Erratum: Nucleon form factors and root-mean-square radii on a (10.8 fm)⁴ lattice at the physical point [Phys. Rev. D **99, 014510 (2019)]**

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In our analysis we employed different normalization factors from Eqs. (20)–(23) in the paper to evaluate the electric, magnetic, axial-vector, induced pseudoscalar, and pseudoscalar form factors in the nonzero q^2 . In some parts of the normalization factors, M_N was used instead of E_N . We corrected the normalization factors in our analysis code, and then recalculated the form factors with the correct normalization factors as in the equations.

The effect of this correction increases as q^2 increases. As a result, $G_E(q^2)$ and $F_A(q^2)$ are overestimated from the experiments in the large q^2 region as presented in Figs. 3, 4, and 14. The root-mean-square (RMS) charge radius and the axial-vector RMS radius become smaller than the ones quoted in the paper, and are underestimated from the experiments as shown in Figs. 5 and 15, respectively. In contrast to these RMS radii, other results obtained from the magnetic, induced pseudoscalar, and pseudoscalar form factors are reasonably consistent with the ones that appeared in the paper.

Figure 23 presents that the RMS charge radius becomes reasonably consistent with the recent lattice results [6–9]. Thus, the finite volume effect could be small in this radius, although we discussed the possibility of its existence in the paper. For the charge radius, it is important to understand the reason of the discrepancy between the lattice results and the experiments for solving the proton size puzzle. Towards understanding of the discrepancy, we plan to measure the RMS charge radius with a finer lattice spacing in the next calculation to estimate discretization error, which was not estimated in the paper.

Our best estimated values for the RMS radii and magnetic moments in Eqs. (29)–(32) of the paper are replaced by the following values,

$$\begin{aligned}\sqrt{\langle r_E^2 \rangle} &= 0.785(17)(21) \text{ [fm]} \quad (\text{isovector}) \\ \sqrt{\langle r_E^2 \rangle} &= 0.766(14)(32) \text{ [fm]} \quad (\text{proton}) \\ \langle r_E^2 \rangle &= -0.047(20)(13) \text{ [fm}^2\text{]} \quad (\text{neutron}),\end{aligned}\tag{1}$$

$$\begin{aligned}\sqrt{\langle r_M^2 \rangle} &= 0.707(36)(311) \text{ [fm]} \quad (\text{isovector}) \\ \sqrt{\langle r_M^2 \rangle} &= 0.710(39)(211) \text{ [fm]} \quad (\text{proton}) \\ \sqrt{\langle r_M^2 \rangle} &= 0.758(33)(286) \text{ [fm]} \quad (\text{neutron}),\end{aligned}\tag{2}$$

$$\begin{aligned}\mu_v &= 4.409(136)(333) \quad (\text{isovector}) \\ \mu_p &= 2.743(92)(148) \quad (\text{proton}), \\ \mu_n &= -1.697(63)(147) \quad (\text{neutron})\end{aligned}\tag{3}$$

$$\sqrt{\langle r_A^2 \rangle} = 0.532(28)(72) \text{ [fm]}.\tag{4}$$

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The corresponding figures and tables related to the corrected form factors are shown below.

FIG. 1. Isovector electric form factor $G_E^v(q^2)$, which is extracted from the ratios of three- to two-point functions of Eq. (20) in the paper, for $t_{\text{sep}}/a = 10, 12, 14, 16$ with four lowest nonzero momentum transfers. The gray-shaded area denotes the fit range in each panel. This figure replaces Fig. 2 in the paper.

FIG. 2. t_{sep} dependence of the isovector electric form factor $G_E^v(q^2)$ with five lowest momentum transfers. The horizontal band represents the fit result of $G_E^v(q^2)$ at $t_{\text{sep}}/a = 12, 14, 16$ for each q^2 . This figure replaces Fig. 3 in the paper.

FIG. 3. q^2 dependence of the isovector electric form factor $G_E^v(q^2)$ obtained by the combined analysis of the results at $t_{\text{sep}}/a = \{12, 14, 16\}$ (circle) and $t_{\text{sep}}/a = \{14, 16\}$ (square). Diamond symbols, which are obtained with $t_{\text{sep}}/a = 15$ on a 96^4 lattice at $m_\pi = 146$ MeV in Ref. [1], are also plotted for comparison. This figure replaces Fig. 4 in the paper.

