Escalating core formation with dark matter self-heating

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Exothermic scatterings of dark matter (DM) produce DM particles with significant kick velocities inside DM halos. In collaboration with DM self-interaction, the excess kinetic energy of the produced DM particles is distributed to the others, which self-heats the DM particles as a whole. The DM self-heating is efficient towards the halo center, and the heat injection is used to enhance the formation of a uniform density core inside halos. The effect of DM self-heating is expected to be more significant in smaller halos for two reasons: 1) the exothermic cross section times the relative velocity, $\langle\sigma_{\text{exo}} v_{\text{rel}}\rangle$, is constant; 2) and the ratio of the injected heat to the velocity dispersion squared gets larger toward smaller-size halos. For the first time, we quantitatively investigate the core formation from DM self-heating for halos in a wide mass range (10^9 – 10^{15} $M_{\odot}$) using the gravothermal fluid formalism. Notably, we demonstrate that the core formation is sharply escalating toward smaller-size halos by taking the self-heating DM (i.e., DM that semiannihilates and self-interacts) as an example. We show that the sharp escalation of the core formation may cause a tension in simultaneously explaining the observed central mass deficit of Milky Way satellites and field dwarf/low surface brightness spiral galaxies. We expect DM self-heating to be present also in other models that exhibit exothermic scatterings and self-interaction of DM, which can appreciably contribute to the core formation of DM halos.

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I. INTRODUCTION

Dark matter (DM) shapes the matter distribution of our Universe. It provides gravitational potential wells, where galaxies are formed to evolve and merge. While their existence and distribution is probed through their gravitation, the particle nature of DM remains unknown. Perhaps one of the most motivated candidates is a weakly interacting massive particle (WIMP), and there have been extensive efforts to detect them by probing their interactions with the Standard Model (SM). However, there has been no conclusive signal yet [1,2].

In light of this situation, there have been a great deal of efforts to look for new benchmarks for DM. Many of the attempts in this direction invoke new mechanisms that stabilize DM, which accompany additional particle contents and DM interactions within a dark sector. Such secluded DM interactions may leave an imprint on the distribution of DM in our Universe [3]. Thus, looking for

distinctive consequences of DM interactions on their spatial distribution could provide hints that may be difficult to achieve through traditional particle physics probes, i.e., large hadron collider (LHC) [4–6] and direct/indirect detection searches [7–9].

The discrepancies between the predictions of the standard CDM paradigm and the astrophysical observations in small-scale structures [10] (small-scale issues) may be hinting at the existence of dark-sector interactions. While it is not clear if subgrid astrophysical processes could entirely resolve the issues [11–13], it is an interesting possibility that the secluded DM interactions are responsible for the discrepancies. A prominent aspect of the issues is that observed galaxies prefer cored central density profile, rather than cuspy [14,15]. Such a central mass deficit is reported to be halo-mass dependent¹; galaxy clusters (10^{14} – 10^{15} $M_{\odot}$) [17,18] exhibit milder cores than dwarf spheroidal galaxies in the field [19,20] and low surface brightness (LSB) spiral galaxies [21–23] (10^{10} – 10^{11} $M_{\odot}$).

Self-interacting DM (SIDM) is one of the well-known frameworks for explaining the central mass deficit of halos [24], and the halo-mass dependent cores may be explained by the velocity dependent self-interaction of DM particles [25]. One of the realizations (see Refs. [26–31] for the other

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¹Central DM mass deficits have also been inferred for high surface brightness spiral galaxies [16].

references) for the velocity dependence is the exothermic self-interaction; kinematically unsuppressed exothermic cross section exhibits $\sigma_{\text{exo}} \propto 1/v_{\text{rel}}$ in the nonrelativistic limit [27].

Meanwhile, the energy released from such exothermic DM scatterings² give rise to yet another scale dependent effect: the injected energy self-heats the DM particles, which could enhance the core formation toward smaller-size halos [28]. See Refs. [29,41–43] for exothermic DM models, where the DM self-heating effect may be relevant. Despite the recent interests in exothermic DM scenarios as an explanation for the observed halo-mass dependence, there has been no quantitative study that sharpens the prediction of DM self-heating effect on halo structures and directly compares the predicted halo-mass dependence with observations.

In this work, we clarify these points by studying the impact of DM self-heating on isolated halos. We adopt the gravothermal fluid formalism [44–53], which allows us to follow the evolutions of halos as small as the Milky Way (MW) satellites ($10^9 M_\odot$) at a small computational expense, which is difficult to achieve in N -body simulations. Exploiting the method, we investigate the effect of DM self-heating on halos in a wide mass range (10^9 – $10^{15} M_\odot$) for the first time. Interestingly, we find that the core formation effect from DM self-heating is sharply escalating toward smaller-size halos. We demonstrate that this sharp halo-mass dependence may cause a tension between parameters that explain the central mass deficit of MW satellite dwarf spheroidal galaxies [54] and those of field dwarf/LSB galaxies, although the mass difference between the two is just an order of magnitude.

