Fragmentation function of gluon into spin-singlet P -wave quarkonium

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(Received 10 June 2018; accepted 18 June 2020; published 22 July 2020)

Following the operator definition of the fragmentation function developed by Collins and Soper, we compute the gluon-to- h_c fragmentation function at the lowest order in the velocity expansion in nonrelativistic QCD factorization approach. Utilizing some modern technique developed in the area of multiloop calculation, we are able to analytically deduce the infrared-finite color-singlet short-distance coefficient associated with the fragmentation function. The fragmentation probability for gluon into h_c is estimated to be of the order of 10^{-6} .

DOI: [10.1103/PhysRevD.102.014038](https://doi.org/10.1103/PhysRevD.102.014038)

Like parton distribution functions (PDFs), fragmentation functions (FFs) constitute one of the fundamental probes to uncover the nonperturbative partonic structure related to a hadron. According to the QCD factorization theorem [1], in a high-energy collision experiment with two colliding beams composed of hadrons of type A and B , the inclusive production rate of the identified hadron H at very large transverse momentum is dominated by the fragmentation mechanism:

$$d\sigma[A + B \rightarrow H(P_\perp) + X] = \sum_i d\hat{\sigma}[A + B \rightarrow i(P_\perp/z) + X] \otimes D_{i \rightarrow H}(z, \mu) + \mathcal{O}(1/P_\perp^2), \quad (1)$$

where $d\hat{\sigma}$ denotes the partonic cross section and the fragmentation function $D_{i \rightarrow H}(z)$ characterizes the probability for the parton i to materialize into a complicated multihadron state that contains the hadron H carrying the fractional light-cone momentum z with respect to the parent parton. The sum in (1) is extended over all parton species ($i = q, \bar{q}, g$). Similar to the PDFs, the FFs are nonperturbative but universal objects, whose scale dependence is

governed by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation. Specifically speaking, the scale dependence of the gluon fragmentation function is controlled by

$$\frac{d}{d \ln \mu^2} D_{g \rightarrow H}(z, \mu) = \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ig}(\xi, \alpha_s(\mu)) D_{i \rightarrow H}\left(\frac{z}{\xi}, \mu\right), \quad (2)$$

where $P_{ig}(\xi)$ designate the splitting kernel and μ is also frequently referred to as the QCD factorization scale, since it also enters in the partonic cross section in (1). Once this FF can be deduced at some initial scale μ_0 by some means, one can then determine its form at any other scale μ by solving the evolution equation (2).

In contrast to the fragmentation functions for light hadrons, the functions for a parton to fragmentate into a heavy quarkonium, a nonrelativistic bound state composed of a heavy quark and heavy antiquark, need not be viewed as genuinely nonperturbative objects. In fact, owing to the weak QCD coupling at the length scale $\sim 1/m$ (m represents the heavy quark mass) as well as the nonrelativistic nature of quarkonium, the nonrelativistic QCD (NRQCD) factorization [2] can be invoked to refactorize the quarkonium FFs as the sum of products of short-distance coefficients (SDCs) and long-distance yet universal NRQCD matrix elements [3,4]. To some extent, the profiles of quarkonium FFs are largely determined by perturbative QCD, which renders the NRQCD approach a particularly predictive theoretical framework. Recently, armed with various fragmentation functions computed in the NRQCD approach, a phenomenological analysis based on (1) is

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conducted to confront with copious large- $P_\perp J/\psi, \chi_{cJ}$, and ψ' data accumulated at LHC experiments [5,6].

Since the original computation of the quark and gluon fragmentation into S -wave quarkonium using the NRQCD approach by Braaten and collaborators [3,4], numerous fragmentation functions for quarks and gluons into various quarkonium states have been calculated in the NRQCD approach during the past two decades [7–24]. For a recent compilation of the SDCs associated with various FFs, we refer the interested reader to Ref. [25].

