Coming decade of $h \rightarrow \tau\mu$ and $\tau \rightarrow \mu\gamma$ interplay in τ flavor violation search

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We explore the interplay between the $h(125) \rightarrow \tau\mu$ search at the LHC and $\tau \rightarrow \mu\gamma$ at the upcoming Belle II experiment, in the context of the general two Higgs doublet model with extra Yukawa couplings such as $\rho_{\tau\mu}$. The search for $h \rightarrow \tau\mu$ constrains $\rho_{\tau\mu} \cos \gamma$, where $\cos \gamma$ is the h - H mixing angle of h with the exotic CP -even scalar H . For $\tau \rightarrow \mu\gamma$, we define the “BSM benchmark” by setting the extra top Yukawa coupling as $\rho_{tt} = 1 (\cong \lambda_t)$ in two-loop diagrams, and we set $\cos \gamma = 0$ to decouple h . We show that this leading effect, which is due to H and CP -odd scalars A , can be readily probed by Belle II, even for the conservative value of $\rho_{\tau\mu} = 0.7\lambda_\tau$. We define the subleading “ h benchmark” by setting $\rho_{tt} = 0$ in two-loop diagrams, and we take the conservative maximal value of $\cos \gamma = 0.2$. We show that it falls beyond the reach of Belle II but can interfere with the BSM-benchmark effect, which, in principle, probes the phase of ρ_{tt} . We further show that the one-loop H, A effect, proportional to $\rho_{\tau\mu}\rho_{\tau\tau}$ in amplitude, is beyond the sensitivity of Belle II to probe, even for $\rho_{\tau\tau}$ as large as $3\lambda_\tau$. With the working assumption that $\rho_{32}^f, \rho_{33}^f = \mathcal{O}(\lambda_3^f)$ for all charged fermions f , we find a good discovery potential for both τ lepton flavor violation searches in the coming decade.

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I. INTRODUCTION

Ever since the discovery of the muon and an empirical “muon number” that is separate from the electron number, the issue of lepton number violation has been pursued. After extending to the third generation of leptons, the era of the B factory closed with the bound [1],

$$\mathcal{B}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8} \quad (\text{PDG18}), \quad (1)$$

which is from the BABAR experiment [2] and based on $\sim 0.96 \times 10^9$ τ decays. An earlier result from the Belle experiment [3] was 4.5×10^{-8} , based on $\sim 0.48 \times 10^9$ τ decays, but somehow this result has not been updated. The Belle II experiment, which commenced running of B physics, aims at improving the bound by a factor of 100, which we take conservatively as 10^{-9} [4]. Thus, there is the potential for discovery in the coming decade.

The discovery of the 125 GeV scalar boson h [5] completes the last piece of the Standard Model (SM), and it is a triumph of the Large Hadron Collider (LHC). With LHC run 1 data at 8 TeV collision energy, the CMS experiment found [6] an intriguing 2σ hint for the τ lepton

flavor violating (τ LFV) $h \rightarrow \tau\mu$ process, which subsequently disappeared [7] with 13 TeV data at run 2,

$$\mathcal{B}(h \rightarrow \tau\mu) < 0.25\% \quad (\text{CMS18}). \quad (2)$$

Recently, with a similar amount of data at $\sim 36 \text{ fb}^{-1}$, the ATLAS experiment reported [8] a consistent bound of 0.28%. As this is still less than 1/3 of the full run 2 data at hand for each experiment, updates are expected. Furthermore, the scheduled run 3 for 2021–2024 would likely add twice as much data as run 2. Thus, scaling naively by statistics and assuming that ATLAS and CMS would make a combined analysis before the start of the High Luminosity LHC (HL-LHC) targeted for 2028—especially if there is some hint—the limit could reach 0.05%, with a corresponding discovery potential.

Thus, there is much to look forward to in the coming decade on the τ LFV front. This paper aims at elucidating the relevant contributions and parameters of importance, enhancing what has been discussed already.

For $\tau\mu h$ couplings, the framework is a two Higgs doublet model (2HDM) [9] without Z_2 symmetry to forbid flavor changing neutral Higgs (FCNH) couplings; this model was dubbed “model III” [10] (distinct from models I and II under Z_2 symmetry) of the 2HDM. There is a vast amount of theoretical work on τ LFV that we cannot possibly give in detail, and we refer to the recent minireview of Vicente [11]. Here, let us trace some major steps in the phenomenological development.

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FIG. 1. One-loop fermion, two-loop fermion, and two-loop W diagrams for $\tau \rightarrow \mu\gamma$.

