Will the subprocesses $\rho(770, 1450)^0 \rightarrow K^+ K^-$ contribute large branching fractions for $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays?

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We analyze the quasi-two-body decays $B^\pm \rightarrow \pi^\pm \rho(770, 1450)^0 \rightarrow \pi^\pm K^+ K^-$ in the perturbative QCD approach. The results in this work do not support that large branching fractions contributed by the resonances $\rho(770, 1450)^0$ in the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays. The virtual contribution for $K^+ K^-$ from the tail of the resonance $\rho(770)^0$ which has been ignored in the experimental studies is about 1.5 times of the $\rho(1450)^0 \rightarrow K^+ K^-$ contribution, with the predicted branching fractions $\mathcal{B}_v = (1.31 \pm 0.27) \times 10^{-7}$ and $\mathcal{B} = (8.96 \pm 2.61) \times 10^{-8}$, respectively, for these two subprocesses in the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays. The absence of $\rho(770)^0 \rightarrow K^+ K^-$ for the decay amplitude of a three-body hadronic B decay involving charged kaon pair could probably result in a larger proportion for the contribution from the resonance $\rho(1450)^0$ in experimental analysis.

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I. INTRODUCTION

Strong dynamics in the charmless three-body hadronic B meson decays is related to the short-distance processes like in the two-body cases, and it also involves the hadron-hadron interactions, the three-body effects [1,2], and the rescattering processes [3,4] in the final states. In experimental studies, the strong interactions along with the weak processes of a three-body B meson decay are always described in its decay amplitude as the coherent sum of the resonant and nonresonant contributions in the isobar formalism [5–7]. The isobar expression with or without certain resonances will certainly have impacts on the fit fractions, and then influence the observables such as the branching ratios and CP violations for the resonant and the nonresonant contributions of the three-body process in view of the explicit distribution of the experimental events in the Dalitz plot [8].

Very recently, LHCb Collaboration presented a surprising large fit fraction $(30.7 \pm 1.2 \pm 0.9)\%$ of the total three-body result in [9] for the subprocess $\rho(1450)^0 \rightarrow K^+ K^-$ in the amplitude analysis of the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays. Considering the branching fraction $(5.2 \pm 0.4) \times 10^{-6}$ for $B^+ \rightarrow K^+ K^- \pi^+$ in the *Review of Particle Physics* [10] which was averaged from the results $(5.38 \pm 0.40 \pm 0.35) \times 10^{-6}$ in [11] provided by Belle and $(5.0 \pm 0.5 \pm$

$0.5) \times 10^{-6}$ in [12] presented by BABAR, one has the branching fraction $(1.60 \pm 0.15) \times 10^{-6}$ for the quasi-two-body decay $B^+ \rightarrow \pi^+ \rho(1450)^0 \rightarrow \pi^+ K^+ K^-$ from LHCb's fit fraction. While the contributions of $B^\pm \rightarrow \pi^\pm \rho(1450)^0$ were found quite small in the ρ^0 dominant decay modes $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ in [13,14] by LHCb recently.

The resonance contributions for the charged kaon pair in the three-body decays $B^\pm \rightarrow \pi^\pm K^+ K^-$ are associated with the low energy scalar, vector and tensor states, such as $f_0(980)$, $\phi(1020)$, $f'_2(1525)$, etc., which have been noticed in [15–18], while the contributions for $K^+ K^-$ also come from the P -wave resonances $\rho(770)$, $\omega(782)$ and their excited states [19,20]. The natural mode for $\rho(770)^0$ decays into $K^+ K^-$ is blocked because of its pole mass which is below the threshold of kaon pair, but the virtual contribution [21,22] from the tail of the Breit-Wigner (BW) formula [23] of $\rho(770)$ could be indispensable for $K^+ K^-$ in the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays, which could be deduced from the works for the processes of $\pi^- p \rightarrow K^- K^+ n$ and $\pi^+ n \rightarrow K^- K^+ p$ [24,25], $e^+ e^- \rightarrow K^+ K^-$ [26–30] and $\pi\pi \rightarrow K\bar{K}$ scattering [31].

We shall analyze the $\rho(770, 1450)^0 \rightarrow K^+ K^-$ contributions for the three-body decays $B^\pm \rightarrow \pi^\pm K^+ K^-$ with the quasi-two-body framework based on the perturbative QCD (PQCD) approach [32–35]. The final state interaction effect was found to be suppressed for the $\rho^0 \rightarrow K^+ K^-$ process [36] and will be neglected in the calculation in this work. For the quasi-two-body decays $B^\pm \rightarrow \pi^\pm \rho^0 \rightarrow \pi^\pm K^+ K^-$, the intermediate states $\rho(770)^0$ and $\rho(1450)^0$ are generated in the hadronization of the light quark-antiquark pair $q\bar{q}$, with $q = (u, d)$, as demonstrated in the Fig. 1, in which the factorizable and nonfactorizable diagrams have been

