \end{pmatrix}. \quad (\text{B1})$$

The $\mathcal{T}$ matrix for $\Delta S = -1$ processes assuming the particular form of dim-6 tree Hamiltonian described in Eq. (11),

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^+) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \pi^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^0) \end{pmatrix} = \begin{pmatrix} -\frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & -\frac{2C_{1,2}^+}{15} & \frac{1}{3}\sqrt{\frac{2}{5}}C_{1,2}^- & 0 \\ \frac{C_{1,2}^+}{6\sqrt{2}} & \frac{C_{1,2}^+}{20} & \frac{2C_{1,2}^+}{15} & -\frac{1}{3}\sqrt{\frac{2}{5}}C_{1,2}^- & 0 \\ -\frac{C_{1,2}^+}{6\sqrt{2}} & \frac{C_{1,2}^+}{4} & \frac{C_{1,2}^+}{6} & -\frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ 0 & -\frac{C_{1,2}^+}{10} & -\frac{C_{1,2}^+}{10} & 0 & \frac{C_{1,2}^-}{\sqrt{30}} - \frac{C_{1,2}^+}{2\sqrt{15}} \\ 0 & -\frac{\sqrt{3}C_{1,2}^+}{10} & \frac{C_{1,2}^+}{5\sqrt{3}} & -\frac{C_{1,2}^-}{\sqrt{30}} & 0 \\ \frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ \frac{C_{1,2}^+}{6\sqrt{2}} & \frac{C_{1,2}^+}{4} & -\frac{C_{1,2}^+}{6} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^-}{\sqrt{30}} - \frac{C_{1,2}^+}{2\sqrt{15}} \\ \frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ -\frac{C_{1,2}^+}{3\sqrt{2}} & \frac{C_{1,2}^+}{5} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ \frac{C_{1,2}^+}{6} & \frac{3C_{1,2}^+}{10\sqrt{2}} & -\frac{C_{1,2}^+}{30\sqrt{2}} & -\frac{C_{1,2}^-}{6\sqrt{5}} & \frac{C_{1,2}^-}{2\sqrt{15}} - \frac{C_{1,2}^+}{2\sqrt{30}} \\ -\frac{C_{1,2}^+}{12} & -\frac{9C_{1,2}^+}{20\sqrt{2}} & -\frac{C_{1,2}^+}{30\sqrt{2}} & -\frac{C_{1,2}^-}{6\sqrt{5}} & \frac{C_{1,2}^-}{2\sqrt{15}} - \frac{C_{1,2}^+}{2\sqrt{30}} \\ \frac{C_{1,2}^+}{4\sqrt{3}} & \frac{1}{20}\sqrt{\frac{3}{2}}C_{1,2}^+ & -\frac{C_{1,2}^+}{10\sqrt{6}} & -\frac{C_{1,2}^-}{2\sqrt{15}} & \frac{C_{1,2}^-}{2\sqrt{5}} - \frac{C_{1,2}^+}{2\sqrt{10}} \\ \frac{C_{1,2}^+}{6\sqrt{2}} & -\frac{C_{1,2}^+}{20} & \frac{C_{1,2}^+}{30} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^+}{2\sqrt{15}} - \frac{C_{1,2}^-}{\sqrt{30}} \\ \frac{C_{1,2}^+}{2\sqrt{6}} & -\frac{\sqrt{3}C_{1,2}^+}{20} & \frac{C_{1,2}^+}{10\sqrt{3}} & \frac{C_{1,2}^-}{\sqrt{30}} & \frac{C_{1,2}^+}{2\sqrt{5}} - \frac{C_{1,2}^-}{\sqrt{10}} \\ 0 & \frac{\sqrt{2}C_{1,2}^+}{5} & -\frac{C_{1,2}^+}{10\sqrt{2}} & -\frac{C_{1,2}^-}{6\sqrt{5}} & \frac{C_{1,2}^+}{2\sqrt{30}} - \frac{C_{1,2}^-}{2\sqrt{15}} \\ 0 & \frac{C_{1,2}^+}{10} & \frac{C_{1,2}^+}{10} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^-}{\sqrt{30}} - \frac{C_{1,2}^+}{2\sqrt{15}} \\ 0 & -\frac{2C_{1,2}^+}{5} & \frac{C_{1,2}^+}{10} & \frac{C_{1,2}^-}{3\sqrt{10}} & \frac{C_{1,2}^-}{\sqrt{30}} - \frac{C_{1,2}^+}{2\sqrt{15}} \\ 0 & -\frac{C_{1,2}^+}{10\sqrt{2}} & -\frac{C_{1,2}^+}{10\sqrt{2}} & -\frac{C_{1,2}^-}{6\sqrt{5}} & \frac{C_{1,2}^+}{2\sqrt{30}} - \frac{C_{1,2}^-}{2\sqrt{15}} \\ 0 & \frac{1}{10}\sqrt{\frac{3}{2}}C_{1,2}^+ & \frac{1}{10}\sqrt{\frac{3}{2}}C_{1,2}^+ & \frac{C_{1,2}^-}{2\sqrt{15}} & \frac{C_{1,2}^-}{2\sqrt{5}} - \frac{C_{1,2}^+}{2\sqrt{10}} \\ 0 & -\frac{\sqrt{3}C_{1,2}^+}{10} & -\frac{\sqrt{3}C_{1,2}^+}{10} & -\frac{C_{1,2}^-}{\sqrt{30}} & \frac{C_{1,2}^+}{2\sqrt{5}} - \frac{C_{1,2}^-}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} \langle 10 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 27 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 8 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 8 \| \bar{\mathbf{6}} \| \bar{3} \rangle \\ \langle 8 \| \mathbf{3} \| \bar{3} \rangle \end{pmatrix}. \quad (\text{B2})$$