The template for discussing $\tau \rightarrow \mu\gamma$ decay can be traced to the work of Chang, Hou, and Keung [12], who studied the $\mu \rightarrow e\gamma$ transition in the context of 2HDM III. The paper stressed that the top contribution to the two-loop Bjorken-Weinberg (or Barr-Zee) mechanism, by bringing in the intrinsically larger extra top Yukawa coupling, can be much larger than the one-loop effect (middle and left diagrams of Fig. 1). One just changes the labels in the formulas from $\mu \rightarrow e$ to $\tau \rightarrow \mu$, as was done in all subsequent work.

The $h \rightarrow \tau\mu$ process was proposed by Han and Marfatia [13] at the start of Tevatron run II, also in the context of 2HDM III. As the Tevatron era came to an end, and at the dawn of the LHC, Davidson and Grenier [14] took interest in $h \rightarrow \tau\mu$ at colliders and emphasized the link with the $\tau \rightarrow \mu\gamma$ bound from B factories as an important constraint. However, the work was oriented towards the lepton perspective. Extending earlier and more general work [15], the authors defined $\tan\beta_\tau = \rho_{\tau\tau}/\lambda_\tau$, where $\rho_{\tau\tau}$ is the extra diagonal τ Yukawa coupling and $\lambda_\tau = \sqrt{2}m_\tau/v$ ($v \cong 246$ GeV) is the τ Yukawa coupling of the SM, and used $\tan\beta_\tau$ in place of the familiar $\tan\beta$ of the 2HDM with Z_2 (e.g., the well-known 2HDM II). Knowing that without Z_2 symmetry, $\tan\beta$, as the ratio of VEVs of the two Higgs doublets, is not a physical parameter, the authors sought a substitute in language and usage, but it should be clear that the ratio of Yukawa couplings is a quite different matter. The authors further extended $\tan\beta_\tau$ into the quark sector, which is a strong assumption. Adopting this assumption, the early work of Aristizabal Sierra and Vicente [16] addressing the CMS hint of $h \rightarrow \tau\mu$ excess [6] allowed $\tan\beta_\tau = \rho_{\tau\tau}/\lambda_\tau$ to be as large as 40; i.e., the extra τ Yukawa coupling could be almost half the strength of the top Yukawa coupling. We will not take this lepton-biased view and instead let extra top Yukawa couplings be independent parameters.

The CMS study [6] showed an excess and was in fact inspired by the work of Harnik, Kopp, and Zupan [17]. While using the formulas of Ref. [12] as usual to study the $\tau \rightarrow \mu\gamma$ constraint on the $\tau\mu h$ coupling, they showed that a direct search for $h \rightarrow \tau\mu$ at the LHC would quickly become more sensitive. However, the paper used the language of Cheng and Sher [18], which was also adopted in the CMS papers. While capturing the mass-mixing hierarchy suppression (model III [10]) of the FCNH for low energy processes, the Cheng-Sher ansatz missed one element, that

the FCNH couplings are associated with the exotic (non-mass-giving) Higgs doublet and would enter the coupling of the SM-like h to, e.g., $\tau\mu$ by the h - H mixing angle between the two CP -even scalars. Thus, the $\tau\mu h$ coupling reads as $\rho_{\tau\mu} \cos(\beta - \alpha)$, where, for the time being, we retain the familiar notation of 2HDM II.

The latter approach was adopted by Omura, Senaha, and Tobe [19] in correlating the $h \rightarrow \tau\mu$ excess with predictions for $\tau \rightarrow \mu\gamma$, where they entertained $\rho_{\tau\mu}$, $\rho_{\tau\tau}$ up to $10\lambda_\tau$ for $c_{\beta-\alpha} \equiv \cos(\beta - \alpha) \simeq 0.01$. The point is that when the CMS excess disappears with more data, it could simply be due to the smallness of $c_{\beta-\alpha}$ (the phenomenon of alignment), rather than demanding $\rho_{\tau\mu}$ to be small. In other words, the proposed search [20] for $H, A \rightarrow \tau\mu$ (where A is the pseudoscalar) is not suppressed by alignment, or small $c_{\beta-\alpha}$. The process has already been searched for by CMS [21], setting bounds.

