Erratum: Two nucleon systems at $m_\pi \sim 450$ MeV from lattice QCD [Phys. Rev. D **92**, 114512 (2015)]

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In the process of NPLQCD's ongoing reanalysis of the baryon-baryon correlation functions at $m_\pi \sim 450$ MeV, a subset of which were the subject of the paper, an error was identified. In particular, it was found that while the effective mass plots and fits shown in Fig. 11 of the paper are correct, an incorrect value of the binding energy and associated uncertainties was propagated through parts of the downstream analysis. The deuteron binding energy obtained from the $L = 24$ ensemble at a pion mass of $m_\pi \sim 450$ MeV that was published and analyzed was (incorrectly) $B_d = 19.6(1.2)(1.6)(0.2)$ MeV, which when combined with results obtained from the $L = 32, 48$ ensembles, incorrectly produced an infinite-volume extrapolated binding energy of $B_d^{(\infty)} = 14.4^{+(1.6)(2.7)(0.2)(0.2)}_{-(1.8)(1.8)(0.2)(0.2)}$ MeV. The correct value of the deuteron binding energy in the $L = 24$ volume is $B_d = 20.9(1.2)(3.3)(0.2)$ MeV, which leads to an infinite volume extrapolated binding energy of $B_d^{(\infty)} = 14.3^{+(1.8)(3.7)(0.2)(0.2)}_{-(2.0)(3.7)(0.2)(0.2)}$ MeV. The effect of this correction is to increase the systematic uncertainty of the deuteron binding energy extracted from the $L = 24$ volume used in the downstream analysis by approximately a factor of 2.

The corrected systematic uncertainty has been propagated through the downstream analysis, and fortunately, it does not invalidate conclusions or discussions related to the results of these lattice QCD calculations. A complete and detailed summary of the impact of this error on the published tables and figures are detailed in the lists that follow.

The reanalysis also uncovered a small number of typographical errors (transcription errors) that also appear in the published paper. These errors are corrected in the last section.

I. THE $^3S_1$ - $^3D_1$ COUPLED CHANNELS AND THE DEUTERON

A. The deuteron

The following lists the correct values and figures in this subsection:

- (1) Published numbers presented in the first and fourth rows of Table I are incorrect. The caption is unchanged. The revised Table I can be found below.
- (2) The region in the $c_1 - B_d^{(\infty)}$ parameter space bounded by $\chi^2 \rightarrow \chi^2_{\min} + 1$ is slightly modified. The figure caption is unchanged. The revised Fig. 12 can be found below.
- (3) Equation (12) should be

$$B_d^{(\infty)} = 14.3^{+(1.8)(3.7)(0.2)(0.2)}_{-(2.0)(3.7)(0.2)(0.2)} \text{ MeV.} \quad (1)$$

TABLE I. The deuteron binding energies extracted from plateaus in the EMPs shown in Fig. 11, along with the infinite-volume extrapolated value. The size of the FV effects is characterized by $e^{-\kappa L}$, shown in the last column. The first uncertainty corresponds to the statistical uncertainty associated with the fit, the second corresponds to the systematic uncertainty associated with the selection of the fitting interval (determined by varying this range). In the case of dimensionful quantities, the third uncertainty is associated with scale setting. For the infinite-volume values of the binding energy, the last uncertainty is introduced by the finite-volume extrapolation in Eq. (11) and is estimated by considering the effect of omitted terms scaling as $e^{-2\kappa_0 L}/L$.