FIG. 4. The same as Fig. 3 for the proton (left) and neutron (right). Results are obtained without the disconnected diagram. This figure replaces Fig. 5 in the paper.

FIG. 5. Electric RMS radius $\sqrt{\langle r_E^2 \rangle}$ for the isovector (left) and proton (right) obtained by linear, dipole, quadratic and z -expansion fits for the combined data. Horizontal bands represent the experimental results from ep scattering (upper) and μH spectroscopy (lower). The results for the proton channel are obtained without the disconnected diagram. This figure replaces Fig. 6 in the paper.

TABLE I. Results for the electric RMS charge radius $\sqrt{\langle r_E^2 \rangle}$ in the isovector, proton and neutron channels. In the row of “This work” we present our best estimates, where the first error is statistical and the second one is systematic as explained in the text. Results for the proton and neutron are obtained without the disconnected diagram. Our previous work was performed on a 96^4 lattice at $m_\pi = 146$ MeV in Ref. [1], where only the statistical errors are presented. This table replaces Table III in the paper.

Fit type	q_{cut}^2 [GeV 2]	t_{sep}/a	Isovector		Proton		Neutron	
			$\sqrt{\langle r_E^2 \rangle}$ [fm]	χ^2/dof	$\sqrt{\langle r_E^2 \rangle}$ [fm]	χ^2/dof	$\langle r_E^2 \rangle$ [fm 2]	χ^2/dof
Linear	0.013	{12, 14, 16}	0.764(26)	...	0.765(22)	...	...	...
		{14, 16}	0.806(35)	...	0.791(28)	...	...	...
Dipole	0.102	{12, 14, 16}	0.785(17)	1.2(8)	0.766(14)	0.8(7)	...	...
		{14, 16}	0.806(26)	0.6(6)	0.788(20)	0.6(6)	...	...
Quadrature	0.102	{12, 14, 16}	0.785(19)	1.0(8)	0.773(16)	0.4(5)	−0.038(18)	2.2(1.9)
		{14, 16}	0.783(30)	0.7(7)	0.780(19)	0.8(8)	−0.029(23)	2.6(2.2)
z-exp ($k_{\text{max}} = 3$)	0.102	{12, 14, 16}	0.776(28)	1.2(9)	0.781(24)	0.6(7)	−0.047(20)	1.8(1.7)
		{14, 16}	0.796(37)	0.8(8)	0.798(25)	0.5(6)	−0.034(25)	2.4(2.1)
This work			0.785(17)(21)		0.766(14)(32)		−0.047(20)(13)	
PACS'18 [1]								
Dipole	0.215	15	0.822(63)	...	...	...	...	...
Quadratic	0.215	15	0.851(62)	...	...	...	...	...
z-exp ($k_{\text{max}} = 3$)	0.215	15	0.914(101)	...	...	...	...	...
Experimental value								
ep scattering			0.939(6)		0.875(6)		−0.1161(22)	
μH atom			0.907(1)		0.8409(4)		...

FIG. 6. The same as Fig. 1 for the ratio of Eq. (21) in the paper to extract the isovector magnetic form factor $G_M^v(q^2)$. This figure replaces Fig. 7 in the paper.

FIG. 7. The same as Fig. 2 for the isovector magnetic form factor $G_M^v(q^2)$ with four lowest nonzero momentum transfers. This figure replaces Fig. 8 in the paper.

FIG. 8. The same as Fig. 3 for the isovector magnetic form factor $G_M^v(q^2)$. This figure replaces Fig. 9 in the paper.

FIG. 9. The same as Fig. 3 for the proton (left) and neutron (right) magnetic form factor $G_M(q^2)$. The results are obtained without the disconnected diagram. This figure replaces Fig. 10 in the paper.

FIG. 10. Magnetic moment μ for the isovector (top), proton (middle) and neutron (bottom) channels obtained by the fitting with the linear, dipole, quadratic forms and the z -expansion method for the combined data. Horizontal bands represent the experimental results. Two types of symbols denote the results with two choices of the combined t_{sep} ranges. Results for the proton and neutron channels are obtained without the disconnected diagram. This figure replaces Fig. 11 in the paper.

FIG. 11. The same as Fig. 10 for the magnetic RMS radius $\sqrt{\langle r_M^2 \rangle}$. This figure replaces Fig. 12 in the paper.

