Neutrino phenomenology in a left-right D_4 symmetric model

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(Received 22 March 2020; accepted 30 July 2020; published 7 August 2020)

We present a minimal left-right symmetric flavor model and analyze the predictions for the neutrino sector. In this scenario, the Yukawa sector is shaped by the dihedral D_4 symmetry which leads to correlations for the neutrino mixing parameters. We end up with the four possible solutions within this model. We further analyzed the impact of the upcoming long-baseline neutrino oscillation experiment, DUNE. Due to its high sensitivity, it has been observed that the DUNE will be able to rule out two of the solutions. Finally, the predictions for the neutrinoless double beta decay and the lepton flavor violating process $\mu \rightarrow e\gamma$ for the model have also been examined.

DOI: [10.1103/PhysRevD.102.036006](https://doi.org/10.1103/PhysRevD.102.036006)

I. INTRODUCTION

A number of phenomenal experimental evidences over the past two decades have established the fact that neutrinos oscillate through their propagation path [1–4], which implies nonzero neutrino masses and mixings. This fact provides an undoubtedly motivation for the existence of physics beyond the Standard Model (SM), as neutrinos are

massless in the SM. Furthermore, the experimental efforts in understanding the neutrino properties have determined the two mass-squared differences and three sufficiently large leptonic mixing angles. From the global analysis of neutrino oscillation data [5] (other global analysis can be found in [6,7]), the best fit values and the 1σ intervals for a normal neutrino mass ordering (NO) are given by¹

$$|\Delta m_{\text{sol}}^2| = 7.55_{-0.16}^{+0.20} \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{\text{atm}}^2| = 2.50 \pm 0.03 \times 10^{-3} \text{ eV}^2, \\ \theta_{12}/^\circ = 34.5_{-1.0}^{+1.2}, \quad \theta_{13}/^\circ = 8.45_{-0.14}^{+0.16}, \quad \theta_{23}/^\circ = 47.7_{-1.7}^{+1.2}, \quad \text{and} \quad \delta_{CP}/^\circ = 218_{-27}^{+38}. \quad (1)$$

Moreover, the theory behind the dynamical origin of the masses of neutrinos along with their flavor mixing patterns and whether they are Majorana or Dirac fermions, is yet unanswered. The simplest idea behind these shortcomings relies on the assumption that the neutrinos are Majorana particles and their tiny masses are generated

through a seesaw mechanism [8–13]. Interesting extensions of the SM featured by the inherent new physics signatures are those that consider a left-right (LR) symmetric nature [14–17]. For instance, the LR symmetric models have the virtue of accounting for the small neutrino masses from the contribution of two mechanisms, the type-I and type-II seesaw, which implies the existence of new particles.

The simplest LR symmetric model is dictated by the gauge symmetry group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. In this case, the fermion fields have the following charge assignments [18],

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¹Note here that the neutrino oscillation experiments are sensitive to (mass)²-differences and hence, the possibility of a massless neutrino is not excluded.

$$l_L \simeq (\mathbf{1}, \mathbf{2}, \mathbf{1}, -1), \quad l_R \simeq (\mathbf{1}, \mathbf{1}, \mathbf{2}, -1),$$

$$Q_L \simeq (\mathbf{3}, \mathbf{2}, \mathbf{1}, \mathbf{1}/3) \quad \text{and} \quad Q_R \simeq (\mathbf{3}, \mathbf{1}, \mathbf{2}, \mathbf{1}/3), \quad (2)$$

whereas the scalar potential is formed by the two triplets and one bidoublet whose LR charges are

$$\Delta_L \simeq (\mathbf{1}, \mathbf{3}, \mathbf{1}, \mathbf{2}), \quad \Delta_R \simeq (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{2}) \quad \text{and} \quad \Phi \simeq (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{0}). \quad (3)$$

If the LR breaking scale and the masses of the new scalar fields are of $\mathcal{O}(\text{TeV})$, this minimal setup produces sizeable contributions to the lepton flavor violating (LFV) decays, lepton number violation as well as CP violating processes [19–25]. Therefore, this scenario turns out to be very appealing for the experimental searches among the low-energy LFV processes [26]. Further constraints apply to this model from the LHC searches of new physics [27–31]. On the other hand, the LR symmetry is also possible to be broken at higher energies, such as the grand unification theory (GUT) scale, leading to the gauge coupling unification [32,33]. This makes the LR models interesting frameworks from the perspective of the GUTs like $SO(10)$ [34,35].

On top of gauge symmetries, one can impose additional global symmetries that relate the flavor structure of the SM. In the past decade, there have been numerous amount of works in this direction, for reviews see [36,37]. Nevertheless, it is particularly interesting to examine the interplay between the LR symmetry and a discrete flavor symmetry. This combination shapes and correlates the Yukawa sector, giving predictions for the flavor observables, i.e., neutrino masses and mixings [38–42]. In this work, we study the effects of combining a non-Abelian discrete flavor symmetry group D_4 with the LR symmetry. It is worthwhile to mention here that the D_4 flavor symmetry group has been explored in literature in [43–56], not in combination with a LR symmetric model to the best of our knowledge. It is to be noted further that a Z_2 symmetry has also been augmented with the D_4 flavor group in the model. We like to stress here that our intention is to keep the left-right scalar sector as the one in the standard LR model, namely a bidoublet and two triplets (left and right). Hence, in order to break the flavor symmetry we include the flavon fields that are charged under Z_2 symmetry as well as the bidoublet which also carries Z_2 charge to obtain the desired Lagrangian in this model.