Ensemble	ΔE (l.u.)	B_d (MeV)	$e^{-\kappa L}$
$24^3 \times 64$	$-0.0123(07)(19)$	$20.9(1.2)(3.3)(0.2)$	0.103
$32^3 \times 96$	$-0.01037(89)(96)$	$17.5(1.5)(1.6)(0.2)$	0.063
$48^3 \times 96$	$-0.0078(12)(19)$	$13.3(2.0)(3.2)(0.2)$	0.027
$L = \infty$	$-0.0084^{+(10)(22)(01)}_{-(12)(22)(01)}$	$14.3^{+(1.8)(3.7)(0.2)(0.2)}_{-(2.0)(3.7)(0.2)(0.2)}$	

FIG. 12. The region in c_1 - $B_d^{(\infty)}$ parameter space defined by $\chi^2 \rightarrow \chi_{\min}^2 + 1$. The inner region is defined by the statistical uncertainty, while the outer region is defined by the statistical and systematic uncertainties combined in quadrature.

FIG. 14. The pion mass dependence of the deuteron binding energy calculated with LQCD. The NPLQCD anisotropic-clover result is from Ref. [10], the Yamazaki *et al.* results are from Refs. [14, 20], and the NPLQCD isotropic-clover results are from this work and Ref. [13]. The black disk corresponds to the experimental binding energy.

FIG. 18. Scattering in the ${}^3S_1$ - ${}^3D_1$ coupled channels. The left panel shows $k^* \cot \delta_{1\alpha} / m_\pi$ as a function of k^{*2} / m_π^2 , while the right panel shows the phase shift as a function of momentum in MeV, assuming that $\delta_{1\beta}$ and the D-wave and higher partial-wave phase shifts vanish. The thick (thin) region of each result corresponds to the statistical (statistical and systematic uncertainties combined in quadrature). The black circle in the right panel corresponds to the known result from Levinson's theorem, while the dashed gray curve corresponds to the phase shift extracted from the Nijmegen partial-wave analysis of experimental data [61].

FIG. 19. Scattering in the $^3S_1$ - $^3D_1$ coupled channels below the start of the t-channel cut, $k^{*2} < m_\pi^2/4$, assuming that $\delta_{1\beta}$ and the D-wave and higher partial-wave phase shifts vanish. The left panel shows solid region corresponding to linear fits associated with the statistical uncertainty and the statistical and systematic uncertainties combined in quadrature. The right panel shows the scattering parameters, $1/a(^3S_1)$ and $r(^3S_1)$ determined from fits to scattering results below the t-channel cut. The solid circle corresponds to the experimental values.

- (4) The deuteron binding energy given in the text below Eq. (12) should be $B_d^{(\infty)} = 14.3^{+(4.1)(0.3)}_{-(4.2)(0.3)} \text{ MeV}$.

1. The mixing parameter, ϵ_1

No changes to this subsection.

2. A compilation of deuteron binding energies from LQCD

In this subsection, the only correction, resulting from the previously discussed error, is to the deuteron binding energy at this pion mass shown in Fig. 14. The figure caption is unchanged. The revised Fig. 14 can be found above.

B. Scattering in the $^3S_1$ - $^3D_1$ coupled channels

As a result of the previously described error, the value of the deuteron binding energy and binding momentum shown in tables and figures in this subsection are incorrect.

- (1) The deuteron binding momentum given in Table VII is modified (in the row labeled as ensemble “All”). The table caption is unchanged. The corrected value is $i0.284^{+(28)(24)}_{-(30)(34)}$, as shown in Table III.
- (2) The binding momentum and $k \cot \delta$ values shown in the left panel of Fig. 18 are modified. There are no other changes to the figure or caption. The revised Fig. 18 is shown above.
- (3) The correct value of the deuteron binding momentum leads to changes to the corresponding point in the left panel shown in Fig. 19, and the associated fit regions, and to the region of scattering parameters shown in the right panel of the same figure. The caption is unchanged. The revised Fig. 19 is shown above.
- (4) Equation (17) should be replaced with

$$\begin{aligned} (m_\pi a(^3S_1))^{-1} &= -0.03^{+(0.07)(0.07)}_{-(0.10)(0.18)}, & m_\pi r(^3S_1) &= 7.9^{+(2.5)(3.8)}_{-(1.6)(1.8)}, \\ (a(^3S_1))^{-1} &= -0.08^{+(0.16)(0.17)}_{-(0.24)(0.42)} \text{ fm}^{-1}, & r(^3S_1) &= 3.5^{+(1.1)(1.7)}_{-(0.7)(0.8)} \text{ fm}. \end{aligned} \quad (17)$$