The $\mathcal{P}$ matrix for $\Delta S = 0$ processes assuming the particular form of a dim-6 gluonic and electroweak penguin Hamiltonian described in Eq. (12) and Eq. (13),

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^- \pi^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \pi^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^+) \\ \mathcal{A}(\Xi_b^- \rightarrow \Delta'^0 K^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Delta'^- \bar{K}^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \pi^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^0) \end{pmatrix} = \begin{pmatrix} \frac{C_{9,10}^+}{2\sqrt{2}} & -\frac{3C_{9,10}^+}{10} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ -\frac{C_{9,10}^+}{8} & \frac{9C_{9,10}^+}{40\sqrt{2}} & \frac{C_{9,10}^+}{10\sqrt{2}} & \frac{C_{9,10}^-}{2\sqrt{5}} & \frac{2D}{\sqrt{15}} \\ \frac{\sqrt{3}C_{9,10}^+}{8} & \frac{3}{8}\sqrt{\frac{3}{2}}C_{9,10}^+ & 0 & 0 & 0 \\ -\frac{1}{4}\sqrt{\frac{3}{2}}C_{9,10}^+ & \frac{3\sqrt{3}C_{9,10}^+}{40} & -\frac{\sqrt{3}C_{9,10}^+}{20} & -\frac{1}{2}\sqrt{\frac{3}{10}}C_{9,10}^- & -\sqrt{\frac{2}{5}}D \\ -\frac{C_{9,10}^+}{4} & -\frac{9C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^-}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ \frac{C_{9,10}^+}{4\sqrt{2}} & -\frac{3C_{9,10}^+}{8} & -\frac{C_{9,10}^+}{4} & \frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & -\frac{3C_{9,10}^+}{8} & \frac{C_{9,10}^+}{4} & -\frac{C_{9,10}^-}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\ \frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & \frac{C_{9,10}^+}{5} & -\frac{C_{9,10}^-}{\sqrt{10}} & 0 \\ 0 & \frac{3C_{9,10}^+}{10} & -\frac{3C_{9,10}^+}{40} & \frac{3C_{9,10}^-}{4\sqrt{10}} & \frac{D}{\sqrt{30}} \\ 0 & 0 & \frac{\sqrt{3}C_{9,10}^+}{8} & -\frac{1}{4}\sqrt{\frac{3}{10}}C_{9,10}^- & \frac{D}{\sqrt{10}} \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & -\frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{5} & \frac{C_{9,10}^-}{\sqrt{10}} & 0 \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ 0 & -\frac{3C_{9,10}^+}{5} & \frac{3C_{9,10}^+}{20} & \frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ 0 & -\frac{3\sqrt{3}C_{9,10}^+}{20} & -\frac{3\sqrt{3}C_{9,10}^+}{20} & -\frac{1}{2}\sqrt{\frac{3}{10}}C_{9,10}^- & \sqrt{\frac{2}{5}}D \\ 0 & \frac{3C_{9,10}^+}{5\sqrt{2}} & -\frac{3C_{9,10}^+}{20\sqrt{2}} & -\frac{C_{9,10}^-}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\ 0 & \frac{3C_{9,10}^+}{20\sqrt{2}} & \frac{3C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^-}{4\sqrt{5}} & -\frac{D}{\sqrt{15}} \\ 0 & -\frac{3}{20}\sqrt{\frac{3}{2}}C_{9,10}^+ & -\frac{3}{20}\sqrt{\frac{3}{2}}C_{9,10}^+ & -\frac{1}{4}\sqrt{\frac{3}{5}}C_{9,10}^- & \frac{D}{\sqrt{5}} \\ 0 & \frac{3C_{9,10}^+}{20} & \frac{3C_{9,10}^+}{20} & \frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \end{pmatrix} \begin{pmatrix} \langle 10 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 27 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 8 \| \mathbf{15} \| \bar{3} \rangle \\ \langle 8 \| \bar{\mathbf{6}} \| \bar{3} \rangle \\ \langle 8 \| \mathbf{3} \| \bar{3} \rangle \end{pmatrix}. \quad (\text{B3})$$