Among many of the consequences of this model, a noteworthy outcomes of the model is the appearance of the two-zero texture of the neutrino mass matrix. Under the Glashow-Frampton-Marfatia classification [57] for the two-zero texture Majorana neutrino mass matrices, we get an A_2 type texture zero matrix. This model also

TABLE I. Matter content and charge assignments of the left-right D_4 model, where $D(S)$ stands for D_4 doublet (singlet). Three lepton families are arranged in a doublet and a singlet of D_4 , respectively. Notice also that only the SM Higgs doublet Φ and the flavon fields η, χ are kept charged under the Z_2 symmetry.

	$\ell_{L_{D(S)}}$	$\ell_{R_{D(S)}}$	Δ_L	Δ_R	Φ	η	χ
$SU(2)_L$	2	1	3	1	2	1	1
$SU(2)_R$	1	2	1	3	2	1	1
$U(1)_{B-L}$	-1	-1	2	2	0	0	0
D_4	$\mathbf{2} \oplus \mathbf{1}$	$\mathbf{2} \oplus \mathbf{1}$	1	1	1	2	1
Z_2	1	1	1	1	-1	-1	-1

predicts a nondiagonal mass matrix for the charged leptons.

The outline of the paper is as follows: in Sec. II we present the model and the necessary charge assignments. In this section, we also give the invariant Lagrangian of the theory and derive the leptonic mass matrices. We explain the procedure of our analysis in Sec. III as well as show our results for the neutrino predictions within the model. Our final comments and summary are given in Sec. IV. Appendix A summarizes the D_4 algebra, whereas the necessary equations for the LFV processes are given in Appendix B.

II. LEFT-RIGHT D_4 SYMMETRIC MODEL

We consider an extension of the minimal left-right symmetric model by adding a D_4 flavor symmetry. Besides postulating a symmetry that shapes the Yukawa sector, we add two flavon fields, χ and η transforming as a singlet and doublet under D_4 , respectively. In Table I we provide the matter content and charge assignments of the model. In this framework, the symmetry breaking goes like

$$\text{LRSM} \otimes G_F \xrightarrow{\eta, \chi} \text{LRSM} \xrightarrow{\Delta} \text{SM} \xrightarrow{\Phi} \text{SU}(3)_C \otimes \text{U}(1)_{em},$$

where $G_F = D_4 \otimes Z_2$ and its breaking is associated to the nonzero vacuum expectation values (vevs) of the flavon fields $\langle \chi \rangle$ and $\langle \eta \rangle$.

We assume the following sequential symmetry breaking $\Lambda_F \gg \Lambda_{LR} \gg \Lambda_{EW}$, where Λ_F is the flavor breaking scale and Λ_{LR} is the left-right symmetry breaking scale.²

Given the matter content shown in Table I, the Yukawa Lagrangian (up to dimension-5) for the leptons can be expressed as

²With this assumption the flavon fields decouple from the theory having only an impact on the Yukawa couplings. Then, in this energy regime the scalar potential is approximate to the minimal LRSM one [18]. A detailed discussion of the LFV process in this model has been presented in the next section.

$$\begin{aligned} \mathcal{L}_Y \supset & \bar{\ell}_{L_D} \left(y_1 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_1 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_D} + \bar{\ell}_{L_D} \left(y_2 \frac{\eta}{\Lambda_F} \Phi + \tilde{y}_2 \frac{\eta}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_s} \\ & + \bar{\ell}_{L_s} \left(y_3 \frac{\eta}{\Lambda_F} \Phi + \tilde{y}_3 \frac{\eta}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_D} + \bar{\ell}_{L_s} \left(y_4 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_4 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_s} \\ & + \frac{Y_{L_1}}{2} \ell_{L_D}^T C(i\sigma_2) \Delta_L \ell_{L_D} + \frac{Y_{L_2}}{2} \ell_{L_s}^T C(i\sigma_2) \Delta_L \ell_{L_s} + \frac{Y_{R_1}}{2} \ell_{R_D}^T C(i\sigma_2) \Delta_R \ell_{R_D} + \frac{Y_{R_2}}{2} \ell_{R_s}^T C(i\sigma_2) \Delta_R \ell_{R_s} + \text{H.c.} \end{aligned} \quad (4)$$

where $\ell_{L_D} = (\ell_{L_1}, \ell_{L_2})^T$, $(\ell_{R_D} = (\ell_{R_1}, \ell_{R_2})^T)$ represents left (right)-handed D_4 doublet, $\ell_{L_s} = \ell_{L_3}$ ($\ell_{R_s} = \ell_{R_3}$) left (right)-handed D_4 singlet and the bi-doublet Φ can be read as

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}. \quad (5)$$

In Appendix A we give a detailed discussion of the D_4 algebra that has been utilized to derive the Lagrangian as given in Eq. (4). An explicit Lagrangian has also been presented using the D_4 product rules that are used to derive

Eq. (4). Moreover, it is worth mentioning that the other two possible choices to assign the matter fields namely, doublets and singlets of D_4 are equivalent to the one in Table I and a detailed discussion has been given in Appendix A.