We have mentioned quite a few parameters in our retracing of the development of $h \rightarrow \tau\mu$ and $\tau \rightarrow \mu\gamma$ decay in the literature. The main goal of this paper is to elucidate the relevant vs less relevant parameters, as the coming decade unfolds for the search for these two important τ LFV processes, to clarify the landscape. Another motivation for this work arose from the recent $H, A \rightarrow \tau\mu$ study [20], where constraints on ρ_{tt} (an extra top Yukawa coupling that enters $\tau \rightarrow \mu\gamma$ at two loops) and $\rho_{\tau\mu}$ from, e.g., $\tau \rightarrow \mu\gamma$ were extracted by assuming ρ_{tt} to be real, for simplicity. While this is a common, prevailing assumption, a couple of years earlier, and before the hint for $h \rightarrow \tau\mu$ evaporated, it was pointed out [22] that the complexity of ρ_{tt} could drive the baryon asymmetry of the Universe (BAU). With such big issues at stake, this paper explores the possible effect of $\varphi_{tt} = \arg \rho_{tt}$, which has not been explored before. We call the 2HDM III, or the 2HDM without Z_2 symmetry and where extra Yukawa couplings are allowed, the general 2HDM (g2HDM).

II. PARAMETERS AND FORMULAS IN THE GENERAL 2HDM

In this paper, we take the masses of the physical CP -even scalars h, H , the CP -odd scalar A , and the charged scalar $H^\pm$ as given, and are not concerned with details of the Higgs potential, which can be found, e.g., in Ref. [23]. The Yukawa couplings are [15,23]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{\sqrt{2}} \sum_{f=u,d,\ell} \bar{f}_i [(\lambda_i^f \delta_{ij} s_\gamma + \rho_{ij}^f c_\gamma) h \\ & + (\lambda_i^f \delta_{ij} c_\gamma - \rho_{ij}^f s_\gamma) H - \text{sgn}(Q_f) \rho_{ij}^f A] R f_j \\ & - \bar{u}_i [(V \rho^d)_{ij} R - (\rho^{u\dagger} V)_{ij} L] d_j H^+ \\ & - \bar{\nu}_i \rho_{ij}^\ell R \ell_j H^+ + \text{H.c.}, \end{aligned} \quad (3)$$

where i, j are generation indices that are summed over, $L, R = (1 \mp \gamma_5)/2$ are projection operators, and V is the Cabibbo-Kobayashi-Maskawa matrix. Due to the very near degeneracy of the neutrinos for our processes, the corresponding matrix in the lepton sector is taken as unity. The shorthand notation of $c_\gamma \equiv \cos \gamma$ (and $s_\gamma \equiv \sin \gamma$) is the h - H mixing angle, which corresponds to the usual $\cos(\beta - \alpha)$ in 2HDM II nomenclature. The emergent *alignment* phenomenon, that h so closely resembles the SM Higgs boson [24], implies that c_γ is rather small. But we do not quite know its value, which is especially true in the g2HDM, where more parameters exist compared with the 2HDM II. In the alignment limit of $c_\gamma \rightarrow 0$, the couplings of h , including to vector bosons, approach the SM. But as shown in Ref. [23], small c_γ need not imply small Higgs quartic couplings. Thus, the prerequisite [22] of $\mathcal{O}(1)$ Higgs quartics, for the first order electroweak phase transition for generating BAU, can be sustained.

The off-diagonal coupling $\rho_{\tau\mu}$ (and $\rho_{\mu\tau}$) enters the $\tau \rightarrow \mu\gamma$ and $h \rightarrow \tau\mu$ processes of interest. Note that the first FCNH parameter studied directly at the LHC is ρ_{tc} via $t \rightarrow ch$ decay [1], which was pointed out already in Ref. [10] and reemphasized [25] shortly after the $h(125)$ discovery. Whether $h \rightarrow \tau\mu$ or $t \rightarrow ch$, the SM-like h boson picks up the FCNH coupling via a factor of c_γ , or h - H mixing. In hindsight, as discussed in Ref. [23], the alignment phenomenon that emerged with full run 1 data can account for the absence of $t \rightarrow ch$ and $h \rightarrow \tau\mu$, without the need for overly suppressing extra FCNH Yukawa couplings ρ_{tc} or $\rho_{\tau\mu}$. But since

$$\mathcal{B}(h \rightarrow \tau\mu) = \frac{m_h c_\gamma^2}{16\pi\Gamma_h} (|\rho_{\tau\mu}|^2 + |\rho_{\mu\tau}|^2), \quad (4)$$

the bound of Eq. (2) places the constraint

$$|\rho_{\tau\mu} c_\gamma| \lesssim 0.0014 \simeq 0.14 \lambda_\tau, \quad (5)$$

where we have taken $|\rho_{\mu\tau}| = |\rho_{\tau\mu}|$ to simplify. The two chiral couplings do not interfere.