The $\mathcal{P}$ matrix for $\Delta S = -1$ processes assuming the particular form of a dim-6 gluonic and electroweak penguin Hamiltonian described in Eq (12) and Eq (13),

$$\begin{pmatrix} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \pi^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 \eta_8) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \eta_8) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+) \\ \mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^+) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \pi^-) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \pi^0) \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \eta_8) \\ \mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^0) \end{pmatrix} = \begin{pmatrix} \frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & \frac{C_{9,10}^+}{5} & -\frac{C_{9,10}^-}{\sqrt{10}} & 0 \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & -\frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{5} & \frac{C_{9,10}^-}{\sqrt{10}} & 0 \\ \frac{C_{9,10}^+}{4\sqrt{2}} & -\frac{3C_{9,10}^+}{8} & -\frac{C_{9,10}^+}{4} & \frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ 0 & \frac{3C_{9,10}^+}{20} & \frac{3C_{9,10}^+}{20} & 0 & \sqrt{\frac{2}{15}}D \\ 0 & \frac{3\sqrt{3}C_{9,10}^+}{20} & -\frac{\sqrt{3}C_{9,10}^+}{10} & \frac{1}{2}\sqrt{\frac{3}{10}}C_{9,10}^- & 0 \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & -\frac{3C_{9,10}^+}{8} & \frac{C_{9,10}^+}{4} & -\frac{C_{9,10}^-}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ \frac{C_{9,10}^+}{2\sqrt{2}} & -\frac{3C_{9,10}^+}{10} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ -\frac{C_{9,10}^+}{4} & -\frac{9C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^-}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\ \frac{C_{9,10}^+}{8} & \frac{27C_{9,10}^+}{40\sqrt{2}} & \frac{C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^-}{4\sqrt{5}} & \frac{D}{\sqrt{15}} \\ -\frac{\sqrt{3}C_{9,10}^+}{8} & -\frac{3}{40}\sqrt{\frac{3}{2}}C_{9,10}^+ & \frac{1}{20}\sqrt{\frac{3}{2}}C_{9,10}^+ & \frac{1}{4}\sqrt{\frac{3}{5}}C_{9,10}^- & \frac{D}{\sqrt{5}} \\ -\frac{C_{9,10}^+}{4\sqrt{2}} & \frac{3C_{9,10}^+}{40} & -\frac{C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & -\sqrt{\frac{2}{15}}D \\ -\frac{1}{4}\sqrt{\frac{3}{2}}C_{9,10}^+ & \frac{3\sqrt{3}C_{9,10}^+}{40} & -\frac{\sqrt{3}C_{9,10}^+}{20} & -\frac{1}{2}\sqrt{\frac{3}{10}}C_{9,10}^- & -\sqrt{\frac{2}{5}}D \\ 0 & -\frac{3C_{9,10}^+}{5\sqrt{2}} & \frac{3C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^-}{4\sqrt{5}} & -\frac{D}{\sqrt{15}} \\ 0 & -\frac{3C_{9,10}^+}{20} & -\frac{3C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\ 0 & \frac{3C_{9,10}^+}{5} & -\frac{3C_{9,10}^+}{20} & -\frac{C_{9,10}^-}{2\sqrt{10}} & \sqrt{\frac{2}{15}}D \\ 0 & \frac{3C_{9,10}^+}{20\sqrt{2}} & \frac{3C_{9,10}^+}{20\sqrt{2}} & \frac{C_{9,10}^-}{4\sqrt{5}} & -\frac{D}{\sqrt{15}} \\ 0 & -\frac{3}{20}\sqrt{\frac{3}{2}}C_{9,10}^+ & -\frac{3}{20}\sqrt{\frac{3}{2}}C_{9,10}^+ & -\frac{1}{4}\sqrt{\frac{3}{5}}C_{9,10}^- & \frac{D}{\sqrt{5}} \\ 0 & \frac{3\sqrt{3}C_{9,10}^+}{20} & \frac{3\sqrt{3}C_{9,10}^+}{20} & \frac{1}{2}\sqrt{\frac{3}{10}}C_{9,10}^- & -\sqrt{\frac{2}{5}}D \end{pmatrix} \begin{pmatrix} \langle 10 || \mathbf{15} || \bar{3} \rangle \\ \langle 27 || \mathbf{15} || \bar{3} \rangle \\ \langle 8 || \mathbf{15} || \bar{3} \rangle \\ \langle 8 || \bar{\mathbf{6}} || \bar{3} \rangle \\ \langle 8 || \mathbf{3} || \bar{3} \rangle \end{pmatrix}. \quad (\text{B4})$$

The $SU(3)$ decomposition of decay amplitudes without assuming $SU(3)$ -breaking for $\bar{\mathbf{3}} \rightarrow \mathbf{10} \otimes \mathbf{1}$ decay modes are given below,

$\Delta S = 0$ processes,

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \eta_1) &= -\frac{C_{1,2}^+}{\sqrt{6}} \left(\lambda_u^d - \frac{3}{2} \kappa \lambda_t^d \right) \langle 10 || \mathbf{15} || \bar{3} \rangle \\ \mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \eta_1) &= 0, \end{aligned} \quad (\text{B5})$$

$\Delta S = -1$ processes,

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^0 \eta_1) &= -\frac{C_{1,2}^+}{2\sqrt{3}} \left(\lambda_u^s - \frac{3}{2} \kappa \lambda_t^s \right) \langle 10 || \mathbf{15} || \bar{3} \rangle \\ \mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \eta_1) &= 0. \end{aligned} \quad (\text{B6})$$