Note that the Dirac lepton mass matrices stem from the dimension-5 operators. Hence, from Eq. (4) after the spontaneous symmetry breaking (SSB), one gets that the mass matrix for the charged leptons as

$$M_\ell = \frac{1}{\sqrt{2}} (Y'_L v_2 + \tilde{Y}'_L v_1), \quad (6)$$

where

$$Y'_L = \frac{1}{\Lambda_F} \begin{pmatrix} 0 & y_1 v_\chi & y_2 v_{\eta_2} \\ y_1 v_\chi & 0 & y_2 v_{\eta_1} \\ y_3 v_{\eta_2} & y_3 v_{\eta_1} & y_4 v_\chi \end{pmatrix} \quad \text{and} \quad \tilde{Y}'_L = \frac{1}{\Lambda_F} \begin{pmatrix} 0 & \tilde{y}_1 v_\chi & \tilde{y}_2 v_{\eta_2} \\ \tilde{y}_1 v_\chi & 0 & \tilde{y}_2 v_{\eta_1} \\ \tilde{y}_3 v_{\eta_2} & \tilde{y}_3 v_{\eta_1} & \tilde{y}_4 v_\chi \end{pmatrix}, \quad (7)$$

with $\langle \chi \rangle \equiv v_\chi$, $\langle \eta \rangle \equiv (v_{\eta_1}, v_{\eta_2})^T$ and Φ

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}. \quad (8)$$

Assuming a vev alignment $\langle \eta \rangle \sim (1, 0)^T$, the mass matrix for the charged leptons becomes

$$M_\ell = \frac{1}{\sqrt{2}\Lambda_F} \begin{pmatrix} 0 & (y_1 v_2 + \tilde{y}_1 v_1) v_\chi & 0 \\ (y_1 v_2 + \tilde{y}_1 v_1) v_\chi & 0 & (y_2 v_2 + \tilde{y}_2 v_1) v_{\eta_1} \\ 0 & (y_3 v_2 + \tilde{y}_3 v_1) v_{\eta_1} & (y_4 v_2 + \tilde{y}_4 v_1) v_\chi \end{pmatrix}. \quad (9)$$

The matrix M_ℓ can be diagonalized by a biunitary transformation as

$$\text{diag}(m_e, m_\mu, m_\tau) = U_\ell M_\ell V_\ell^\dagger, \quad (10)$$

and the neutrino mass matrix is given by

$$m_\nu = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}, \quad (11)$$

where $m_L = \sqrt{2} Y_L v_L$ and $m_R = \sqrt{2} Y_R v_R$, with

$$Y_{L(R)} = \begin{pmatrix} 0 & Y_{L_1(R_1)} & 0 \\ Y_{L_1(R_1)} & 0 & 0 \\ 0 & 0 & Y_{L_2(R_2)} \end{pmatrix}. \quad (12)$$

In this scenario, the Dirac neutrino mass matrix turns out to be

$$m_D = \frac{1}{\sqrt{2}} (Y'_L v_1 + \tilde{Y}'_L v_2). \quad (13)$$

After the SSB, the light neutrino eigenstates acquire their masses through the type-I and type-II seesaw mechanism. Hence, the light-neutrino mass matrix is given by,

$$M_\nu^{\text{light}} = m_L - m_D m_R^{-1} m_D^T, \quad (14)$$

where $v_R \gg v_L$, v_1, v_2 has been assumed.

The left-right symmetric nature of the theory demands a relation between the Yukawa couplings mediating the interaction of leptons with the scalar triplets, i.e., $Y_R = Y_L$. The left-right exchange symmetry can be realized through either C or P transformations. Here we choose to use P -transformations, which demand the Hermiticity of Dirac type fermion mass matrices, that is

$$M_\ell = M_\ell^\dagger, \quad m_D = m_D^\dagger. \quad (15)$$

III. NEUTRINO PHENOMENOLOGY

Having discussed the theoretical framework in the previous section, here we aim to discuss various phenomenological importance of the model. In doing so, we notice from Eq. (9) that the mass matrix for charged leptons is nondiagonal. The left-right symmetry gives further relations for leptonic Yukawas, as mentioned in the previous section. Using this fact, the mass matrix for charged leptons, as given by Eq. (9), can be recasted as

$$M_\ell = \begin{pmatrix} 0 & a_\ell & 0 \\ a_\ell^* & 0 & b_\ell \\ 0 & b_\ell^* & c_\ell \end{pmatrix}. \quad (16)$$

The phases of this matrix can be absorbed in a pair of diagonal phase matrices (P and P'), this will lead to a real charged-lepton matrix as

$$O_\ell = \begin{pmatrix} 0.998 & -\text{sgn}(a_\ell)0.070 & \text{sgn}(a_\ell b_\ell)0.001 \\ \text{sgn}(a_\ell)0.068 & 0.969 & \text{sgn}(b_\ell)0.236 \\ -\text{sgn}(a_\ell b_\ell)0.017 & -\text{sgn}(b_\ell)0.235 & 0.972 \end{pmatrix}. \quad (20)$$

Note that O_ℓ in Eq. (20) is determined up to sign combinations of the parameters a_ℓ and b_ℓ .