II. TYPOGRAPHICAL ERRORS

A number of typographical errors in the published paper have been identified. Typographical errors were found in Tables II, IV, VII, and X. These errors were made in preparing for publication and do not appear in the analysis pipeline from which physics conclusions were deduced. The corrections to Tables II and IV of the 2015 paper are given in Table II, corrections to Table VII are given in Table III, and corrections to Table X are given in Table IV.

TABLE II. Correcting typographical errors appearing in Tables II and IV of our 2015 paper.

Meson	Ensemble	Entries in 2015 paper		Correct entries	
		$ \mathbf{n} = 0$ (l.u.)	MeV	$ \mathbf{n} = 0$ (l.u.)	MeV
$\pi^\pm$	$L = \infty$ (Table II)	0.26606(14)(08)	...	0.26607(13)(11)	...
	$L = \infty$ (Table IV)	0.26614(15)(15)	449.9(0.3)(0.3)(4.6)	0.26607(13)(11)	449.9(0.2)(0.2)(4.6)
$K^\pm$	$L = 48$ (Table II)	0.35236(16)(25)	...	0.35236(16)(05)	...
	$L = \infty$ (Table IV)	0.35241(12)(11)	595.9(0.2)(0.2)(6.1)	0.35240(11)(03)	595.4(0.2)(0.1)(6.1)

TABLE III. Correcting typographical errors appearing in Table VII of our 2015 paper. Corrected values are highlighted in bold for convenience. The entries highlighted in bold italics result from the error described in the previous section and are not typographical in origin.

Ensemble	$ \mathbf{P}_{\text{tot}} $ (l.u.)	Entries in 2015 paper			Correct entries		
		k^*/m_π	$k^* \cot \delta_{1\alpha}/m_\pi$	$\delta_{1\alpha}$ (degrees)	k^*/m_π	$k^* \cot \delta_{1\alpha}/m_\pi$	$\delta_{1\alpha}$ (degrees)
All	0	$i0.294^{+(17)(27)}_{-(18)(24)}$	$-0.294^{+(17)(27)}_{-(18)(24)}$	...	<i>$i0.284^{+(28)(24)}_{-(30)(34)}$</i>	<i>$-0.284^{+(28)(24)}_{-(30)(34)}$</i>	...
$24^3 \times 64$	0	$0.9754^{+(44)(98)}_{-(45)(99)}$	...	3.1(1.7)(3.7)	$0.9754^{+(44)(98)}_{-(45)(99)}$	...	3.1(1.7)(3.7)
$32^3 \times 96$	0	$0.702^{+(10)(23)}_{-(10)(24)}$	$2.3^{+(1.0)(5.7)}_{-(0.55)(0.89)}$	17(5)(11)	$0.702^{+(10)(23)}_{-(10)(24)}$	$2.3^{+(1.0)(5.7)}_{-(0.55)(0.89)}$	17(5)(11)
$32^3 \times 96$	0	$1.065^{+(07)(16)}_{-(08)(17)}$	$-5.4^{+(1.4)(2.1)}_{-(2.9)(29.5)}$	-11.1(3.8)(8.5)	$1.065^{+(07)(16)}_{-(08)(17)}$	<i>$-5.4^{+(1.1)(1.8)}_{-(3.4)(18.7)}$</i>	<i>$-11.1^{+(4.3)(7.5)}_{-(2.8)(6.5)}$</i>
$32^3 \times 96$	1	$0.270^{+(26)(29)}_{-(40)(51)}$	$0.35^{+(24)(15)}_{-(59)(20)}$	$38^{+(13)(23)}_{-(11)(16)}$	<i>$0.270^{+(25)(38)}_{-(23)(44)}$</i>	<i>$0.35^{+(21)(50)}_{-(13)(18)}$</i>	$38^{+(13)(23)}_{-(11)(16)}$
$48^3 \times 96$	0	$0.426(03)(12)$	$0.45^{+(67)(34)}_{-(26)(08)}$	$44^{+(21)(07)}_{-(21)(08)}$	<i>$0.426(35)(12)$</i>	$0.45^{+(67)(34)}_{-(26)(08)}$	$44^{+(21)(07)}_{-(21)(08)}$
$48^3 \times 96$	0	$0.662(08)(29)$	$0.35^{+(0.14)(3.0)}_{-(0.09)(0.21)}$	$26^{+(07)(25)}_{-(07)(22)}$	$0.662(08)(29)$	<i>$1.35^{+(0.51)(11.2)}_{-(0.36)(0.79)}$</i>	$26^{+(07)(25)}_{-(07)(22)}$