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FIG. 1. Feynman diagrams for the processes $B^+ \rightarrow \pi^+ \rho^0 \rightarrow \pi^+ K^+ K^-$, with $\otimes$ is the weak vertex, $\times$ denotes possible attachments for the hard gluons and the rectangle represents the resonance $\rho(770)^0$ and its excited states.

merged for the sake of simplicity and Fig. 1(b) and (d) will generate different decay amplitudes because of the different position of the bachelor and the intermediate states. The subprocesses $\rho(770, 1450)^0 \rightarrow K^+ K^-$, which cannot be calculated in the PQCD approach, could be introduced into the distribution amplitudes of the kaon pair system by the timelike form factor of kaon. The quasi-two-body framework based on PQCD approach has been discussed in detail in [37], and has been adopted in some works for the quasi-two-body B meson decays [38–43] recently.

This paper is organized as follows: In Sec. II, we briefly review the electromagnetic timelike form factor for the charged kaon, we introduce the P -wave $K^+ K^-$ system distribution amplitudes and give the expression of differential branching fractions for the quasi-two-body decays $B^\pm \rightarrow \pi^\pm \rho^0 \rightarrow \pi^\pm K^+ K^-$. In Sec. III, we provide numerical results for the concerned decay processes and give some necessary discussions. The conclusions are presented in Sec. IV.

II. FRAMEWORK

In the light-cone coordinates, the momentum p_B and the light spectator quark's momentum k_B for B^+ are defined as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad k_B = \left(\frac{m_B}{\sqrt{2}}x_B, 0, k_{BT}\right), \quad (1)$$

in the rest frame of B meson, where m_B is the mass of B^+ . The resonance ρ has the momentum $p = \frac{m_B}{\sqrt{2}}(\eta, 1, 0)$, with $\eta = s/m_B^2$ and $s = p^2$, its spectator quark got the momentum $k = (0, \frac{m_B}{\sqrt{2}}z, k_T)$. For the bachelor final state pion and its spectator quark, the momenta p_3 and k_3 have the definitions

$$p_3 = \frac{m_B}{\sqrt{2}}(1 - \eta, 0, 0_T), \quad k_3 = \left(\frac{m_B}{\sqrt{2}}(1 - \eta)x_3, 0, k_{3T}\right). \quad (2)$$

The x_B , x_3 and z are the momentum fractions which run from zero to one in the calculation.

The timelike form factor F_K for the charged kaon is related to the electromagnetic form factor and defined by [44]

$$\langle K^+(p_1) K^-(p_2) | j_\mu^{em} | 0 \rangle = (p_1 - p_2)_\mu F_K(s), \quad (3)$$

with the squared invariant mass $s = (p_1 + p_2)^2$ for kaon pair and the constraint $F_K(0) = 1$. The electromagnetic current $j_\mu^{em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$ is carried by the light quarks [45]. With the BW formula for the resonances ω and ϕ and the Gounaris-Sakurai (GS) model [46] for ρ , we have the timelike form factor [44]

$$F_K(s) = \frac{1}{2} \sum_\rho c_\rho^K \text{GS}_\rho(s) + \frac{1}{6} \sum_\omega c_\omega^K \text{BW}_\omega(s) + \frac{1}{3} \sum_\phi c_\phi^K \text{BW}_\phi(s), \quad (4)$$

where the $\sum$ means the summation of the resonances ρ , ω or ϕ and their corresponding excited states, the explicit expressions and auxiliary functions for BW and GS are referred to Refs. [46,47]. The parameters $c_{\rho(\omega,\phi)}^K$ have been fitted to the data in Refs. [44,48,49].

For the concerned subprocesses $\rho(770, 1450)^0 \rightarrow K^+ K^-$, the P -wave $K^+ K^-$ system distribution amplitudes are organized into [50]

$$\phi_{KK}^{P\text{-wave}}(z, s) = \frac{-1}{\sqrt{2N_c}} [\sqrt{s} \phi_L^0(z, s) + \phi_L \not{p} \phi^t(z, s) + \sqrt{s} \phi^s(z, s)]. \quad (5)$$

The momentum $p = p_1 + p_2$, and ϵ_L is the longitudinal polarization vector for the resonances. We have the distribution amplitudes as [37]

$$\phi^0(z, s) = \frac{3F_K^\rho(s)}{\sqrt{N_c}} z(1-z)[1 + a_2^0 C_2^{3/2}(1-2z)], \quad (6)$$

$$\phi^t(z, s) = \frac{3F_K^t(s)}{2\sqrt{N_c}} (1-2z)^2[1 + a_2^t C_2^{3/2}(1-2z)], \quad (7)$$

$$\phi^s(z, s) = \frac{3F_K^s(s)}{2\sqrt{N_c}} (1-2z)[1 + a_2^s(1-10z+10z^2)], \quad (8)$$

where the F_K^ρ is the ρ term of Eq. (4), the Gegenbauer polynomial $C_2^{3/2}(\chi) = 3(5\chi^2 - 1)/2$. The Gegenbauer moments $a_2^0 = 0.25 \pm 0.10$, $a_2^t = -0.60 \pm 0.20$ and $a_2^s = 0.75 \pm 0.25$ which are the same as they in [37] for the intermediate state ρ . For the subprocess $\rho^0 \rightarrow K^+ K^-$, we adopt $F_K^{s,t}(s) \approx (f_\rho^T/f_\rho) F_K^\rho(s)$ as in [37] for the channel $\rho^0 \rightarrow \pi^+ \pi^-$.