The goal of this work is to compute the FF of gluon into the spin-singlet P -wave quarkonium, exemplified by the $h_{c,b}$ states. To date, these P -wave quarkonium states have been observed only in the e^+e^- collision experiments via hadronic transitions from higher vector quarkonium states [26,27]. It is conceivable that they will be established at LHC experiments in the future, owing to the enormous partonic luminosity there. For this purpose, it is desirable if one can make accurate predictions for the fragmentation functions for the $h_{c,b}$ states.

Because of the odd C parity of h_c , the gluon fragmentation remnants must involve two additional gluons in the color-singlet channel, considerably more complicated than gluon-to- χ_{cJ} FF considered in Refs. [9,11], which involves only one additional gluon in the fragmentation products. Although the Feynman diagrams are topologically identical to those for gluon-to- J/ψ FF [13], the actual calculation is much more challenging, due to the occurrence of the IR divergence in the former case, whereas the latter is free from IR singularity, at least up to the relative order v^2 [17].

The gluon-to- h_c fragmentation function was originally calculated by Hao, Zuo, and Qiao in 2009 [28]. Unfortunately, the authors of Ref. [28] seem not to employ a gauge-invariant regulator to regularize the encountered IR singularity in the course of their calculation. Also, their final results are expressed in terms of a twofold integral with a rather complicated integrand.

Recently, with the aid of some powerful technique concerning multibody phase-space integration [29,30], the authors of Ref. [24] were able to compute the gluon-to- J/ψ fragmentation function in a closed form, which was unimaginable from the angle of the conventional method [13,17,19].

Stimulated by the advance made in Ref. [24], we feel that it is the time to revisit the gluon-to- h_c fragmentation function. We will start from the gauge-invariant operator definition for this FF [31] and employ dimensional regularization as the gauge-invariant IR regulator. Ultimately, we will also be able to achieve the analytical expression for this FF.

We choose to evaluate this gluon fragmentation function in a frame such that the h_c has vanishing transverse momentum. It is customary to adopt the light-cone

coordinates in calculating FF. Any four-vector $A^\mu = (A^0, A^1, A^2, A^3)$ can be recast in a light-cone format $A^\mu = (A^+, A^-, \mathbf{A}_\perp)$, with $A^\pm \equiv \frac{1}{\sqrt{2}}(A^0 \pm A^3)$ and $\mathbf{A}_\perp \equiv (A^1, A^2)$. The scalar product of two four-vectors A and B thus becomes $A \cdot B = A^+ B^- + A^- B^+ - \mathbf{A}_\perp \cdot \mathbf{B}_\perp$. Specifically speaking, the four-momentum of the h_c meson can be written as $P^\mu = (P^+, P^- \equiv M_{h_c}^2/(2P^+), \mathbf{0}_\perp)$, where $M_{h_c} \approx 2m_c$ signifies the mass of the h_c meson.

A gauge-invariant operator definition for the fragmentation functions was formulated by Collins and Soper in 1981 [31]. This definition was first utilized by Ma to compute the quarkonium FFs in NRQCD [8]. For the intended g -to- h_c fragmentation function, the operator definition is given by [31] (also see [17,19])

$$D_{g \rightarrow h_c}(z, \mu) = \frac{-g_{\mu\nu} z^{D-3}}{2\pi k^+ (N_c^2 - 1)(D-2)} \int_{-\infty}^{+\infty} dx^- e^{-ik^+ x^-} \times \langle 0 | G_c^{+\mu}(0) \Phi^\dagger(0, 0, \mathbf{0}_\perp)_{cb} \sum_X |h_c(P, \lambda) + X\rangle \langle h_c(P, \lambda) + X | \Phi(0, x^-, \mathbf{0}_\perp)_{ba} \times G_a^{+\nu}(0, x^-, \mathbf{0}_\perp) | 0 \rangle, \quad (3)$$