As elucidated by Davidson and Grenier [14] (from the template of Ref. [12] for $\mu \rightarrow e\gamma$), there are three distinct types of diagrams contributing to $\tau \rightarrow \mu\gamma$: the one-loop diagram that pairs the necessary FCNH $\rho_{\tau\mu}$ coupling with a diagonal τ Yukawa coupling, be it the λ_τ of the SM or the extra $\rho_{\tau\tau}$; the two-loop Bjorken-Weinberg/Barr-Zee type of diagrams with a top Yukawa, be it λ_t or ρ_{tt} ; and the

two-loop W diagram. The three type of diagrams are illustrated in Fig. 1. The H^+ effect is unimportant. In these diagrams, we have labeled the vertices with the compact notation of Ref. [19] (similar to Davidson and Grenier), $-y_{\phi ij}^f \bar{f}_i R f_j \phi$ (H.c. implied) for $f = u, d, \ell$ and $\phi = h, H, A$, where $y_{\phi ij}^f$ can be read off from Eq. (3). One can now see that the two-loop mechanism constitutes an insertion of $\phi \rightarrow \gamma\gamma$ (we neglect the Z contribution), which is similar to the gg fusion production of ϕ , hence connecting with the $h \rightarrow \tau\mu$ and $H, A \rightarrow \tau\mu$ searches.

The branching fraction for $\tau \rightarrow \mu\gamma$ can be written as

$$\frac{\mathcal{B}(\tau \rightarrow \mu\gamma)}{\mathcal{B}(\tau \rightarrow \mu\nu\bar{\nu})} = \frac{48\pi^3\alpha}{G_F^2} (|A_L|^2 + |A_R|^2), \quad (6)$$

where $\mathcal{B}(\tau \rightarrow \mu\nu\bar{\nu}) = 17.39\%$ [1], and the chiral amplitudes A_L and A_R , which do not interfere, contribute equally under our simplifying assumption of $\rho_{\tau\mu} = \rho_{\mu\tau}$. The A_L amplitude corresponding to the three types of diagrams in Fig. 1 are (three separate sums)

$$\begin{aligned} A_L \simeq & \sum_{\phi=h,H,A} \frac{\hat{y}_{\phi\tau\mu}^* \hat{y}_{\phi\tau\tau}^*}{8\pi^2 v^2} x_{\tau\phi} (\log x_{\phi\tau} - 3/2) \\ & - \sum_{\phi=h,H,A} \frac{N_c Q_f^2 \alpha}{4\pi^3 v^2} \hat{y}_{\phi\tau\mu}^* [\text{Re}(\hat{y}_{\phi tt}) F_H(x_{t\phi}) - i \text{Im}(\hat{y}_{\phi tt}) F_A(x_{t\phi})] \\ & + \sum_{\phi=h,H} \frac{\alpha \tilde{g}_{\phi WW} \hat{y}_{\phi\tau\mu}^*}{32\sqrt{2}\pi^3 v^2} \{12 F_H(x_{W\phi}) + 23 F_A(x_{W\phi}) \\ & + 3G(x_{W\phi}) + 2x_{\phi W} [F_H(x_{W\phi}) - F_A(x_{W\phi})]\}, \end{aligned} \quad (7)$$

where $\hat{y}_{\phi\tau j} = y_{\phi\tau j}/\lambda_\tau$ (and likewise $\hat{y}_{\phi tt} = y_{\phi tt}/\lambda_t$), $x_{ab} = m_a^2/m_b^2$, N_c is the number of colors, $\tilde{g}_{hWW} = s_\gamma$, $\tilde{g}_{HWW} = c_\gamma$, and the loop functions F_H , F_A , and G can be found in, e.g., Ref. [12]. We include only the $\phi\gamma\gamma$ vertex contributions and neglect $\phi Z\gamma$ vertex terms, as these are suppressed by $(1 - 4\sin^2 \theta_W)$, which amounts to $\sim 10\%$ variation in our results. The b and τ contributions in the second sum are suppressed by loop functions, as $x_{b\phi}$ and $x_{\tau\phi}$ are rather small.

