Surface/state correspondence and $T\bar{T}$ deformation

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The surface/state correspondence suggests that the bulk codimensional two surface could be dual to the quantum state in the holographic conformal field theory (CFT). Inspired by the cutoff-anti-de Sitter (AdS)/ $T\bar{T}$ -deformed-CFT correspondence, we propose that the quantum states of the two-dimensional $T\bar{T}$ -deformed holographic CFT are dual to some particular surfaces in the AdS_3 gravity. In particular, the time slice of the cutoff surface is dual to the ground state of the $T\bar{T}$ -deformed CFT. We examine our proposal by studying the entanglement entropy and complexity. We find that the complexity of the ground state in the deformed theory is consistent with the one of a particular cMERA and the holographic complexity via CV or CA prescription.

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I. INTRODUCTION

The study of the black hole thermodynamics inspired 't Hooft and Susskind [1,2] to propose holography as the guiding principle of quantum gravity. One concrete realization of the holographic principle is the AdS/CFT correspondence, which states that quantum gravity theory in anti-de Sitter (AdS) spacetime could be dual to a conformal field theory (CFT) living on the asymptotical AdS boundary [3]. Among various issues in AdS/CFT, the emergence of bulk geometry is outstanding. One promising proposal is the surface/state correspondence [4,5], which claims that any codimension two convex surface Σ corresponds to a quantum state described by a density matrix $\rho(\Sigma)$ in the dual Hilbert space. The proposal is based on the recent development of holographic entanglement entropy [6,7] and an interesting connection between the AdS/CFT and the tensor network [8,9]. The physical picture of the surface/state correspondence is intuitive and has inspired

many new discoveries, such as the holographic entanglement of purification [10] and holographic slice [11]. However, the concrete form of the correspondence is still far from clear.

In this paper, we study the surface/state correspondence in the context of newly discovered cutoff-AdS/ $T\bar{T}$ -deformed-CFT (cAdS/dCFT) correspondence [12]. The study of $T\bar{T}$ deformation was pioneered by Smirnov and Zamolodchikov [13,14]. They studied the deformation triggered by the composite operator $T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2$ in two-dimensional (2D) quantum field theory and found that an integrable quantum field theory is still integrable after the $T\bar{T}$ deformation. More interestingly, it was conjectured that the $T\bar{T}$ -deformed holographic CFT₂ in the large central charge limit could be dual to the cutoff AdS_3 gravity, provided that the sign of the deformation parameter is chosen suitably [12]. For some related works, see [15–20]. As the boundary moves into the bulk, one gets a series of codimension-one hypersurfaces, whose codimension-two time slices are where the quantum states of the $T\bar{T}$ -deformed CFTs dwell. This fact inspires us to propose the surface/state correspondence in the cAdS/dCFT correspondence: these time slices in AdS_3 correspond to the vacuum states of the $T\bar{T}$ -deformed CFTs, while those in the Banados-Teitelboim-Zanelli (BTZ) background correspond to the mixed states at finite temperature. We will examine this proposal quantitatively by studying the entanglement entropy and complexity in both field theory and bulk gravity.

Entanglement entropy plays an important role in the recent study of quantum gravity. The authors in [6,7]

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proposed that the entanglement entropy in holographic CFT can be calculated by the area of the bulk minimum surface anchoring on the boundary entanglement surface. The appearance of a minimal surface, the so-called Ryu-Takayanagi (RT)-surface, encouraged people to conjecture that the spacetime may emerge from the quantum entanglement [21]. Moreover, the analog between tensor network [22] and the RT surface [8] led to the works on the spacetime reconstruction from tensor network [23], which further led to the surface/state correspondence [4,5].

Though entanglement plays a crucial role in studying the dual gravity, it is not enough [24], especially when one tries to understand the interior of the black hole. The complexity of a quantum state was thus introduced as a dual probe to investigate the growth rate of the Einstein-Rosen bridge [24,25]. There have been two conjectures about the holographic complexity: the CV conjecture [26] and CA conjecture [27]. These two conjectures share some similarities but differ in many ways [28,29]. More importantly, how to define a complexity in quantum field theory is still under intense investigations [30–38]. Since we are studying the surface/state correspondence which was inspired from the tensor network, we will adopt the definition from [31,35].

Our study could be seen from another point of view. In [39], the quantum information metric in a CFT with respect to a small marginal deformation has been studied. Here, we study the quantum information metric with respect to an irrelevant deformation.

II. SURFACE/STATE CORRESPONDENCE, $T\bar{T}$ DEFORMATION, AND ENTANGLEMENT

A. Surface/state correspondence

Let us briefly review the surface/state correspondence [4,5]. It claims that any codimension two convex surface Σ corresponds to a quantum state described by a density matrix $\rho(\Sigma)$ in the dual Hilbert space. When this surface is closed and topologically trivial, the state is given by a pure state $|\Phi(\Sigma)\rangle$, as shown in Fig. 1(a). Especially, if Σ is a time slice of the AdS boundary, $|\Phi(\Sigma)\rangle$ is simply the ground state of the dual CFT.

A subregion A of Σ has its own dual state ρ_A , and we can define the entanglement entropy of A as $S_A^\Sigma = -\text{tr}_A \rho_A \log \rho_A$. It was proposed [4,5] that S_A^Σ satisfies the generalized Ryu-Takayanagi formula, $S_A^\Sigma = \frac{\text{Area}(\gamma_A^\Sigma)}{4G_N}$, where γ_A^Σ is a minimal surface in the bulk whose boundary is given by ∂A , as shown in Fig. 1(b).