We have used the following shorthand notation to express the tree ($\mathcal{T}$) and penguin ($\mathcal{P}$) matrices in a convenient form:

$$\begin{aligned}(C_{10} \pm C_9) &= C_{9,10}^\pm, \\ (C_1 \pm C_2) &= C_{1,2}^\pm, \\ (C_3 \pm C_4) &= C_{3,4}^\pm, \\ (C_5 \pm C_6) &= C_{5,6}^\pm,\end{aligned}$$

and D is given by the particular combination of Wilson coefficients,

$$D = -\frac{1}{4\sqrt{2}}(C_{9,10}^+ + \sqrt{2}C_{9,10}^-) + \{\sqrt{2}C_{3,4}^+ - C_{3,4}^- + \sqrt{2}C_{5,6}^+ - C_{5,6}^-\}. \quad (\text{B7})$$

APPENDIX C

The flavor flow in the five topologies [50] given in Fig. 1 is described below,

$$\begin{aligned}E_1: & B_{[ij]} D^{\{kmj\}} M_k^l H_{lm}^i \\ E_2: & B_{[ij]} D^{\{kmj\}} M_k^l H_{ml}^i \\ P_u: & B_{[ij]} D^{\{kjl\}} M_k^i H_{ml}^m \\ T: & B_{[ij]} D^{\{lmj\}} M_k^i H_{ml}^k \\ E_3: & B_{[ij]} D^{\{klm\}} M_m^j H_{kl}^i,\end{aligned}$$

where $B_{[ij]}$, $D^{\{klm\}}$, M_y^x are flavor wave functions of the initial antitriplet ($\bar{\mathbf{3}}$) b baryon, the final state decuplet baryon ($\mathbf{10}$), and the octet meson ($\mathbf{8}$), respectively. The H_{ab}^i mediates the quark transition and for $\Delta S = -1$, $\Delta S = 0$ processes, the only nonzero contribution comes from the elements $H_{13}^1 = 1$ and $H_{12}^1 = 1$, respectively. According to the phase convention chosen for quark (q_i) and antiquark ($\bar{q}_i$) flavor states in Eq. (10), the meson wave functions are given as

$$\begin{aligned}K^+ &= u\bar{s}, \quad K^- = -s\bar{u}, \quad K^0 = d\bar{s}, \quad \bar{K}^0 = s\bar{d}, \\ \pi^+ &= u\bar{d}, \quad \pi^- = -d\bar{u}, \quad \pi^0 = \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u}), \\ \eta_8 &= -\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \quad \eta_1 = -\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}).\end{aligned}$$

The initial b -baryon antitriplet $B_{[ij]}$ consists of the following states:

$$\begin{aligned}\Lambda_b^0 &= \frac{1}{\sqrt{2}}(ud - du)b \\ \Xi_b^0 &= \frac{1}{\sqrt{2}}(us - su)b \\ \Xi_b^- &= \frac{1}{\sqrt{2}}(ds - sd)b.\end{aligned}$$

The decuplet baryons have a completely symmetric flavor wave function,

$$\begin{aligned}\Delta^{++} &= uuu, \quad \Delta^- = ddd \\ \Delta^+ &= \frac{1}{\sqrt{3}}(uud + udu + duu) \\ \Delta^0 &= \frac{1}{\sqrt{3}}(udd + ddu + dud) \\ \Sigma'^+ &= \frac{1}{\sqrt{3}}(uus + usu + suu) \\ \Sigma'^0 &= \frac{1}{\sqrt{6}}(uds + usd + dus \\ &\quad + dsu + sud + sdu) \\ \Sigma'^- &= \frac{1}{\sqrt{3}}(dds + dsd + sdd) \\ \Xi'^0 &= \frac{1}{\sqrt{6}}(uss + sus + ssu) \\ \Xi'^- &= \frac{1}{\sqrt{3}}(dss + ssd + sds) \\ \Omega^- &= sss.\end{aligned}$$

APPENDIX D

Considering only the tree operators in the effective Hamiltonian,

$$\mathcal{H}^{\text{eff}} = C_1 O_1 + C_2 O_2, \quad (\text{D1})$$

that can be recast as [97–101]

$$\mathcal{H}^{\text{eff}} = C^- O^- + C^+ O^+, \quad (\text{D2})$$

where $C^- = C_1 - C_2$, $C^+ = C_1 + C_2$ and

$$\begin{aligned}O^- &= \frac{1}{2}(O_1 - O_2) \\ &= \frac{1}{2}((\bar{u}_L^i \gamma^\mu b_L^j)(\bar{d}_L^j \gamma_\mu u_L^i) - (\bar{u}_L^i \gamma^\mu b_L^i)(\bar{d}_L^j \gamma_\mu u_L^j))\end{aligned} \quad (\text{D3})$$

$$\begin{aligned}O^+ &= \frac{1}{2}(O_1 + O_2) \\ &= \frac{1}{2}((\bar{u}_L^i \gamma^\mu b_L^j)(\bar{d}_L^j \gamma_\mu u_L^i) + (\bar{u}_L^i \gamma^\mu b_L^i)(\bar{d}_L^j \gamma_\mu u_L^j)).\end{aligned} \quad (\text{D4})$$

