Vacuum stability and spontaneous violation of the lepton number at a low-energy scale in a model for light sterile neutrinos

João Paulo Pinheiro[✉] and C. A. de S. Pires[✉]

*Departamento de Física, Universidade Federal da Paraíba,
Caixa Postal 5008, 58051-970 João Pessoa, Paraíba, Brasil*

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It is well known that the standard model of electroweak interactions rests on a metastable vacuum. This can be fixed only by means of new physics. Presently neutrino physics provides the most intriguing framework to formulate new physics. This is so because, in addition to the problem with lightness for the active standard neutrinos, current MiniBooNE experimental results may indicate that sterile neutrinos exist and are light, too. In this case, it is reasonable to expect that the framework that yields light active and sterile neutrinos could stabilize the vacuum, too. To achieve this goal, we consider an extension of the standard model which involves new fermions in the form of right-handed neutrinos (ν_R) and new scalars in the form of triplets (Δ) and singlets (σ). Within this framework, tiny masses are obtained when we consider that lepton number is spontaneously broken at low-energy scale, which means that Δ and σ both develop very small vacuum expectation values. We investigate whether this setting leads to a stable vacuum. For this, we obtain the whole set of conditions over the quartic terms of the potential that ensures that the model is bounded from below and evaluate the renormalization group equation of the self-coupling of the Higgs. We show that in such a scenario the quartic coupling κ of the interaction ($\Phi^T \Delta \Phi \sigma + \text{H.c.}$), where Φ is the standard Higgs doublet, is responsible for the stability of the electroweak vacuum up to Planck scale. We also extract constraints over the parameters of the potential by means of lepton flavor violating processes and from invisible decay of the standardlike Higgs.

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I. INTRODUCTION

Although researchers have devoted considerable attention to the study of extensions of the standard Higgs sector [1–3], relative little attention has been given to a Higgs sector involving triplets (Δ), doublets (Φ), and singlets (σ) of scalars [4–6]. From now on, we shall refer to this case as the 3-2-1 model. This model is interesting in its own right. However, it becomes even more interesting when right-handed neutrinos are introduced, too. This is so because in this case the 3-2-1 model yields the most general neutrino mass matrix involving Majorana and Dirac mass terms for both neutrinos. Hence, when we assume that the lepton number is spontaneously violated at low-energy scale, right-handed neutrinos acquire light masses and may explain the recent MiniBooNE experimental results [7] by means of neutrino oscillation.

In this work, we derive the complete set of conditions that guarantee the potential of the 3-2-1 model to be

bounded from below. For the specific case in which lepton number is spontaneously broken at low-energy scale, we obtain the spectrum of scalars of the model and discuss the stability of the vacuum by evaluating the renormalization group equation (RGE) of the self-coupling of the standardlike Higgs up to Planck scales. This case is particularly interesting because it encompasses a Majoron and a light CP -even scalar in their spectrum of scalars. We discuss the contributions of these scalars for the invisible decay channels of the standardlike Higgs and of the neutral gauge boson Z . We also obtain the constraints that lepton flavor violation (LFV) puts over the parameters of the potential. Regarding neutrino physics, we provide a solution, i.e., a set of values for the Yukawa couplings, that recovers the standard neutrino sector and provides at least one right-handed neutrino with mass resting on the eV scale and robustly mixed with the standard neutrinos in such a way that it accommodates current MiniBooNE results by means of neutrino oscillation and is in agreement with cosmological data.

This work is organized as follows. In Sec. II, we develop the main aspect of the model including neutrino masses, while in Sec. III we develop the scalar sector. In Sec. IV, we discuss the stability of the vacuum. In Sec. V, we present our final remarks.

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II. THE 3-2-1 MODEL

The leptonic sector of the model is composed of the standard content plus right-handed neutrinos in the singlet form,

$$L_i = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}_L; \quad \ell_{i_R}; \quad \nu_{i_R}, \quad (1)$$

where $i = e, \mu, \tau$.

The scalar sector is composed of the standard Higgs doublet, Φ , one Higgs triplet, Δ , and one Higgs singlet, σ , presented below,

$$\Delta = \begin{pmatrix} \Delta^0 & \frac{\Delta^+}{\sqrt{2}} \\ \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \end{pmatrix}; \quad \Phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}; \quad \sigma. \quad (2)$$

The quark sector is the standard one.

The most general potential involving this scalar content that conserves lepton number is composed of the following terms:

$$\begin{aligned} V(\sigma, \Phi, \Delta) = & \mu_1^2 \sigma^* \sigma + \mu_2^2 \Phi^\dagger \Phi + \mu_3^2 \text{tr}(\Delta^\dagger \Delta) \\ & + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 [\text{tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 \Phi^\dagger \Phi \text{tr}(\Delta^\dagger \Delta) \\ & + \lambda_4 \text{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta) + \lambda_5 (\Phi^\dagger \Delta^\dagger \Delta \Phi) \\ & + \beta_1 (\sigma^* \sigma)^2 + \beta_2 \Phi^\dagger \Phi \sigma^* \sigma + \beta_3 \text{tr}(\Delta^\dagger \Delta) \sigma^* \sigma \\ & - \kappa (\Phi^T \Delta \Phi \sigma + \text{H.c.}). \end{aligned} \quad (3)$$

With such lepton and scalar content, the Yukawa interactions that generate mass for all neutrinos of the model is given by

$$\mathcal{L}_Y^\nu = \frac{1}{2} Y_{ij}^L L_i^T \Delta L_j + Y_{ij}^D \bar{L}_i \tilde{\phi} \nu_{Rj} + \frac{1}{2} Y_{ij}^R \bar{\nu}_{Ri}^C \nu_{Rj} \sigma + \text{H.c.} \quad (4)$$

The Yukawa interactions of the charged fermions are the standard ones.

When the neutral scalars of the model develop vacuum expectation values (VEVs) other than zero, i.e., $\langle \sigma \rangle = \frac{v_1}{\sqrt{2}}$, $\langle \phi \rangle = \frac{v_2}{\sqrt{2}}$, and $\langle \Delta \rangle = \frac{v_3}{\sqrt{2}}$, the Yukawa interactions in Eq. (4) provide the following mass terms for the neutrinos:

$$\mathcal{L}_{\text{mass}}^{D+M} = \frac{1}{2} \bar{\nu}_L^C M_L \nu_L + \bar{\nu}_L M_D \nu_R + \frac{1}{2} \bar{\nu}_R^C M_R \nu_R + \text{H.c.}, \quad (5)$$

with $\nu_L = (\nu_{e_L} \nu_{\mu_L} \nu_{\tau_L})^T$ and $\nu_R = (\nu_{e_R} \nu_{\mu_R} \nu_{\tau_R})^T$.