where z denotes the fraction of the $+$ momentum carried by h_c with respect to the gluon, $D = 4 - 2\epsilon$ signifies the space-time dimensions, and $N_c = 3$ is the number of colors. $G_{\mu\nu}$ is the matrix-valued gluon field-strength tensor in the adjoint representation of $SU(N_c)$, and $k^+ = P^+/z$ is the $+$ -component momentum of injected by the gluon field-strength operator. μ is the renormalization scale for this composite operator. The insertion of the intermediate states implies that, in the asymptotic future, one needs only project out those out states that contain a h_c meson carrying definite momentum P^μ and polarization λ , plus any unobserved hadrons, which are collectively denoted by the symbol X . Note that the summation on X also implicitly indicates that the three polarizations of h_c are summed over.

The gauge link (eikonal factor) $\Phi(0, x^-, \mathbf{0}_\perp)$ in (3) is a path-ordered exponential of the gluon field, whose role is to ensure the gauge invariance of the FF:

$$\Phi(0, x^-, \mathbf{0}_\perp)_{ba} = \text{P exp} \left[ig_s \int_{x^-}^{\infty} dy^- n \cdot A(0^+, y^-, \mathbf{0}_\perp) \right]_{ba}, \quad (4)$$

where P implies the path ordering, g_s is the QCD coupling constant, and A^μ designates the matrix-valued gluon field in the adjoint representation. $n^\mu = (0, 1, \mathbf{0}_\perp)$ is a reference null 4-vector. Actually, due to the C -odd property of the h_c state, to our concerned perturbative order, we do not need to consider the complication for gluons to attach to the eikonal line.

According to the NRQCD factorization, the gluon fragmentation function for h_c can be expressed as [28]

FIG. 1. Representative Feynman diagrams for the gluon fragmentation function $D_{g \rightarrow h_c}(z)$. The double line signifies the eikonal line, and the vertical dashed curve implies the Cutkosky cut. The cap represents the insertion of the operator $G_a^{+\mu}$, whose Feynman rule reads $+i(g^{\mu a} - \frac{Q^{\mu} n^a}{Q \cdot n})\delta_{ab}$ (left to the cut), where Q represents the momentum of the gluon flowing out of the cap.

$$D_{g \rightarrow h_c}(z) = \frac{d_1(z, \mu_\Lambda)}{m^5} \langle 0 | \mathcal{O}_1^{h_c}(^1P_1) | 0 \rangle + \frac{d_8(z)}{m^3} \langle 0 | \mathcal{O}_8^{h_c}(^1S_0)(\mu_\Lambda) | 0 \rangle + \dots, \quad (5)$$

where both color-singlet and color-octet channels contribute at lowest order in v . The corresponding h_c production operators in NRQCD are defined as¹ [2]

$$\mathcal{O}_1^{h_c}(^1P_1) = \chi^\dagger \left(-\frac{i}{2} \mathbf{D} \right) \psi \sum_X |h_c + X\rangle \cdot \langle h_c + X | \psi^\dagger \left(-\frac{i}{2} \mathbf{D} \right) \chi, \quad (6a)$$

$$\mathcal{O}_8^{h_c}(^1S_0) = \chi^\dagger T^a \psi \sum_X |h_c + X\rangle \langle h_c + X | \psi^\dagger T^a \chi. \quad (6b)$$

The μ_Λ in (5) refers to the NRQCD factorization scale,² which lies in the range $mv \leq \mu_\Lambda \leq m$. These two production operators are linked by the following renormalization group equation in NRQCD [2]:

$$\frac{d}{d \ln \mu_\Lambda^2} \langle \mathcal{O}_8^{h_c}(^1S_0)(\mu_\Lambda) \rangle = \frac{2\alpha_s C_F}{3\pi N_c m^2} \langle \mathcal{O}_1^{h_c}(^1P_1) \rangle, \quad (7)$$

where $C_F = (N_c^2 - 1)/2N_c$.