Rewriting O_1 and O_2 we get

$$O_1 = \frac{1}{4} [(\bar{u}^i)_\alpha [\gamma^\mu (1 - \gamma_5)]_{\alpha\beta} (b^l)_\beta] \times [(\bar{d}^j)_\sigma [\gamma_\mu (1 - \gamma_5)]_{\sigma\lambda} (u^k)_\lambda] \delta^{lj} \delta^{ik}, \quad (D5)$$

$$O_2 = \frac{1}{4} [(\bar{u}^i)_\alpha [\gamma^\mu (1 - \gamma_5)]_{\alpha\beta} (b^l)_\beta] \times [(\bar{d}^j)_\sigma [\gamma_\mu (1 - \gamma_5)]_{\sigma\lambda} (u^k)_\lambda] \delta^{il} \delta^{jk} \quad (D6)$$

Using the following identity:

$$\frac{1}{2} \epsilon^{aij} \epsilon^{alk} = (\delta^{il} \delta^{jk} - \delta^{ik} \delta^{jl}),$$

one can write O^- as

$$O^- = -\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} \epsilon^{aij} \epsilon^{alk} [(\bar{u}^i)_\alpha [\gamma^\mu (1 - \gamma_5)]_{\alpha\beta} (b^l)_\beta] \times [(\bar{d}^j)_\sigma [\gamma_\mu (1 - \gamma_5)]_{\sigma\lambda} (u^k)_\lambda]. \quad (D7)$$

Using the Firez transformation [116], one can recast Eq. (D7) as [99],

$$O^- = -\frac{1}{4} \frac{1}{2} \epsilon^{alk} \underbrace{[(\bar{b}^l)^C (1 - \gamma_5) (u^k)]}_{(bu)_{\bar{3}}} \underbrace{\epsilon^{aij} [(\bar{u}^i) (1 + \gamma_5) (d^j)^C]}_{(ud)_{\bar{3}}^\dagger}, \quad (D8)$$

where $q^C = \mathbf{C}q^*$, $\mathbf{C}$ being the charge-conjugation operator. According to the diquark mechanism [99,100], the diquark $\epsilon^{aij} [(\bar{u}^i) (1 + \gamma_5) (d^j)^C]$ has a total spin 0, and it transforms as a $\bar{\mathbf{3}}$ and $\bar{\mathbf{3}}$ under $SU(3)_F$ and $SU(3)_{\text{color}}$, respectively. This diquark ending up completely in the final state baryon cannot produce² a decuplet baryon as any quark pair inside the decuplet baryon transforms as $\mathbf{6}$ and $\bar{\mathbf{3}}$ under $SU(3)_F$ and $SU(3)_{\text{color}}$ with a total spin equaling 1. On the other hand, the operator Q^+ , which is a product of $SU(3)_{\text{color}}$

sextet currents, cannot produce a color singlet baryon. In diagrams T and E_3 , the quark pair antisymmetric in color and flavor [97,98] originating from weak interaction ends up in the decuplet baryon which is forbidden by the above mentioned argument [102–108]. Since the quark pair $(ud)_{\bar{3}}$ is in an isospin 0 state,³ the total quark transition obeys the $\Delta I = 1/2$ rule. In contrast, in diagrams E_1 and E_2 , the diquark argument is not applicable as only one of the quarks from a weak interaction form the final state baryon, while the other ends up in the meson.

As a consequence, the number of independent diagrams reduces to three. If we demand an equivalent description of all possible decays in terms of $SU(3)$ -reduced amplitudes, some of the reduced amplitudes can no longer be independent. The relation between the remaining diagrams and the $SU(3)$ -reduced amplitudes is given below,

$$E_1 = \frac{\sqrt{3}a_{10}}{4} - \frac{2b_8}{\sqrt{15}}, \quad (D9)$$

$$E_2 = \frac{\sqrt{3}a_{10}}{4} + \frac{2b_8}{\sqrt{15}}, \quad (D10)$$

$$P = -\frac{\sqrt{3}a_{10}}{16} + \frac{c_8}{\sqrt{10}} - \frac{b_8}{\sqrt{15}}, \quad (D11)$$

where $c_8, b_8, a_8, a_{10}, a_{27}$ are defined in Eq. (15).

Equivalently, the following relations hold for the $SU(3)$ -reduced matrix elements:

$$a_{27} = 0 \quad (C_1 \neq 0, C_2 \neq 0) \quad (D12)$$

$$a_8 = \frac{5}{8\sqrt{2}} a_{10}. \quad (D13)$$

The reduction in the number of independent $SU(3)$ -reduced amplitudes also imply additional amplitude relations that are enclosed in the box,

$$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ K^-) \boxed{= -\sqrt{2} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^0 K^0) = -\sqrt{2} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^0 \bar{K}^0)}, \quad (D14)$$

$$\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ K^-) = \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^+ \pi^-) = -\mathcal{A}(\Lambda_b^0 \rightarrow \Delta^0 \bar{K}^0) = -\mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 K^0), \quad (D15)$$

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- \pi^+) &= \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^- K^+) = \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- \pi^+) = \frac{1}{\sqrt{3}} \mathcal{A}(\Xi_b^0 \rightarrow \Omega^- K^+) \\ &= \frac{1}{\sqrt{3}} \mathcal{A}(\Lambda_b^0 \rightarrow \Delta^+ \pi^-) = \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^- K^+) = \mathcal{A}(\Xi_b^0 \rightarrow \Sigma'^- \pi^+) = \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^- K^+) \\ &\boxed{= -\sqrt{2} \mathcal{A}(\Xi_b^0 \rightarrow \Xi'^0 \pi^0)} \end{aligned} \quad (D16)$$

²Assuming the baryon made of the diquark and a third quark transforming as a $\mathbf{3}$ and $\mathbf{3}$ under $SU(3)_F$ and $SU(3)_{\text{color}}$, the final state transforms as $SU(3)_{\text{color}}: \bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}$, $SU(3)_F: \bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}$. Since baryons are color singlet, a $\mathbf{6}$ of $SU(3)_{\text{color}}$ diquark can never form a baryon.