Considering the basis $\nu = (\nu_L \nu_R^C)^T$, we can simplify Eq. (5) to

$$\mathcal{L}_{\text{mass}}^{D+M} = \frac{1}{2} \bar{\nu}^C M^{D+M} \nu + \text{H.c.}, \quad (6)$$

and the 6×6 symmetric mass matrix is given by

$$M^{D+M} = \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}, \quad (7)$$

where $M_L = Y^L \langle \Delta \rangle$, $M_D = Y^D \langle \phi \rangle$, and $M_R = Y^R \langle \sigma \rangle$. M^{D+M} is the most general neutrino mass matrix. It involves Dirac and Majorana mass terms for both left- and right-handed neutrinos. The 3-2-1 model is the simplest model that generates this mass matrix in the case of spontaneous violation of the lepton number.

The relation between the flavor basis, ν , and the physical ones, $N = (N_1 N_2 N_3 N_4 N_5 N_6)^T$, is given by $N = U \nu$, with U being the unitary matrix that diagonalizes M^{D+M} ,

$$U^T M^{D+M} U = M = \text{diag}(M_1 M_2), \quad (8)$$

where $M_1 = \text{diag}(m_1 m_2 m_3)^T$ and $M_2 = \text{diag}(m_4 m_5 m_6)^T$.

To go further, we need to obtain information about the VEVs v_1 , v_2 , and v_3 . For this, we have to develop the scalar sector of the model.

First, we expand the neutral scalar fields around their respective VEVs,

$$\begin{aligned} \sigma &= \frac{v_1}{\sqrt{2}} + \frac{R_1 + I_1}{\sqrt{2}}, \\ \phi^0 &= \frac{v_2}{\sqrt{2}} + \frac{R_2 + I_2}{\sqrt{2}}, \\ \Delta^0 &= \frac{v_3}{\sqrt{2}} + \frac{R_3 + I_3}{\sqrt{2}}, \end{aligned} \quad (9)$$

and obtain the set of minimum conditions required by the potential above to allow spontaneous breaking of the symmetries of the model which include the global $B - L$ symmetry,

$$\begin{aligned} v_1 \left(\mu_1^2 + \beta_1 v_1^2 + \frac{1}{2} \beta_2 v_2^2 + \frac{1}{2} \beta_3 v_3^2 \right) - \frac{1}{2} \kappa v_2^2 v_3 &= 0, \\ v_2 \left(\mu_2^2 + \lambda_1 v_2^2 + \frac{1}{2} \lambda_3 v_3^2 + \frac{1}{2} \lambda_5 v_3^2 + \frac{1}{2} \beta_2 v_1^2 - \kappa v_1 v_3 \right) &= 0, \\ v_3 \left(\mu_3^2 + \lambda_2 v_3^2 + \frac{1}{2} \lambda_3 v_2^2 + \lambda_4 v_3^2 + \frac{1}{2} \lambda_5 v_2^2 + \frac{1}{2} \beta_3 v_1^2 \right) - \frac{\kappa v_1 v_2^2}{2} &= 0. \end{aligned} \quad (10)$$

Upon analyzing this set of constraints, observe that the first and third relations yield

$$v_1 \approx \frac{\mu_3}{\mu_1} v_3. \quad (11)$$

The parameters μ_1 , μ_3 , and v_1 are free to take any value, while v_3 is constrained by the rho parameter. This is so because v_3 gives different contributions to the mass of the standard gauge bosons $W^\pm$ and Z^0 modifying, in this way, the standard expression of the Veltman ρ parameter [8] to

$\frac{1+2(v_3/v_2)^2}{1+4(v_3/v_2)^2}$. As consequence of the fact that the experimental value of the ρ parameter is near unity, we have the constraint $\frac{v_3}{v_2} \ll 1$. Then the direct proportionality among v_1 and v_3 provided by Eq. (11) hints that $v_1 < v_2$, which implies that right-handed neutrinos may be light particles, too. That is very nice.

A strong reason for the existence of light right-handed neutrinos is the explanation of short-baseline neutrino results (LSND and MiniBooNE) [7,9] by means of neutrino oscillation. In this case, the adequate value for v_1 is one that accommodates at least one right-handed neutrino with mass around eV with robust mixing with the standard neutrinos and that is in conciliation with cosmology. We follow this line of reasoning here.

The current scenario of neutrino physics involving MiniBooNE and LSND experimental results may be accommodated within our model with the following set of values for the VEVs,

$$v_1 = 10^5 \text{ eV}; \quad v_2 = 246 \text{ GeV}; \quad v_3 = 1 \text{ eV}, \quad (12)$$

and the following set of values for the Yukawa couplings,

$$Y_D = \begin{pmatrix} 3,6 \times 10^{-13} & 5,74 \times 10^{-13} & 5,74 \times 10^{-14} \\ -2,21 \times 10^{-13} & -3,45 \times 10^{-10} & 5,75 \times 10^{-13} \\ -7,07 \times 10^{-13} & 5,75 \times 10^{-12} & 5,75 \times 10^{-13} \end{pmatrix}, \quad (13)$$

$$Y_L = \begin{pmatrix} 6,20 \times 10^{-3} & -4,11 \times 10^{-3} & -1,25 \times 10^{-2} \\ -4,11 \times 10^{-3} & 3,90 \times 10^{-1} & 1,95 \times 10^{-2} \\ -1,25 \times 10^{-2} & 1,95 \times 10^{-2} & 3,83 \times 10^{-2} \end{pmatrix}, \quad (14)$$

$$Y_R = \begin{pmatrix} 1,40 \times 10^{-5} & 4,75 \times 10^{-12} & 4,52 \times 10^{-12} \\ 4,75 \times 10^{-12} & 10^{-1} & 5,08 \times 10^{-15} \\ 4,52 \times 10^{-12} & 5,08 \times 10^{-15} & 10^{-1} \end{pmatrix}. \quad (15)$$

On substituting all of these values into M^{D+M} , given in Eq. (7), we have that its diagonalization provides

$$m_1 = 2 \times 10^{-4} \text{ eV}; \quad m_2 = 8,6 \times 10^{-3} \text{ eV}; \quad m_3 = 5 \times 10^{-2} \text{ eV}, \\ m_4 = 1,4 \text{ eV}; \quad m_5 = 10^4 \text{ eV}; \quad m_6 = 10^4 \text{ eV}. \quad (16)$$