To demonstrate the escalating core formation from the self-heating effect, we take self-heating DM (SHDM) [55,56] as an example, where DM particles semiannihilate ($\text{DM} + \text{DM} \rightarrow \text{DM} + \phi$, where ϕ is a light particle) and self-interact. We find that (sub-)MeV scale SHDM can simultaneously explain the observed central mass deficit of field dwarf/LSB galaxies and galaxy clusters for the following *constant* cross sections:

$$\langle \sigma_{\text{semi}} v_{\text{rel}} \rangle \simeq 6 \times 10^{-26} \text{ cm}^3/\text{s}, \quad \sigma_{\text{self}}/m \simeq 0.1 \text{ cm}^2/\text{g}, \quad (1)$$

as discussed in Sec. III. We note that (sub-)MeV scale SHDM may be subject to warm DM (WDM) constraints from matter distribution at $z = 3$ – 5 probed by the Lyman- α forest data [55,56]. If SHDM decouples from a dark plasma in the early Universe, the SHDM mass needs to be heavier than $m \gtrsim 40 \text{ MeV}((1/(T_{\text{DR}}/T_{\text{SM}})_{\text{eq}}); (T_{\text{DR}}/T_{\text{SM}})_{\text{eq}}$ is the

temperature ratio between the dark plasma and the SM one around the matter-radiation equality. Therefore, in order for (sub-)MeV scale SHDM to be consistent with the Lyman- α forest data, the dark plasma may need to be much colder than the SM.

II. MODELING DARK MATTER SELF-HEATING

We study the effect of DM self-heating by following the gravothermal evolution of isolated halos that consist of SHDM. We numerically solve the gravothermal fluid equations that are analogous to the ones adopted to describe halos of pure SIDM [47–53] but with an additional heat injection term that models the DM self-heating. We first review the gravothermal fluid model for the pure SIDM and discuss our implementation of DM self-heating.

We consider a spherically symmetric initial halo that is formed around the time t_* that is determined by its virial mass, based on a spherical collapse model; see the Appendix A for details on our assignment of t_* . We *assume* that DM self-interaction and DM self-heating are unimportant during the process of halo formation; we use the initial halo profiles predicted from CDM N -body simulations. Then the initial halo density profile is approximated by the Navarro-Frenk-White (NFW) profile [57,58], $\rho_{\text{NFW}}(r) = \rho_s / [(r/r_s)(1 + (r/r_s))^2]$, where r_s (ρ_s) is the NFW scale radius (density); see Appendix C for our analysis with *cored* initial halo profiles. We use the NFW scale parameters determined from the halo virial mass (M_{200}), using the concentration-mass relation inferred from CDM N -body simulations [59],

$$\begin{aligned} \rho_s &\simeq 0.011 M_\odot/\text{pc}^3 \left(\frac{10^{10} M_\odot}{M_{200}} \right)^{0.24}, \\ r_s &\simeq 3.43 \text{ kpc} \left(\frac{M_{200}}{10^{10} M_\odot} \right)^{0.44}. \end{aligned} \quad (2)$$

The gravothermal fluid formalism approximates the DM particles as an ideal fluid that is described completely by their mass density ρ and fluid pressure p . The initial halo is assumed isotropic in pressure, and we define the one-dimensional velocity dispersion as $\nu(r, t) = \sqrt{p(r, t)/\rho(r, t)}$. In the case of pure SIDM, gravothermal evolution of DM fluid is governed by the following equations [47–53]:

$$\frac{\partial(\rho\nu^2)}{\partial r} + \frac{GM\rho}{r^2} = 0, \quad (3a)$$

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho, \quad (3b)$$

$$\frac{3}{\nu} \left(\frac{\partial \nu}{\partial t} \right)_M - \frac{1}{\rho} \left(\frac{\partial \rho}{\partial t} \right)_M = \frac{1}{\nu^2} \frac{\delta u_{\text{cond}}}{\delta t}, \quad (3c)$$

²In this work, we consider a case where only the DM self-heating via exothermic self-scattering is relevant. Meanwhile, it is also possible for DM particles to lose kinetic energy, e.g., through endothermic up scattering in multistate DM scenarios [32–40].

where $M(r, t)$ is the fluid mass enclosed within radius r , and G is the Newton's constant. $(\partial_t)_M = \partial_t + \mathbf{V} \cdot \nabla$ is the Lagrangian time derivative, where $\mathbf{V}$ is the fluid bulk velocity; it refers to changes within the fluid element as it changes its state and location. We note that $\mathbf{V}$ is determined by the continuity equation, $\partial_t \rho + \nabla \cdot (\rho \mathbf{V}) = 0$. Meanwhile, the terms associating $\mathbf{V}$ in the fluid momentum conservation equation (Euler equation) are assumed to be negligible [Eq. (3a)]; we require the halo to be in quasistatic equilibrium. See Appendix B for the justification of this assumption. Equation (3b) defines the enclosed fluid mass M .

Equation (3c) is the first law of thermodynamics. The details of particle physics appear in the right-hand side, which is the rate of heat gain of a fluid element; fluid elements conduct heat with neighboring sites in the radial direction through DM self-interaction. The net heat gain from heat conduction is used to increase the velocity dispersion (the first term of the left-hand side) and to decrease the mass density (the second term of the left-hand side) by expanding the volume of the DM fluid. The heat conduction is modeled with a heat diffusion equation,

$$\frac{\delta u_{\text{cond}}}{\delta t} = \frac{m}{\rho} \nabla \cdot (\kappa \nabla \nu^2), \quad (4)$$

where u is the specific energy (energy per DM mass), and κ is the thermal conductivity. We follow the implementation of κ in Refs. [47, 50]. We naively interpolate κ in the limits of the short/long mean free path (SMFP/LMFP) regimes: $\kappa^{-1} = \kappa_{\text{SMFP}}^{-1} + \kappa_{\text{LMFP}}^{-1}$. In the SMFP [60, 61] (LMFP [44]) regime, the free-streaming length of a DM particle, $\lambda = 1/(\rho \sigma_{\text{self}}/m)$, is much shorter (longer) than the system size, $H = \sqrt{\nu^2/4\pi G\rho}$. In the SMFP (LMFP) regime, Eq. (4) is analytically (empirically) given as

$$\kappa_{\text{SMFP}} = \frac{75\sqrt{\pi}}{64} \frac{\rho \lambda^2}{a m t_{\text{self}}}, \quad \kappa_{\text{LMFP}} = \frac{3C}{2} \frac{\rho H^2}{m t_{\text{self}}}, \quad (5)$$

where $a = \sqrt{16/\pi}$, and $C \simeq 0.75$ is a constant that is calibrated with N -body simulations in the LMFP regime [48, 50]. t_{self} is the self-scattering time scale given as

$$t_{\text{self}} = (a \rho \nu \sigma_{\text{self}}/m)^{-1} \simeq 310 \text{ Gyr} \left(\frac{0.1 \text{ cm}^2/\text{g}}{\sigma_{\text{self}}/m} \right) \left(\frac{10^9 \text{ M}_\odot}{M_{200}} \right)^{0.08}. \quad (6)$$