We find that the extra Yukawa couplings can always be normalized against the Yukawa couplings in the SM, namely,

$$\hat{\rho}_{3j}^f = \rho_{3j}^f / \lambda_3^f, \quad (8)$$

and perhaps nature hints at such “normalization.” After all, the extra Yukawa matrix ρ^f can be viewed as the orthogonal combination of two Yukawa matrices with respect to the mass matrix. Along this line, throughout this work, we take

$$\rho_{32}^f, \rho_{33}^f = \mathcal{O}(\lambda_3^f) \quad (9)$$

FIG. 2. Comparison of the three benchmark scenarios (see text for details). Black curves are for the degenerate case of $m_H = m_A$. Red (blue) curves show variation in $m_H(m_A)$, with the other scalar mass $m_A(m_H)$ heavier by 10, 100, 200 GeV.

as our working assumption, which is the most reasonable one without tuning, given that λ_3^f and ρ^f emerge from the procedure of diagonalizing the mass matrix. The two Yukawa matrices should share the mass-mixing hierarchy structure [23]. For this reason, we illustrate, with $\rho_{\tau\mu}$ and $\rho_{\tau\tau}$, values not exceeding $3\lambda_\tau$. We remark that $\hat{\rho}_{\tau\tau}$ is precisely $\tan\beta_\tau$ as defined in Ref. [14], up to a sign. But Eq. (9) should make it clear that, while $\hat{\rho}_{\tau\tau}$ and $\hat{\rho}_{tt}$ are both $\mathcal{O}(1)$, their actual values could differ by an order of magnitude and should be determined by experiment.

Thus, besides scalar masses, the parameters that enter are $\rho_{\tau\mu}$ (overall and factorized), $\rho_{\tau\tau}$ (one-loop), ρ_{tt} (two-loop), and the h - H mixing parameter c_γ . Although c_γ is expected to be small, its uncertain value is relevant in bringing in the extra Yukawa couplings of h that can interfere with the leading two-loop top effect, as we now elucidate. We conservatively take $c_\gamma = 0.2$ as its maximal value.

III. (LESS) RELEVANT CONTRIBUTIONS

Having clarified the natural setting of $\rho_{\tau\mu} = \mathcal{O}(\lambda_\tau)$ and $|c_\gamma| \lesssim 0.2$, we see that the $h \rightarrow \tau\mu$ search at the LHC would continue to probe this space. Still, as there are multiple parameters that enter $\tau \rightarrow \mu\gamma$, one needs to discern relevant from less relevant parameters and processes. It is well known [12] that, as long as $\rho_{tt} \sim \lambda_t \cong 1$ [Eq. (9)], the two-loop mechanism is by far the leading effect. But what about the other two types of diagrams in Fig. 1? We propose two benchmarks to elucidate the leading and subleading effects, clarifying that, in contrast to the much larger $\rho_{\tau\mu}$, $\rho_{\tau\tau}$ values taken in the past, the one-loop diagram cannot really be probed by Belle II under the rule of thumb of Eq. (9).

We define the “BSM benchmark” as setting $c_\gamma = 0$ in the two-loop mechanism to decouple h , and take $\rho_{tt} = 1$, as

larger values tend to run into flavor constraints [20,26], which we shall not explore in detail here. This benchmark captures the BSM effect from extra top Yukawa couplings of H , A , and could stand alone in the alignment limit, when the ρ_{tt} phase no longer matters. We plot the $\sqrt{\mathcal{B}(\tau \rightarrow \mu\gamma)}$ in Fig. 2, where we set $\rho_{\tau\mu} = \rho_{\mu\tau} = 0.7\lambda_\tau$ (which will be clarified below), which is conservative. The current bound on $\tau \rightarrow \mu\gamma$ is the shaded region, while the (conservatively) projected Belle II limit of 10^{-9} is the horizontal solid line. It is interesting that, even for the conservative value of $\rho_{\tau\mu} = 0.7\lambda_\tau$, this BSM benchmark can itself be readily probed by Belle II.

Conversely, if we set ρ_{tt} to zero, then the leading two-loop effect vanishes, but the two-loop top mechanism still has an amplitude proportional to $\lambda_t c_\gamma$ coming from the h boson, and similarly through the two-loop W diagram, also with c_γ dependence. We combine these m_h -dependent effects and call it the h benchmark; its $\sqrt{\mathcal{B}(\tau \rightarrow \mu\gamma)}$ is also plotted in Fig. 2 as the dotted line, taking the conservative maximal value of $c_\gamma = 0.2$, which implies $\rho_{\tau\mu} = 0.7\lambda_\tau$ as maximally allowed by Eq. (5). We see from Fig. 2 that, if it is a stand-alone case, this h benchmark is out of the reach of Belle II. This actually does not depend on detailed values of c_γ or $\rho_{\tau\mu}$, but it depends only on the bound of Eq. (5), which follows from Eq. (2), the current bound [7] on $h \rightarrow \tau\mu$. This is because the h benchmark is also proportional to $|\rho_{\tau\mu} c_\gamma|^2$. Thus, the CMS bound on $h \rightarrow \tau\mu$ excludes the possibility of observing the two-loop effect without the extra top Yukawa coupling ρ_{tt} . We enlarge this branching ratio region and display it in Fig. 2 (right panel), which can be used to understand our numerical discussion in the next section.