One has the differential branching fractions ($\mathcal{B}$) for the quasi-two-body decays $B^\pm \rightarrow \pi^\pm \rho^0 \rightarrow \pi^\pm K^+ K^-$ as [22,51,52]

$$\frac{d\mathcal{B}}{d\eta} = \tau_B \frac{|\vec{q}_\pi|^3 |\vec{q}_K|^3}{12\pi^3 m_B^5} |\mathcal{A}|^2, \quad (9)$$

where τ_B being the B meson mean lifetime. It should be noted that the Eqs. (6)–(8) and Eq. (9) are slightly different from the corresponding expressions in [37]. These differences are caused by the introduction of the Zemach tensor $-2\vec{q}_\pi \cdot \vec{q}_K$ [53] in this work as did in Refs. [51,52], this tensor is employed to describe the angular distribution for the decay of spin 1 resonances. The magnitudes of the kaon and pion momenta $|\vec{q}_K|$ and $|\vec{q}_\pi|$ are written, in the center-of-mass frame of the kaon pair, as

$$|\vec{q}_K| = \frac{1}{2} \sqrt{s - 4m_K^2}, \quad (10)$$

$$|\vec{q}_\pi| = \frac{1}{2\sqrt{s}} \sqrt{(m_B^2 - m_\pi^2)^2 - 2(m_B^2 + m_\pi^2)s + s^2}, \quad (11)$$

with the pion mass m_π and the kaon mass m_K .

The decay amplitudes $\mathcal{A}$ for the quasi-two-body decays $B^+ \rightarrow \pi^+ \rho(770, 1450)^0 \rightarrow \pi^+ K^+ K^-$ dependent only on the quark structures of the hadronic matrix elements for B^+ to $\pi^+ \rho(770, 1450)^0$ transitions and have the same expressions as the decays $B^+ \rightarrow \pi^+ \rho(770, 1450)^0 \rightarrow \pi^+ \pi^+ \pi^-$ in [40,41] except the replacement of the form factor $F_\pi \rightarrow F_K$. The amplitudes in $\mathcal{A}$ according to the diagrams in Fig. 1 could be found in the Appendix of [37].

III. RESULTS

In the numerical calculation, we adopt the decay constant $f_B = 0.189$ GeV [54] and the mean lifetime $\tau = (1.638 \pm 0.004) \times 10^{-12}$ s [10] for the B^+ meson. The masses and the decay constant f_π for the relevant particles in the numerical calculation in this work, the full widths for $\rho(770)$ and $\rho(1450)$, and the Wolfenstein parameters of the CKM matrix are presented in Table I.

The ratio between the f_ρ^T and f_ρ for $\rho(770)$ has been computed in lattice QCD, we choose the result $f_\rho^T/f_\rho = 0.687$ at the scale $\mu = 2$ GeV [55] for our numerical calculation. The decay constant f_ρ for $\rho(770)$ can be extracted from the processes $\tau^- \rightarrow \rho^- \nu_\tau$ and $\rho^0 \rightarrow e^+ e^-$, we take the value 0.216 GeV [56]. The $c_{\rho(770)}^K$ was fitted to be 1.139 ± 0.010 and 1.195 ± 0.009 with the unconstrained and constrained fit procedure in [44], respectively, which are consistent with the values 1.138 ± 0.011 and 1.120 ± 0.007 in [48]. We employ the result 1.139 ± 0.010 for the quasi-two-body decay $B^+ \rightarrow \pi^+ \rho(770)^0 \rightarrow \pi^+ K^+ K^-$. As for the c^K for $\rho(1450)$, we adopt the value -0.124 ± 0.012 [44] for the subprocess $\rho(1450)^0 \rightarrow K^+ K^-$, which is close to the constrained fit result -0.112 ± 0.010 in [44] and the

TABLE I. Masses, decay constant f_π , full widths of $\rho(770)$ and $\rho(1450)$ (in units of GeV) and Wolfenstein parameters [10].

$m_{B^\pm} = 5.279$	$m_{\pi^\pm} = 0.14$	$m_{K^\pm} = 0.494$
$m_{\rho(770)} = 0.775$	$m_{\rho(1450)} = 1.465$	$f_\pi = 0.131$
$\Gamma_{\rho(770)} = 0.149$	$\Gamma_{\rho(1450)} = 0.400 \pm 0.060$	
$\lambda = 0.22453 \pm 0.00044$	$A = 0.836 \pm 0.015$	
$\bar{\rho} = 0.122_{-0.017}^{+0.018}$	$\bar{\eta} = 0.355_{-0.011}^{+0.012}$	

unconstrained fit result -0.107 ± 0.010 in [48]. We need to stress that different choices for $c_{\rho(770)}^K$ and $c_{\rho(1450)}^K$ with the values in [44,48] can not significantly change the numerical results and will not change our conclusions in this work.