We will proceed to compute the two SDCs using the standard perturbative matching technique, by replace the

¹It was pointed out by Nayak, Qiu, and Sterman [32,33] in 2005 that the original definition of the NRQCD color-octet production operator is not gauge invariant, and the correct definition necessitates the inclusion of eikonal lines that run from the quark or antiquark field to infinity. To the perturbative order we are concerned, this nuisance can be safely neglected, so we stick to the conventional definition.

²The scale μ_Λ here should not be confused with the QCD factorization scale μ introduced in (3), which enters the DGLAP equation.

physical h_c state by the free $c\bar{c}(^1P_1^{(1)})$ and $c\bar{c}(^1S_0^{(8)})$ states, respectively, in (5). The representative Feynman diagrams are shown in Fig. 1. Since (3) is manifestly gauge invariant, for simplicity, we adopt the Feynman gauge. Dimensional regularization will be used to regularize both UV and IR divergences. It is convenient to employ the well-known covariant projector technique to expedite the calculation [34]. Since one does not bother to consider the situation where gluons attach to the eikonal line, it becomes rather straightforward to generate the squared quark-level amplitude. We employ the package QGRAF [35] to generate the corresponding Feynman diagrams and amplitudes and then use the packages FeynCalc and FormLink [36,37] to conduct the Dirac and color trace calculation.

The final state phase space implicit in (3) assumes the specific form [17,19]

$$\begin{aligned} d\Phi_n &= \frac{4\pi M_{h_c}}{S_n} \delta\left(k^+ - P^+ - \sum_{i=1}^n k_i^+\right) \prod_{i=1}^n \frac{dk_i^+}{2k_i^+} \frac{d^{D-2}k_{i\perp}}{(2\pi)^{D-1}} \\ &\quad \times \theta(k_i^+) \\ &= \frac{4\pi M}{S_n} \delta\left(k^+ - P^+ - \sum_{i=1}^n k_i^+\right) \prod_{i=1}^n \frac{d^D k_i}{(2\pi)^D} 2\pi \delta_+(k_i^2), \end{aligned} \quad (8)$$

where k_i stands for the momentum of the i th gluon in the final state and S_n is the statistical factor for n identical gluons. For our purpose, it suffices to know $S_1 = 1$ and $S_n = 2$.

To efficiently cope with a multibody phase-space integral, an ingenious generalized cut propagator method (also commonly referred to as the method of inverse unitarity) [29,30] turns out to be very powerful. Recently, this technique has been applied to analytically compute the gluon-to- J/ψ fragmentation function [24]. The key is to replace each δ function in (8) by invoking the following identity:

$$\delta(x) = \frac{1}{2\pi i} \left[\frac{1}{x - i\epsilon} - \frac{1}{x + i\epsilon} \right]. \quad (9)$$

One can then apply the integration-by-part (IBP) method to the phase-space integration just as in loop integration [29], since the differentiation operation involved in IBP identities is insensitive to the $i\epsilon$. Therefore, one can also utilize the packages APART [38] and FIRE [39] to conduct partial fraction and the corresponding IBP reduction and finally end up with a set of master integrals (MIs), which are much easier to manipulate than the original integrand.³ Finally, we end up with nine MIs in the color-singlet channels and two MIs in the color-octet channels. The former class of MIs can be parameterized as

³Note that, in order to accomplish the overwhelmingly challenging calculation of the next-to-next-to-leading-order QCD correction of the η_c hadronic width [40], it is crucial to utilize this powerful trick.

$$F(i, j, m, n) = \int d\Phi_2 \frac{1}{E_1^i} \frac{1}{E_2^j} \frac{1}{E_3^m} \frac{1}{E_4^n}, \quad (10)$$

where the propagators E_i ($i = 1, \dots, 4$) are

$$E_{1,2} = (k_{1,2} + p)^2 - m^2, \quad E_3 = (k_1 + k_2 + p)^2 - m^2, \quad E_4 = (k_1 + k_2 + 2p)^2, \quad (11)$$

respectively. $p = P/2$ is the half of the h_c momentum. The nine encountered MIs in the singlet channel are labeled by the indices

$$\begin{aligned} (i, j, m, n) &= (0, 0, 1, 0), & (0, 0, 0, 1), & (1, 1, 0, 0), & (0, 1, 2, 0), & (0, 1, 1, 0), \\ &= (0, 1, 0, 2), & (0, 1, 0, 1), & (0, 0, 1, 1), & (1, 1, 0, 1). \end{aligned} \quad (12)$$