B. $T\bar{T}$ -deformed CFT

When a two-dimensional quantum field theory is deformed by the $T\bar{T}$ operator, we obtain a family of theories $\mathcal{T}^{(\mu)}$. Moving infinitesimally from $\mathcal{T}^{(\mu)}$ to $\mathcal{T}^{(\mu+\delta\mu)}$, the action changes as

FIG. 1. (a) A codimension two surface Σ corresponds to a quantum state $|\Phi(\Sigma)\rangle$ in surface/state correspondence. (b) Generalized Ryu-Takayanagi formula.

$$S^{(\mu)} \rightarrow S^{(\mu+\delta\mu)} = S^{(\mu)} + \delta\mu \int d^2x O^{(\mu)}, \quad (1)$$

$$O^{(\mu)} \equiv T^{(\mu)} \bar{T}^{(\mu)} - \Theta^{(\mu)2}, \quad (2)$$

where $T^{(\mu)} = -2\pi T_{zz}^{(\mu)}$, $\bar{T}^{(\mu)} = -2\pi T_{\bar{z}\bar{z}}^{(\mu)}$, $\Theta^{(\mu)} = 2\pi T_{z\bar{z}}^{(\mu)}$ are the energy-momentum tensor of theory $\mathcal{T}^{(\mu)}$. The deformation parameter μ has dimension $(\text{length})^2$, and this deformation is irrelevant. The original theory sits at $\mu = 0$. If the quantum field theory is a holographic CFT, the cAdS/dCFT correspondence claims that the deformed theory $\mathcal{T}^{(\mu)}$ is dual to a gravitational theory living in a finite region in AdS_3 with a radial cutoff at $r = r_c$, where the metric can be written as

$$ds^2 = \frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2} g_{\alpha\beta} dx^\alpha dx^\beta, \quad (r \leq r_c). \quad (3)$$

Here, L is the AdS radius. Within our convention, μ and r_c are related by

$$\mu = \frac{6L^4}{\pi c r_c^2}. \quad (4)$$

The theory $\mathcal{T}^{(\mu)}$ lives on the new boundary at $r = r_c$, which is a codimension one surface. The time slice S^{r_c} of the new boundary is a codimension two surface on which the quantum states of $\mathcal{T}^{(\mu)}$ dwell, as shown in Fig. 2.

FIG. 2. Quantum state $|\Omega^{(\mu)}\rangle$ of $\mathcal{T}^{(\mu)}$ lives on S^{r_c} .

C. Entanglement entropy

The similarity between Figs. 1(a) and 2 inspires our proposal: cAdS/dCFT is one kind of realization of the surface/state correspondence. The state corresponding to the surface $\mathcal{S}^{r_c}$ is the quantum state of theory $\mathcal{T}^{(\mu)}$. Especially, when the $T\bar{T}$ deformation is performed on the CFT ground state, the surface $\mathcal{S}^{r_c}$ will correspond to the ground state $|\Omega^{(\mu)}\rangle$ of $\mathcal{T}^{(\mu)}$, which could be expressed as

$$\langle\phi|\Omega^{(\mu)}\rangle = \frac{\int_{\phi(t=0)=\phi} D\phi e^{-\int_{-\infty}^0 dt \int dx \mathcal{L}^{(\mu)}}}{\left(\int D\phi e^{-\int dt \int dx \mathcal{L}^{(\mu)}}\right)^{\frac{1}{2}}}. \quad (5)$$

Here, $\mathcal{L}^{(\mu)}$ is the Euclidean Lagrangian density of $\mathcal{T}^{(\mu)}$; t and x are the Euclidean coordinates. The boundary condition should satisfy $\phi(t=0) = \phi$. Note that through $T\bar{T}$ deformation, we can only obtain the corresponding states of surfaces like $\mathcal{S}^{r_c}$, which comes from the uniformly moving inward of the original boundary. The corresponding states of more general codimension two surfaces are still unknown.

One remarkable point is that in our proposal the surface states parametrized by μ are actually corresponding to the vacuum states of different theories on the boundary, rather than different states in a single theory. In the usual surface/state correspondence, the states corresponding to different codimension two surfaces live in the same state space of a single theory. Conventionally, the parameters ξ_i of the state space characterize the states in a single theory. However, the states $|\Omega^{(\mu)}\rangle$ we considered here are in different theories in a flow parametrized by μ .

In our study, we do not need to require the existence of a BTZ black hole in the bulk. This means that we focus on the ground state of $T\bar{T}$ -deformed holographic CFT at zero temperature. Without the black hole, the metric (3) is invariant under the scaling $r \rightarrow \lambda r, t \rightarrow t/\lambda, x \rightarrow x/\lambda$, and the cutoff theory is invariant under the rescaling $r_c \rightarrow r_c/\lambda, t \rightarrow \lambda t, x \rightarrow \lambda x$. Since $\mu \propto r_c^{-2}$, we see that (μ, t, x) is equivalent to $(\lambda^2\mu, \lambda t, \lambda x)$. We may certainly include a BTZ black hole in the bulk. In this case, the time slice $\mathcal{S}^{r_c}$ corresponds to a mixed state describing the $T\bar{T}$ -deformed CFT at a finite temperature.

In [40], it was shown that the RT formula still holds for cAdS/dCFT to the leading order of the deformation parameter. More works on the entanglement entropy showed that the RT formula still works in cAdS/dCFT [41–47]. Now the holographic entanglement entropy is captured by the length of the geodesic ending on the cutoff surface. This is exactly the generalized RT formula in the surface/state correspondence [5].

