Comment on “Gravitational transition form factors of $N(1535) \rightarrow N^*$ ”

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In a recent paper [1], U. Özdem and K. Azizi studied the form factors of the quark part of the energy momentum tensor (EMT) for the transition $N \rightarrow N^*(1535)$ in the framework of the light-cone QCD sum rules. The corresponding form factors are defined in Eq. (4) of their paper. The authors introduced four independent factors. Unfortunately, the parametrization of EMT matrix element used in Ref. [1] is not complete. One can see this from the fact that presented in [1] parametrization does not satisfy EMT conservation. One may argue that in Ref. [1] only the quark part of EMT (nonconserved) was considered and one does not need to impose the conservation conditions. However, it is clear that parametrization of the form factors of the total (conserved) EMT should contain *less* number of

structures. The reader can easily check that it is impossible to reproduce the conservation conditions nullifying some of form factors (or their combination) in Eq. (4) of [1].

Another inconsistency in Eq. (4) of [1] can be seen if to note that in the limit of the equal masses of the final and initial state the Lorentz structure corresponding to the form factor $J(t)$ is (up to constant factor) equal to the structure of corresponding to the $A(t)$ form factor. Therefore, parametrization of Ref. [1] contains a redundancy in the limit of equal masses.

Using the Lorentz invariance, discrete space-time symmetries, and the equations of motion (Gordon identities), the $N \rightarrow N^*(1535)$ ($1/2^+ \rightarrow 1/2^-$) EMT form factors can be parametrized as follows¹:

$$\begin{aligned} \langle N^*(p') | T^{\mu\nu}(0) | N(p) \rangle = & F_1(t) \bar{u}(p') \left\{ \Delta^2 P^{\{\mu} P^{\nu\}} - (m_*^2 - m^2) \Delta^{\{\mu} P^{\nu\}} + \frac{(m_*^2 - m^2)^2}{4} g^{\mu\nu} \right\} \gamma_5 u(p) \\ & + F_2(t) \bar{u}(p') \left\{ \Delta^2 \gamma^{\{\mu} P^{\nu\}} - (m_* + m) \Delta^{\{\mu} P^{\nu\}} - \frac{(m_*^2 - m^2)}{2} (\gamma^{\{\mu} \Delta^{\nu\}} - (m_* + m) g^{\mu\nu}) \right\} \gamma_5 u(p) \\ & + F_3(t) \{ \Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2 \} \bar{u}(p') \gamma_5 u(p). \end{aligned} \quad (1)$$

Here $P = (p + p')/2$, $\Delta = p' - p$, $t = \Delta^2$, and m, m_* are masses of the nucleon and of the $N^*(1535)$, respectively. The symmetrization is defined as $X^{\{\mu} Y^{\nu\}} = (X^\mu Y^\nu + X^\nu Y^\mu)/2$. One can easily check that the conservation of EMT $\partial_\mu T^{\mu\nu}(x) = 0$ is satisfied automati-

cally. We note that our parametrization (1) for $m_* = m$ coincides (after use of the Gordon identity and trivial renaming of the form factors) with the parity violating gravitational form factors for spin-1/2 particles is introduced by Kobzarev and Okun in Ref. [2] [see their Eq. (24)].

If one considers the partial EMT (say, the quark part of the EMT) which is not conserved, one has to add to Eq. (1) three other terms,

$$\begin{aligned} \langle N^*(p') | \delta T^{\mu\nu}(0) | N(p) \rangle = & \bar{u}(p') [\bar{C}_1(t) g^{\mu\nu} + \bar{C}_2(t) \gamma^{\{\mu} P^{\nu\}} \\ & + \bar{C}_3(t) \gamma^{\{\mu} \Delta^{\nu\}}] \gamma_5 u(p). \end{aligned} \quad (2)$$

We see that in total the quark part of EMT is parameterized by six form factors. The total number of EMT form factors and the type of the Lorentz structures in Eqs. (1) and (2) are at variance with those in basic Eq. (4) of Ref. [1].

¹The form factors introduced here are dimensional; they can be easily made dimensionless, using the mass scales in the problem. E.g., it could be the nucleon mass, or various types of the averaged masses (e.g. $(m + m_*)/2$, or $\sqrt{mm_*}$, etc.), or it could be the f_π if one would consider the chiral properties of GFFs. The choice of the mass scale depends on specific physics problem one considers and surely does not affect the classification of the Lorentz structures.

In conclusion, the parametrization of the EMT transition $N \rightarrow N^*(1535)$ matrix element used in Ref. [1] is not complete and contains redundancy in the limit $m_* \rightarrow m$. Therefore, the sum rules of Ref. [1] should be checked for consistency and considerably modified.

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[1] U. Özdem and K. Azizi, *Phys. Rev. D* **101**, 054031 (2020).

[2] I. Y. Kobzarev and L. B. Okun, *Zh. Eksp. Teor. Fiz.* **43**, 1904 (1962) [*Sov. Phys. JETP* **16**, 1343 (1963)].