The mixing matrix, U , responsible for the diagonalization of M^{D+M} and relating the basis ν to N , as in Eq. (8), is given by

$$U = \begin{pmatrix} 0,83 & 0,54 & -0,12 & 0,045 & 10^{-5} & 10^{-6} \\ -0,25 & 0,59 & 0,72 & -0,03 & -6 \times 10^{-3} & 10^{-5} \\ 0,44 & -0,6 & 0,69 & -0,09 & 10^{-4} & 10^{-5} \\ -0,045 & 0,03 & 0,09 & 1 & \sim 0 & \sim 0 \\ 10^{-6} & 10^{-4} & 10^{-4} & \sim 0 & 1 & \sim 0 \\ 10^{-6} & 10^{-5} & 10^{-4} & \sim 0 & \sim 0 & 1 \end{pmatrix}. \quad (17)$$

The values of m_1 , m_2 , and m_3 given in Eq. (16) and the upper left 3×3 submatrix of U accommodate the current solar and atmospheric neutrino oscillation data. A nice thing to observe is that the mixing angles among N_4 , ν_μ , and ν_e , together with the mass value of m_4 , are in such a way that they allow the explanation of neutrino anomalies suggested by the data from short-baseline (SBL) neutrino experiments by means of neutrino oscillation. Finally, observe in U that N_5 and N_6 practically decouple from the other neutrinos. In other words, this case recovers the $3 + 1$ sterile neutrino scenario.

A problem with models involving an eV sterile neutrino is that they create tension with current cosmological data [10]. We discuss this point later.

III. SCALAR SECTOR

We saw in the previous section that the scenario we are developing here is capable of accommodating neutrino physics including SBL anomalies such as LSND and MiniBooNE. This provides a strong reason for us to go deep into the development of such a case. Thus, in this section we perform a careful analysis of the spectrum of scalars of the model that is in consonance with the previous section, which means that we should consider the VEVs respecting the hierarchy $v_3 \ll v_1 \ll v_2$.

A. Spectrum of scalars

We now start developing the CP -even sector. Considering the basis $(R_1 R_3 R_2)$, the potential above together with the minimum conditions provides

$$M_R^2 = \begin{pmatrix} 2\beta_1 v_1^2 + \frac{1}{2}\kappa v_2^2 \frac{v_3}{v_1} & \beta_3 v_1 v_3 - \frac{1}{2}\kappa v_2^2 & \beta_2 v_1 v_2 - \kappa v_2 v_3 \\ \beta_3 v_1 v_3 - \frac{1}{2}\kappa v_2^2 & 2(\lambda_2 + \lambda_4) v_3^2 + \frac{1}{2}\kappa v_2^2 \frac{v_1}{v_3} & (\lambda_5 + \lambda_3) v_2 v_3 - \kappa v_1 v_2 \\ \beta_2 v_1 v_2 - \kappa v_2 v_3 & (\lambda_5 + \lambda_3) v_2 v_3 - \kappa v_1 v_2 & 2\lambda_1 v_2^2 \end{pmatrix}. \quad (18)$$

The complexity of this mass matrix prevents us from achieving a closed analytic form of the eigenvalues/eigenvectors. In this case, the best that we can do is a simplification based on our assumptions regarding the parameter space. In this way, according to the hierarchy of the VEVs that we have assumed here, this matrix may be approximated by

$$M_R^2 \approx \begin{pmatrix} \frac{1}{2}\kappa v_2^2 \frac{v_3}{v_1} & -\frac{1}{2}\kappa v_2^2 & \sim 0 \\ -\frac{1}{2}\kappa v_2^2 & \frac{1}{2}\kappa v_2^2 \frac{v_1}{v_3} & \sim 0 \\ \sim 0 & \sim 0 & 2\lambda_1 v_2^2 \end{pmatrix}. \quad (19)$$

This means that R_2 decouples from the other ones, while R_1 and R_3 mix among themselves to form H_1 and H_3 according to the following relation:

$$\begin{pmatrix} H_1 \\ H_3 \end{pmatrix} = U_R \begin{pmatrix} R_1 \\ R_3 \end{pmatrix}; \quad R_2 = H_2, \quad (20)$$

where

$$U_R \approx \begin{pmatrix} 1 & \epsilon \\ -\epsilon & 1 \end{pmatrix}; \quad \epsilon \approx \frac{v_3}{v_1}. \quad (21)$$

The masses are given by

$$m_{H_1}^2 \approx \frac{2\beta_2^2 v_1^2}{\kappa}, \quad m_{H_3}^2 \approx \frac{\kappa v_1 v_2^2}{2v_3}, \quad m_{H_2}^2 \approx 2\lambda_1 v_2^2. \quad (22)$$

Observe that, for the hierarchy of the VEVs assumed here, we have that H_2 will play the role of the standard Higgs, while H_3 is a heavy Higgs, with mass at around the TeV scale, and H_1 is a light one with mass at the eV scale.

In the CP -odd sector, things are much simpler, and the mass matrix in the basis (I_1, I_2, I_3) is given by

$$M_I^2 = \begin{pmatrix} \frac{1}{2}\kappa v_2^2 \frac{v_3}{v_1} & \kappa v_2 v_3 & \frac{1}{2}\kappa v_2^2 \\ \kappa v_2 v_3 & 2\kappa v_1 v_3 & \kappa v_1 v_2 \\ \frac{1}{2}\kappa v_2^2 & \kappa v_1 v_2 & \frac{1}{2}\kappa v_2^2 \frac{v_1}{v_3} \end{pmatrix}. \quad (23)$$

Its diagonalization leads to a Goldstone boson, G , that is dominantly I_2 and will be eaten by the standard gauge boson Z , a massless pseudoscalar, J , which we call the Majoron, and a heavy pseudoscalar, A , which is dominantly I_3 . The relation among these pseudoscalars with the basis is given by

$$\begin{pmatrix} J \\ G \\ A \end{pmatrix} = U_I \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}, \quad (24)$$

where U_I is given by

$$U_I \approx \begin{pmatrix} 1 & -2\frac{v_3^2}{v_1 v_2} & -\epsilon \\ 0 & 1 & -2\frac{v_3}{v_2} \\ \epsilon & 2\frac{v_3}{v_2} & 1 \end{pmatrix}. \quad (25)$$

For the case of interest here, the Majoron is related to the basis in the following way,

$$J \approx I_1 - 2\frac{v_3^2}{v_2 v_1} I_2 - \epsilon I_3, \quad (26)$$

which allows us to conclude that it is dominantly singlet. In this case, its coupling to the standard neutrinos may be approximated by $\sim i g_{\alpha\beta} \bar{\nu}_\alpha \gamma_5 \nu_\beta J$, where $g_{\alpha\beta} = Y_{L_{\alpha\beta}} \epsilon$. For $\epsilon = 10^{-5}$ and for the values of $Y_{L_{\alpha\beta}}$ given in Eq. (14), we have $g_{e\mu} \sim 10^{-8}$. This value for $g_{e\mu}$ is in agreement with supernova bounds, $\beta\beta 0\nu$ decay, and neutrino decays, as discussed in [11]. Another source of constraint on the Majoron arises from star cooling, which, for our hierarchy of VEVs, puts the following constraint over them [see Eq. (36) of the first paper in Ref. [5]]:

$$\frac{2v_3^2}{v_1 v_2} \leq 10^{-6}, \quad (27)$$

which is completely obeyed by our choice in Eq. (12).