It is illuminating to compare the cutoff surfaces with the layers in cMERA. The coarse graining in cMERA is reflected in the quantum states in different layers. On the other hand, the cutoff surfaces corresponding to different

$T\bar{T}$ deformations play a similar role, as shown in the holographic RG flow argument [12]. In this sense, our proposal is similar to the one in [11].

III. SURFACE/STATE CORRESPONDENCE, $T\bar{T}$ DEFORMATION, AND COMPLEXITY

Complexity captures the information beyond entanglement. In this section, we study the complexity of the ground state of $T\bar{T}$ deformed CFT and compare it with the complexity of the cMERA, which has been studied in [31,35]. The holographic complexity of the ground state is studied as well.

We follow the definition of complexity for field theory from [31,35]. For two quantum states, $|\Phi(\xi)\rangle$ and $|\Phi(\xi + d\xi)\rangle$, where $\xi = (\xi_1, \xi_2, \dots, \xi_n)$ are the parameters of the state space, the (squared) Hilbert-Schmidt distance is defined by

$$d_{HS}^2(\xi) \equiv 1 - |\langle\Phi(\xi + d\xi)|\Phi(\xi)\rangle|^2 = g_{ij}d\xi_i d\xi_j, \quad (6)$$

where g_{ij} is the Fisher information metric. For two points $|\Phi(0)\rangle$ and $|\Phi(\lambda_{\max})\rangle$ in the state space, one can find a curve $|\Phi(\lambda)\rangle$ connecting them, which has the minimum length under the Fisher information metric. Supposing the reference state $|\Psi\rangle = |\Phi(0)\rangle$ and the target state $|\Upsilon\rangle = |\Phi(\lambda_{\max})\rangle$, then the complexity between $|\Upsilon\rangle$ and $|\Psi\rangle$ is defined by

$$C_{(|\Psi\rangle, |\Upsilon\rangle)} = \int_0^{\lambda_{\max}} d_{HS}(\lambda) = \int_0^{\lambda_{\max}} \sqrt{g_{\lambda\lambda}} d\lambda. \quad (7)$$

Note that (7) is invariant under the reparametrization of λ .

A. The complexity of cMERA

The definition (7) seems good but difficult to apply due to the very high dimensions of the state space. Fortunately, the situation is simplified for cMERA. The layers in cMERA can be labeled by one parameter u and the corresponding quantum states by $|\Phi(u)\rangle$. We suppose that the curve composed by $|\Phi(u)\rangle$ is exactly the minimum curve for computing the complexity, and the metric along this curve is just the Fisher information metric, even though more choices exist. According to [35], one can write the line element as

$$d_{HS}^2(u) \equiv 1 - |\langle\Phi(u + du)|\Phi(u)\rangle|^2 = \mathcal{N}(u) g_{uu} du^2. \quad (8)$$

Here,

$$\mathcal{N}(u) = \text{Vol}_d \cdot \int_{|k| \leq \Lambda e^u} d^d k \quad (9)$$

is the volume of the effective phase space at u th layer, in which d is the spacial dimension, Vol_d is the volume of R^d ,

k is the momentum, $\Lambda = 1/\epsilon$ is the ultraviolet cutoff, and Λe^u is the momentum cutoff for the u th layer. Using $\mathcal{N}(u)$, one can define a series of complexity of cMERA [31,35] by

$$C^{(n)} = \int \mathcal{N}(u)^{\frac{1}{n}} \sqrt{g_{uu}} du. \quad (10)$$

Note that $C^{(2)}$ is just the complexity defined by Fisher information metric. For the ground state of 1 + 1 dimensional massless free scalar, its cMERA complexity has been worked out as

$$C_{\text{cMERA}}^{(1)} = \frac{\text{Vol}_1 \cdot \Lambda}{2}, \quad C_{\text{cMERA}}^{(2)} = \sqrt{\text{Vol}_1 \cdot \Lambda}, \quad (11)$$

where Λ is the momentum cutoff of cMERA. Λ can also be understood as the momentum cutoff at different layer u in the same cMERA. So we have

$$C_{|\Phi(u)\rangle}^{(1)} = \frac{\text{Vol}_1 \cdot \Lambda e^u}{2}, \quad C_{|\Phi(u)\rangle}^{(2)} = \sqrt{\text{Vol}_1 \cdot \Lambda e^u}, \quad (12)$$

where Λe^u is the momentum cutoff at u th layer.

B. The complexity of the ground state of $T\bar{T}$ deformed CFT

For the $T\bar{T}$ -deformed CFT, the Hilbert-Schmidt distance between the ground states of theories $\mathcal{T}^{(\mu)}$ and $\mathcal{T}^{(\mu+d\mu)}$ is

$$d_{\text{HS}}^2(\mu) \equiv 1 - |\langle \Omega^{(\mu+d\mu)} | \Omega^{(\mu)} \rangle|^2 = \mathcal{N}(\mu) g_{\mu\mu} d\mu^2, \quad (13)$$

where $\mathcal{N}(\mu) = \text{Vol}_1 / \sqrt{\mu}$ with $1/\sqrt{\mu}$ being the momentum cutoff of $\mathcal{T}^{(\mu)}$. Then the complexity of the ground state of $T\bar{T}$ -deformed CFT could be defined as

$$C^{(n)}(\mu) = \int \mathcal{N}(\mu)^{\frac{1}{n}} \sqrt{g_{\mu\mu}} d\mu. \quad (14)$$