We see from Fig. 2 that, if one has relatively light extra neutral scalars ($\lesssim 300$ GeV), then the effect from the BSM

benchmark tends to predominate. However, as the extra scalar mass increases, say, beyond 500–600 GeV, on one hand, it would require a larger fraction of full Belle II data to probe; on the other hand, the interference between the BSM benchmark and h benchmark becomes important. As the latter is real in amplitude, the phase $\varphi_{tt} = \arg \rho_{tt}$ is important, as is the value of $|\rho_{tt}|$, which affects the extra Higgs two-loop effect, and the value of c_γ , which controls the effect of h .

Finally, we show the $\sqrt{\mathcal{B}(\tau \rightarrow \mu\gamma)}$ of the one-loop τ effect in Fig. 2, where $\rho_{\tau\tau}$ can also carry a phase, and we take the nominally largest value of $|\rho_{\tau\tau}| = 3\lambda_\tau$ that satisfies Eq. (9). It is known that the H and A effects strongly cancel each other when degenerate (black dashed curve), but the cancellation weakens when degeneracy is lifted. We give three sets of dashed curves, where red (blue) corresponds to m_H (m_A) on the real axis, with the other neutral scalar being heavier by 10, 100, 200 GeV (this is also done for the two-loop BSM benchmark, where the effect is minor). For a given scalar m_H (m_A) mass, the one-loop effect varies by more than 1 order of magnitude as the splitting increases. In general, the amplitude is far below even the h benchmark, except for rather light scalars ($\lesssim 300$ GeV). Thus, we see that Belle II would not have the ability to probe the one-loop τ contribution, as it is more than a nuisance effect. In the next section, we neglect the one-loop effect in our illustrations, as it just smears the projections at small $|\rho_{tt}|$, but it cannot be discerned by Belle II.

IV. INTERPLAY OF $h \rightarrow \tau\mu$ AND $\tau \rightarrow \mu\gamma$

In Fig. 2, we have exhibited the BSM benchmark, which illustrates the two-loop effect from H and A with near maximal $|\rho_{tt}| = 1$, and the h benchmark, which illustrates the two-loop effect of h with near maximal $c_\gamma = 0.2$. The strength $|\rho_{tt}|$ (and phase φ_{tt}) and value of c_γ (proximity to alignment limit) together determine the strength of interference between the leading and subleading effects. We have shown that the one-loop τ effect is less than subleading, which we ignore in the following numerical illustration.

Of course, the strength of $\rho_{\tau\mu}$ determines the overall scale for the branching fraction, as it factorizes and one cannot probe its phase. Together with c_γ , $|\rho_{\tau\mu}|$ is constrained by the bound on $\mathcal{B}(h \rightarrow \tau\mu)$, Eq. (5). For instance, our near maximal value of $c_\gamma = 0.2$ for the h benchmark allows only $\rho_{\tau\mu} \lesssim 0.7\lambda_\tau$, while $c_\gamma = 0.1, 0.05$ can allow the larger ranges of $\rho_{\tau\mu} \lesssim 1.4\lambda_\tau, 2.8\lambda_\tau$, respectively. For the alignment limit case of $c_\gamma = 0$, one recovers the BSM benchmark, which scales with $|\rho_{\tau\mu}\rho_{tt}|^2$ and can be read off from Fig. 2.

To illustrate the interference effect between the leading H, A and subleading h contributions and the role played by φ_{tt} , we plot in Fig. 3 the future reach of Belle II data at 3, 10, and 50 ab^{-1} in the $|\rho_{tt}|$ - φ_{tt} plane, for the three values of $\rho_{\tau\mu} = 0.7\lambda_\tau, 1.4\lambda_\tau, 2.8\lambda_\tau$, respectively. As seen from

Fig. 2, the BSM benchmark does not depend strongly on m_H - m_A splitting, so we use a common $m_{H,A}$ mass value, taken as 300 and 500 GeV. It is illustrated, e.g., in Ref. [27] that large parameter space in the Higgs potential is allowed by the electroweak precision T parameter and other considerations.