Regarding the neutrino mass matrix, this is obtained by using Eqs. (12)–(14) and turns out to be,

$$\tilde{M}_\nu = \begin{pmatrix} 0 & a_\nu & 0 \\ a_\nu & d_\nu & b_\nu \\ 0 & b_\nu & c_\nu \end{pmatrix}, \quad (21)$$

where a_ν , b_ν , c_ν , and d_ν are complex entries. Therefore, the diagonalization of $\tilde{M}_\nu$ as given by Eq. (21) leads to the

³Notice that the $\det(M_\ell)$ is negative and hence one of the masses carries negative sign. Here we choose $m_\mu = -|m_\mu|$.

$$\tilde{M}_\ell = P M_\ell P'. \quad (17)$$

In this basis the neutrino mass matrix becomes

$$\tilde{M}_\nu = P^T M_\nu P, \quad (18)$$

where M_ν is the neutrino mass matrix in the interaction basis as given by Eq. (14). Since $\tilde{M}_\ell$ is symmetric, it can be diagonalized as

$$\text{diag}(m_e, m_\mu, m_\tau) = O_\ell \tilde{M}_\ell O_\ell^T,$$

where O_ℓ is an orthogonal matrix and one can easily get the expressions for a_ℓ , b_ℓ , and c_ℓ in terms of the charged-lepton masses. This is done by computing the invariants of the charged-lepton mass matrix, namely $\text{Tr}(M_\ell)$, $\text{Tr}(M_\ell^2)$ and $\det(M_\ell)$.³ Then, the matrix elements in Eq. (16) as functions of the masses can be read as

$$\begin{aligned} a_\ell &= \pm \frac{\sqrt{m_e m_\mu m_\tau}}{\sqrt{m_e - m_\mu + m_\tau}}, \\ b_\ell &= \pm \frac{\sqrt{-m_\mu + m_\tau} \sqrt{-m_e^2 + m_e m_\mu - m_e m_\tau + m_\mu m_\tau}}{\sqrt{m_e - m_\mu + m_\tau}}, \\ c_\ell &= m_e - m_\mu + m_\tau. \end{aligned} \quad (19)$$

With this information one can compute the rotation matrix O_ℓ as

active light neutrino masses. This mass matrix and the neutrino mass eigenstates are related as follows,

$$\begin{aligned} \tilde{M}_\nu &= U_\nu^*(\theta_{12}^0, \theta_{23}^0, \theta_{13}^0, \delta_0) \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \\ &\times U_\nu^\dagger(\theta_{12}^0, \theta_{23}^0, \theta_{13}^0, \delta_0), \end{aligned} \quad (22)$$

where m_{ν_i} are the light neutrino masses and the unitary matrix U_ν follows the PDG parametrization [58]. Therefore, in this model, the lepton mixing matrix (also known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [59,60]) is defined by,⁴

⁴Similar structure for charged leptons and neutrinos was obtained in the context of S_3 flavor symmetry [61,62].

FIG. 1. Allowed parameter space in $(\sin^2 \theta_{23} - \delta)$ plane for the four-solutions. Various colors viz, light-red, light-blue, light-green, and light-pink show correlation for $A = (+, -)$, $B = (-, -)$, $C = (+, +)$, and $D = (-, +)$, respectively. The solid contours for the left (right)-panel depicts the allowed region for “global-fit data” (“simulated results of DUNE”) corresponding the latest best-fit value as shown by black-dot.

$$V_L(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = O_\ell^T U_\nu(\theta_{12}^0, \theta_{23}^0, \theta_{13}^0, \delta_0) K, \quad (23)$$

where the angles θ_{ij} ($i < j = 1, 2, 3$) correspond to the mixing angles determined by neutrino oscillation experiments, δ represents the Dirac CP -violating phase and K is the diagonal Majorana phase matrix.

A. Results

According to the latest global analysis of the neutrino oscillation data [5–7], among the six neutrino oscillation parameters, the atmospheric mixing angle θ_{23} and the Dirac CP -phase δ are the two least known parameters. Hence, we show results of our numerical scan in the $(\sin^2 \theta_{23} - \delta)$ plane in Fig. 1. While performing our numerical scan, we have varied the unphysical mixing angles θ_{ij}^0 in the range $(0, \pi/2)$, whereas the unphysical Dirac CP phase δ_0 has been scanned over the range $(0, 2\pi)$ of matrix U_ν [see Eq. (23)] for the A_2 texture. Moreover, the neutrino mass squared differences as computed by the global analysis of the neutrino oscillation data [5] have also been utilized. Then we obtain the physical lepton mixing angles using Eq. (23), and compare with their experimentally allowed values from the global fit. Note that depending on the sign of (a_l, b_l) , see Eq. (20), there are four possible solutions which correlate the atmospheric angle θ_{23} and the Dirac type CP -violating phase δ . These are denoted as: in light-red $A = (+, -)$; in light-blue $B = (-, -)$; in light-green $C = (+, +)$; and, in light-pink $D = (-, +)$. In the left-panel, the $(\sin^2 \theta_{23} - \delta)$ plane depicts the allowed regions considering the latest global analysis of neutrino oscillation data [5] at 1, 3, and 5 σ C.L., respectively. These contours are shown using the red, orange, and yellow colors, respectively. The best-fit value has been marked with a “black-dot.” It can be seen from the left-panel that the solution A is ruled out by the present data at 5 σ C.L.,

whereas the solution D is marginally allowed at 3 σ C.L., but only for the CP -conserving values, namely around $\delta = 0, 2\pi$. We also notice that the solutions B and C are allowed at 1 σ C.L. Furthermore, it can be seen that among the four cases only the solution C is able to explain the latest best-fit value of neutrino oscillation data.