Regarding long-range force, the Majoron is so weakly coupled to matter that it may well have escaped detection [12].

The mass of the pseudoscalar A takes the following expression,

$$m_A^2 \approx \frac{\kappa v_1 v_2^2}{2v_3}, \quad (28)$$

which allows us to conclude that it is a heavy particle even for the set of VEVs considered here.

Regarding the charged scalars, in considering the basis (Δ^+, ϕ^+) , we have the following mass matrix for these scalars:

$$M_{H^\pm}^2 = \begin{pmatrix} \kappa v_1 v_3 - \frac{1}{2}\lambda_5 v_3^2 & \frac{1}{2\sqrt{2}} v_2 (\lambda_5 v_3 - 2\kappa v_1) \\ \frac{1}{2\sqrt{2}} v_2 (\lambda_5 v_3 - 2\kappa v_1) & \frac{1}{4v_3} v_2^2 (2\kappa v_1 - \lambda_5 v_3) \end{pmatrix}. \quad (29)$$

We can easily diagonalize this matrix and find the physical fields

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = U_\pm \begin{pmatrix} \phi^\pm \\ \Delta^\pm \end{pmatrix}, \quad (30)$$

$$U_\pm \approx \begin{pmatrix} 1 & \frac{\sqrt{2}v_3}{v_2} \\ -\frac{\sqrt{2}v_3}{v_2} & 1 \end{pmatrix}. \quad (31)$$

We see that there is not any relevant mixing between the charged fields. $G^\pm$ is the Goldstone while $H^\pm$ is

the simply charged scalar whose mass expression is given by

$$m_{H^\pm}^2 = \frac{1}{4v_3} (2\kappa v_1 - \lambda_5 v_3)(v_2^2 + 2v_3^2) \approx \frac{\kappa v_1 v_2^2}{2v_3}. \quad (32)$$

Observe that it must be heavy for the choice of the VEVs used here.

The doubly charged scalar acquires the following mass expression,

$$m_{\Delta^\pm}^2 = \frac{1}{2v_3} (\kappa v_1 v_2^2 - 2\lambda_4 v_3^2 - \lambda_5 v_2^2 v_3) \approx \frac{\kappa v_1 v_2^2}{2v_3}, \quad (33)$$

which must be heavy, too.

Thus, we see have that, although the VEVs v_1 and v_3 are much smaller than v_2 , we have that the scalars that belong to the triplet Δ are heavier than the standardlike Higgs and their masses are practically determined by the parameter κ . This is a consequence of the hierarchy of the VEVs. It is curious that the same hierarchy among the VEVs does the opposite with regard to the scalars belonging to the singlet σ . The scenario predicts a light scalar H_1 . The heavy scalars may be probed at the LHC, while the massless J and light H_1 will contribute to the invisible decay channels of the Higgs and Z .

B. Some constraints

The coupling constants κ , β_2 , and $\lambda_{3,5}$ will play an important role in the RGE of the quartic coupling of the standardlike Higgs λ_1 . Thus, information on these parameters in the form of constraints is mandatory for us to determine whether or not the vacuum of the 3-2-1 model in the regime of low-energy scale is stable. But before we address this issue, let us investigate the contributions of the light scalars to the invisible decay of the standard neutral gauge boson Z .

Regarding the invisible decay of Z , the Lagrangian of interest is given by

$$\mathcal{L}_{R_3 I_3 Z} \supset -\frac{g}{c_W} Z^\mu [R_3 \partial_\mu I_3 - I_3 \partial_\mu R_3]. \quad (34)$$

Because R_3 mixes with R_1 to compose H_1 and I_3 mixes with I_1 to compose J , we have that this Lagrangian generates an interaction among Z , H_1 , and J modulated by the following vertex:

$$V_{ZH_1(P_1)J(P_2)} \approx \frac{g\epsilon^2}{c_W} (P_1 - P_2)_\mu, \quad (35)$$

where g is the $SU(2)$ coupling constant and $c_W = \cos(\theta_W)$, with θ_W being the Weinberg angle. ϵ is given in Eq. (21). The current data give $\Gamma(Z)_{\text{inv}} = 500.1 \pm 1.9$ MeV [13]. Because $M_{H_1} \ll M_Z$, the vertex above provides the following expression for the decay width $Z \rightarrow H_1 J$,

$$\Gamma(Z \rightarrow JH_1) = \frac{M_Z \epsilon^4 G_F}{16\sqrt{2}\pi} \left(M_Z - \frac{M_{H_1}^2}{M_Z} \right)^2 \approx \frac{M_Z^3 \epsilon^4 G_F}{16\sqrt{2}\pi}. \quad (36)$$

The expression for the decay width of Z in two neutrinos is given by

$$\Gamma(Z \rightarrow \bar{\nu}\nu) = \frac{G_F M_Z^3}{12\sqrt{2}\pi}. \quad (37)$$

Upon substituting in the current values of the standard parameters entered in the expression above, i.e., $M_Z = 91.18$ GeV, $G_F = 1.1663787 \times 10^{-5}$ GeV⁻², we obtain $\Gamma(Z \rightarrow \bar{\nu}\nu) \approx 166$ MeV. In view of this, the window for new physics is established by $\Gamma(Z)_{\text{inv}} - 3 \times \Gamma(Z \rightarrow \bar{\nu}\nu) \approx 2.1$ MeV. In other words, all new contributions to the invisible decay of Z must lie within 2.1 MeV.

Observe that Eqs. (36) and (37) provide

$$\frac{\Gamma_{Z \rightarrow JH_1}}{\Gamma_{Z \rightarrow \bar{\nu}\nu}} \approx 0.75\epsilon^4 \rightarrow \Gamma_{Z \rightarrow JH_1} \approx 124.5\epsilon^4 \text{ MeV}. \quad (38)$$

According to this, we have that $\Gamma_{Z \rightarrow JH_1}$ must be smaller than 2.1 MeV. Once $\frac{v_3}{v_1} = \epsilon$, at the end of the day we get

$$\epsilon < 0.36 \rightarrow v_1 > 2.77v_3. \quad (39)$$

This result confirms the hierarchy among the VEVs that we are considering here.