³Only allowed isospin values in a $\bar{\mathbf{3}}$ of $SU(3)_F$ is 0 or 1/2.

$$\begin{aligned}
\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \bar{K}^0) &= -\sqrt{2}\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \pi^0) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- \eta_8) = -\frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \rightarrow \Omega^- K^0) \\
&= \frac{1}{\sqrt{3}}\mathcal{A}(\Xi_b^- \rightarrow \Delta'^- \bar{K}^0) = \sqrt{\frac{2}{3}}\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \eta_8) = -\mathcal{A}(\Xi_b^- \rightarrow \Xi'^- K^0) = -\sqrt{2}\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^- \pi^0) \\
&= -\mathcal{A}(\Xi_b^0 \rightarrow \Delta^+ K^-) = \mathcal{A}(\Xi_b^0 \rightarrow \Delta^0 \bar{K}^0) = -\mathcal{A}(\Lambda_b^0 \rightarrow \Sigma'^+ \pi^-) = \mathcal{A}(\Lambda_b^0 \rightarrow \Xi'^0 K^0) \\
&= -\mathcal{A}(\Xi_b^- \rightarrow \Delta^0 K^-) = \sqrt{2}\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 \pi^-) = -\sqrt{2}\mathcal{A}(\Xi_b^- \rightarrow \Sigma'^0 K^-) = \mathcal{A}(\Xi_b^- \rightarrow \Xi'^0 \pi^-). \tag{D17}
\end{aligned}$$

By measuring the phase-space corrected branching fractions and δ_{CP} , one can test the diquark picture in bottom baryon decays.

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- [1] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **100**, 031102 (2019).
 - [2] T. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **106**, 181802 (2011).
 - [3] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **10** (2012) 037.
 - [4] T. A. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **113**, 242001 (2014).
 - [5] R. Aaij *et al.* (LHCb Collaboration), *Nat. Phys.* **13**, 391 (2017).
 - [6] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **02** (2018) 098.
 - [7] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **08** (2018) 039.
 - [8] R. Aaij *et al.* (LHCb Collaboration), *Phys. Lett. B* **787**, 124 (2018).
 - [9] R. Aaij *et al.* (LHCb Collaboration), *Eur. Phys. J. C* **79**, 745 (2019).
 - [10] R. Aaij *et al.* (LHCb Collaboration), *arXiv:1912.10741*.
 - [11] D. Zeppenfeld, *Z. Phys. C* **8**, 77 (1981).
 - [12] M. J. Savage and M. B. Wise, *Phys. Rev. D* **39**, 3346 (1989); **40**, 3127(E) (1989).
 - [13] M. Gronau and D. London, *Phys. Rev. Lett.* **65**, 3381 (1990).
 - [14] L. L. Chau, H. Y. Cheng, W. K. Sze, H. Yao, and B. Tseng, *Phys. Rev. D* **43**, 2176 (1991); **58**, 019902(E) (1998).
 - [15] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, *Phys. Rev. D* **50**, 4529 (1994).
 - [16] N. G. Deshpande and X. G. He, *Phys. Rev. Lett.* **74**, 26 (1995); **74**, 4099(E) (1995).
 - [17] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, *Phys. Rev. D* **52**, 6374 (1995).
 - [18] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, *Phys. Rev. D* **52**, 6356 (1995).
 - [19] B. Grinstein and R. F. Lebed, *Phys. Rev. D* **53**, 6344 (1996).
 - [20] N. G. Deshpande, B. Dutta, and S. Oh, *Phys. Lett. B* **473**, 141 (2000).
 - [21] N. G. Deshpande, B. Dutta, and S. Oh, *Phys. Rev. D* **57**, 5723 (1998).
 - [22] M. Gronau, D. Pirjol, and T. M. Yan, *Phys. Rev. D* **60**, 034021 (1999); **69**, 119901(E) (2004).
 - [23] M. Neubert, *Phys. Lett. B* **424**, 152 (1998).
 - [24] D. Atwood and A. Soni, *Phys. Rev. D* **58**, 036005 (1998).
 - [25] A. J. Buras, R. Fleischer, and T. Mannel, *Nucl. Phys. B* **533**, 3 (1998).
 - [26] A. J. Buras and L. Silvestrini, *Nucl. Phys. B* **569**, 3 (2000).
 - [27] A. F. Falk, A. L. Kagan, Y. Nir, and A. A. Petrov, *Phys. Rev. D* **57**, 4290 (1998).
 - [28] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, *Nucl. Phys. B* **591**, 313 (2000).
 - [29] X. G. He, J. Y. Leou, and C. Y. Wu, *Phys. Rev. D* **62**, 114015 (2000).
 - [30] N. G. Deshpande, X. G. He, and J. Q. Shi, *Phys. Rev. D* **62**, 034018 (2000).
 - [31] M. Gronau and D. Pirjol, *Phys. Rev. D* **62**, 077301 (2000).
 - [32] H. K. Fu, X. G. He, Y. K. Hsiao, and J. Q. Shi, *Chin. J. Phys.* **41**, 601 (2003), <https://arxiv.org/abs/hep-ph/0206199>.
 - [33] G. Paz, *arXiv:hep-ph/0206312*.
 - [34] M. A. Dariescu, N. G. Deshpande, X. G. He, and G. Valencia, *Phys. Lett. B* **557**, 60 (2003).
 - [35] Y. L. Wu and Y. F. Zhou, *Eur. Phys. J. Direct* **5**, 014 (2003); *Eur. Phys. J. C* **32**, s179 (2004).
 - [36] M. Beneke and M. Neubert, *Nucl. Phys. B* **675**, 333 (2003).
 - [37] Y. Grossman, Z. Ligeti, Y. Nir, and H. Quinn, *Phys. Rev. D* **68**, 015004 (2003).
 - [38] C. W. Chiang, M. Gronau, and J. L. Rosner, *Phys. Rev. D* **68**, 074012 (2003).
 - [39] X. G. He and B. H. McKellar, *arXiv:hep-ph/0410098*.
 - [40] C. W. Chiang, M. Gronau, J. L. Rosner, and D. A. Suprun, *Phys. Rev. D* **70**, 034020 (2004).
 - [41] A. J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab, *Nucl. Phys. B* **697**, 133 (2004).
 - [42] M. Gronau, Y. Grossman, G. Raz, and J. L. Rosner, *Phys. Lett. B* **635**, 207 (2006).
 - [43] A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang, and Y. M. Wang, *Phys. Rev. D* **76**, 074018 (2007).