In the second equality, we took $\nu_{\text{NFW}}(r_s) \simeq 0.3 \sqrt{4\pi G \rho_s r_s^2}$ and $\rho_{\text{NFW}}(r_s) = \rho_s/4$, which are, respectively, the velocity dispersion and density of the initial NFW halo at the scale radius $r = r_s$, and used the concentration-mass relation given in Eq. (2); hereafter, we will refer to t_{self} as this value.

We implement the effects of DM self-heating by introducing another heating term in the right-hand side of

Eq. (3c). In the case of SHDM, a tiny fraction of the boosted DM particles produced from DM semiannihilations is captured before escaping the halo through DM self-interaction. (For simplicity, we assume that the produced ϕ 's escape from the halo and plays no role in halo evolution.) The high kinetic energy of the captured boosted DM particle, $\delta E = m/4$, is then distributed to the others, i.e., DM particles self-heat. We effectively describe this process as injecting heat to the local DM fluid element with the following rate:

$$\frac{\delta u_{\text{heat}}}{\delta t} = \frac{\rho \langle \sigma_{\text{semi}} v_{\text{rel}} \rangle}{m} \frac{\xi \delta E}{m}, \quad (7)$$

where ξ is the efficiency to capture a boosted DM particle. The realistic modeling of the capture efficiency ξ is nontrivial; the production site of the captured boosted DM particle may not be the local fluid element, but some remote site from the captured region. Instead, we use a constant and uniform profile for ξ ,

$$\xi = b \times \frac{r}{\lambda} \Big|_{r=r_s} \simeq 0.0002b \left(\frac{r_s}{3.43 \text{ kpc}} \right) \left(\frac{\rho_s}{0.011 \text{ M}_\odot/\text{pc}^3} \right) \left(\frac{\sigma_{\text{self}}/m}{0.1 \text{ cm}^2/\text{g}} \right), \quad (8)$$

where b is a fudge factor and $\lambda = 1/(\rho \sigma_{\text{self}}/m)$ is the free-streaming length of a DM particle. We expect b to be an order of unity and set $b = 1$ from now on. Nevertheless, b could be changed by a factor of order unity. For a given $\langle \sigma_{\text{semi}} v_{\text{rel}} \rangle$ and σ_{self}/m , rescaling b is equivalent to rescaling the SHDM mass, as will be discussed in the next section. For other exothermic DM models, we may replace the parameters $\langle \sigma_{\text{exo}} v_{\text{rel}} \rangle/m \times \xi \delta E/m$ in Eq. (7) with the corresponding values. Taking into account the DM self-heating represented by Eq. (7), we numerically solve Eqs. (3) using the methods described in Refs. [48, 51–53]. We find that DM self-heating leads to a formation of uniform cores inside halos (see Fig. 1).

III. HALO-MASS DEPENDENCE OF THE CORE FORMATION

The core formation from the self-heating effect becomes stronger toward smaller-size halos. This is understood by observing the DM self-heating term in the right-hand side of Eq. (3c),

$$\frac{1}{\nu^2} \frac{\delta u_{\text{heat}}}{\delta t} = \left(\frac{\nu_{\text{NFW}}(r_s)}{\nu} \right)^2 \left(\frac{\rho}{\rho_{\text{NFW}}(r_s)} \right) \frac{1}{t_{\text{heat}}}. \quad (9)$$

t_{heat} is the representative heating time scale of the initial NFW halo; it is roughly the time scale required for a DM particle to absorb energy comparable to its kinetic energy

$\sim m\nu^2$ by capturing boosted DM particles produced from semiannihilation,

$$t_{\text{heat}} = \left(\frac{\rho_{\text{NFW}}(r_s) \langle \sigma_{\text{semi}} v_{\text{rel}} \rangle}{m} \frac{\xi \delta E}{m \nu_{\text{NFW}}^2(r_s)} \right)^{-1} \\ \simeq 42 \text{ Gyr} \left(\frac{M_{200}}{10^9 \text{ M}_\odot} \right)^{0.68} \left(\frac{0.1 \text{ cm}^2/\text{g}}{\sigma_{\text{self}}/m} \right) \\ \times \left(\frac{6 \times 10^{-26} \text{ cm}^3/\text{s}}{\langle \sigma_{\text{semi}} v_{\text{rel}} \rangle} \right) \left(\frac{m/b}{0.3 \text{ MeV}} \right) \left(\frac{1/4}{\delta E/m} \right), \quad (10)$$

where M_{200} is the virial mass of a halo. In the second equality, we used Eq. (8) and the concentration-mass relation for the NFW scale parameters [Eq. (2)]. In Eq. (9), $\nu(r)/\nu_{\text{NFW}}(r_s)$ and $\rho(r)/\rho_{\text{NFW}}(r_s)$ for a given r/r_s do not depend much on halo mass, while t_{heat} is significantly shorter in smaller halos. Thus, the core formation from self-heating is increasing toward smaller-size halos. This feature is illustrated in Fig. 1. In the left panel, we show the present density profiles of a galaxy cluster (green), a field dwarf/LSB galaxy (blue), and a MW satellite (red). We define the core density ρ_{core} to be the density at the innermost radius. In the $m = 0.3 \text{ MeV}$ case (solid), $\rho_{\text{core}}/\rho_s$ decreases by a factor of $\simeq 2.3$ from green to blue and by a factor of $\simeq 5$ from blue to red, which clearly shows that the core formation is efficient toward smaller-size halos. The latter is a sharp decrease given that their

mass difference is just an order of magnitude. This is because DM self-heating starts to dominate the core formation effect from the scales of field dwarf/LSB galaxies, and the core formation escalates toward smaller-size halos.