FIG. 12. The same as Fig. 1 for the axial-vector form factor $F_A(q^2)$ extracted from the ratio of Eq. (22) in the paper. This figure replaces Fig. 15 in the paper.

FIG. 13. The same as Fig. 2 for $F_A(q^2)$ with four lowest nonzero momentum transfers. This figure replaces Fig. 16 in the paper.

TABLE II. Results for the magnetic moments μ and magnetic RMS radius $\sqrt{\langle r_M^2 \rangle}$ for the isovector, proton and neutron channels. In the row of “This work” we present our best estimates, where the first error is statistical and the second one is systematic as explained in the text. Results for the proton and neutron are obtained without the disconnected diagram. Previous work was performed on a 96^4 lattice at $m_\pi = 146$ MeV in Ref. [1], where only the statistical errors are presented. This table replaces Table IV in the paper.

Fit type	q_{cut}^2 [GeV 2]	t_{sep}/a	Isovector		
			μ_v	$\sqrt{\langle r_M^2 \rangle}$ [fm]	χ^2/dof
Linear	0.039	{12, 14, 16}	4.489(179)	0.750(62)	0.8(1.9)
		{14, 16}	4.672(209)	0.828(63)	0.4(1.2)
Dipole	0.102	{12, 14, 16}	4.409(136)	0.707(36)	0.8(7)
		{14, 16}	4.685(236)	0.818(51)	3.6(3.8)
Quadrature	0.102	{12, 14, 16}	4.438(163)	0.723(65)	1.0(9)
		{14, 16}	4.700(237)	0.868(71)	0.9(1.8)
z-exp ($k_{\text{max}} = 3$)	0.102	{12, 14, 16}	4.468(177)	0.748(104)	0.9(9)
		{14, 16}	4.742(232)	1.018(93)	0.9(9)
This work			4.409(136)(333)	0.707(36)(311)	
PACS'18 [1]					
Dipole	0.215	15	3.96(46)	0.656(133)	...
Quadratic	0.215	15	4.24(52)	0.852(130)	...
z-exp ($k_{\text{max}} = 3$)	0.215	15	4.86(82)	1.495(437)	...
Experimental value			4.70589	0.862(14)	

Fit type	q_{cut}^2 [GeV 2]	t_{sep}/a	Proton			Neutron		
			μ_p	$\sqrt{\langle r_M^2 \rangle}$ [fm]	χ^2/dof	μ_n	$\sqrt{\langle r_M^2 \rangle}$ [fm]	χ^2/dof
Linear	0.039	{12, 14, 16}	2.769(117)	0.708(73)	0.2(9)	-1.701(71)	0.729(84)	2.4(3.0)
		{14, 16}	2.881(153)	0.816(68)	0.1(4)	-1.803(85)	0.832(82)	0.5(1.6)
Dipole	0.102	{12, 14, 16}	2.743(92)	0.710(39)	0.2(4)	-1.697(63)	0.758(43)	0.3(6)
		{14, 16}	2.777(150)	0.715(51)	1.6(1.7)	-1.815(94)	0.861(64)	1.3(2.5)
Quadrature	0.102	{12, 14, 16}	2.759(109)	0.724(71)	0.3(4)	-1.691(67)	0.686(73)	1.0(9)
		{14, 16}	2.862(157)	0.829(78)	0.8(1.7)	-1.750(81)	0.850(91)	1.3(1.1)
z-exp ($k_{\text{max}} = 3$)	0.102	{12, 14, 16}	2.763(120)	0.727(118)	0.3(4)	-1.684(72)	0.622(160)	0.9(8)
		{14, 16}	2.891(163)	0.921(121)	0.5(1.3)	-1.844(103)	1.044(134)	0.7(8)
This work			2.743(92)(148)	0.710(39)(211)		-1.697(63)(147)	0.758(33)(286)	
Experimental value			2.79285	0.776(38)		-1.91304	0.864(9)

FIG. 14. The same as Fig. 3 for the axial-vector form factor $F_A(q^2)$. Experimental line is obtained by the dipole form with the value of dipole mass [2,3] and g_A [4]. This figure replaces Fig. 17 in the paper.

FIG. 15. The axial-vector RMS radius $\sqrt{\langle r_A^2 \rangle}$ obtained by the fit for the combined data with the linear, dipole, quadratic forms and the z-expansion method. Horizontal bands represent the experimental values. This figure replaces the right panel of Fig. 18 in the paper.