The distance (13) can be calculated by using the method in [39]. We consider the $T\bar{T}$ -deformed CFT living on the plane. Then $\langle \Omega^{(\mu+d\mu)} | \Omega^{(\mu)} \rangle$ can be expressed by the path integral,

$$\frac{\int D\phi \exp[-\int dx (\int_{-\infty}^0 dt \mathcal{L}^{(\mu)} + \int_0^{\infty} dt \mathcal{L}^{(\mu+d\mu)})]}{\sqrt{Z^{(\mu)} Z^{(\mu+d\mu)}}}, \quad (15)$$

where $Z^{(\mu)}$ is the partition function of $\mathcal{T}^{(\mu)}$. According to the definition of $T\bar{T}$ deformation, we have

$$\mathcal{L}^{(\mu+d\mu)} - \mathcal{L}^{(\mu)} \equiv \delta\mathcal{L} = d\mu \cdot O^{(\mu)}(t, x). \quad (16)$$

Introducing a UV cutoff ϵ , the ground state becomes

$$|\tilde{\Omega}^{(\mu+d\mu)}(\epsilon)\rangle \equiv \frac{e^{-\epsilon H^{(\mu)}} |\Omega^{(\mu+d\mu)}\rangle}{(\langle \Omega^{(\mu+d\mu)} | e^{-2\epsilon H^{(\mu)}} | \Omega^{(\mu+d\mu)} \rangle)^{1/2}}, \quad (17)$$

where $H^{(\mu)}$ is the Hamiltonian of $\mathcal{T}^{(\mu)}$. The cutoff ϵ depends on μ implicitly $\epsilon \sim \sqrt{\mu}$. Then we get

$$\langle \tilde{\Omega}^{(\mu+d\mu)}(\epsilon) | \Omega^{(\mu)} \rangle = \frac{\langle \exp[-\int_{\epsilon}^{\infty} dt \int dx \delta\mathcal{L}] \rangle}{(\langle \exp[-(\int_{-\infty}^{-\epsilon} + \int_{\epsilon}^{\infty}) dt \int dx \delta\mathcal{L}] \rangle)^{1/2}}, \quad (18)$$

where $\langle \dots \rangle$ denotes the expectation value in the ground state $|\Omega^{(\mu)}\rangle$. Expanding the above formula to the second order of $d\mu$, we have

$$1 - |\langle \tilde{\Omega}^{(\mu+d\mu)}(\epsilon) | \Omega^{(\mu)} \rangle|^2 = d\mu^2 \int_{\epsilon}^{\infty} dt \int_{-\infty}^{-\epsilon} dt' \int dx dx' \langle O^{(\mu)}(t, x) O^{(\mu)}(t', x') \rangle. \quad (19)$$

Here, the time reversal symmetry $\langle \delta\mathcal{L}(-t, x) \delta\mathcal{L}(-t', x') \rangle = \langle \delta\mathcal{L}(t, x) \delta\mathcal{L}(t', x') \rangle$ was used.

The general expression of $\langle O^{(\mu)}(t, x) O^{(\mu)}(t', x') \rangle$ is unknown. However, its behavior can be obtained by dimensional analysis. Since (μ, t, x) is equivalent to $(\lambda^2\mu, \lambda t, \lambda x)$, which is concluded from the analysis of the reparametrization of the gravity side, we find that the correlation function takes the form,

$$\langle O^{(\mu)}(t, x) O^{(\mu)}(t', x') \rangle = \frac{c^2 [1 + \sum_{i=1}^{\infty} a_i (\frac{\mu}{(t-t')^2 + (x-x')^2})^i]}{4((t-t')^2 + (x-x')^2)^4}, \quad (20)$$

where the coefficients a_i are constants that cannot be determined. See Appendix A for details.

Plugging (20) into (19), we get

$$1 - |\langle \tilde{\Omega}^{(\mu+d\mu)}(\epsilon) | \Omega^{(\mu)} \rangle|^2 = \mathcal{N}(\mu) g_{\mu\mu} d\mu^2 = N \cdot \text{Vol}_1 \cdot \mu^{-5/2} d\mu^2, \quad (21)$$

with N a dimensionless constant. Since $\mathcal{N}(\mu) = \text{Vol}_1 / \sqrt{\mu}$, we have $g_{\mu\mu} = N/\mu^2$. Then,

$$C^{(n)}(\mu) = \int \mathcal{N}(\mu)^{\frac{1}{n}} \sqrt{g_{\mu\mu}} d\mu \propto \left(\frac{\text{Vol}_1}{\sqrt{\mu}} \right)^{\frac{1}{n}}. \quad (22)$$

Since $1/\sqrt{\mu}$ is the momentum cutoff of theory $\mathcal{T}^{(\mu)}$, these results are consistent with those in cMERA (11) given $\Lambda \leftrightarrow 1/\sqrt{\mu}$. A different ground state $|\Omega^{(\mu)}\rangle$ in $T\bar{T}$ deformed CFT corresponds to $|\Phi(u)\rangle$ at different layer u in cMERA. This supports our proposal that cAdS/dCFT is a realization of surface/state correspondence.

C. Holographic complexity

There are two conjectures about the holographic complexity: CV conjecture and CA conjecture. For the complexity of a quantum state on the boundary time slice Ξ , the

CV conjecture claims it is proportional to the maximum volume of the spacelike hypersurface B , which meets the boundary on Ξ : $C_V(\Xi) = \max_{\Xi=\partial B} \frac{\mathcal{V}(B)}{G_N L}$, while the CA conjecture claims it is given by $C_A(\Xi) = I_{\text{WdW}}/\pi\hbar$, where I_{WdW} is the on shell action in the Wheeler-de Witt (WdW) patch which is the closure of all spacelike surfaces with boundary Ξ .