Similarly, in the right-panel of Fig. 1, we show the compatibility of the model by considering the simulated results of the next generation long baseline oscillation experiment, DUNE [63]. The allowed parameter space of DUNE in the $(\sin^2 \theta_{23} - \delta)$ plane is found using the latest best-fit value of neutrino oscillation data. For the numerical simulation of DUNE, the GLOBES package was used [64,65] along with the auxiliary files in Ref. [66]. A running time of 3.5 years was assumed in both neutrino and antineutrino modes for DUNE, i.e., DUNE[3.5 + 3.5]. The detailed numerical procedures that have been followed to simulate data coincides with the one performed in [67,68]. Notice from the right-panel that DUNE results would significantly improve the precision of both the parameters. It is observed that $\sin^2 \theta_{23}$ is constrained to values between (0.45, 0.58), whereas δ is restricted to the range $(0.95, 1.88)\pi$ at 5 σ C.L. after DUNE[3.5 + 3.5] running time. Therefore, one can infer that the precise measurement of both parameters (θ_{23} and δ) by DUNE, the solution D will be ruled out at 5 σ C.L., still allowed by the latest global-fit data.

In this model we also have a prediction for the lepton number violating processes such as the neutrinoless double beta decay ($0\nu\beta\beta$). Ongoing experiments that are looking for the signatures of $0\nu\beta\beta$ decays are namely, GERDA Phase-II [69], CUORE [70], SuperNEMO [71], KamLAND-Zen [72] and EXO [73]. The half-life of these processes can be expressed as [74,75],

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |M_{0\nu}(A, Z)|^2 |\langle m \rangle_{ee}|^2, \quad (24)$$

where $G_{0\nu}$ represents the two-body phase-space factor, $M_{0\nu}$ is the nuclear matrix element and $|\langle m \rangle_{ee}|$ is the effective Majorana neutrino mass. The expression of $|\langle m \rangle_{ee}|$ is given by,

$$|\langle m \rangle_{ee}| = \left| \sum_{i=1}^3 m_i V_{Lei}^2 \right|, \quad (25)$$

where V_L stands for lepton mixing matrix as mentioned in Eq. (22). We like to point out here that in the diagonal charged lepton basis, the A_2 type two zero texture for the Majorana neutrino mass matrix gives $(M_\nu)_{11} = 0 = (M_\nu)_{13}$. Hence, leads to zero predictions for the effective Majorana neutrino mass $|\langle m \rangle_{ee}|$ as discussed in Ref. [57], where the PMNS mixing matrix has been completely determined by the neutrino sector. However, in our formalism the charged-lepton mass matrix is nondiagonal [see Eq. (16)], while the Majorana neutrino mass matrix respects A_2 type two zero texture as given by Eq. (21). Thus, one can notice that contribution to the PMNS mixing matrix arises from both the leptonic mass matrices as given by Eq. (23). Therefore, we end-up with nonzero predictions for the effective Majorana neutrino mass in this study as given by Eq. (25).

Figure 2 shows the prediction for $0\nu\beta\beta$ decay. For comparison, we first show the allowed 3σ parameter space in $(m_{\text{light}} - |\langle m \rangle_{ee}|)$ -plane using the latest global analysis of neutrinos oscillation data [5], as shown by the gray color. We proceed to compute the effective Majorana neutrino mass in Eq. (25) for the allowed solutions, namely for B, C, and D. The color code of the prediction remains same as the

FIG. 2. The effective Majorana neutrino mass $|\langle m \rangle_{ee}|$ vs the lightest neutrino mass m_{light} . The prediction for the solutions B, C, and D are shown by the color codes, which are same as the one used in Fig. 1. Moreover, the latest upper bound on $|\langle m \rangle_{ee}|$ from the KamLAND-Zen collaboration are shown by the dark-yellow horizontal band. Also, the current results on the lightest neutrino mass is shown by the light green-vertical band from the Planck Collaboration which gives $\sum m_\nu < 0.12$ eV at the 95% C.L.

one used in Fig. 1. The current upper bound on $|\langle m \rangle_{ee}|$ comes from the KamLAND-Zen collaboration [72] which is read as $|\langle m \rangle_{ee}| < (61-165)$ meV at 90% C.L. by taking into account the uncertainty in the estimation of the nuclear matrix elements. This is given by the dark-yellow horizontal band. The two black lines on this band corresponds to the uncertainty of the nuclear matrix element, $|M_{0\nu}|$ in Eq. (24). In addition, the light green-vertical band represents the bound on m_{light} coming from the cosmological limit on the sum of neutrino masses provided by the Planck Collaboration, namely $\sum m_\nu < 0.12$ eV at the 95% C.L. [76,77]. Furthermore, as pointed out before, from the left panel of Fig. 1 one can observe that the DUNE can rule out solution D. This also has an impact for the prediction of $0\nu\beta\beta$. As a final remark, notice that the allowed solutions are compatible only with the normal neutrino mass ordering.