TABLE III. Results for the axial-vector coupling $g_A = F_A(0)$ and axial-vector RMS radius $\sqrt{\langle r_A^2 \rangle}$. In the row of “This work” we present our best estimates, where the first error is statistical and the second one is systematic as explained in the text. Results for the proton and neutron are obtained without the disconnected diagram. Previous work was performed on a 96^4 lattice at $m_\pi = 146$ MeV in Ref. [1], where only the statistical errors are presented. This table replaces Table V in the paper.

Fit type	q_{cut}^2 GeV ²	t_{sep}/a	$F_A(0)$	$\sqrt{\langle r_A^2 \rangle}$ [fm]	χ^2/dof
Linear	0.039	{12, 14, 16}	1.278(23)	0.482(60)	1.1(1.5)
	0.052	{14, 16}	1.262(29)	0.584(54)	0.2(1.2)
Dipole	0.102	{12, 14, 16}	1.288(20)	0.532(28)	1.2(8)
	0.077	{14, 16}	1.256(28)	0.589(49)	0.2(7)
Quadrature	0.102	{12, 14, 16}	1.287(20)	0.513(46)	1.4(1.0)
	0.077	{14, 16}	1.258(28)	0.603(83)	0.4(1.5)
z-exp ($k_{\max} = 3$)	0.102	{12, 14, 16}	1.287(20)	0.517(58)	1.5(1.1)
	0.077	{14, 16}	1.257(28)	0.604(94)	0.4(1.4)
This work				0.532(28)(72)	
PACS'18 [1]					
Dipole	0.215	15	...	0.40(12)	...
Quadratic	0.215	15	...	0.22(49)	...
z-exp ($k_{\max} = 3$)	0.215	15	...	0.46(11)	...
Experimental value				0.67(1)

FIG. 16. The same as Fig. 1 for the induced pseudoscalar form factor $F_P(q^2)$ extracted from the ratio of Eq. (22) in the paper. This figure replaces Fig. 19 in the paper.

FIG. 17. The same as Fig. 2 for the normalized induced pseudoscalar form factor $2M_N F_P(q^2)$ with four lowest nonzero momentum transfers. This figure replaces Fig. 20 in the paper.

FIG. 18. q^2 dependence of the normalized induced pseudoscalar form factor $2M_N F_P(q^2)$. We also plot the results in the previous work [1] for comparison. The colored band shows the function of the pion pole dominance model using the fit result of $F_A(q^2)$ with the dipole form in Fig. 14. This figure replaces Fig. 21 in the paper.

FIG. 19. The same as Fig. 1 for the pseudoscalar form factor $G_P(q^2)$ extracted from the ratio of Eq. (23) in the paper. This figure replaces Fig. 22 in the paper.

FIG. 20. q^2 dependence for the pseudoscalar form factor $G_P(q^2)$ at $t_{\text{sep}}/a = 14, 16$. We also plot the results in [1] for comparison. This figure replaces Fig. 23 in the paper.

FIG. 21. Ratio of the pseudoscalar form factor $G_P(q^2)$ over the induced one $F_P(q^2)$ as a function of q^2 at each t_{sep} . This figure replaces Fig. 24 in the paper.

FIG. 22. q^2 dependence (left) and t_{sep} dependence (right) of the quark mass defined by the generalized GT relation of Eq. (9) in the paper. The data in the right panel is obtained at Q1 corresponding to $|n|^2 = 1$. Horizontal band denotes the quark mass obtained from the PCAC relation in Ref. [5]. This figure replaces Fig. 25 in the paper.

FIG. 23. (Top) Comparison of recent LQCD results for the isovector nucleon electric and the magnetic RMS radii, and magnetic moment obtained by CLS-Mainz [6], PNDME [7], ETMC [8], Hasan *et al.* [9] and PACS [1]. (Bottom) The same as the top panels for the axial RMS radius and the axial-vector coupling obtained by CLS-Mainz [10], Green *et al.* [11], PNDME [12,13], ETMC [14], CallLat [15], and PACS [1]. Those errors are total one combined with statistical and systematic errors in the quadrature. Vertical bands denote experimental values. This figure replaces Fig. 26 in the paper, except for the figure of g_A .