1. The CV conjecture

The $T\bar{T}$ deformed CFT living on a two-dimensional plane is dual to the Poincare patch of AdS_3 with a boundary at finite cutoff,

$$ds^2 = -\frac{r^2}{L^2} dt^2 + \frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2} dx^2, \quad (r \leq r_c). \quad (23)$$

The deformed theory $\mathcal{T}^{(\mu)}$ lives on the new boundary $r = r_c$, satisfying (4). Its Penrose diagram is shown in Fig. 3.

We are only interested in the complexity of the ground state $|\Omega^{(\mu)}\rangle$ in the theory $\mathcal{T}^{(\mu)}$. Since $|\Omega^{(\mu)}\rangle$ is just the quantum state at $t = 0$ in theory $\mathcal{T}^{(\mu)}$, the boundary Ξ is at $(t = 0, r = r_c)$. Due to the symmetry, B is the hypersurface with $t = 0$. Its volume is simply $\mathcal{V}(B) = \text{Vol}_1 \cdot r_c$, so the complexity is

$$C_V(|\Omega^{(\mu)}\rangle) = \frac{\text{Vol}_1 \cdot r_c}{G_N L} \propto \frac{\text{Vol}_1}{\sqrt{\mu}}, \quad (24)$$

where the relation (4) was used. Comparing with the result (22) from field theory, we find

$$C_V(|\Omega^{(\mu)}\rangle) \sim C^{(1)}(\mu). \quad (25)$$

This implies $C^{(1)}$ is a more suitable definition of complexity for field theory.

FIG. 3. The WdW patch for the Poincare patch of AdS_3 . We introduced a regularized surface $r = \epsilon$.

2. The CA conjecture

The calculation of CA conjecture is more complicated but straightforward. The corresponding WdW patch is shown in Fig. 3, and the final result is

$$C_A(|\Omega^{(\mu)}\rangle) = \frac{\text{Vol}_1 \cdot r_c}{8\pi^2 \hbar G_N L} \log \frac{\tilde{L}^2}{L^2}. \quad (26)$$

Here, $\tilde{L}$ is a constant introduced to remove the ambiguity associated with the normalization of the tangent vectors of the null boundaries [48]. The details are shown in Appendix B.

Comparing with the result from CV conjecture, we find that they are consistent up to a constant factor $\frac{1}{8\pi} \log \frac{\tilde{L}^2}{L^2}$. We should require $\tilde{L} > L$ to ensure they have the similar behavior. Comparing the result of CA conjecture with that from field theory, we find

$$C_A(|\Omega^{(\mu)}\rangle) \sim C^{(1)}(\mu). \quad (27)$$

Thus, for the ground state of $T\bar{T}$ deformed CFT, the complexity defined by $C^{(1)}$ is consistent with the corresponding holographic CA and CV conjectures.

IV. CONCLUSIONS AND DISCUSSIONS

The surface/state correspondence is an interesting proposal to study the emergence of spacetime. In this paper, we propose that cAdS/dCFT correspondence is a realization of surface/state correspondence. More concretely, when the surface is a time slice of the moved boundary, the corresponding state is just the ground state of the $T\bar{T}$ -deformed CFT. This proposal is novel and reveals new connections in holography. The study of the entanglement entropy of the $T\bar{T}$ -deformed CFT gives us the first evidence for our proposal. We further tested our proposal from the perspective of the complexity. The complexity of the ground state of $T\bar{T}$ -deformed CFT is consistent with that of the cMERA for the ground state of a massless free scalar and also with the holographic complexity.

As a by-product, we find that the complexity of a quantum state might be defined by using the $T\bar{T}$ deformation. To test our proposal, it would be interesting to generalize our study to thermofield double states in the future and check whether it is consistent with the holographic complexity of the eternal BTZ black hole [49,50].

Due to this interesting relation, we also expect that cAdS/dCFT may be useful to study the bulk reconstruction.

Note that the time slice of the moved boundary is just a subset of the surfaces in surface/state correspondence. So the surface/state correspondence is not fully realized by the cAdS/dCFT correspondence. We leave it for the future.

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APPENDIX A

Now let us consider the general $\langle O^{(\mu)}(t, x) O^{(\mu)}(t', x') \rangle$. According to the definition, we have

$$\begin{aligned} \langle O^{(\mu)}(t, x) O^{(\mu)}(t', x') \rangle \\ = \frac{\int D\phi e^{-\int dt dx \mathcal{L}^{(\mu)}} O^{(\mu)}(t, x) O^{(\mu)}(t', x')}{\int D\phi e^{-\int dt dx \mathcal{L}^{(\mu)}}}. \end{aligned} \quad (\text{A1})$$

Here, $\mathcal{L}^{(\mu)}$ is the Lagrangian density of theory $\mathcal{T}^{(\mu)}$. It has dimension $[L^{-2}]$ and the parameter μ has dimension $[L^2]$. Since the original CFT does not include parameters with dimension, μ is the only one has dimension in $\mathcal{L}^{(\mu)}$. Based on the dimension analysis, we suppose that $\mathcal{L}^{(\mu)}$ can be expressed in general as

$$\mathcal{L}^{(\mu)} = \frac{1}{\mu} F(\mu X_1, \mu^2 X_2, \dots, \mu^n X_n). \quad (\text{A2})$$

Here, X_i are quantities that are constructed from operator $\mathcal{O}$ and primary fields. F is a function of dimensionless quantities $\mu X_1, \mu^2 X_2, \dots, \mu^n X_n$. They depend on coordinates implicitly. All examples in [17] satisfy this formal expression.