Most of them can be readily worked out by the standard method, that is, rewriting the propagators in (11) in terms of light-cone variables and conducting two-loop integration in $D - 2$ -dimensional transverse momentum $\mathbf{k}_{1,2\perp}$, then followed by a one-dimensional parametric integration. The last MI, which involves three propagators, is, however, somewhat challenging. Fortunately, with the aid of the differential equation technique, its analytic expression has already been unravelled in Ref. [24].

The two MIs in the color-octet channel are rather rudimentary:

$$\int d\Phi_1 \frac{1}{(k_1 + p)^2}, \quad \int d\Phi_1 \frac{1}{(k_1 + p)^2 - m^2}. \quad (13)$$

NRQCD provides a systematic framework to enable one to factor the IR divergence encountered in the color-singlet

channel into the color-octet production matrix element $\langle \mathcal{O}_8^{h_c}({}^1S_0) \rangle$, as originally demonstrated in B meson decay to P -wave charmonium [41].

After some straightforward but tedious manipulation, we can obtain the “bare” short-distance coefficient associated with the ${}^1P_1^{(1)}$ channel in closed form. Some individual MIs may be UV-divergent, but the final expression is free from UV divergence, as it should be. Nevertheless, this bare color-singlet short-distance coefficient still contains residual logarithmic IR singularity. One can invoke the standard color-octet mechanism in NRQCD [2] to factor this IR divergence into the ${}^1S_0^{(8)}$ NRQCD production matrix element (for more details, see [42,43]). If we choose the $\overline{\text{MS}}$ factorization scheme, the color-singlet short-distance coefficient becomes IR finite, yet logarithmically depending on the NRQCD factorization scale μ_Λ :

$$\begin{aligned} d_1(z, \mu_\Lambda) &= \frac{\alpha_s^3 B_F}{3\pi N_c^2} \left\{ [(3 - 2z)z + 2(1 - z) \ln(1 - z)] \ln\left(\frac{4m^2}{\mu_\Lambda^2}\right) - \frac{14}{15}(6 - 5z) \text{Li}_2\left(\frac{1 - z}{2 - z}\right) \right. \\ &\quad + \frac{2}{15}(33 - 17z - 9z^2) \text{Li}_2\left(\frac{2 - 2z}{2 - z}\right) + \frac{1}{15}(18 - 25z) \text{Li}_2(1 - z) - \frac{3 + z}{8} I_9 \\ &\quad - \frac{c_1}{60(2 - z)^2(1 - z)(1 - 2z)^3} - \frac{c_2 \ln(1 - z)}{60(1 - 2z)^4} + \frac{c_3(1 - z) \ln^2(1 - z)}{120(1 - 2z)^5} \\ &\quad - \frac{c_4 z \ln(2 - z)}{30(2 - z)^2(1 - z)^2} - \frac{c_5 \ln^2(2 - z)}{120(1 - z)^3} - \frac{c_6 z \ln z}{60(2 - z)^2(1 - z)^2(1 - 2z)^4} \\ &\quad \left. + \frac{c_7 z \ln(1 - z) \ln z}{60(1 - 2z)^5} - \frac{c_8 z^2 \ln(2 - z) \ln z}{60(1 - z)^3} - \frac{c_9 z \ln^2 z}{120(1 - z)^3(1 - 2z)^5} \right\}, \end{aligned} \quad (14a)$$

$$d_8(z) = \frac{\alpha_s^2 B_F}{N_c C_F} \left[(1 - z) \ln(1 - z) + \frac{1}{2}(3 - 2z)z \right], \quad (14b)$$