Finally, we proceed to discuss the possible lepton flavor violating (LFV) processes in the model. As we notice from the structure of the Yukawa couplings in Eq. (4), the charged leptons have nondiagonal couplings with the triplet scalar fields $\Delta_{L,R}$ and with the flavon fields η and χ . This may induce undesirably large tree level LFV processes such as $\mu \rightarrow e\gamma$ decay. Additionally, one-loop level diagrams with $W_{L,R}$ bosons also contribute to these processes. This processes may be used to delimit the parameter space of the LR gauge bosons and scalars. For example, the $\mu \rightarrow e\gamma$ branching ratio is given by [78]

$$B(\mu \rightarrow e\gamma) = 384\pi^2 e^2 (|A_L|^2 + |A_R|^2), \quad (26)$$

where the functions $A_{L/R}$ are given in Appendix B for completeness. Using the Yukawa couplings in Eq. (4) we find $\text{BR}(\mu \rightarrow e\gamma) \leq 10^{-15}$, well below the experimental bound $\text{BR}(\mu \rightarrow e\gamma)_{\text{exp}} \leq 4.2 \times 10^{-13}$ [79] with the following LR parameters

$$\begin{aligned} M_{W_R} &= 6 \text{ TeV}, & M_{N_{1,2}} &= 5 \text{ TeV}, & M_{\delta_{L/R}^{++}} &= 4 \text{ TeV}, \\ M_{H^+} &= 4 \text{ TeV}, & v_R &= 8 \text{ TeV}, & v_L &= 0.8 \text{ eV}, \\ v_\chi/\Lambda_F &= 0.1, & v_\eta/v_\chi &= 0.4. \end{aligned} \quad (27)$$

Notice that the benchmark values that have been considered here to calculate the branching ratio are in accordance with the neutrino oscillation results as discussed in Figs. 1,2. Moreover, one may also have contributions from the tree level flavon mediated processes. Given the hierarchy of scales of the model we expect them to be suppressed by the high scale of the flavon masses.

IV. CONCLUSIONS

We have constructed a minimal left-right symmetric model with the addition of a flavor symmetry, the non-Abelian discrete group D_4 . We notice that besides the

relations in the lepton Yukawas due to the left-right symmetry there are further correlations due to the additional family symmetry behind the theory. For this reason, there are a few free parameters left that can be written in terms of the leptonic observables, namely masses and their mixing angles. This can be observed from the computation of the charged lepton mass matrix as well as the corresponding rotation matrix. The simplicity of the model leads to clear predictions for the neutrino sector. Further, the model turns out to be compatible only with the normal neutrino mass ordering and provides a correlation between the atmospheric angle θ_{23} and the leptonic CP -violating phase δ . Given the possible solutions of the model there is one, namely the solution A is ruled out by the current neutrino oscillation data. More importantly, due to the high potential of the DUNE experiment, which can improve the precision of θ_{23} and to probe δ , it gives further restrictions to the parameter space as shown in the right-panel in Fig. 1. Using this, the DUNE will be able to rule out two of the solutions, namely A and D. We have also provided the prediction for neutrinoless double beta decay in terms of the lightest neutrino mass for a mass range of $\sim(10^{-3}-10^{-2})$ eV, which we summarize in Fig. 2. Finally, we have estimated the parameters of the LR sector where the LFV processes are under control. Branching ratio of $\mu \rightarrow e\gamma$ of the order of $\sim 10^{-15}$ has been calculated, which is well below the latest experimental bound.

ACKNOWLEDGMENTS

The authors would like to thank Manuel E. Krauss, Toby Opferkuch, and Werner Porod for collaboration in the

early stages of this project. This work is supported by the German-Mexican research collaboration Grant No. SP 778/4-1 (DFG) and No. 278017 (CONACYT), CONACYT No. CB-2017-2018/A1-S-13051 (México) and DGAPA-PAPIIT No. IN107118. NN is supported by the postdoctoral fellowship program DGAPA-UNAM. The work of C.B. was also supported by the Universidad Católica del Norte (UCN) grant “Neutrino mass generation and BSM” No. 20190803029. C.B. would like to thank IFUNAM, Universidad de Colima-DCPIHP, and Instituto de Física Corpuscular (CSIC) for the hospitality while part of this work was carried out. R. F. and L. M. G. D. L. V. are supported by CONACYT.

APPENDIX A: BASICS OF D_4 GROUP

The dihedral group D_4 is a non-Abelian group of order eight and contains five irreducible representations (irreps), denoted as $\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{1}'''$, and $\mathbf{2}$, respectively. The two group generators A and B are chosen as [47]

$$A = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (\text{A1})$$

For irreps $\mathbf{2}$, they satisfy the following relations

$$A^4 = \mathbb{I}, \quad B^2 = \mathbb{I}, \quad \text{and} \quad ABA = B. \quad (\text{A2})$$

where $\mathbb{I}$ is identity matrix.