According to the definition, we have

$$\begin{aligned} O^{(\mu)} &= \frac{\partial \mathcal{L}^{(\mu)}}{\partial \mu} = \frac{1}{\mu^2} \left(\sum_i \frac{\partial F}{\partial (\mu^i X_i)} \cdot i \mu^i X_i - F \right) \\ &\equiv \frac{1}{\mu^2} Q(\mu X_1, \mu^2 X_2, \dots, \mu^n X_n), \end{aligned} \quad (\text{A3})$$

where Q is a dimensionless function of $\mu X_1, \mu^2 X_2, \dots, \mu^n X_n$ and is determined by F . For convenience, we write them as $F(\mu^i X_i)(t, x)$, $Q(\mu^i X_i)(t, x)$ in which the first parentheses stands for the dependence on cutoff μ and the second on coordinates t, x .

Taking the transformation $t \rightarrow \tilde{t} = \lambda t$, $x \rightarrow \tilde{x} = \lambda x$, the primary field ϕ with dimension $-\Delta$ transforms as $\phi(t, x) \rightarrow \tilde{\phi}(\tilde{t}, \tilde{x}) = \lambda^{-\Delta} \phi(t, x)$. Especially, $X_i(t, x) \rightarrow \tilde{X}_i(\tilde{t}, \tilde{x}) = \lambda^{-2i} X_i(t, x)$. The action becomes

$$\begin{aligned} \tilde{S}^{(\mu)} &\equiv \int dt dx \frac{F(\mu^i \tilde{X}_i)(t, x)}{\mu} = \int d\tilde{t} d\tilde{x} \frac{F(\mu^i \tilde{X}_i)(\tilde{t}, \tilde{x})}{\mu} \\ &= \int dt dx \frac{F((\mu/\lambda^2)^i X_i)(t, x)}{(\mu/\lambda^2)} = S^{(\mu/\lambda^2)}. \end{aligned} \quad (\text{A4})$$

One may suspect the above arguments. But this can be settled for the holographic $T\bar{T}$ deformed CFT. According to the cAdS/dCFT, the metric of the Poincare patch of AdS_3 is

$ds^2 = -\frac{r^2}{L^2} dt^2 + \frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2} dx^2$. Under the coordinate transformation $r' = \lambda r, t' = t/\lambda, x' = x/\lambda$, the metric becomes $ds'^2 = -\frac{r'^2}{L^2} dt'^2 + \frac{L^2}{r'^2} dr'^2 + \frac{r'^2}{L^2} dx'^2$. It is obvious that the boundary $(r = r_c, t, x)$ is equivalent to $(r' = r_c, t', x')$. The boundary $(r' = r_c, t', x')$ becomes $(r = r_c/\lambda, \lambda t, \lambda x)$ after coordinate transformation. Thus, the theory is invariant under rescaling $r_c \rightarrow r_c/\lambda, t \rightarrow \lambda t, x \rightarrow \lambda x$. Since $\mu \propto r_c^{-2}$, we see that (μ, t, x) is equivalent to $(\lambda^2 \mu, \lambda t, \lambda x)$. This is consistent with (A4). Here is the detailed version of the argument for the equivalence between (μ, t, x) and $(\lambda^2 \mu, \lambda t, \lambda x)$ via the reparametrization of the gravity side.

From (A1) and (A4), one can derive that

$$\langle O^{(\mu)}(\lambda t, \lambda x) O^{(\mu)}(\lambda t', \lambda x') \rangle = \frac{\langle O^{(\frac{\mu}{\lambda^2})}(t, x) O^{(\frac{\mu}{\lambda^2})}(t', x') \rangle}{\lambda^8}. \quad (\text{A5})$$

This formula relates the $\langle O^{(\mu)}(\lambda t, \lambda x) O^{(\mu)}(\lambda t', \lambda x') \rangle$ at different μ . However, we only know the results at $\mu = 0$. When $\lambda \rightarrow \infty$, there is $\lambda^8 \langle O^{(\mu)}(\lambda t, \lambda x) O^{(\mu)}(\lambda t', \lambda x') \rangle = \langle O^{(0)}(t, x) O^{(0)}(t', x') \rangle = \frac{c^2/4}{((t-t')^2 + (x-x')^2)^4}$. Thus, by dimension analysis, we can get

$$\langle O^{(\mu)}(t, x) O^{(\mu)}(t', x') \rangle = \frac{c^2 [1 + \sum_{i=1}^{\infty} a_i (\frac{\mu}{(t-t')^2 + (x-x')^2})^i]}{4((t-t')^2 + (x-x')^2)^4}, \quad (\text{A6})$$

where the coefficients a_i are constants that cannot be determined by above analysis.