where $B_F = (N_c^2 - 4)/(4N_c)$, c_i ($i = 1, \dots, 9$) are defined as

$$\begin{aligned} c_1 &= \pi^2(112 - 960z + 3292z^2 - 5684z^3 + 4972z^4 - 1628z^5 - 504z^6 + 496z^7 - 96z^8) \\ &\quad + 1584z - 14052z^2 + 54785z^3 - 119579z^4 + 158947z^5 - 131887z^6 + 66880z^7 - 18980z^8 + 2304z^9, \end{aligned} \quad (15a)$$

$$c_2 = 210 - 2093z + 7890z^2 - 13501z^3 + 5326z^4 + 16632z^5 - 26920z^6 + 15408z^7 - 3072z^8, \quad (15b)$$

$$c_3 = 408 - 4097z + 16233z^2 - 30962z^3 + 21898z^4 + 20284z^5 - 57056z^6 + 54864z^7 - 28256z^8 + 6144z^9, \quad (15c)$$

$$c_4 = 38 + 920z - 4752z^2 + 9402z^3 - 9621z^4 + 5417z^5 - 1595z^6 + 192z^7, \quad (15d)$$

$$c_5 = 336 - 1288z + 1668z^2 + 4z^3 - 2770z^4 + 4061z^5 - 2991z^6 + 1171z^7 - 192z^8, \quad (15e)$$

$$c_6 = 896 - 10108z + 54243z^2 - 175908z^3 + 380634z^4 - 582264z^5 + 651071z^6 - 535798z^7 + 316152z^8 - 125336z^9 + 29488z^{10} - 3072z^{11}, \quad (15f)$$

$$c_7 = 45 - 210z - 645z^2 + 10180z^3 - 41842z^4 + 89500z^5 - 111920z^6 + 83120z^7 - 34400z^8 + 6144z^9, \quad (15g)$$

$$c_8 = 180 - 1180z + 3050z^2 - 4061z^3 + 2991z^4 - 1171z^5 + 192z^6, \quad (15h)$$

$$c_9 = 45 - 525z + 3100z^2 - 10610z^3 + 22054z^4 - 28190z^5 + 21555z^6 - 8960z^7 + 1500z^8 + 32z^9, \quad (15i)$$

and I_9 is defined as

$$I_9 = \text{Li}_3\left(\frac{z}{z-1}\right) + \text{Li}_3\left(\frac{2z-1}{z-1}\right) + \text{Li}_3\left(\frac{2z-1}{z}\right) + \text{Li}_2\left(\frac{2z-1}{z-1}\right) \ln\left(\frac{1-z}{z}\right) - \text{Li}_2(z) \ln\left(\frac{1-z}{z}\right) + \frac{1}{6} \ln^3\left(\frac{1-z}{z}\right) - \frac{1}{2} \ln(1-z) \ln z \ln\left(\frac{1-z}{z}\right) - \zeta(3). \quad (16)$$

From these analytic expressions, it is not difficult to find the asymptotic behavior of these short-distance coefficients near $z = 1$:

$$d_1(z) \rightarrow \frac{\alpha_s B_F}{3\pi N_c^2} \left[\ln\left(\frac{4m^2}{\mu_\Lambda^2}\right) + 2 \ln(1-z) + \frac{1}{3} \right] + \mathcal{O}[(1-z) \ln^2(1-z)], \quad (17a)$$

$$d_8(z) \rightarrow \frac{\alpha_s^2 B_F}{2N_c C_F} + \mathcal{O}((1-z) \ln(1-z)). \quad (17b)$$

Notice that the mild logarithmic singularity of $d_1(z)$ is developed in the $z \rightarrow 1$ limit.