Utilizing the differential branching fractions the Eq. (9), we have the branching fractions

$$\begin{aligned} \mathcal{B}(B^\pm \rightarrow \pi^\pm \rho(1450)^0 \rightarrow \pi^\pm K^+ K^-) \\ = (8.96 \pm 1.58(a_2^s + a_2^0 + a_2^t) \pm 0.83(\omega_B) \\ \pm 0.79(m_0^\pi + a_2^\pi) \pm 1.73(c_{\rho(1450)}^K)) \times 10^{-8}, \end{aligned} \quad (12)$$

$$\begin{aligned} \mathcal{B}_v(B^\pm \rightarrow \pi^\pm \rho(770)^0 \rightarrow \pi^\pm K^+ K^-) \\ = (1.31 \pm 0.22(a_2^s + a_2^0 + a_2^t) \pm 0.12(\omega_B) \\ \pm 0.11(m_0^\pi + a_2^\pi) \pm 0.02(c_{\rho(770)}^K)) \times 10^{-7}. \end{aligned} \quad (13)$$

The subscript v for $\mathcal{B}$ of $B^\pm \rightarrow \pi^\pm \rho(770)^0 \rightarrow \pi^\pm K^+ K^-$ means the virtual contribution [21,22] which is integrated of the Eq. (9) from the threshold of kaon pair. The first error for the two branching fractions above comes from the uncertainty of the Gegenbauer moments a_2^s, a_2^0 and a_2^t in the $K^+ K^-$ system distribution amplitudes, the second error is induced by the shape parameter $\omega_B = 0.40 \pm 0.04$ for B^+ , the third one is contributed by the chiral mass $m_0^\pi = 1.40 \pm 0.10$ GeV and the Gegenbauer moment $a_2^\pi = 0.25 \pm 0.15$ for pion and the fourth one due to the variation of c^K in the form factor F_K . There are other errors come from the uncertainties of the Wolfenstein parameters of the CKM matrix, the parameters in the distribution amplitudes of bachelor pion, the masses and the decay constants of the initial and final states, etc. are small and have been neglected.

From the fit fractions in [9], one has a branching fraction $\mathcal{B} \approx (1.60 \pm 0.15) \times 10^{-6}$ for the quasi-two-body decay $B^+ \rightarrow \pi^+ \rho(1450)^0 \rightarrow \pi^+ K^+ K^-$, which is about 18 times larger than the PQCD prediction shown in Eq. (12). The value $(1.60 \pm 0.15) \times 10^{-6}$ is close to the result $\mathcal{B} = 1.4_{-0.9}^{+0.6} \times 10^{-6}$ for the decay $B^+ \rightarrow \rho(1450)^0 \pi^+$ with $\rho(1450)^0 \rightarrow \pi^+ \pi^-$ from BABAR Collaboration [10,57]. Recently, in the amplitude analysis of the $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ decay, LHCb Collaboration provided a CP -averaged fit fraction $(5.2 \pm 0.3 \pm 0.2 \pm 1.9)\%$ [13,14] for the $\rho(1450)^0$ component with the isobar model, implying a branching ratio $\mathcal{B} = (7.9 \pm 3.0) \times 10^{-7}$ for the quasi-two-body process $B^+ \rightarrow \pi^+ \rho(1450)^0 \rightarrow \pi^+ \pi^+ \pi^-$ with the

data $\mathcal{B} = (1.52 \pm 0.14) \times 10^{-5}$ [10] for the three-body $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ decay. With the same expressions for $B^+ \rightarrow \pi^+ \rho(1450)^0 \rightarrow \pi^+ K^+ K^-$ but the subprocess $\rho(1450)^0 \rightarrow \pi^+ \pi^-$, we have the prediction

$$\begin{aligned} \mathcal{B}(B^\pm \rightarrow \pi^\pm \rho(1450)^0 \rightarrow \pi^\pm \pi^+ \pi^-) \\ = (9.97 \pm 1.81(a_2^s + a_2^0 + a_2^i) \pm 0.98(\omega_B) \\ \pm 0.91(m_0^\pi + a_2^\pi)) \times 10^{-7}, \end{aligned} \quad (14)$$

which agree with the results $(7.9 \pm 3.0) \times 10^{-7}$ and $1.4_{-0.9}^{+0.6} \times 10^{-6}$.