Let us start with the upper left plot in Fig. 3, which is for the conservative value of $\rho_{\tau\mu} = 0.7\lambda_\tau$ and relatively light $m_H, m_A \simeq 300$ GeV. From Fig. 2, one can easily understand that the current bound on $\tau \rightarrow \mu\gamma$, Eq. (1), does not put a constraint on the displayed parameter space, but it can be probed as data accumulates at Belle II, where the three sets of curves correspond to 3, 10, and 50 ab^{-1} . Each set of curves is further illustrated with three curves that correspond to $c_\gamma = 0.05, 0.1, 0.2$ allowed by Eq. (5), i.e., the bound from Eq. (2). The curves are all of similar shape, and the dependence on φ illustrates the interference of H, A with the h effects, which is richer than the real value of ρ_{tt} assumed in Ref. [20]. For the larger value of $\rho_{\tau\mu} = 1.4\lambda_\tau$, $c_\gamma = 0.2$ becomes excluded, so we illustrate this with two curves for each projected data value. The smaller c_γ means the h effect is reduced; hence, the interference weakens, while the current bound of Eq. (1) starts to cut into the $|\rho_{tt}|$ parameter space as an effect of the BSM benchmark. For the near maximal $\rho_{\tau\mu} = 2.8\lambda_\tau$, only the small $c_\gamma = 0.05$ is allowed; hence, we show only one curve for each data value in the right figure, and the current bound of Eq. (1) now cuts deeper into $|\rho_{tt}|$ parameter space.

The lower plots of Fig. 3 are for heavier $m_H, m_A = 500$ GeV; hence, the contribution from the BSM benchmark is weakened, resulting in stronger interference due to the relative importance of the h -benchmark contribution. For $\rho_{\tau\mu}$ at the conservative $0.7\lambda_\tau$, $\tau \rightarrow \mu\gamma$ does not yet start to probe the $|\rho_{tt}|$ parameter space even with 3 ab^{-1} . For $\rho_{\tau\mu} = 1.4\lambda_\tau$, the current bound is still ineffective, but 3 ab^{-1} would cut into $|\rho_{tt}|$ parameter space; for the relatively large $\rho_{\tau\mu} = 2.8\lambda_\tau$, even the current bound excludes some $|\rho_{tt}|$ parameter space.

Our figures project the discovery potential of $\tau \rightarrow \mu\gamma$ by Belle II, as constrained by $h \rightarrow \tau\mu$ under our working assumption of $\rho_{\tau\mu} = \mathcal{O}(\lambda_\tau) \sim 0.01$. The parameter space is substantial, so long as $\rho_{\tau\mu}/\lambda_\tau$ is not far below 1 and the extra Higgs mass scale does not approach decoupling.

V. DISCUSSION AND SUMMARY

The constraint of Eq. (5), which arises from a $h \rightarrow \tau\mu$ search at the LHC, should improve in the next couple of years when the full run 2 data are analyzed. It would likely drop further, which would imply that our h -benchmark line in Fig. 2 would drop. This means the interference effect, as exhibited in Fig. 3, would shrink further, and one would have less access to the phase φ_{tt} . However, it is not impossible that a hint emerges for $h \rightarrow \tau\mu$, which would

FIG. 3. For $m_{H,A} = 300$ (500) GeV, the upper (lower) plots are for the 3, 10, and 50 ab^{-1} Belle II data reach, plotted in the $|\rho_{tt}|$ - ϕ_{tt} plane. For the lower $\rho_{\tau\mu} = 0.7\lambda_\tau$ value, three curves for the allowed c_γ values are illustrated, which reduce to just one low c_γ value for the larger $\rho_{\tau\mu} = 2.8\lambda_\tau$. The shaded region is excluded by Eq. (1). See text for further discussion.

suggest that neither $\rho_{\tau\mu}$ nor c_γ vanishes, and it would heighten the interest in a $\tau \rightarrow \mu\gamma$ search at Belle II. Assuming there is no hint for the signal, combining the full run 2 + 3 data set of ATLAS and CMS and scaling naively by statistics, one can probe down to 0.05%, compared with 0.25% in Eq. (5). One would then be close to the BSM-benchmark scenario. If we are rather close to the alignment limit, then the constraint on $\rho_{\tau\mu}$ is alleviated, with $\tau \rightarrow \mu\gamma$ probing $|\rho_{\tau\mu}\rho_{tt}|^2$, and Belle II would still have a wide discovery potential.