To check that our scenario obeys the constraint put by the invisible decay of Z as discussed above, see that for $v_1 = 10^5$ eV and $v_3 = 1$ eV we get $\Gamma(Z \rightarrow JH_1) = 124.5 \times 10^{-20}$ MeV, which is much smaller than 2.1 MeV. The other possible contribution to $\Gamma(Z)_{\text{inv}}$ is $\Gamma(Z^0 \rightarrow JJJ)$. However, we must have that $\Gamma(Z^0 \rightarrow JH_1) > \Gamma(Z \rightarrow JJJ)$ because the later decay is obtained from the first by means of the decay $H_1 \rightarrow JJ$. Thus, we conclude here that the invisible Z decay is not a threat to our model.

Now let us extract constraints over the parameters of the potential by means of the invisible Higgs decay channels and the LFV process $\mu \rightarrow e\gamma$.

Let us consider the contributions that our case gives to the invisible decay of the standardlike Higgs H_2 . We consider the following contributions: $\Gamma(H_2 \rightarrow H_1 H_1)$ and $\Gamma(H_2 \rightarrow JJ)$. Their decay widths take the expression [14]

$$\Gamma(H_2 \rightarrow H_1 H_1) \approx \frac{\beta_2^2 v_2}{128\sqrt{2}\pi} \quad \text{and} \quad \Gamma(H_2 \rightarrow JJ) \approx \frac{(\lambda_3 + \lambda_5)^2 v_2}{128\sqrt{2}\pi}. \quad (40)$$

The prediction for the total decay width of the standard Higgs is around 4 MeV, with $\sim 20\%$ being invisible decay rates [$\text{BR}(H_2 \rightarrow \text{inv}) = 0.26 \pm 0.17$]. This allows us to

conclude that β_2 , λ_3 , and λ_5 are constrained to lie at around 10^{-2} or smaller.

Thus, we conclude here that the 3-2-1 model lies in the regime of the low-energy scale. Although it has a Majoron, which is a massless pseudoscalar, and a light CP -even scalar, it is a safe model for the invisible decay of the standard neutral gauge boson Z . As a nice fact, we have that our particular case gives reasonable contribution to the invisible decay of the standard Higgs through the channels $\Gamma(H_2 \rightarrow H_1 H_1)$ and $\Gamma(H_2 \rightarrow JJ)$. In other words, our case may be constrained by future improvements of the data relating to Higgs physics.

Regarding LFV processes, the muon decay channel $\mu \rightarrow e\gamma$ provides the strongest constraint on the parameters of the potential. This is so because, in our case, $\text{BR}(\mu \rightarrow 3e) \sim \frac{\text{BR}(\mu \rightarrow e\gamma)}{160}$; see Eq. (70) of Ref. [15].

In one-loop order, we have the following expression for the branching ratio of this process [16]:

$$\begin{aligned} \text{BR}(\mu \rightarrow \gamma e) \\ \approx \frac{27\alpha |(Y_L)_{11}(Y_L)_{12} + (Y_L)_{13}(Y_L)_{32} + (Y_L)_{12}(Y_L)_{22}|^2}{64\pi G_F^2 M_{\Delta^{++}}^4}, \end{aligned} \quad (41)$$

where α is the fine structure constant and $G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}$.

Upon substituting in the expression of the mass of the doubly charged scalar given in Eq. (33), we have for the fixed values of Y_L 's given in Eq. (14) and of the VEVs given in Eq. (12) that the upper bound $\text{BR}(\mu \rightarrow \gamma e) < 5.7 \times 10^{-13}$ [17] translates to the following lower bound over κ :

$$\frac{7 \times 10^{-19}}{\kappa^2} < 5.7 \times 10^{-13} \rightarrow \kappa > 1.1 \times 10^{-3}. \quad (42)$$

In addition to this lower bound over κ , there is a natural upper bound over κ , too, that arises from the constraint that all dimensionless parameters in any scalar potential are required to be less than $\sqrt{4\pi}$ to fulfill the perturbativity condition. In summary, we have

$$1.1 \times 10^{-1} < \kappa < \sqrt{4\pi}. \quad (43)$$

With this set of constraints in hand, we are ready to analyze the RGE of the quartic coupling of the standardlike Higgs λ_1 .

IV. VACUUM STABILITY

Now that we have developed the scalar sector by finding the spectrum of scalars for a particular set of values of the VEVs and obtained some constraints over the

parameters of the potential due to Higgs invisible decay and lepton flavor violation, it is time to investigate the stability of the vacuum by finding the bound from below conditions and calculating the running of the self-coupling of the Higgs.

A. Bound from below conditions

To assure that the scalar potential of the 3-2-1 model is bounded from below at large field strength, where the potential is generically dominated by the quartic terms, we need to find the set of conditions that guarantee that the parameters of the quartic couplings of the potential are positive when the fields go to infinity. We find the whole set of conditions and pave the way for similar models. We follow the techniques employed in [18].

First, we separate the quartic couplings of the potential,

$$\begin{aligned} V^4 = & \lambda_1(\Phi^\dagger \Phi)^2 + \lambda_2[\text{tr}(\Delta^\dagger \Delta)]^2 + \lambda_3 \Phi^\dagger \Phi \text{tr}(\Delta^\dagger \Delta) \\ & + \lambda_4 \text{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta) + \lambda_5(\Phi^\dagger \Delta^\dagger \Delta \Phi) \\ & + \beta_1(\sigma^* \sigma)^2 + \beta_2 \Phi^\dagger \Phi \sigma^* \sigma + \beta_3 \text{tr}(\Delta^\dagger \Delta) \sigma^* \sigma \\ & - \kappa(\Phi^T \Delta \Phi \sigma + \text{H.c.}), \end{aligned} \quad (44)$$

and we then build the following parametrization:

$$\begin{aligned} r^2 &= \Phi^\dagger \Phi + \text{tr}(\Delta^\dagger \Delta) + \sigma^* \sigma, \\ \Phi^\dagger \Phi &= r^2 \cos^2 \gamma \sin^2 \theta, \\ \text{tr}(\Delta^\dagger \Delta) &= r^2 \sin^2 \gamma \sin^2 \theta, \\ \sigma^* \sigma &= r^2 \cos^2 \theta, \end{aligned} \quad (45)$$

where $0 \leq r \leq \infty$, $0 \leq \gamma \leq \frac{\pi}{2}$, and $0 \leq \theta \leq \frac{\pi}{2}$.