From the predictions for the quasi-two-body decays $B^\pm \rightarrow \pi^\pm \rho(1450)^0 \rightarrow \pi^\pm K^+ K^-$ and $B^\pm \rightarrow \pi^\pm \rho(1450)^0 \rightarrow \pi^\pm \pi^+ \pi^-$ in this work, we have the ratio $R_{\rho(1450)}$ between their branching fractions as

$$R_{\rho(1450)} = \frac{\mathcal{B}(\rho(1450)^0 \rightarrow K^+ K^-)}{\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+ \pi^-)} = 0.090 \pm 0.017, \quad (15)$$

with the factorization relation $\Gamma(B^\pm \rightarrow \rho^0 \pi^\pm \rightarrow h^+ h^- \pi^\pm) \approx \Gamma(B^\pm \rightarrow \rho^0 \pi^\pm) \mathcal{B}(\rho^0 \rightarrow h^+ h^-)$, here $h = (\pi, K)$. The only error for $R_{\rho(1450)}$ comes from the uncertainty of $c_{\rho(1450)}^K$ because of the cancellation between the other errors of the two branching fractions in Eq. (12) and Eq. (14). That is to say, the increase or the decrease of the parameters that caused the errors will result in nearly identical change of the weight for the numerator and denominator of $R_{\rho(1450)}$. The ratio $R_{\rho(1450)}$ can also be estimated from the coupling constants $g_{\rho(1450)^0 \pi^+ \pi^-}$ and $g_{\rho(1450)^0 K^+ K^-}$. With the relation $g_{\rho(1450)^0 K^+ K^-} \approx \frac{1}{2} g_{\rho(1450)^0 \pi^+ \pi^-}$ [44] one has

$$\begin{aligned} R_{\rho(1450)} &= \frac{\mathcal{B}(\rho(1450)^0 \rightarrow K^+ K^-)}{\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+ \pi^-)} \\ &\approx \frac{g_{\rho(1450)^0 K^+ K^-}^2 (m_{\rho(1450)}^2 - 4m_K^2)^{3/2}}{g_{\rho(1450)^0 \pi^+ \pi^-}^2 (m_{\rho(1450)}^2 - 4m_\pi^2)^{3/2}} \\ &= 0.107. \end{aligned} \quad (16)$$

This value is consistent with the result in Eq. (15).

The ratio $R_{\rho(1450)}$ has been measured by BABAR Collaboration with the result

$$\begin{aligned} R_{\rho(1450)} &= \frac{\mathcal{B}(\rho(1450)^0 \rightarrow K^+ K^-)}{\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+ \pi^-)} \\ &= 0.307 \pm 0.084(\text{stat}) \pm 0.082(\text{sys}), \end{aligned} \quad (17)$$

in the Dalitz plot analyses of $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ and $J/\psi \rightarrow K^+ K^- \pi^0$ decays in Ref. [58]. This result is quite large comparing with the values in Eqs. (15)–(16). But one should note that the virtual contribution of $\rho(770)^0 \rightarrow K^+ K^-$ has been ignored for the channel $J/\psi \rightarrow K^+ K^- \pi^0$ in [58]. We argue that the large value for $R_{\rho(1450)}$ from BABAR could be understood as that the virtual contribution of

$\rho(770)^0$ for $K^+ K^-$ has probably been taken into account for the process $\rho(1450)^0 \rightarrow K^+ K^-$.

The virtual contribution for $K^+ K^-$ from the tail of the resonance $\rho(770)^0$, which has not been taken into the decay amplitudes of $B^\pm \rightarrow \pi^\pm K^+ K^-$ and other charmless three-body B meson decays involving kaon pair in the experimental studies, is about 1.5 times of the contribution from $\rho(1450)^0$ for the charged kaon pair as shown in Eq. (13) for the branching fractions. The predicted result $(1.31 \pm 0.27) \times 10^{-7}$ is roughly 2.5% of the total branching fraction of the corresponding three-body decay and larger than the fit fraction $(0.3 \pm 0.1 \pm 0.1)\%$ in the same channel for $\phi(1020)$ in [9], and should not be ignored in the studies of the three-body decays like $B^\pm \rightarrow \pi^\pm K^+ K^-$. The large virtual contribution of $\rho(770)^0$ for the charged kaon pair is mainly due to its large decay width which result in a relative dispersive distribution of the differential branching fraction as shown in Fig. 2, where the red short dash line is for the decay $B^\pm \rightarrow \pi^\pm \rho(1450)^0 \rightarrow \pi^\pm K^+ K^-$, the blue dash-dot line is for the process $B^\pm \rightarrow \pi^\pm \rho(770)^0 \rightarrow \pi^\pm K^+ K^-$ and the black solid line is for $B^\pm \rightarrow \pi^\pm (\rho(770)^0 + \rho(1450)^0) \rightarrow \pi^\pm K^+ K^-$.