The multiplication rules for the 1-dimensional irreps are the following

$$\mathbf{1}^a \times \mathbf{1} = \mathbf{1}^a, \quad \mathbf{1} \times \mathbf{1}^a = \mathbf{1}^a, \quad \mathbf{1}' \times \mathbf{1}'' = \mathbf{1}''', \quad \mathbf{1}' \times \mathbf{1}''' = \mathbf{1}'' \quad \text{and} \quad \mathbf{1}'' \times \mathbf{1}''' = \mathbf{1}'.$$

For $(s_1, s_2, s_3, s_4) \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{1}''')$ and $(x_1, x_2)^T \sim \mathbf{2}$ we find

$$\begin{pmatrix} s_1 x_1 \\ s_1 x_2 \end{pmatrix} \sim \mathbf{2}, \quad \begin{pmatrix} s_2 x_1 \\ -s_2 x_2 \end{pmatrix} \sim \mathbf{2}, \quad \begin{pmatrix} s_3 x_2 \\ s_3 x_1 \end{pmatrix} \sim \mathbf{2} \quad \text{and} \quad \begin{pmatrix} s_4 x_2 \\ -s_4 x_1 \end{pmatrix} \sim \mathbf{2}.$$

The product of a two-dimensional irreps $\mathbf{2} \times \mathbf{2}$ decomposes into the four singlets. Taking, for instance, $(x_1, x_2)^T \sim \mathbf{2}$ and $(y_1, y_2)^T \sim \mathbf{2}$ one finds

$$x_1 y_2 + x_2 y_1 \sim \mathbf{1}, \quad x_1 y_2 - x_2 y_1 \sim \mathbf{1}', \quad x_1 y_1 + x_2 y_2 \sim \mathbf{1}'' \quad \text{and} \quad x_1 y_1 - x_2 y_2 \sim \mathbf{1}'''.$$

Now we show here the explicit form of Eq. (4) using the D_4 product rules. We express the Lagrangian as

$$\begin{aligned} \mathcal{L}_Y \supset & \bar{\ell}_{L_1} \left(y_1 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_1 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_2} + \bar{\ell}_{L_2} \left(y_1 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_1 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_1} + \bar{\ell}_{L_1} \left(y_2 \frac{\eta_2}{\Lambda_F} \Phi + \tilde{y}_2 \frac{\eta_2}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_3} \\ & + \bar{\ell}_{L_2} \left(y_2 \frac{\eta_1}{\Lambda_F} \Phi + \tilde{y}_2 \frac{\eta_1}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_3} + \bar{\ell}_{L_3} \left(y_3 \frac{\eta_1}{\Lambda_F} \Phi + \tilde{y}_3 \frac{\eta_1}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_2} + \bar{\ell}_{L_3} \left(y_3 \frac{\eta_2}{\Lambda_F} \Phi + \tilde{y}_3 \frac{\eta_2}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_1} \\ & + \bar{\ell}_{L_3} \left(y_4 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_4 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_3} + \frac{Y_{L_1}}{2} \ell_{L_1}^T C(i\sigma_2) \Delta_L \ell_{L_2} + \frac{Y_{L_1}}{2} \ell_{L_2}^T C(i\sigma_2) \Delta_L \ell_{L_1} \\ & + \frac{Y_{L_2}}{2} \ell_{L_3}^T C(i\sigma_2) \Delta_L \ell_{L_3} + \frac{Y_{R_1}}{2} \ell_{R_1}^T C(i\sigma_2) \Delta_R \ell_{R_2} + \frac{Y_{R_1}}{2} \ell_{R_2}^T C(i\sigma_2) \Delta_R \ell_{R_1} + \frac{Y_{R_2}}{2} \ell_{R_3}^T C(i\sigma_2) \Delta_R \ell_{R_3} + \text{H.c.} \end{aligned} \quad (\text{A3})$$

The mass matrices for the charged-leptons [see Eq. (9)] and neutrinos [see Eq. (14)] are obtained from the Lagrangian (A3) after spontaneous symmetry breaking of the gauge group.

We also show here that if one consider ℓ_{L_2} , and ℓ_{R_2} to be the D_4 singlets, the following Lagrangian can be obtained

$$\begin{aligned} \mathcal{L}_Y \supset & \bar{\ell}_{L_1} \left(y_1 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_1 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_3} + \bar{\ell}_{L_3} \left(y_1 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_1 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_1} \\ & + \bar{\ell}_{L_1} \left(y_2 \frac{\eta_2}{\Lambda_F} \Phi + \tilde{y}_2 \frac{\eta_2}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_2} + \bar{\ell}_{L_3} \left(y_2 \frac{\eta_1}{\Lambda_F} \Phi + \tilde{y}_2 \frac{\eta_1}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_2} \\ & + \bar{\ell}_{L_2} \left(y_3 \frac{\eta_1}{\Lambda_F} \Phi + \tilde{y}_3 \frac{\eta_1}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_3} + \bar{\ell}_{L_2} \left(y_3 \frac{\eta_2}{\Lambda_F} \Phi + \tilde{y}_3 \frac{\eta_2}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_1} \\ & + \bar{\ell}_{L_2} \left(y_4 \frac{\chi}{\Lambda_F} \Phi + \tilde{y}_4 \frac{\chi}{\Lambda_F} \tilde{\Phi} \right) \ell_{R_2} + \frac{Y_{L_1}}{2} \ell_{L_1}^T C(i\sigma_2) \Delta_L \ell_{L_3} + \frac{Y_{L_1}}{2} \ell_{L_3}^T C(i\sigma_2) \Delta_L \ell_{L_1} \\ & + \frac{Y_{L_2}}{2} \ell_{L_2}^T C(i\sigma_2) \Delta_L \ell_{L_2} + \frac{Y_{R_1}}{2} \ell_{R_1}^T C(i\sigma_2) \Delta_R \ell_{R_3} + \frac{Y_{R_1}}{2} \ell_{R_3}^T C(i\sigma_2) \Delta_R \ell_{R_1} \\ & + \frac{Y_{R_2}}{2} \ell_{R_2}^T C(i\sigma_2) \Delta_R \ell_{R_2} + \text{H.c.} \end{aligned} \quad (\text{A4})$$