This halo-mass dependence of the core formation may be used to explain why the observed central mass deficit is more appreciable in smaller halos. In Fig. 2, we demonstrate that the core formation of (sub-)MeV scale SHDM may address this issue even with constant $\langle \sigma_{\text{semi}} v_{\text{rel}} \rangle \simeq 6 \times 10^{-26} \text{ cm}^3/\text{s}$ and $\sigma_{\text{self}}/m \simeq 0.1 \text{ cm}^2/\text{g}$. There, we define the core radius r_{core} to be the radius, where the density is $\rho(r_{\text{core}}) = \rho_{\text{core}}/2$. We find that, for $b = 1$, the SHDM mass $m \simeq 0.3 \text{ MeV}$ (solid orange curve in Fig. 2) is preferred to explain the observed core density/size of dwarf/LSB galaxies. We take these parameters as a benchmark for SHDM. We note that rescaling b is equivalent to rescaling the benchmark value for m ; t_{heat} (hence, the core formation) is equivalent among the same values of m/b .

As shown in Fig. 2, for the benchmark parameters of SHDM (solid orange), the core formation of halos sharply escalates towards smaller-size halos; the core density $\rho_{\text{core}}/\rho_s$ (core radius r_{core}/r_s) decreases (increases) towards smaller-size halos as $\propto M_{200}^{0.68}$ ($\propto M_{200}^{-0.39}$). This sharp halo-mass dependence begins from the scales of the dwarf/LSB galaxies ($M_{200} \simeq 10^{10} \text{ M}_\odot$), where the core formation effect from DM self-heating starts to dominate over that from the DM self-interaction, i.e., $t_{\text{heat}} < t_{\text{self}}$. In this

FIG. 1. *Left panel*—Present density profiles of SHDM halos. Colored lines represent different masses of halos; a galaxy cluster (green), a field dwarf/LSB galaxy (blue), and a MW satellite galaxy (red). The NFW profile (dotted black) is used as initial condition; we use the concentration-mass relation from Ref. [59] to determine ρ_s and r_s . We take the fudge factor to be $b = 1$. Solid lines represent the SHDM mass ($m = 0.3 \text{ MeV}$) that reproduce a similar core size as the inferred pure SIDM cross section for field dwarf/LSB galaxies, $\sigma_{\text{self}}/m = 1.9^{+0.6}_{-0.4} \text{ cm}^2/\text{g}$ [25]. Note that for given cross sections, rescaling b is equivalent to rescaling the SHDM mass m [see Eq. (10)]; e.g., if we take $b = 10$, the solid curves represent the result for $m = 3 \text{ MeV}$. The core formation is efficient toward smaller-size halos. For galaxy clusters, the resultant profiles for $m = 0.3 \text{ MeV}$ and $m = 3 \text{ MeV}$ are almost identical; this is because the effect of DM self-heating is negligible, and the core formation is predominantly determined by the effect of DM self-scattering. In the case of $m = 3 \text{ MeV}$, the core density of the galaxy cluster is lower than that of the field dwarf/LSB galaxy. This is because the core formation is still dominated by the self-scattering effect, and the self-scattering time scale [Eq. (6)] is longer toward smaller-size halos, $t_{\text{self}} \propto M_{200}^{-0.08}$. *Right panel*—Same as the left panel but for the one-dimensional velocity dispersion. $t_G(r_s) = 1/\sqrt{4\pi G \rho_{\text{NFW}}(r_s)}$ is the gravitational time scale with $\rho_{\text{NFW}}(r_s) = \rho_s/4$.

FIG. 2. *Left panel*—Halo-mass dependence of resultant core densities of SHDM halos (in orange). We took the evolution time that is determined by the virial mass of a halo [see Appendix A]. Gray curves represent the case of pure SIDM; the gray curves can be understood by noting that t_{self} [Eq. (6)] is shorter for larger-size halos, while $(t_{\text{age}} - t_*)$ is also shorter for larger-size halos. The data points are the core densities predicted from the inferred values of pure σ_{self}/m from the observations on field dwarf (red)/LSB (blue) galaxies, and galaxy clusters (green) [25]. The core formation is sharply escalating toward smaller-size halos, which appears to disfavor SHDM as a solution to small-scale issues; while the SHDM mass $m = 0.3$ MeV (solid orange) is preferred to explain the core size of field dwarf/LSB galaxies, the central densities of MW satellites [54] (black circle) require the SHDM mass to be heavier than 3 MeV (dashed orange). This is in contrast to the pure SIDM; there is a lower limit on the core density since the SIDM halos undergo the gravothermal collapse at present for $\sigma_{\text{self}}/m \gtrsim 10$ cm²/g. The red line represents the analytic estimation given in Eq. (11). Note that for given cross sections, rescaling b would be equivalent to rescaling the SHDM mass m [see Eq. (10)]; e.g., if we take $b = 10$, the solid orange curve would represent the result for $m = 3$ MeV. *Right panel*—Same as the left panel but for the resultant core radius.

regime, the following empirical expressions approximates the core density/radius well³ (solid red in Fig. 2):

$$\begin{aligned} \rho_{\text{core}}(t_{\text{age}}) &\simeq 0.2 \times \rho_s \left(\frac{t_{\text{heat}}}{t_{\text{age}} - t_*} \right), \\ r_{\text{core}}(t_{\text{age}}) &\simeq 1.4 \times r_s \left(\frac{t_{\text{age}} - t_*}{t_{\text{heat}}} \right)^{0.57}, \end{aligned} \quad (11)$$

where t_* is the time when the initial virialized halo was formed, and $t_{\text{age}} \simeq 13.8$ Gyr is the age of the Universe.