In Fig. 2, the peak of the differential branching fraction for $B^\pm \rightarrow \pi^\pm \rho(1450)^0 \rightarrow \pi^\pm K^+ K^-$ is around the pole mass of the $\rho(1450)$ resonance as expected. There is also a bump at about 1.35 GeV of the invariant mass of the kaon pair for the process $B^\pm \rightarrow \pi^\pm \rho(770)^0 \rightarrow \pi^\pm K^+ K^-$. This bump is generated by the tail of the BW formula for $\rho(770)$ along with the phase space factors $\vec{q}_K$ and $\vec{q}_\pi$ in the Eqs. (10)–(11). When we investigate the total contributions from $\rho(1450)^0$ plus $\rho(770)^0$, the bump from $\rho(770)^0$ will disappear and an enhanced peak around 1.4 GeV is the only one existed as shown by the solid line in Fig. 2. This makes us conclude that the absence of the virtual contribution

FIG. 2. The differential branching fractions for the quasi-two-body decays $B^\pm \rightarrow \pi^\pm \rho(770)^0 \rightarrow \pi^\pm K^+ K^-$, $B^\pm \rightarrow \pi^\pm \rho(1450)^0 \rightarrow \pi^\pm K^+ K^-$, and $B^\pm \rightarrow \pi^\pm (\rho(770)^0 + \rho(1450)^0) \rightarrow \pi^\pm K^+ K^-$.

from $\rho(770)^0$ for the decay amplitudes of $B^\pm \rightarrow \pi^\pm K^+ K^-$ could probably enhance the experimental result for the $K^+ K^-$ from the resonance $\rho(1450)^0$.

In principle, the interference between $\rho(770)^0$ and $\rho(1450)^0$ can increase or decrease the total contributions from these two resonances. Then we adopt the same expressions and parameters of timelike form factor for the $\rho(770)^0$ and $\rho(1450)^0$ components in Eq. (4), but add a phase θ_K between these two components, namely

$$F_K^{\rho(770+1450)} = \frac{1}{2} [c_{\rho(770)}^K \text{GS}_{\rho(770)} + e^{i\theta_K} c_{\rho(1450)}^K \text{GS}_{\rho(1450)}]. \quad (18)$$

When $\theta_K = 1.3\pi$, we find the largest interferential branching fraction, with $\mathcal{B} \approx 3.64 \times 10^{-7}$ for the $(\rho(770)^0 + \rho(1450)^0) \rightarrow K^+ K^-$ of the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays. The differential branching fraction curves for $B^\pm \rightarrow \pi^\pm (\rho(770)^0 + \rho(1450)^0) \rightarrow \pi^\pm K^+ K^-$ with $\theta_K = \{0, 0.5\pi, \pi, 1.3\pi, 1.5\pi\}$ are shown in the Fig. 3, the red solid line is corresponding to $\theta_K = 1.3\pi$.

The resonance $\omega(782)$, different from $\rho(770)$, has a narrow full width $\Gamma = 8.49 \pm 0.08$ MeV. In spite of the relation $g_{\omega(782)K^+K^-} = g_{\rho(770)^0K^+K^-}$ [44], one can expect a small contribution from the tail of $\omega(782)$ for the $K^+ K^-$ in the $B^\pm \rightarrow \pi^\pm K^+ K^-$ decays. As a result, we predict the branching fraction to be $\mathcal{B}_v \approx 2.32 \times 10^{-8}$ for the decay $B^+ \rightarrow \pi^+ \omega(782) \rightarrow \pi^+ K^+ K^-$, which is much smaller than the branching fractions for the decays $B^\pm \rightarrow \pi^\pm K^+ K^-$ with the subprocesses $\rho(770)^0 \rightarrow K^+ K^-$ and $\rho(1450)^0 \rightarrow K^+ K^-$. The excited state $\rho(1700)^0$ has been studied in the quasi-two-body decays in PQCD approach with $\rho(1700)^0 \rightarrow \pi^+ \pi^-$ [40,42]. With the Eq. (16) and the relation $g_{\rho K^+K^-} \approx \frac{1}{2} g_{\rho \pi^+ \pi^-}$ but the replacement $\rho(1450)^0 \rightarrow \rho(1700)^0$, we estimate the ratio between the

branching fractions of $\rho(1700)^0 \rightarrow K^+ K^-$ and $\rho(1700)^0 \rightarrow \pi^+ \pi^-$ as

$$\begin{aligned} R_{\rho(1700)} &= \frac{\mathcal{B}(\rho(1700)^0 \rightarrow K^+ K^-)}{\mathcal{B}(\rho(1700)^0 \rightarrow \pi^+ \pi^-)} \\ &\approx \frac{g_{\rho(1700)^0 K^+ K^-}^2 (m_{\rho(1700)}^2 - 4m_K^2)^{3/2}}{g_{\rho(1700)^0 \pi^+ \pi^-}^2 (m_{\rho(1700)}^2 - 4m_\pi^2)^{3/2}} \\ &= 0.143, \end{aligned} \quad (19)$$

with $m_{\rho(1700)} = 1.72$ GeV [10]. In consideration of the $\mathcal{B} = 2.81_{-0.66}^{+0.63} \times 10^{-7}$ in [40] for the decay $B^+ \rightarrow \pi^+ \rho(1700)^0 \rightarrow \pi^+ \pi^+ \pi^-$, one could expect a branching fraction $\mathcal{B} \approx 4.02_{-0.94}^{+0.90} \times 10^{-8}$ for the decay $B^+ \rightarrow \pi^+ \rho(1700)^0$ with the subprocess $\rho(1700)^0 \rightarrow K^+ K^-$, which is close to half of the contribution from $\rho(1450)^0 \rightarrow K^+ K^-$. Then we have about 5% of the total branching fraction, but still much less than the $(30.7 \pm 1.2 \pm 0.9)\%$ from LHCb [9], of the three-body decays $B^\pm \rightarrow \pi^\pm K^+ K^-$ from the resonances $\rho(770)^0$, $\rho(1450)^0$, and $\rho(1700)^0$ when neglecting the interferences between them.