APPENDIX B

The action for CA conjecture including all boundary terms can be written as [48]

$$\begin{aligned} I_{\text{WdW}} &= \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^{d+1}x \sqrt{-g} \left(R + \frac{d(d-1)}{L^2} \right) \\ &\quad + \frac{1}{8\pi G_N} \int_B d^d x \sqrt{|h|} K \\ &\quad - \frac{1}{8\pi G_N} \int_{\mathcal{B}'} d\lambda d^{d-1} \theta \sqrt{\gamma} \kappa + \frac{1}{8\pi G_N} \int_{\Sigma} d^{d-1} x \sqrt{\sigma} a \\ &\quad + \frac{1}{8\pi G_N} \int_{\mathcal{B}'} d\lambda d^{d-1} \theta \sqrt{\gamma} \Theta \log \frac{|\tilde{L}\Theta|}{d-1}. \end{aligned} \quad (\text{B1})$$

Here, $\int_{\mathcal{M}}$ is the bulk integration of the Einstein-Hilbert action and the cosmological constant term. $\int_B$ and $\int_{\mathcal{B}'}$ is the boundary integration of the extrinsic curvature (the Gibbons-Hawking-York term). $\mathcal{B}$ is the d dimensional timelike or spacelike boundary, and $\mathcal{B}'$ is the null boundary. $\int_{\Sigma}$ is the joint term defined on the intersections of the boundaries. The last term is introduced to cancel the ambiguity from reparametrization of λ .

The contribution from the bulk action is

$$I_{\text{bulk}} = \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^{d+1}x \sqrt{-g} \left(R + \frac{d(d-1)}{L^2} \right). \quad (\text{B2})$$

From the Einstein equation, there is $R = -d(d+1)/L^2$. When $d = 2$, the integration becomes

$$I_{\text{bulk}} = \frac{-\text{Vol}}{4\pi G_N L} \left(r_c - 2\epsilon + \frac{\epsilon^2}{r_c} \right), \quad (\text{B3})$$

where Vol is the length of R^1 , the regularization surface is $r = \epsilon$. When $\epsilon \rightarrow 0$, we get

$$I_{\text{bulk}} = \frac{-\text{Vol} \cdot r_c}{4\pi G_N L}. \quad (\text{B4})$$

The WdW patch has three boundaries: two null boundaries and one timelike boundary. Let us consider the timelike boundary $\mathcal{B}$, which lives on $r = \epsilon$ first. Its contribution is

$$I_{\text{GHY}} = \frac{1}{8\pi G_N} \int_{\mathcal{B}} d^d x \sqrt{|h|} K. \quad (\text{B5})$$

Here, h is the induced metric on $\mathcal{B}$ and K is the extrinsic curvature. For the Poincare patch of AdS_3 , the unit normal vector of $\mathcal{B}$ is

$$\mathbf{t} = t_\mu dx^\mu = -\frac{dr}{\sqrt{f(\epsilon)}}. \quad (\text{B6})$$

We assumed that the normal vector points the outside of WdW patch. The extrinsic curvature is

$$K = \frac{t_r}{2} \left(\partial_r f(r) + \frac{2}{r} f(r) \right). \quad (\text{B7})$$

Then we get

$$\begin{aligned} I_{\text{GHY}} &= \frac{1}{8\pi G_N} \int dx \int_{t_{\min}}^{t_{\max}} dt \sqrt{f(r)} \frac{r^2}{L^2} K \Big|_{r=\epsilon} \\ &= -\frac{\text{Vol} \cdot \epsilon^2}{2\pi G_N L} \left(\frac{1}{\epsilon} - \frac{1}{r_c} \right). \end{aligned} \quad (\text{B8})$$

Here, $t_{\max} = -t_{\min} = v_c - r^*(\epsilon)$ is used. Taking $\epsilon \rightarrow 0$, we find

$$I_{\text{GHY}} = 0. \quad (\text{B9})$$

Now let us consider the two null boundaries. We take the future null boundary $\mathcal{B}'_f$, for example. It lives on $v = v_c = -\frac{L^2}{r_c}$. Its contribution is

$$I_{\text{future null surface}} = -\frac{1}{8\pi G_N} \int_{\mathcal{B}'_f} d\lambda d\theta \sqrt{\gamma} \kappa, \quad (\text{B10})$$

where (λ, θ) are the coordinates on $\mathcal{B}'_f$, which was constituted by light rays. θ labels different light rays, and λ is the parameter of each light ray. The points on $\mathcal{B}'_f$ can be expressed as $x^\mu = x^\mu(\lambda, \theta)$. γ is the induced metric, and κ is defined as

$$k^\mu \nabla_\mu k_\nu = \kappa k_\nu, \quad \text{with} \quad k^\mu = \frac{\partial x^\mu}{\partial \lambda}. \quad (\text{B11})$$

If we take $\theta = x$ and $\lambda = r$ as the affine parameter, we get

$$k^\mu = \frac{\partial x^\mu}{\partial \lambda} = \frac{\partial x^\mu}{\partial r} \Rightarrow \mathbf{k} = k_\mu dx^\mu = dv|_{v=v_c}. \quad (\text{B12})$$

It is easy to show that $k^\mu \nabla_\mu k_\nu = 0$, namely, $\kappa = 0$. Thus,

$$I_{\text{future null surface}} = 0, \quad (\text{B13})$$

provided the parameter along the null direction is affine. Similarly, the contribution from the past null boundary is also zero for the affine parameter.

There are three joint terms in the WdW patch here: the two joint terms between the timelike boundary $\mathcal{B}$ and two null boundaries, the joint term between the future null boundary $\mathcal{B}'_f$ and past null boundary $\mathcal{B}'_p$. Their contribution can be written as

$$I_{\text{jnt}} = \frac{1}{8\pi G_N} \int_{\Sigma} d^{d-1}x \sqrt{\sigma} a, \quad (\text{B14})$$

in which σ is the induced metric on intersection Σ and a is defined as

$$a = \begin{cases} \eta \log |k \cdot s|, & \text{for spacelike-null joint,} \\ \eta \log |k \cdot t|, & \text{for timelike-null joint,} \\ \eta \log |k \cdot \tilde{k}/2|, & \text{for null-null joint.} \end{cases} \quad (\text{B15})$$

Here, s_μ , t_μ are the unit normal vector of the spacelike boundary and timelike boundary, respectively. k^μ , $\tilde{k}^\mu$ are tangent vector of the null boundary. The value of η depends on the relative position of the WdW patch, null boundary, and the joint: when the WdW patch appears in the future/past of the null boundary and the joint at the past/future of the null boundary, $\eta = 1$. Otherwise, $\eta = -1$. It is obvious that a depends on the normalization of k^μ and $\tilde{k}^\mu$.