TABLE IV. Correcting typographical errors appearing in Table X of our 2015 paper. Corrected values are highlighted in bold for convenience.

Ensemble	$ \mathbf{P}_{\text{tot}} $ (l.u.)	Entries in 2015 paper			Correct entries		
		k^*/m_π	$k^* \cot \delta^{(1S_0)}/m_\pi$	$\delta^{(1S_0)}$ (degrees)	k^*/m_π	$k^* \cot \delta^{(1S_0)}/m_\pi$	$\delta^{(1S_0)}$ (degrees)
All	0	$i0.274^{+(19)(26)}_{-(20)(44)}$	$-0.274^{+(19)(26)}_{-(20)(44)}$	...	$i0.274^{+(19)(26)}_{-(20)(44)}$	$-0.274^{+(19)(26)}_{-(20)(44)}$	...
$24^3 \times 64$	0	$0.954^{+(08)(18)}_{-(08)(19)}$	$5.0^{+(2.0)(10.0)}_{-(1.1)(1.8)}$	$10.8^{+(3.0)(6.5)}_{-(3.0)(6.7)}$	<i>$0.963^{+(08)(11)}_{-(08)(11)}$</i>	<i>$7.1^{+(4.2)(13.2)}_{-(2.0)(2.1)}$</i>	<i>$7.7^{+(2.8)(4.1)}_{-(2.8)(4.2)}$</i>
$32^3 \times 96$	0	$0.691^{+(09)(16)}_{-(09)(16)}$	$1.7^{+(0.5)(1.1)}_{-(0.3)(0.5)}$	$22.0^{+(4.2)(7.0)}_{-(4.2)(7.2)}$	$0.691^{+(09)(16)}_{-(09)(16)}$	$1.7^{+(0.5)(1.1)}_{-(0.3)(0.5)}$	$22.0^{+(4.2)(7.0)}_{-(4.2)(7.2)}$
$32^3 \times 96$	0	$1.079^{+(05)(10)}_{-(05)(10)}$	$-3.3^{+(0.4)(0.7)}_{-(0.6)(1.5)}$	-18.3(2.6)(5.2)	$1.079^{+(05)(10)}_{-(05)(10)}$	$-3.3^{+(0.4)(0.7)}_{-(0.6)(1.5)}$	-18.3(2.6)(5.2)
$32^3 \times 96$	1	$0.220^{+(28)(32)}_{-(32)(42)}$	$0.13^{+(10)(14)}_{-(08)(08)}$	$60^{+(14)(20)}_{-(12)(14)}$	$0.220^{+(28)(32)}_{-(32)(42)}$	$0.13^{+(10)(14)}_{-(08)(08)}$	$60^{+(14)(20)}_{-(12)(14)}$
$48^3 \times 96$	0	$0.453(11)(29)$	$0.89^{+(39)(3.7)}_{-(23)(44)}$	$27^{+(07)(18)}_{-(07)(20)}$	$0.453(11)(29)$	$0.89^{+(39)(3.7)}_{-(23)(44)}$	$27^{+(07)(18)}_{-(07)(20)}$

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