After the SSB of the gauge group, we obtain the charged-lepton mass matrix $M_\ell^{(2)}$ from Eq. (A4), whereas using the Dirac and Majorana neutrino mass matrices in Eq. (14) one obtains the light neutrino mass matrix $M_\nu^{(2)}$. These can be read as

$$M_\ell^{(2)} = \begin{pmatrix} 0 & 0 & a_\ell \\ 0 & c_\ell & b_\ell \\ a_\ell^* & b_\ell^* & 0 \end{pmatrix}, \quad M_\nu^{(2)} = \begin{pmatrix} 0 & 0 & a_\nu \\ 0 & c_\nu & b_\nu \\ a_\nu & b_\nu & d_\nu \end{pmatrix}, \quad (\text{A5})$$

where a_ν , b_ν , c_ν , and d_ν are complex entries.

Similarly, if we choose ℓ_{L_1} , and ℓ_{R_1} to be the D_4 singlets, then we find the following leptonic mass matrices

$$M_\ell^{(3)} = \begin{pmatrix} c_\ell & b_\ell & 0 \\ b_\ell^* & 0 & a_\ell \\ 0 & a_\ell^* & 0 \end{pmatrix}, \quad M_\nu^{(3)} = \begin{pmatrix} c_\nu & b_\nu & 0 \\ b_\nu & d_\nu & a_\nu \\ 0 & a_\nu & 0 \end{pmatrix}. \quad (\text{A6})$$

One can also show that

$$M_\ell^{(2)} = R_{32} M_\ell R_{32}^T, \quad M_\nu^{(2)} = R_{32} M_\nu R_{32}^T, \quad (\text{A7})$$

$$M_\ell^{(3)} = R_{31} M_\ell R_{31}^T, \quad M_\nu^{(3)} = R_{31} M_\nu R_{31}^T. \quad (\text{A8})$$

Here, R_{31} , and R_{32} are the permutation matrices and are defined as

$$R_{31} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad R_{32} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (\text{A9})$$

It can be observed from Eq. (A7) that the matrix $R_{32} O_\ell$ diagonalizes $M_\ell^{(2)}$ in analogous to Eqs. (16), (17), whereas $R_{32} U_\nu$ helps to diagonalize $M_\nu^{(2)}$ [see Eqs. (18), (22)]. Therefore, the leptonic mixing matrix remains same as given by Eq. (23). Note that a similar conclusion also holds true for Eq. (A8).

APPENDIX B: $\mu \rightarrow e\gamma$ DECAY IN LEFT-RIGHT MODELS

In LR models the LFV processes are induced at tree level by the scalar triplets and at the loop level by the charged gauge bosons. Following the notation in [78], the branching ratio for $\mu \rightarrow e\gamma$ decay can be written as

$$B(\mu \rightarrow e\gamma) = 384\pi^2 e^2 (|A_L|^2 + |A_R|^2), \quad (\text{B1})$$

where

$$A_L = \frac{1}{16\pi^2} \sum_{n=\text{heavy}} (K_R^\dagger)_{en} (K_R)_{n\mu} \left[\frac{M_{W_L}^2}{M_{W_R}^2} S_3(x_n) - \frac{x_n}{3} \frac{M_{W_L}^2}{M_{\delta_R^{++}}^2} \right], \quad (\text{B2})$$

$$A_R = \frac{1}{16\pi^2} \sum_{n=\text{heavy}} (K_R^\dagger)_{en} (K_R)_{n\mu} x_n \left[-\frac{1}{3} \frac{M_{W_L}^2}{M_{\delta_L^{++}}^2} - \frac{1}{24} \frac{M_{W_L}^2}{M_{H_1^+}^2} \right]. \quad (\text{B3})$$

Also,

$$x_n = \left(\frac{M_n}{M_{W_R}} \right)^2, \quad (\text{B4})$$

$$S_3(x) = -\frac{x(1+2x)}{8(1-x)^2} + \frac{3x^2}{4(1-x)^2} (S_4(x) + 1), \quad (\text{B5})$$

$$S_4(x) = \frac{x}{(1-x)^2} (1-x + \ln x), \quad (\text{B6})$$

$$K_R = (V_R^\nu)^\dagger V_R^l, \quad (\text{B7})$$

where $V_R^{\nu/l}$ are the right handed neutrino/charged lepton mixing matrices respectively. We have neglected the small mixing of the charged gauge bosons, given the parameters of the LR gauge theory that were used in this prescription.

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