It is also straightforward to obtain the fragmentation probability:

$$\int_0^1 D_{g \rightarrow h_c}(z, \mu) dz = \frac{\alpha_s^2 B_F}{6N_c C_F} \frac{\langle \mathcal{O}_8^{h_c}(^1S_0) \rangle_{\mu_\Lambda}}{m^3} + \frac{\alpha_s^3 B_F}{3\pi N_c^2} \frac{\langle \mathcal{O}_1^{h_c}(^1S_0) \rangle}{m^5} \left[\frac{1}{3} \ln\left(\frac{4m^2}{\mu_\Lambda^2}\right) - 1.1341 \right]. \quad (18)$$

For concreteness, we take the following input parameters [43]:

$$m_c = 1.5 \text{ GeV}, \quad \langle \mathcal{O}_1^{h_c}(^1P_1) \rangle = 0.322 \text{ GeV}^5, \quad \langle \mathcal{O}_8^{h_c}(^1S_0)(m) \rangle = 0.02 \text{ GeV}^3, \\ \mu_\Lambda = m_c, \quad \alpha_s(2m_c) = 0.26. \quad (19)$$

Substituting these values into (18), we then find the total fragmentation probability is about (4.50×10^{-6}) . The profile of the gluon fragmentation function, with an projected uncertainty band, is displayed in Fig. 2.

FIG. 2. The gluon fragmentation function $D_{g \rightarrow h_c}(z)$ evaluated with NRQCD factorization scale $\mu_\Lambda = m_c$, including individual color-singlet and -octet contributions. Each uncertainty band is obtained by varying the renormalization scale μ from m_c to $4m_c$, with the central value evaluated at $\mu = 2m_c$.

Inspired by the recent technical advance in computing the quarkonium fragmentation function [24], in this work, we revisit the gluon-to- h_c fragmentation function and, for the first time, achieve the analytic, gauge-invariant expression for this FF. Our study might shed some light on the future establishing of the h_c and h_b states at the LHC experiment.

The work of F.F. is supported by the National Natural Science Foundation of China under Grants No. 11875318 and No. 11505285 and by the Yue Qi Young Scholar Project in China University of Mining and Technology. The work of S.I., Y.J., and J.-Y.Z. is supported in part by the National Natural Science Foundation of China under Grants No. 11925506, No. 11875263, and No. 11621131001 (CRC110 by DFG and NSFC). S.I. also acknowledges the financial support from the CAS-TWAS President's Fellowship Program. The Feynman diagrams in this paper are prepared by using JaxoDraw [44].