- [44] H. Y. Cheng and S. Oh, *J. High Energy Phys.* **09** (2011) 024.
- [45] X. G. He, S. F. Li, and H. H. Lin, *J. High Energy Phys.* **08** (2013) 065.
- [46] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, *Phys. Rev. D* **91**, 014011 (2015).
- [47] B. Grinstein, D. Pirtskhalava, D. Stone, and P. Uttayarat, *Phys. Rev. D* **89**, 114014 (2014).
- [48] Y. K. Hsiao, C. F. Chang, and X. G. He, *Phys. Rev. D* **93**, 114002 (2016).
- [49] X. G. He and W. Wang, *Chin. Phys. C* **42**, 103108 (2018).
- [50] X. G. He, Y. J. Shi, and W. Wang, *Eur. Phys. J. C* **80**, 359 (2020).
- [51] D. Wang, C. P. Jia, and F. S. Yu, [arXiv:2001.09460](https://arxiv.org/abs/2001.09460).
- [52] I. Hinchliffe and T. A. Kaeding, *Phys. Rev. D* **54**, 914 (1996).
- [53] B. Bhattacharya and J. L. Rosner, *Phys. Rev. D* **77**, 114020 (2008).
- [54] D. Pirtskhalava and P. Uttayarat, *Phys. Lett. B* **712**, 81 (2012).
- [55] T. Feldmann, S. Nandi, and A. Soni, *J. High Energy Phys.* **06** (2012) 007.
- [56] G. Hiller, M. Jung, and S. Schacht, *Phys. Rev. D* **87**, 014024 (2013).
- [57] B. Bhattacharya, M. Gronau, and J. L. Rosner, *Phys. Rev. D* **85**, 054014 (2012); **85**, 079901 (2012).
- [58] Y. Grossman and D. J. Robinson, *J. High Energy Phys.* **04** (2013) 067.
- [59] M. Gronau, *Phys. Rev. D* **91**, 076007 (2015).
- [60] S. Müller, U. Nierste, and S. Schacht, *Phys. Rev. D* **92**, 014004 (2015).
- [61] S. Müller, U. Nierste, and S. Schacht, *Phys. Rev. Lett.* **115**, 251802 (2015).
- [62] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, *Phys. Rev. D* **93**, 114010 (2016).
- [63] Y. Grossman and S. Schacht, *J. High Energy Phys.* **07** (2019) 020.
- [64] C. D. Lu, Y. M. Wang, H. Zou, A. Ali, and G. Kramer, *Phys. Rev. D* **80**, 034011 (2009).
- [65] M. Gronau and J. L. Rosner, *Phys. Rev. D* **89**, 037501 (2014); **91**, 119902(E) (2015).
- [66] Y. K. Hsiao and C. Q. Geng, *Phys. Rev. D* **91**, 116007 (2015).
- [67] X. G. He and G. N. Li, *Phys. Lett. B* **750**, 82 (2015).
- [68] M. He, X. G. He, and G. N. Li, *Phys. Rev. D* **92**, 036010 (2015).
- [69] J. Zhu, H. W. Ke, and Z. T. Wei, *Eur. Phys. J. C* **76**, 284 (2016).
- [70] Y. K. Hsiao, Y. Yao, and C. Q. Geng, *Phys. Rev. D* **95**, 093001 (2017).
- [71] M. J. Savage and R. P. Springer, *Phys. Rev. D* **42**, 1527 (1990).
- [72] S. Pakvasa, S. P. Rosen, and S. F. Tuan, *Phys. Rev. D* **42**, 3746 (1990).
- [73] M. J. Savage, *Phys. Lett. B* **257**, 414 (1991).
- [74] C. D. Lü, W. Wang, and F. S. Yu, *Phys. Rev. D* **93**, 056008 (2016).
- [75] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *J. High Energy Phys.* **11** (2017) 147.
- [76] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *Phys. Rev. D* **99**, 073003 (2019).
- [77] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *Eur. Phys. J. C* **78**, 593 (2018).
- [78] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *Phys. Rev. D* **97**, 073006 (2018).