FIG. 24. Summary plot for the proton electric and magnetic RMS radii and magnetic moment (top) and the neutron electric and magnetic RMS radii and magnetic moment (bottom). We also plot the results of the ETM collaboration [8] for comparison. Vertical bands denote experimental values. Our results are obtained without the disconnected diagram. This figure replaces Fig. 27 in the paper.

TABLE IV. q^2 dependence of the isovector form factors obtained by the constant fit for $t_{\text{sep}}/a = \{12, 14, 16\}$ and $t_{\text{sep}}/a = \{14, 16\}$. In the previous work [1] the results are obtained with $t_{\text{sep}}/a = 15$ on a 96^4 lattice at $m_\pi = 146$ MeV. This table replaces Table VII in the paper.

$q^2 [\text{GeV}^2]$	$t_{\text{sep}}/a = \{12, 14, 16\}$			$t_{\text{sep}}/a = \{14, 16\}$		
	$G_E^v(q^2)$	$G_M^v(q^2)$	$F_A(q^2)$	$G_E^v(q^2)$	$G_M^v(q^2)$	$F_A(q^2)$
0.000	0.997(1)	...	1.273(24)	0.999(1)	...	1.269(34)
0.013	0.964(2)	4.312(163)	1.251(22)	0.962(2)	4.546(215)	1.249(33)
0.026	0.934(3)	4.189(140)	1.237(22)	0.930(5)	4.347(185)	1.226(39)
0.039	0.905(5)	4.045(125)	1.223(21)	0.900(6)	4.170(176)	1.213(42)
0.052	0.873(6)	3.923(123)	1.190(21)	0.870(8)	3.976(178)	1.174(30)
0.064	0.848(6)	3.839(112)	1.189(22)	0.843(9)	3.926(158)	1.157(42)
0.077	0.825(7)	3.734(104)	1.176(22)	0.817(10)	3.804(158)	1.144(43)
0.102	0.778(8)	3.557(96)	1.146(23)	0.766(12)	3.627(148)	1.103(51)

$q^2 [\text{GeV}^2]$	PACS'18 [1] $t_{\text{sep}}/a = 15$		
	$G_E^v(q^2)$	$G_M^v(q^2)$	$F_A(q^2)$
0.000	1.000(4)	...	1.163(75)
0.024	0.924(11)	4.071(456)	1.121(68)
0.048	0.861(19)	3.640(350)	1.137(69)
0.072	0.804(27)	3.333(305)	1.112(64)
0.095	0.774(30)	3.313(344)	1.118(72)

TABLE V. q^2 dependence of the proton and neutron form factors obtained by the constant fit for $t_{\text{sep}}/a = \{12, 14, 16\}$ and $t_{\text{sep}}/a = \{14, 16\}$. This table replaces Table VIII in the paper.

$q^2 [\text{GeV}^2]$	$t_{\text{sep}}/a = \{12, 14, 16\}$				$t_{\text{sep}}/a = \{14, 16\}$			
	Proton		Neutron		Proton		Neutron	
	$G_E^p(q^2)$	$G_M^p(q^2)$	$G_E^n(q^2)$	$G_M^n(q^2)$	$G_E^p(q^2)$	$G_M^p(q^2)$	$G_E^n(q^2)$	$G_M^n(q^2)$
0.000	0.9988(8)	...	0.0016(7)	...	1.000(0)	...	0.002(1)	...
0.013	0.966(1)	2.681(105)	0.002(1)	-1.632(63)	0.965(2)	2.778(144)	0.003(1)	-1.764(86)
0.026	0.936(3)	2.609(88)	0.003(1)	-1.598(55)	0.933(3)	2.669(130)	0.004(2)	-1.682(69)
0.039	0.907(4)	2.532(77)	0.004(2)	-1.543(50)	0.904(5)	2.564(122)	0.005(4)	-1.613(63)
0.052	0.879(5)	2.447(77)	0.008(3)	-1.489(49)	0.877(6)	2.441(120)	0.014(11)	-1.535(67)
0.064	0.854(5)	2.401(66)	0.008(3)	-1.457(46)	0.850(7)	2.428(110)	0.013(9)	-1.502(58)
0.077	0.830(6)	2.342(61)	0.008(3)	-1.411(43)	0.825(7)	2.345(108)	0.012(8)	-1.463(58)
0.102	0.786(7)	2.229(55)	0.010(4)	-1.334(43)	0.778(9)	2.255(98)	0.015(8)	-1.372(62)

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