We note that the approximations for the escalating core formation given in Eq. (11) may be significantly modified when the t_{heat} or t_{self} is as short as the formation time of halos t_* . In such case, DM self-heating or self-interaction along the halo formation may significantly modify the initial profile of halos; the effects would make the initial profile to be more cored from the NFW profile. Since t_{heat} has a sharp power-law halo-mass dependence, the modification of initial halo profiles would become more significant as we consider smaller-size halos. Nevertheless, for the SHDM benchmark parameters, we expect that the

³For the considered SIDM cross section in our analysis [$\sigma_{\text{self}}/m = \mathcal{O}(0.1)$ cm²/g], the halo evolution takes place in the LMFP regime. In this regime, the right-hand side of Eq. (3c) is dominated by the effects of self-heating ($\delta u_{\text{heat}}/\delta t > \delta u_{\text{cond}}/\delta t$) for $t_{\text{heat}} < t_{\text{self}}$. We numerically confirm that the estimation given in Eq. (11) works well for any halo mass and for $\sigma_{\text{self}}/m \lesssim 10$ cm²/g, as long as $t_{\text{heat}} < t_{\text{self}}$. This estimate resembles the analytic estimation given in Ref. [28], although Ref. [28] assumes a uniform and constant velocity dispersion profile.

modification of initial halo profiles to have subdominant effect on present halo structures as long as we consider halos of $M_{200} \gtrsim 10^7 M_{\odot}$; see Appendix C for more discussion.

IV. CONSTRAINTS FROM MILKY WAY SATELLITES

As illustrated in Fig. 1 and Fig. 2, the core formation from self-heating effect is stronger toward smaller-size halos, which may address why galaxy clusters exhibit milder cores than dwarf/LSB galaxies. On the other hand, the core formation from self-heating in smaller halos ($M_{200} \lesssim 10^{10} M_{\odot}$) may be too efficient to be compatible with observations. In fact, observed central densities of MW satellite dwarf spheroidal galaxies provide strong restrictions on the efficiency of the self-heating.

The observed central densities of MW satellite dwarf galaxies is in the range of $\rho_{\text{c,obs}} \sim 0.1\text{--}1 M_{\odot}/\text{pc}^3$ [54]. We require the predicted core density to be larger than the observed one ($\rho_{\text{core}}(t_{\text{age}}) \gtrsim 0.1 M_{\odot}/\text{pc}^3$); employing the analytic estimation for core density given in Eq. (11), the condition restricts the heating time scale for MW satellites as

$$t_{\text{heat}} \gtrsim 320 \text{ Gyr} \left(\frac{\rho_{\text{c,obs}}}{0.1 M_{\odot}/\text{pc}^3} \right) \left(\frac{M_{200,\text{infall}}}{10^9 M_{\odot}} \right)^{0.24}. \quad (12)$$

Here, we took the masses of MW satellites as their masses prior to accretion onto the MW, which is estimated using the concentration-mass relation of field halos [62]. Since

progenitor halos of MW satellite galaxies lose their mass along the accretion, our choice of a satellite halo mass is conservative; for example, together with Eq. (10) and given the cross sections, Eq. (12) provides stronger limits to the SHDM mass m for smaller MW satellite halo mass. This is because t_{heat} decreases toward smaller-size halos much faster than the right-hand side of Eq. (12). With the benchmark cross sections, Eq. (12) translates to a rough lower of $m \gtrsim 3$ MeV. The core formation for SHDM with mass of $m = 3$ MeV is shown in Fig. 2 (dashed orange): for these parameters, the core formation may not be efficient enough to explain the inner density profiles of field dwarf/LSB galaxies.

It is interesting that observations on MW satellites provide a significant constraint, although their masses (at infall) are smaller than those of field dwarf/LSB galaxies by just an order of magnitude. This is because the core formation from DM self-heating is sharply escalating toward smaller-size halos, which is represented by the sharp power-law halo-mass dependence of t_{heat} [Eq. (10)] inside Eq. (11). Thus, observations on even smaller halos (i.e., minihalos) would be a good probe to examine the self-heating nature of DM. For example, the observed concentration-mass relation of minihalos may provide a firm test to distinguish SHDM from pure SIDM; SHDM halos would exhibit consistently lower concentration than the pure SIDM halos. This is because the core density of self-heating dominated halos decreases indefinitely until present, as in Eq. (11). This is in contrast to pure SIDM halos, since their core density do not get lower than $\rho_{\text{NFW}}(r_s) = \rho_s/4$ due to the self-similar gravothermal collapse [48].

V. CONCLUDING REMARKS

Using the gravothermal fluid formalism, we have studied the impact of DM self-heating on halo structures. We took SHDM as an example and showed that DM self-heating flattens the central cuspy profile of an initial halo and develops a uniform core. The fluid formalism allowed us to scope the effect of DM self-heating for a wide mass range of halos, and we found that the core formation is sharply escalating toward smaller-size halos. By requiring the SHDM to be consistent with the observed central densities of MW satellites, we showed that the escalating core formation of SHDM appears to be disfavored as an explanation for the observed central mass deficit of field dwarf/LSB galaxies.