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- [1] J. C. Collins, D. E. Soper, and G. F. Sterman, *Adv. Ser. Dir. High Energy Phys.* **5**, 1 (1989).
 - [2] G. T. Bodwin, E. Braaten, and G. P. Lepage, *Phys. Rev. D* **51**, 1125 (1995); **55**, 5853(E) (1997).
 - [3] E. Braaten, K. m. Cheung, and T. C. Yuan, *Phys. Rev. D* **48**, 4230 (1993).
 - [4] E. Braaten and T. C. Yuan, *Phys. Rev. Lett.* **71**, 1673 (1993).
 - [5] G. T. Bodwin, H. S. Chung, U. R. Kim, and J. Lee, *Phys. Rev. Lett.* **113**, 022001 (2014).
 - [6] G. T. Bodwin, K. T. Chao, H. S. Chung, U. R. Kim, J. Lee, and Y. Q. Ma, *Phys. Rev. D* **93**, 034041 (2016).
 - [7] E. Braaten, K. m. Cheung, and T. C. Yuan, *Phys. Rev. D* **48**, R5049 (1993).
 - [8] J. P. Ma, *Phys. Lett. B* **332**, 398 (1994).
 - [9] E. Braaten and T. C. Yuan, *Phys. Rev. D* **50**, 3176 (1994).
 - [10] P. L. Cho and M. B. Wise, *Phys. Rev. D* **51**, 3352 (1995).
 - [11] J. P. Ma, *Nucl. Phys.* **B447**, 405 (1995).
 - [12] J. P. Ma, *Phys. Rev. D* **53**, 1185 (1996).
 - [13] E. Braaten and T. C. Yuan, *Phys. Rev. D* **52**, 6627 (1995).
 - [14] K. m. Cheung and T. C. Yuan, *Phys. Rev. D* **53**, 3591 (1996).
 - [15] C. f. Qiao, F. Yuan, and K. T. Chao, *Phys. Rev. D* **55**, 5437 (1997).
 - [16] E. Braaten and J. Lee, *Nucl. Phys.* **B586**, 427 (2000).
 - [17] G. T. Bodwin and J. Lee, *Phys. Rev. D* **69**, 054003 (2004).
 - [18] W. l. Sang, L. f. Yang, and Y. q. Chen, *Phys. Rev. D* **80**, 014013 (2009).
 - [19] G. T. Bodwin, U. R. Kim, and J. Lee, *J. High Energy Phys.* **11** (2012) 020.
 - [20] P. Artoisenet and E. Braaten, *J. High Energy Phys.* **04** (2015) 121.
 - [21] G. T. Bodwin, H. S. Chung, U. R. Kim, and J. Lee, *Phys. Rev. D* **91**, 074013 (2015).
 - [22] X. Gao, Y. Jia, L. Li, and X. Xiong, *Chin. Phys. C* **41**, 023103 (2017).
 - [23] R. Sepahvand and S. Dadfar, *Phys. Rev. D* **95**, 034012 (2017).
 - [24] P. Zhang, Y. Q. Ma, Q. Chen, and K. T. Chao, *Phys. Rev. D* **96**, 094016 (2017).
 - [25] Y. Q. Ma, J. W. Qiu, and H. Zhang, *Phys. Rev. D* **89**, 094029 (2014).
 - [26] J. L. Rosner *et al.* (CLEO Collaboration), *Phys. Rev. Lett.* **95**, 102003 (2005).
 - [27] I. Adachi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **108**, 032001 (2012).
 - [28] G. Hao, Y. Zuo, and C. F. Qiao, *arXiv:0911.5539*.
 - [29] C. Anastasiou and K. Melnikov, *Nucl. Phys.* **B646**, 220 (2002).
 - [30] A. Gehrmann-De Ridder, T. Gehrmann, and G. Heinrich, *Nucl. Phys.* **B682**, 265 (2004).
 - [31] J. C. Collins and D. E. Soper, *Nucl. Phys.* **B194**, 445 (1982).
 - [32] G. C. Nayak, J. W. Qiu, and G. F. Sterman, *Phys. Lett. B* **613**, 45 (2005).
 - [33] G. C. Nayak, J. W. Qiu, and G. F. Sterman, *Phys. Rev. D* **72**, 114012 (2005).
 - [34] A. Petrelli, M. Cacciari, M. Greco, F. Maltoni, and M. L. Mangano, *Nucl. Phys.* **B514**, 245 (1998).
 - [35] P. Nogueira, *J. Comput. Phys.* **105**, 279 (1993).
 - [36] R. Mertig, M. Bohm, and A. Denner, *Comput. Phys. Commun.* **64**, 345 (1991).
 - [37] F. Feng and R. Mertig, *arXiv:1212.3522*.

- [38] F. Feng, *Comput. Phys. Commun.* **183**, 2158 (2012).
- [39] A. V. Smirnov, *Comput. Phys. Commun.* **189**, 182 (2015).
- [40] F. Feng, Y. Jia, and W. L. Sang, *Phys. Rev. Lett.* **119**, 252001 (2017).
- [41] G. T. Bodwin, E. Braaten, T. C. Yuan, and G. P. Lepage, *Phys. Rev. D* **46**, R3703 (1992).
- [42] M. Beneke, F. Maltoni, and I. Z. Rothstein, *Phys. Rev. D* **59**, 054003 (1999).
- [43] Y. Jia, W. L. Sang, and J. Xu, *Phys. Rev. D* **86**, 074023 (2012).
- [44] D. Binosi and L. Theussl, *Comput. Phys. Commun.* **161**, 76 (2004).