Let us consider the intersection of timelike boundary $\mathcal{B}$ and future null boundary $\mathcal{B}'_f$ first. The unit normal vector of $\mathcal{B}$ is worked out in (B6). From (B12), the tangent vector of $\mathcal{B}'_f$ can be expressed in general as

$$k^\mu = \alpha \cdot \frac{\partial x^\mu}{\partial r} \Rightarrow \mathbf{k} = k_\mu dx^\mu = \alpha \cdot dv|_{v=v_c}, \quad (\text{B16})$$

since $\lambda = r/\alpha + b$ is still the affine parameter. Here, α, b are the free parameters due to the arbitrariness of the normalization of k . Then, we get

$$a = \log |\alpha/\sqrt{f(\epsilon)}| = \log |\alpha L/\epsilon|, \quad (\text{B17})$$

$$I_{\text{jnt}}^1 = \frac{\text{Vol} \cdot \epsilon}{8\pi G_N L} \log |\alpha L/\epsilon|. \quad (\text{B18})$$

Taking $\epsilon \rightarrow 0$, one gets

$$I_{\text{jnt}}^1 = 0. \quad (\text{B19})$$

Similarly, the contribution from the joint term between $\mathcal{B}$ and past null boundary $\mathcal{B}'_p$ is also 0.

Now we consider the joint term between $\mathcal{B}'_f$ and $\mathcal{B}'_p$. The tangent vector of $\mathcal{B}'_p$ can be written as

$$\tilde{\mathbf{k}} = \tilde{k}_\mu dx^\mu = -\beta \cdot du|_{u=u_c}. \quad (\text{B20})$$

Here, β is the free parameter due to the arbitrariness of the normalization of $\tilde{k}$. Then, we get

$$a = -\log |\alpha\beta/f(r_c)| = -\log |\alpha\beta L^2/r_c^2|, \quad (\text{B21})$$

$$I_{\text{jnt}}^{\text{null-null}} = -\frac{\text{Vol} \cdot r_c}{8\pi G_N L} \log |\alpha\beta L^2/r_c^2|. \quad (\text{B22})$$

It is obvious that $I_{\text{jnt}}^{\text{null-null}}$ depends on parameters α and β .

To avoid the unphysical dependence of the result on the normalization of the null vector, the last term is (B1) is introduced,

$$I_{\text{amb}} = \frac{1}{8\pi G_N} \int_{\mathcal{B}'} d\lambda d^{d-1} \theta \sqrt{\gamma} \Theta \log \frac{|\tilde{L}\Theta|}{d-1}. \quad (\text{B23})$$

Here, $\tilde{L}$ is a constant with length dimension. Θ is defined as

$$\Theta = \frac{1}{\sqrt{\gamma}} \frac{\partial \sqrt{\gamma}}{\partial \lambda}. \quad (\text{B24})$$

Let us consider the future null boundary $\mathcal{B}'_f$ first. The affine parameter λ can be taken as $r/\alpha + b$. The corresponding $\Theta = \frac{\alpha}{r}$ and

$$\begin{aligned} I_{\text{amb}}^{\mathcal{B}'_f} &= \frac{1}{8\pi G_N} \int_{\mathcal{B}'_f} d\lambda dx \sqrt{\gamma} \Theta \log |\tilde{L}\Theta| \\ &= \frac{1}{8\pi G_N} \int dx \int_\epsilon^{r_c} \frac{dr}{\alpha} \sqrt{\gamma} \Theta \log |\tilde{L}\Theta| \\ &= \frac{\text{Vol} \cdot r}{8\pi G_N L} \left(1 + \log \left| \frac{\alpha \tilde{L}}{r} \right| \right) \Big|_\epsilon^{r_c}, \end{aligned} \quad (\text{B25})$$

in which we used $d = 2$. Taking $\epsilon \rightarrow 0$, one gets

$$I_{\text{amb}}^{\mathcal{B}'_f} = \frac{\text{Vol} \cdot r_c}{8\pi G_N L} \left(1 + \log \left| \frac{\alpha \tilde{L}}{r_c} \right| \right). \quad (\text{B26})$$

Similarly, the counterterm from past null boundary $\mathcal{B}'_p$ is

$$I_{\text{amb}}^{\mathcal{B}'_p} = \frac{\text{Vol} \cdot r_c}{8\pi G_N L} \left(1 + \log \left| \frac{\beta \tilde{L}}{r_c} \right| \right). \quad (\text{B27})$$

Here, the affine parameter is taken as $\lambda = r/\beta + c$, the corresponding tangent vector is shown in (B20).

Combining all the results above, we finally get

$$I_{\text{wdw}} = \frac{\text{Vol} \cdot r_c}{8\pi G_N L} \log \frac{\tilde{L}^2}{L^2}. \quad (\text{B28})$$

Note that the final result is independent of α, β . The counterterm cancels the dependence on the normalization of the tangent vector of the null boundary. The price is that a new constant $\tilde{L}$ with dimension $[L]$ is introduced.

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