DM self-heating is expected to be a general feature of models, where DM particles undergo exothermic scatterings and self-interaction. DM self-heating can significantly contribute to the core formation of DM halos; we emphasize that the core formation of exothermic DM is different from that of pure SIDM. Specifically, one cannot naively interpret the exothermic cross section as the pure SIDM cross section. In general, exothermic scatterings can have

both direct and indirect effects on the core formation: boosted DM particles from exothermic scatterings escape from a halo and directly decrease central DM density; meanwhile, boosted DM particles deposit energy through elastic scatterings to expand DM fluid and indirectly decreases DM density. We showed that even only the latter can cause the escalating core formation by taking SHDM, where the former is negligible. In other models [27,29,43], both effects can be equally important, and we will study such a case in a future work [63].

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APPENDIX A: HALO FORMATION TIME

The structure formation is hierarchical in the sense that smaller structures form prior to larger ones. Since we consider evolution of halos in a wide mass range ($M_{200} = 10^8\text{--}10^{15} M_\odot$), we take into account the formation time of halos consistent with their mass. In this section, we summarize the assignment of the halo formation time t_* .

We define the overdensity $\delta = \rho_m/\bar{\rho}_m - 1$, where $\bar{\rho}_m$ is the average matter density. For a standard CDM model, the overdensity in the linear regime ($\delta < 1$) grows as $\delta(t) = \delta(t_i)D(t)/D(t_i)$, where $D(t)$ is the growth factor given by

$$D(a(t)) = \frac{H(a) \int_0^a da' / [a' H(a')/H_0]^3}{H_0 \int_0^1 da' / [a' H(a')/H_0]^3}, \quad (\text{A1})$$

which is normalized as $D(a(t_{\text{age}})) = 1$ where $t_{\text{age}} \simeq 13.8$ Gyr is the age of the Universe. According to the spherical collapse model [46] in the flat matter dominated FRW Universe, virialized halos form when their overdensity becomes $\delta(t_*) \sim 200$, where the linear theory is no longer valid. On the other hand, if we assume that the linear theory were valid even when $\delta > 1$, a virialized halo is formed when the overdensity in the linear theory exceeds some critical value $\delta_c \simeq 1.686$ at time t_* : $\delta_L(t_*) \simeq \delta_c$.

Now we would like to assign masses to such virialized halos. To do so, we define a smoothed overdensity δ_M , which is the overdensity after averaging out all density fluctuations on scales less than scale R ,

$$\delta_M(\mathbf{x}) = \int d^3\mathbf{x}' W_R(\mathbf{x}' - \mathbf{x}) \delta_L(\mathbf{x}'), \quad (\text{A2})$$

where we choose the window function as the top-hat filter in real space; $W_R(\mathbf{x}' - \mathbf{x}) = 3/(4\pi R^3)\Theta(R - |\mathbf{x}' - \mathbf{x}|)$. Given a length scale R , we assign a mass scale of the filter as $M = 4\pi R^3 \bar{\rho}_m/3$. Following the Press-Schechter formalism [64], we identify the mass fraction that is contained in halos of mass greater than M with the probability that $\delta_M > \delta_c$;

$$P(\delta_M > \delta_c) = \int_{\delta_c}^{\infty} \frac{\exp(-\frac{\delta_M^2}{2\sigma_M^2})}{\sqrt{2\pi}\sigma_M} d\delta_M, \quad (\text{A3})$$

where σ_M^2 is the variance of δ_M given by

$$\sigma_M^2(t) = D^2(a(t)) \int_0^{\infty} \frac{k^2 dk}{2\pi^2} P_L(k) \tilde{W}_R^2(k), \quad (\text{A4})$$

where $P_L(k)$ is the CDM linear matter power spectrum at present, and $\tilde{W}_R(k)$ is the Fourier transformed top-hat window function. Here, we take $P_L(k) \propto T_{\text{BBKS}}^2(k) P_{\text{prim}}(k)$; $T_{\text{BBKS}}(k)$ is the BBKS fitting transfer function [65] and $P_{\text{prim}}(k) \propto k^{n_s}$. We adopt the following cosmological parameters as in Ref. [66]: $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.7$, $n_s = 0.96$, and $\sigma_8 = 0.8$. The overall normalization for $P_L(k)$ is set by the cosmological parameter σ_8 . From Eq. (A3), the fraction of mass contained in halos with mass greater than M becomes significant when $\sigma_M(t) \sim \delta_c$. We choose the formation time of a halo of mass M to be the moment when $3\sigma_M(t_*) = \delta_c$, where the factor 3 is chosen to reproduce the halo evolution time used in Ref. [25]. The halo-mass dependence of t_* is shown in Fig. 3.

FIG. 3. The halo-mass dependence of the halo formation time t_* . The assignment of the formation time is based on the spherical collapse model, and the observed CDM density fluctuations. The solid curve corresponds to the condition $2\sigma_M(t_*) = \delta_c$. The dashed curve is the one that we use in this work, which is determined by the condition $3\sigma_M(t_*) = \delta_c$.

APPENDIX B: ASSUMPTION OF QUASISTATIC EQUILIBRIUM

In the adopted gravothermal fluid model [Eqs. (3)], we assume that the evolution of a halo is quasistatic: we require that the hydrostatic equilibrium condition [Eq. (3a)] is held at each moment. This may be a good approximation as long as the gravitational time scale, t_G , is the shortest time scale of the system [47,48],

$$t_G = (4\pi G\rho)^{-1/2} \simeq 5.7 \times 10^{-2} \text{ Gyr} \left(\frac{M_{200}}{10^9 M_\odot} \right)^{0.12}, \quad (\text{B1})$$

where in the second equality, we took $\rho = \rho_{\text{NFW}}(r_s) = \rho_s/4$ as the representative value for a halo. In the case of pure SIDM, the approximation is confirmed to be in good agreement with the N -body simulations [50,53].