IV. SUMMARY

We analyzed the quasi-two-body decays $B^+ \rightarrow \pi^+ \rho(770, 1450)^0 \rightarrow \pi^+ K^+ K^-$ in the PQCD approach. Our results and analyses do not support that large branching fraction contributed by the resonance $\rho(1450)^0$ for the decays $B^\pm \rightarrow \pi^\pm K^+ K^-$. The predictions of the branching fractions are $\mathcal{B} = (8.96 \pm 2.61) \times 10^{-8}$ and $\mathcal{B}_v = (1.31 \pm 0.27) \times 10^{-7}$ for the quasi-two-body decays with the subprocesses $\rho(1450)^0 \rightarrow K^+ K^-$ and $\rho(770)^0 \rightarrow K^+ K^-$, respectively.

The virtual contribution from the BW tail of $\rho(770)^0$ for $K^+ K^-$, which has not been taken into the decay amplitudes of the charmless three-body B meson decays involving kaon pair, was found about 1.5 times of the contribution from the resonance $\rho(1450)^0$ in this work, and is roughly 2.5% of the total branching fraction for the three-body decays $B^\pm \rightarrow \pi^\pm K^+ K^-$.

There is a bump, which is generated by the tail of the BW formula and the phase space factors, at about 1.35 GeV of the invariant mass of kaon pair for $B^\pm \rightarrow \pi^\pm \rho(770)^0 \rightarrow \pi^\pm K^+ K^-$. This bump will disappear and the enhancement round 1.4 GeV will be the result when we investigated the total contributions from $\rho(1450)^0$ together with $\rho(770)^0$. This means that the absence of $\rho(770)^0 \rightarrow K^+ K^-$ in the decay amplitude of a three-body B decays could probably result in a larger proportion for the resonance $\rho(1450)^0$ in experimental amplitude analysis.

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FIG. 3. The differential branching fractions for $B^\pm \rightarrow \pi^\pm (\rho(770)^0 + \rho(1450)^0) \rightarrow \pi^\pm K^+ K^-$ with the phases $\theta_K = \{0, 0.5\pi, \pi, 1.3\pi, 1.5\pi\}$.