We also need to develop the following parameters,

$$\begin{aligned} \zeta &= \frac{\text{tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta)}{[\text{tr}(\Delta^\dagger \Delta)]^2}, \\ \xi &= \frac{\Phi^\dagger \Delta \Delta^\dagger \Phi}{\Phi^\dagger \Phi \text{tr}(\Delta^\dagger \Delta)}, \\ \alpha &= \frac{\text{Re}(\Phi^T \Delta \Phi \sigma)}{\text{tr}(\Delta^\dagger \Delta) \sigma^* \sigma + \Phi^\dagger \Phi \sigma^* \sigma + \text{tr}(\Delta^\dagger \Delta) \Phi^\dagger \Phi}, \end{aligned} \quad (46)$$

where $\frac{1}{2} \leq \zeta \leq 1$, $0 \leq \xi \leq 1$, and $-1 \leq \alpha \leq 1$. Two of them are already known in the literature. The third one is a new parameter. We can see in detail in the Appendix how we can limit this parameter.

Let us also define new variables x and y that must vary between 0 and 1 in the following way:

$$\begin{aligned} y &= \sin^2 \theta, & \lambda_1 &> 0, \\ x &= \sin^2 \gamma. & \lambda_2 + \lambda_4 &> 0, \end{aligned} \quad (47)$$

Replacing Eq. (47) with Eq. (44), we get

$$\begin{aligned} \frac{V^4}{r^4} &= y^2[\lambda_1(1-x)^2 + \lambda_2 x^2 + \lambda_3(1-x)x \\ &\quad + \zeta\lambda_4 x^2 + \xi\lambda_5(1-x)x - 2\kappa\alpha x(1-x)] \\ &\quad + (1-y)^2\beta_1 + (1-y)y[\beta_2(1-x) + \beta_3 x - 2\kappa\alpha]. \end{aligned} \quad (48)$$

We manage things such that we can express these quartic terms in the following way:

$$\begin{aligned} \frac{V^4}{r^4} &= A_x y^2 + B_x(1-y)^2 + C_x(1-y)y, \quad \text{where} \\ A_x &= \lambda_1(1-x)^2 + (\lambda_2 + \zeta\lambda_4)x^2 + (\lambda_3 + \xi\lambda_5 - 2\kappa\alpha)(1-x)x, \\ B_x &= \beta_1, \\ C_x &= \beta_2(1-x) + \beta_3 x - 2\kappa\alpha. \end{aligned} \quad (49)$$

We can fix $y = 0$ or $y = 1$ to obtain the cases in which the quartic couplings of the potential is positive. When we do this, we obtain the following conditions:

$$A_x > 0, \quad (50)$$

$$B_x > 0, \quad (51)$$

$$C_x + 2\sqrt{A_x B_x} > 0. \quad (52)$$

For $A_x > 0$, we need to use the same argument as before. Fixing $x = 0$ and $x = 1$, we have similar conditions for the inequalities,

$$\begin{aligned} \lambda_1 &> 0, \\ \lambda_2 + \zeta\lambda_4 &> 0, \\ \lambda_3 + \xi\lambda_5 - 2\kappa\alpha + 2\sqrt{\lambda_1(\lambda_2 + \zeta\lambda_4)} &> 0. \end{aligned} \quad (53)$$

These new conditions depend of the parameters in Eq. (49). They vary in different ranges, but we only need to study the boundary values of these intervals. In this case, the new conditions are

$$\begin{aligned} \lambda_3 + 2\kappa + 2\sqrt{\lambda_1\left(\lambda_2 + \frac{1}{2}\lambda_4\right)} &> 0, \\ \lambda_3 + 2\kappa + 2\sqrt{\lambda_1(\lambda_2 + \lambda_4)} &> 0, \\ \lambda_3 + \lambda_5 + 2\kappa + 2\sqrt{\lambda_1\left(\lambda_2 + \frac{1}{2}\lambda_4\right)} &> 0, \\ \lambda_3 + \lambda_5 + 2\kappa + 2\sqrt{\lambda_1(\lambda_2 + \lambda_4)} &> 0, \\ \lambda_3 - 2\kappa + 2\sqrt{\lambda_1\left(\lambda_2 + \frac{1}{2}\lambda_4\right)} &> 0, \\ \lambda_3 - 2\kappa + 2\sqrt{\lambda_1(\lambda_2 + \lambda_4)} &> 0, \\ \lambda_3 + \lambda_5 - 2\kappa + 2\sqrt{\lambda_1\left(\lambda_2 + \frac{1}{2}\lambda_4\right)} &> 0, \\ \lambda_3 + \lambda_5 - 2\kappa + 2\sqrt{\lambda_1(\lambda_2 + \lambda_4)} &> 0. \end{aligned} \quad (54)$$

Using the same argument as for the condition in Eq. (51), it is easy to see that

$$\beta_1 > 0. \quad (55)$$

Using the condition in Eq. (52) and the same fact that C_x can have $x = 0$ or $x = 1$, we obtain

$$\begin{aligned} \beta_2 - 2\kappa\alpha + 2\sqrt{\beta_1\lambda_1} &> 0, \\ \beta_3 - 2\kappa\alpha + 2\sqrt{\beta_1(\lambda_2 + \zeta\lambda_4)} &> 0. \end{aligned} \quad (56)$$

The first inequality has two different solutions, while the second has four. At the end of the day, we have

$$\begin{aligned} \beta_2 + 2\kappa + 2\sqrt{\beta_1\lambda_1} &> 0, \\ \beta_2 - 2\kappa + 2\sqrt{\beta_1\lambda_1} &> 0, \\ \beta_3 + 2\kappa + 2\sqrt{\beta_1\left(\lambda_2 + \frac{1}{2}\lambda_4\right)} &> 0, \\ \beta_3 + 2\kappa + 2\sqrt{\beta_1(\lambda_2 + \lambda_4)} &> 0, \\ \beta_3 - 2\kappa + 2\sqrt{\beta_1\left(\lambda_2 + \frac{1}{2}\lambda_4\right)} &> 0, \\ \beta_3 - 2\kappa + 2\sqrt{\beta_1(\lambda_2 + \lambda_4)} &> 0. \end{aligned} \quad (57)$$

So, that is the set of conditions that guarantees that the potential in Eq. (3) is bounded from below. In what follows,

FIG. 1. Running of $\lambda_\phi \approx \lambda_1$ at one-loop level as a function of the energy scale μ for $\lambda_5 = \lambda_3 = \beta_2 = 0.001$ with $y_t = 0.9965$, $g_Y = 0.4627$, and $g = 0.6535$. The dashed lines represent the expectation of the standard model, and the red lines represent the expectation for our model for two values of κ .

we obtain the running of the self-coupling related to the standardlike Higgs.

B. RGE of the self-coupling of the standardlike Higgs

The standard model predicts that the self-coupling of the Higgs becomes negative at an energy scale of around $\Lambda = 10^{11}$ GeV. This means that the standard model cannot assure the stability of the vacuum up to the Planck scale. This must be remedied by means of new physics in the form of new particles with appropriate interactions. This issue has been extensively investigated in the literature [19]. In our scenario, we show that the right behavior of the self-coupling of the Higgs that guarantees absolute stability for the electroweak vacuum depends strongly on the coupling κ . We do our analysis by implementing the model in SARAH4.13.0 [20] and evaluating the β function for λ_1 at one-loop level.