Since we are interested in the case where the two other time scales, t_{self} [Eq. (6)] and t_{heat} [Eq. (10)], are comparable to the age of the Universe, the same approximation is expected to be valid in the presence of DM self-heating. Nevertheless, it is good to have an independent check for the case of SHDM. The comparison between our fluid model and the N -body simulations would be the most ideal check, but performing N -body simulations for SHDM is beyond the scope of this work. Instead, we compare the numerical solutions to Eqs. (3) with the solutions *without* assuming the quasistatic equilibrium. We obtain the solutions without the assumption in the Eulerian description of fluid [28,49],

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho V_r) = 0, \quad (\text{B2a})$$

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} = -\frac{\partial \Phi}{\partial r} - \frac{1}{\rho} \frac{\partial (\rho \nu^2)}{\partial r}, \quad (\text{B2b})$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Phi}{\partial r} \right) = 4\pi G\rho, \quad (\text{B2c})$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_r) + \frac{3}{\nu} \left[\frac{\partial \nu}{\partial t} + V_r \frac{\partial \nu}{\partial r} \right] = \frac{1}{\nu^2} \frac{\delta u}{\delta t}, \quad (\text{B2d})$$

where V_r is the fluid bulk velocity in the radial direction, and Φ is the gravitational potential. Remark that quasistatic equilibrium corresponds to taking $V_r = 0$ in Eq. (B2b), which becomes Eq. (3a) in this limit. We solved Eqs. (B2) iteratively, which is summarized into three procedures: 1) assume a uniform and constant ν and integrate Eqs. (B2a) and (B2d) to get solutions for ρ and V_r ; 2) take ρ and V_r from the first procedure and integrate Eq. (B2b) and Eq. (B2c) to get a solution for ν ; and 3) repeat the first

FIG. 4. *Left panel*—Time evolution of DM density ρ of a MW satellite galaxy ($M_{200} = 10^9 M_\odot$). NFW profile ($\rho_s = 0.02 M_\odot/\text{pc}^3$, $r_s = 1.26$ kpc) is used as the initial condition. Heat conduction effect through self-interaction of DM is not taken into account. Numerical solutions from the Lagrangian formulation [Eqs. (3)] and the Eulerian formulation [Eqs. (B2)] are in blue and red, respectively. The reference time scale is $t_0 = 1$ Gyr. We set the fudge factor $b = 1$, and the cross sections are $\langle\sigma_{\text{semi}}v_{\text{rel}}\rangle/m = 6 \times 10^{-26} \text{ cm}^3/\text{s}$ and $\sigma_{\text{self}}/m = 0.1 \text{ cm}^2/\text{g}$ with SHDM mass $m = 0.3$ MeV. We find that two formalisms result in consistent profiles for ρ . The agreement between two formalisms validates the quasistatic equilibrium assumption in the Lagrangian formulation. *Right panel*—Same as the left panel but for the velocity dispersion ν . $t_G(r_s) = 1/\sqrt{4\pi G\rho_{\text{NFW}}(r_s)}$ is the gravitational time scale with $\rho_{\text{NFW}}(r_s) = \rho_s/4$.

procedure but with the updated ν from the second procedure. We repeat this until the solutions converge.

Due to numerical difficulties, we only compare the solutions in the case where the heat conduction effect is absent; we take $\delta u = \delta u_{\text{heat}}$ [Eq. (7)] in both Eq. (3c) and Eq. (B2d) for a comparison. The time evolution of ρ and ν in both formalisms is presented in Fig. 4. We find that both Eqs. (3) (in blue) and Eqs. (B2) (in red) exhibit consistent resultant profiles for ρ and ν . Since the quasistatic equilibrium is not assumed in Eqs. (B2), we conclude that dynamics induced by heat injection from semiannihilation do not spoil the assumption.

APPENDIX C: CORED INITIAL HALO PROFILES

For the fluid simulations results in the main text, we took halos of the NFW profile as initial conditions. In reality, the initial condition would be modified due to the effect of DM self-heating/self-interaction along the formation/virialization of halos. Still, we expect the modification of initial conditions to have a subdominant effect on present halo structures, compared to that along the postformation evolution we followed. This is mainly because the time scale of forming a halo (t_*) is shorter than the heating time scale (t_{heat}) and the self-scattering time scale (t_{self}), which are the physical time scales that are relevant to structural evolution of a halo. For field dwarf/LSB galaxies (10^{10} – $10^{11} M_\odot$), the formation/virialization time of a halo is $t_* \lesssim 1$ Gyr [see Fig. 3], while the heating/self-interaction time scales are as long as the age of the Universe in our parameter space of interest $t_{\text{heat/self}} \gtrsim 13.8$ Gyr.

At least in the case of pure SIDM, the fluid model with the NFW profile as an initial condition reproduces the

present density and velocity dispersion profiles of the *Pippin* halo (a field dwarf/LSB galaxy from a cosmological N -body simulations [67]) within a factor of 2, for various SIDM cross sections $\sigma_{\text{self}}/m = 0.1$ – $50 \text{ cm}^2/\text{g}$ [52]. The corresponding self-interaction time scales for the *Pippin* halo ($M_{200} = 8 \times 10^{10} M_\odot$) are $t_{\text{self}} = \mathcal{O}(1\text{--}100)$ Gyr. Since the heating/self-scattering time scales considered in our work are similar [Eq. (10)], $t_{\text{heat}} \gtrsim 5$ Gyr and $t_{\text{self}} \gtrsim 100$ Gyr, we expect a similar conclusion also in the case of SHDM. Although performing dedicated N -body simulations would provide a direct proof on this time scale discussion, it is beyond the scope of this work. Instead, we provide an indirect proof by checking the initial condition sensitivity of present halo structures. We find that as long as the change in the initial profile is marginal, our results of the present core density/size do not change.