- [1] K. S. F. F. Guimarães, I. Bediaga, A. Delfino, T. Frederico, A. C. dos Reis, and L. Tomio, *Nucl. Phys. B, Proc. Suppl.* **199**, 341 (2010).
- [2] P. C. Magalhães, M. R. Robilotta, K. S. F. F. Guimarães, T. Frederico, W. de Paula, I. Bediaga, A. C. dos Reis, C. M. Maekawa, and G. R. S. Zarnauskas, *Phys. Rev. D* **84**, 094001 (2011).
- [3] I. Bediaga and P. C. Magalhães, *arXiv:1512.09284*.
- [4] I. Bediaga, T. Frederico, and O. Lourenço, *Phys. Rev. D* **89**, 094013 (2014).
- [5] G. N. Fleming, *Phys. Rev.* **135**, B551 (1964).
- [6] D. Morgan, *Phys. Rev.* **166**, 1731 (1968).
- [7] D. Herndon, P. Soding, and R. J. Cashmore, *Phys. Rev. D* **11**, 3165 (1975).
- [8] R. H. Dalitz, *Phys. Rev.* **94**, 1046 (1954).
- [9] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **123**, 231802 (2019).
- [10] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018).
- [11] C.-L. Hsu *et al.* (Belle Collaboration), *Phys. Rev. D* **96**, 031101(R) (2017).
- [12] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **99**, 221801 (2007).
- [13] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **124**, 031801 (2020).
- [14] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **101**, 012006 (2020).
- [15] H. Y. Cheng, C. K. Chua, and A. Soni, *Phys. Rev. D* **76**, 094006 (2007).
- [16] H. Y. Cheng and C. K. Chua, *Phys. Rev. D* **88**, 114014 (2013).
- [17] Z. T. Zou, Y. Li, Q. X. Li, and X. Liu, *Eur. Phys. J. C* **80**, 394 (2020).
- [18] Z. Rui, Y. Li, and H. Li, *Eur. Phys. J. C* **79**, 792 (2019).
- [19] D. Boito, J.-P. Dedonder, B. El-Bennich, R. Escobedo, R. Kamiński, L. Leśniak, and B. Loiseau, *Phys. Rev. D* **96**, 113003 (2017).
- [20] C. K. Chua, W. S. Hou, S. Y. Shiao, and S. Y. Tsai, *Phys. Rev. D* **67**, 034012 (2003).
- [21] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **94**, 072001 (2016).
- [22] W. F. Wang and J. Chai, *Phys. Lett. B* **791**, 342 (2019).
- [23] G. Breit and E. Wigner, *Phys. Rev.* **49**, 519 (1936).
- [24] A. J. Pawlicki, D. S. Ayres, D. Cohen, R. Diebold, S. L. Kramer, and A. B. Wicklund, *Phys. Rev. D* **15**, 3196 (1977).
- [25] D. Cohen, D. S. Ayres, R. Diebold, S. L. Kramer, A. J. Pawlicki, and A. B. Wicklund, *Phys. Rev. D* **22**, 2595 (1980).
- [26] R. R. Akhmetshin *et al.* (CMD-2 Collaboration), *Phys. Lett. B* **669**, 217 (2008).
- [27] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **88**, 032013 (2013).
- [28] M. N. Achasov *et al.*, *Phys. Rev. D* **94**, 112006 (2016).
- [29] E. A. Kozyrev *et al.*, *Phys. Lett. B* **779**, 64 (2018).
- [30] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **99**, 032001 (2019).
- [31] J. R. Peláez and A. Rodas, *Eur. Phys. J. C* **78**, 897 (2018).
- [32] Y. Y. Keum, H. n. Li, and A. I. Sanda, *Phys. Lett. B* **504**, 6 (2001).
- [33] Y. Y. Keum, H. n. Li, and A. I. Sanda, *Phys. Rev. D* **63**, 054008 (2001).
- [34] C. D. Lü, K. Ukai, and M. Z. Yang, *Phys. Rev. D* **63**, 074009 (2001).
- [35] H. n. Li, *Prog. Part. Nucl. Phys.* **51**, 85 (2003).
- [36] X. Liu, B. Zhang, L. L. Shen, and S. L. Zhu, *Phys. Rev. D* **75**, 074017 (2007).
- [37] W. F. Wang and H. n. Li, *Phys. Lett. B* **763**, 29 (2016).
- [38] W. F. Wang, J. Chai, and A. J. Ma, *J. High Energy Phys.* **03** (2020) 162.
- [39] W. F. Wang, *Phys. Lett. B* **788**, 468 (2019).
- [40] Y. Li, A. J. Ma, W. F. Wang, and Z. J. Xiao, *Phys. Rev. D* **96**, 036014 (2017).
- [41] Y. Li, A. J. Ma, W. F. Wang, and Z. J. Xiao, *Phys. Rev. D* **95**, 056008 (2017).
- [42] A. J. Ma, Y. Li, W. F. Wang, and Z. J. Xiao, *Phys. Rev. D* **96**, 093011 (2017).
- [43] A. J. Ma, Y. Li, W. F. Wang, and Z. J. Xiao, *Nucl. Phys. B* **923**, 54 (2017).
- [44] C. Bruch, A. Khodjamirian, and J. H. Kühn, *Eur. Phys. J. C* **39**, 41 (2005).
- [45] J. Gasser and H. Leutwyler, *Nucl. Phys. B* **250**, 517 (1985).
- [46] G. J. Gounaris and J. J. Sakurai, *Phys. Rev. Lett.* **21**, 244 (1968).
- [47] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **86**, 032013 (2012).
- [48] H. Czyż, A. Grzelińska, and J. H. Kühn, *Phys. Rev. D* **81**, 094014 (2010).
- [49] K. I. Beloborodov, V. P. Druzhinin, and S. I. Serednyakov, *J. Exp. Theor. Phys.* **129**, 386 (2019).
- [50] A. Ali, G. Kramer, Y. Li, C.-D. Lü, Y.-L. Shen, W. Wang, and Y.-M. Wang, *Phys. Rev. D* **76**, 074018 (2007).
- [51] B. El-Bennich, A. Furman, R. Kamiński, L. Leśniak, and B. Loiseau, *Phys. Rev. D* **74**, 114009 (2006).
- [52] B. El-Bennich, A. Furman, R. Kamiński, L. Leśniak, B. Loiseau, and B. Moussallam, *Phys. Rev. D* **79**, 094005 (2009); **83**, 039903(E) (2011).
- [53] C. Zemach, *Phys. Rev.* **133**, B1201 (1964).
- [54] A. Bazavov *et al.*, *Phys. Rev. D* **98**, 074512 (2018).
- [55] C. Allton *et al.* (RBC-UKQCD Collaboration), *Phys. Rev. D* **78**, 114509 (2008).
- [56] A. Bharucha, D. M. Straub, and R. Zwicky, *J. High Energy Phys.* **08** (2016) 098.
- [57] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. D* **79**, 072006 (2009).
- [58] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **95**, 072007 (2017).