The main contributions for the beta function of λ_1 involve the following terms:

$$\begin{aligned} \beta_{\lambda_1} = & \frac{27}{100} g_Y^4 + \frac{9}{4} g^4 + \frac{9}{20} g_Y^2 (g^2 - 2\lambda_1) - \frac{9}{5} g^2 \lambda_1 + 12\lambda_1^2 \\ & + 12\lambda_1 y_t^\dagger y_t - 6y_t^\dagger y_t y_t^\dagger y_t \\ & + \beta_2^2 + 3\lambda_3^2 + 3\lambda_3 \lambda_5 + \frac{5}{4} \lambda_5^2 + 2\kappa^2, \end{aligned} \quad (58)$$

where g and g_Y are the gauge couplings of the standard gauge group $SU(2)$ and $U(1)_Y$, while y_t is the Yukawa coupling of the quark top.

Observe that the couplings β_2 , $\lambda_{3,5}$, and κ give positive contributions to the running of λ_1 . However, as showed above, the invisible Higgs decay requires β_2 , $\lambda_{3,5}$ less than 10^{-2} , which turns insignificant their contributions to the RGE. We put our hope on the contribution of the parameter κ . In Fig. 1, we show the plot of the running of λ_1 with an energy scale for two possible values of κ . We see that the running of λ_1 may get positive up to Planck scale for

$\kappa > 0.3$. Thus, the model may have the vacuum remain stable thanks to the contribution of the parameter κ .

For the sake of completeness, in Figs. 2 and 3 we present the running at one-loop level of the other self-quartic

FIG. 2. Running of $\lambda_\Delta \approx \lambda_2 + \lambda_4$ at one-loop level as a function of the energy scale μ for $\lambda_5 = \lambda_3 = \beta_2 = 0.001$ with $y_t = 0.9965$, $g_Y = 0.4627$, and $g = 0.6535$. The dashed lines represent the expectation for our model for three values of κ . The coupling β_3 is important at the same level as κ as for this running.

FIG. 3. Running of $\lambda_\sigma \approx \beta_1$ at one-loop level as a function of the energy scale μ for $\lambda_5 = \lambda_3 = \beta_2 = 0.001$ with $y_t = 0.9965$, $g_Y = 0.4627$, and $g = 0.6535$. The dashed lines represent the expectation for our model for three values of β_3 .

couplings $\lambda_{2,4}$ and β_1 . As we can see in those plots, they do not develop negative values and are weakly influenced by κ .

V. CONCLUDING REMARKS

In this work, we studied stability of the vacuum in the 3-2-1 model with right-handed neutrinos. We restricted our investigation to a specific case where lepton number is spontaneously broken at low-energy scale. The scenario is well motivated since it yields light sterile neutrinos and may explain MiniBooNE results by means of neutrino oscillation. In such a scenario, we obtained the whole set of conditions that guarantee that the model is bounded from below and studied the RGE of the self-coupling of the standardlike Higgs. For our main result, we have that the quartic coupling $\kappa\Phi^T\Delta\Phi\sigma$ plays a central role in the process and that the stability of the vacuum requires $\kappa > 0.3$.

As an interesting consequence, we remark that the model has one Majoron (J) and one light Higgs (H_1) composing the spectrum of scalar of the model. Their contributions to the invisible decay rate of the standardlike Higgs, $H_2 \rightarrow JJ$ and $H_2 \rightarrow H_1 H_1$, were considered and the results are the bounds $\beta_2, \lambda_3, \lambda_5 \leq 10^{-2}$ over the couplings of the potential.

Regarding the neutrino sector, the scenario recovers the $3 + 1$ sterile neutrino model which explain the MiniBooNE experiment by means of neutrino oscillation. However, we know that light sterile neutrinos are strongly disfavored by current cosmological data involving big bang nucleosynthesis, cosmic microwave background anisotropies, and large scale structure [10]. This is so because, in the face of the large mixing required by MiniBooNE, neutrino oscillation may conduct sterile neutrino to thermal equilibrium with the active neutrino even before neutrinos decouple from the primordial plasma. A possible solution for this tension requires the suppression of the production of these neutrinos in the early Universe. This keeps them from thermalizing with the active ones at high temperature. This may be achieved by means of secret interactions [21], which is nothing more than the interaction of the sterile neutrino with a pseudoscalar, I ,

$$\sim g_s \bar{\nu}_S^C \gamma_5 \nu_S I. \quad (59)$$

The solution to the tension requires I to be lighter than the lightest sterile neutrino and g_s to take values in the range 10^{-6} – 10^{-5} . Observe that our scenario recovers this solution. For this, recognize that g is Y_{11}^R , whose value in the matrix in Eq. (15) is $1, 4 \times 10^{-5}$, and I is the Majoron J . To generate a small mass for J , we just need to consider a term like $M\sigma\sigma\sigma$ in the potential. This term will generate a mass term for J proportional to M . Assuming that $M < m_{N_4}$, we have a secret sector that reconciliates an eV sterile neutrino with cosmology as was done in [22].

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APPENDIX

Here we will give a hint of the proof of the limitation of the parameter α . The definition of this parameter is

$$\alpha = \frac{\text{Re}[\Phi^T \Delta \Phi \sigma]}{\Phi^\dagger \Phi \sigma^\dagger \sigma + \text{tr}[\Delta^\dagger \Delta] \Phi^\dagger \Phi + \text{tr}[\Delta^\dagger \Delta] \sigma^\dagger \sigma}. \quad (A1)$$

We can expand this parameter in terms of the components of the fields. We have that

$$\begin{aligned} \text{Numerator} &= \text{Re}[\phi^0 \Delta^0 \phi^0 \sigma + \sqrt{2} \phi^0 \Delta^+ \phi^- \sigma \\ &\quad + \phi^- \Delta^{++} \phi^- \sigma], \\ \text{Denominator} &= (\phi^{0\dagger} \phi^0 + \phi^+ \phi^- + \sigma^\dagger \sigma) \\ &\quad \times (\Delta^{0\dagger} \Delta^0 + \Delta^+ \Delta^- + \Delta^{++} \Delta^{--}) \\ &\quad + \sigma^\dagger \sigma (\phi^{0\dagger} \phi^0 + \phi^+ \phi^-). \end{aligned} \quad (A2)$$