- [79] C. Q. Geng, C. W. Liu, and T. H. Tsai, *Phys. Lett. B* **790**, 225 (2019).
- [80] Y. Grossman and S. Schacht, *Phys. Rev. D* **99**, 033005 (2019).
- [81] Y. K. Hsiao, Y. Yao, and H. J. Zhao, *Phys. Lett. B* **792**, 35 (2019).
- [82] C. Q. Geng, C. W. Liu, and T. H. Tsai, *Phys. Lett. B* **794**, 19 (2019).
- [83] R. M. Wang, M. Z. Yang, H. B. Li, and X. D. Cheng, *Phys. Rev. D* **100**, 076008 (2019).
- [84] D. Wang, *Eur. Phys. J. C* **79**, 429 (2019).
- [85] C. P. Jia, D. Wang, and F. S. Yu, *Nucl. Phys. B* **956**, 115048 (2020).
- [86] A. Dery, M. Ghosh, Y. Grossman, and S. Schacht, *J. High Energy Phys.* **03** (2020) 165.
- [87] B. Grinstein and M. A. Savorov, [arXiv:hep-ph/0408346](https://arxiv.org/abs/hep-ph/0408346).
- [88] S. Roy, R. Sinha, and N. G. Deshpande, *Phys. Rev. D* **101**, 036018 (2020).
- [89] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1957).
- [90] J. J. de Swart, *Rev. Mod. Phys.* **35**, 916 (1963); **37**, 326(E) (1965).
- [91] N. P. Samios, M. Goldberg, and B. T. Meadows, *Rev. Mod. Phys.* **46**, 49 (1974).
- [92] T. A. Kaeding and H. T. Williams, *Comput. Phys. Commun.* **98**, 398 (1996).
- [93] T. A. Kaeding, *At. Data Nucl. Data Tables* **61**, 233 (1995).
- [94] M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, *Nucl. Phys. B* **415**, 403 (1994).
- [95] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).
- [96] A. J. Buras, [arXiv:hep-ph/9806471](https://arxiv.org/abs/hep-ph/9806471).
- [97] J. G. Körner, *Nucl. Phys. B* **225**, 282 (1971).
- [98] J. C. Pati and C. H. Woo, *Phys. Rev. D* **3**, 2920 (1971).
- [99] B. Stech, *Phys. Rev. D* **36**, 975 (1987).
- [100] M. Neubert and B. Stech, *Phys. Rev. D* **44**, 775 (1991).
- [101] M. Neubert and B. Stech, *Adv. Ser. Dir. High Energy Phys.* **15**, 294 (1998).
- [102] Y. Kohara, *Phys. Rev. D* **44**, 2799 (1991).
- [103] Q. P. Xu and A. N. Kamal, *Phys. Rev. D* **46**, 3836 (1992).
- [104] L. L. Chau, H. Y. Cheng, and B. Tseng, *Phys. Rev. D* **54**, 2132 (1996).
- [105] C. Q. Geng, C. W. Liu, T. H. Tsai, and Y. Yu, *Phys. Rev. D* **99**, 114022 (2019).
- [106] W. H. Liang and E. Oset, *Eur. Phys. J. C* **78**, 528 (2018).
- [107] T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, *Phys. Rev. D* **98**, 074011 (2018).
- [108] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and Z. Tyulemissov, *Phys. Rev. D* **99**, 056013 (2019).
- [109] D. A. Egolf, R. P. Springer, and J. Urban, *Phys. Rev. D* **68**, 013003 (2003).

- [110] Y. Grossman, Z. Ligeti, and D. J. Robinson, *J. High Energy Phys.* **01** (2014) 066.
- [111] D. Xu, G. N. Li, and X. G. He, *Int. J. Mod. Phys. A* **29**, 1450011 (2014).
- [112] T. Brown, S. F. Tuan, and S. Pakvasa, *Phys. Rev. Lett.* **51**, 1823 (1983).
- [113] J. F. Donoghue and S. Pakvasa, *Phys. Rev. Lett.* **55**, 162 (1985).
- [114] I. Dunietz, *Z. Phys. C* **56**, 129 (1992).
- [115] M. Gronau, *Phys. Lett. B* **492**, 297 (2000).
- [116] J. F. Nieves and P. B. Pal, *Am. J. Phys.* **72**, 1100 (2004).