DM self-heating/self-interaction along halo formation/virialization would make the initial profile to be slightly cored from the NFW profile. Thus, we use the cored-NFW (CNFW) profiles as different initial conditions; $\rho_{\text{CNFW}}(r) = \rho_s r_s / [(r_c + r_s)(1 + (r/r_s)^2)]$, where r_c is the core size parameter, and r_s and ρ_s are the NFW scale parameters. We obtain solutions to the fluid model [Eqs. (3)] for different values of r_c . In Fig. 5, the results for a field dwarf/LSB galaxy (blue) and a Milky Way (MW) satellite dwarf galaxy (red) are presented. The left panel shows the r_c dependence of the present core size of the halos. We find that for the SHDM benchmark parameters $\langle\sigma_{\text{semi}}v_{\text{rel}}\rangle = 6 \times 10^{-26} \text{ cm}^3/\text{s}$, $\sigma_{\text{self}}/m = 0.1 \text{ cm}^2/\text{g}$, and $m = 0.3$ MeV (solid), present core size of the Milky Way (MW) satellite dwarf galaxy remains the same up to initial core sizes as large as $r_c/r_s \sim 1$. This is because the effect of DM self-heating along the postformation evolution is

FIG. 5. *Left panel*—Present core sizes of a MW satellite galaxy (red) and a field dwarf/LSB galaxy (blue) from initial halos of the CNFW profile with r_c . We took the benchmark cross sections, $\langle \sigma_{\text{semi}} v_{\text{rel}} \rangle = 6 \times 10^{-26} \text{ cm}^3/\text{s}$ and $\sigma_{\text{self}} \simeq 0.1 \text{ cm}^2/\text{g}$, and set the fudge factor to be $b = 1$. We show the results for the benchmark SHDM mass $m = 0.3 \text{ MeV}$ (solid), and the largest SHDM mass allowed from the observed central densities of MW satellites [Eq. (12)], $m = 3 \text{ MeV}$ (dashed). For $r_c \lesssim 10^{-3}$, the present core sizes are identical to the ones from the initial halo of the NFW profile. *Right panel*—Time evolution of core size of a MW satellite galaxy (red) and a field dwarf/LSB galaxy (blue) for $r_c/r_s = 0.01$ (solid) and $r_c/r_s = 0.1$ (dashed). We fix the SHDM mass to be the benchmark value, $m = 0.3 \text{ MeV}$. The other parameters took are the same as the left panel.

dominant over the effect of initial cored profiles, as will be discussed below. On the other hand, DM self-heating is relatively weaker in the field dwarf/LSB galaxy, so that the present core size is mildly increased for $r_c/r_s \gtrsim 0.1$. The same tendency is expected as we consider larger SHDM mass because DM self-heating is weaker. Indeed, for $m = 3 \text{ MeV}$ (dashed), cored initial profiles starts to affect the present core size at smaller r_c .

This understanding of the tendency is confirmed in the right panel of Fig. 5. There, we show the time evolution of the core size for different r_c ; $r_c/r_s = 0.01$ (solid) and $r_c/r_s = 0.1$ (dashed). At the early stage of the evolution, core sizes are larger for larger r_c . But for larger r_c , DM self-heating near the center is weaker due to suppressed DM density. This is why the core radius in the $r_c/r_s = 0.1$ case increases rather slowly at initial stage, compared to that in the $r_c/r_s = 0.01$ case. For the MW satellite (red), the core formation in the $r_c/r_s = 0.01$ case is strong enough so that the core size eventually follow the core size in the $r_c/r_s = 0.1$ case before present. On the other hand, for the field dwarf/LSB galaxy (blue), DM self-heating is not strong enough for the $r_c/r_s = 0.01$ case to follow up the core size in the $r_c/r_s = 0.1$ case, resulting in a slight offset in present core size.

The remaining question is if the initial halo would be so cored to affect the present halos; for the SHDM benchmark parameters, the initial CNFW profile with $r_c/r_s \gtrsim 0.1$ leads to larger present core size of field dwarf/LSB galaxies (10^{10} – $10^{11} M_\odot$), compared to that from the initial NFW profile. But we do not expect the initial halos to have cores as large as $r_c/r_s \gtrsim 0.1$. Again, this is because the corresponding heating time scale [Eq. (10)], $t_{\text{heat}} \sim 200 \text{ Gyr}$, and the self-scattering time scale [Eq. (6)], $t_{\text{self}} \sim 250 \text{ Gyr}$, are much longer than the formation time of halos, $t_* \lesssim 1 \text{ Gyr}$ [see Fig. 3]. A similar argument holds for MW satellite galaxies (10^8 – $10^9 M_\odot$). For these reasons, even if we take into account self-heating/self-interaction along the halo formation, we do not expect that the true behavior of the escalating core formation (at present) to deviate much from our expectations. For halos of mass $M_{200} \lesssim 10^7 M_\odot$, which are not observed and thus are not considered in this work, t_{heat} becomes as short as t_* , and DM self-heating may affect significantly the formation/virialization stage of halos. While qualitatively DM self-heating would still lead to the significant core formation, the present core density/size could quantitatively deviate from our analyses at such small-size halos.

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