Then we can study term by term to see what is the behavior of this parameter, e.g., to see whether or not it is limited. As an example, we choose the first term of the numerator and expand the fields in the real and imaginary parts. Using the following expansion,

$$\begin{aligned} \phi^0 &= R_2 + iI_2, \\ \Delta^0 &= R_3 + iI_3, \\ \sigma &= R_1 + iI_1, \end{aligned} \quad (A3)$$

we obtain the denominator terms (only the real part)

$$\begin{aligned} &R_2^2 R_3 R_1 - R_2 R_3 I_2 I_1 - I_2 I_3 R_2 R_1 + I_1 I_2^2 I_3 - I_2^2 R_1 R_3 \\ &- I_1 I_2 R_2 R_3 - R_1 R_2 R_2 I_3 - R_2^2 I_1 I_3. \end{aligned}$$

The idea here is to look closely at each real function and study their limitation range. For the first term, $R_2^2 R_3 R_1$, we have the following relation (for $R_2 \neq 0$):

$$\begin{aligned} \frac{R_2^2 R_3 R_1}{R_2^2 R_3^2 + R_1^2 R_2^2 + R_1^2 R_3^2 + (\dots)} &\rightarrow \frac{R_3 R_1}{R_3^2 + R_1^2 + \frac{R_1^2 R_3^2}{R_2^2} + (\dots)} \\ &< \frac{R_3 R_1}{R_3^2 + R_1^2}. \end{aligned} \quad (A4)$$

We can see easily that this last term is limited in the range $[-1, 1]$ with polar coordinates. We use similar arguments for the next terms and find that α lies in the range $[-1, 1]$.

- [1] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, *Phys. Rep.* **516**, 1 (2012).
- [2] T. P. Cheng and L.-F. Li, *Phys. Rev. Lett.* **45**, 1908 (1980); G. B. Gelmini and M. Roncadelli, *Phys. Lett. B* **99**, 411 (1981).
- [3] N. V. Krasnikov, *Phys. Lett. B* **291**, 89 (1992); A. S. Joshipura and J. W. F. Valle, *Nucl. Phys.* **B397**, 105 (1993); D. O'Connell, M. J. Ramsey-Musolf, and M. B. Wise, *Phys. Rev. D* **75**, 037701 (2007).
- [4] J. Schechter and J. W. F. Valle, *Phys. Rev. D* **25**, 774 (1982).
- [5] M. A. Diaz, M. A. Garcia-Jareno, D. A. Restrepo, and J. W. F. Valle, *Nucl. Phys.* **B527**, 44 (1998); C. Bonilla, J. C. Romo, and J. W. F. Valle, *New J. Phys.* **18**, 033033 (2016); S. Blunier, G. Cottin, M. Aurelio Daz, and B. Koch, *Phys. Rev. D* **95**, 075038 (2017).
- [6] For an extension of this scenario involving the two-Higgs-doublet model, see D. Cogollo, R. D. Matheus, T. B. de Melo, and F. S. Queiroz, *Phys. Lett. B* **797**, 134813 (2019).
- [7] A. Aguilar-Arevalo *et al.*, *Phys. Rev. Lett.* **121**, 221801 (2018).
- [8] M. J. G. Veltman, *Nucl. Phys.* **B123**, 89 (1977).
- [9] A. Aguilar-Arevalo *et al.*, *Phys. Rev. D* **64**, 112007 (2001).
- [10] G. Steigman, *Adv. High Energy Phys.* **2012**, 268321 (2012); N. Aghanim *et al.* (Planck Collaboration), *arXiv:1807.06209*; J. Hamann, S. Hannestad, G. G. Raffelt, and Y. Y. Wong, *J. Cosmol. Astropart. Phys.* **09** (2011) 034.
- [11] A. P. Lessa and O. L. G. Peres, *Phys. Rev. D* **75**, 094001 (2007).
- [12] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, *Phys. Lett.* **98B**, 265 (1981); F. Ferrer and M. Nowakowski, *Phys. Rev. D* **59**, 075009 (1999).
- [13] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018) and 2019 update.
- [14] C. Bonilla, J. C. Romo, and J. W. F. Valle, *New J. Phys.* **18**, 033033 (2016).
- [15] Y. Kuno and Y. Okada, *Rev. Mod. Phys.* **73**, 151 (2001).
- [16] A. G. Akeroyd, M. Aoki, and H. Sugiyama, *Phys. Rev. D* **79**, 113010 (2009); For a general formula for $f_1 \rightarrow f_2 \gamma$, see L. Lavoura, *Eur. Phys. J. C* **29**, 191 (2003).
- [17] MEG Collaboration, *Phys. Rev. Lett.* **110**, 201801 (2013).
- [18] A. Arhrib, R. Benbrik, M. El Kacimi, L. Rahili, and S. Semlali, *Eur. Phys. J. C* **80**, 13 (2020); D. A. Camargo, A. G. Dias, T. B. de Melo, and F. S. Queiroz, *J. High Energy Phys.* **04** (2019) 129; G. Chauhan, *J. High Energy Phys.* **12** (2019) 137.
- [19] I. Gogoladze, N. Okada, and Q. Shafi, *Phys. Rev. D* **78**, 085005 (2008); B. He, N. Okada, and Q. Shafi, *Phys. Lett. B* **109**, 197202 (2012); J. Chakraborty, M. Das, and S. Mohanty, *Mod. Phys. Lett. A* **28**, 1350032 (2013); A. Kobakhidze and A. Spencer-Smith, *J. High Energy Phys.* **08** (2013) 036; J. N. Ng and A. de la Puente, *Eur. Phys. J. C* **76**, 122 (2016); I. Garg, S. Goswami, K. N. Vishnudath, and N. Khan, *Phys. Rev. D* **96**, 055020 (2017); S. Ipek, A. D. Plascencia, and J. Turner, *J. High Energy Phys.* **12** (2018) 111; C. Bonilla, R. M. Fonseca, and J. W. F. Valle, *Phys. Rev. D* **92**, 075028 (2015).
- [20] F. Staub, *Comput. Phys. Commun.* **185**, 1773 (2014).
- [21] S. Hannestad, R. S. Hansen, and T. Tram, *Phys. Rev. Lett.* **112**, 031802 (2014); M. Archidiacono, S. Gariazzo, C. Giunti, S. Hannestad, R. Hansen, M. Laveder, and T. Tram, *J. Cosmol. Astropart. Phys.* **08** (2016) 067; B. Dasgupta and J. Kopp, *Phys. Rev. Lett.* **112**, 031803 (2014); X. Chu, B. Dasgupta, M. Dentler, J. Kopp, and N. Saviano, *J. Cosmol. Astropart. Phys.* **11** (2018) 049.
- [22] C. A. de S. Pires, *Phys. Lett. B* **800**, 135135 (2020); D. Croon, N. Fernandez, D. McKeen, and G. White, *J. High Energy Phys.* **06** (2019) 098.