It should be noted that exotic Higgs bosons as light as 300 GeV are *not* ruled out [28]. There is in fact a mild hint for a pseudoscalar A around 400 GeV [29], interfering with the $gg \rightarrow t\bar{t}$ QCD background. It could be caused by the gg fusion production and decay of A via ρ_{tt} , or even H that is produced via a purely imaginary ρ_{tt} coupling [30]. The exotic Higgs spectrum for the g2HDM is largely unknown, but 300 to 600 GeV is a preferred target zone, if [23] the inertial mass scale of the second (non-mass-giving) doublet is not far above the weak scale, which would be the tuned case of decoupling. Besides $gg \rightarrow H, A \rightarrow t\bar{t}$ [30], $t\bar{t}$ [26], proposed

searches such as $cg \rightarrow tH/A \rightarrow t\bar{t}\bar{c}, t\bar{t}\bar{t}$ [31] and the recently proposed $cg \rightarrow bH^+$ [27] process give rise to signatures of same-sign top with jets, triple top, and single top with two b jets. Some of these processes have also been discussed in [32,33]. In particular, if the mass scale is below 400 GeV, we hope to be able to find the mass spectrum in the coming years. Note that the three signatures above all require sizable ρ_{tc} for production, which is in line with our working assumption of Eq. (9). Furthermore, ρ_{tc} at $\mathcal{O}(1)$ can *also* drive BAU [22]. Thus, the program is well motivated.

We have illustrated that the discovery potential at Belle II does not actually depend on whether a hint for $h \rightarrow \tau\mu$ emerges at the LHC, which is in part regulated by the strength of c_γ . The actual value of c_γ , however, may be hard to extract. Although ATLAS and CMS have fitted for $\cos(\beta - \alpha)$ in the context of the 2HDM II [24,34], with many more parameters in the g2HDM, such a fit may not be feasible until we know more about some parameters related to the extra Higgs, such as the mass spectrum. We have conservatively taken the maximal value of 0.2 for c_γ , but we do not view $c_\gamma = 0.3$ as ruled out in the g2HDM.

Processes that do not depend on c_γ , such as electroweak baryogenesis, i.e., generating BAU, are therefore of interest. On Earth, we note that H^+ and A couplings do not depend on c_γ . Thus, $B^- \rightarrow \mu^- \bar{\nu}$, where the flavor of $\bar{\nu}$ is not detected, probes the product of $\rho_{t\mu}\rho_{\tau\mu}$ [35]. Although we do not advocate that $\rho_{t\mu}$ should also satisfy some relation similar to Eq. (9), we have rather poor knowledge of its value. Reference [35] suggests that the ratio of $\mathcal{B}(B \rightarrow \mu\bar{\nu})/\mathcal{B}(B \rightarrow \tau\bar{\nu})$ in the g2HDM may deviate from the SM expectation of 0.0045, a value that is shared by the 2HDM II. If such a result is found, which could emerge relatively early with Belle II, it would imply nonvanishing $\rho_{\tau\mu}$ and hence would also heighten the interest in $\tau \rightarrow \mu\gamma$ (as well as the pursuit of the tuh coupling). One could also probe $\rho_{\tau\mu}$ via searching for heavy $H, A \rightarrow \tau\mu$ [20]. While such a search is clearly worth exploring [21], it goes against the branching ratio suppression due to the likely dominance of $t\bar{t}$ and $t\bar{c}$ decay modes in the g2HDM. Although we do not think that Belle II could effectively probe $\rho_{\tau\tau}$ through the one-loop $\tau \rightarrow \mu\gamma$ effect, $\rho_{\tau\tau}$ can be probed at the LHC, in principle, both via deviations from the SM rate for $h \rightarrow \tau\tau$ by $h-H$ mixing and by a search for heavy $H, A \rightarrow \tau\tau$ [36,37], but the result might not be better than the $\tau\mu$ final state.

In summary, we analyzed the outlook for the τ LFV search via the $h \rightarrow \tau\mu$ and $\tau \rightarrow \mu\gamma$ processes, which appears quite promising in the general 2HDM. The $h \rightarrow \tau\mu$ process probes the product $\rho_{\tau\mu}c_\gamma$, where $\rho_{\tau\mu}$ is the extra flavor changing neutral Higgs coupling and c_γ is the CP -even Higgs mixing angle, which is expected to be small because of the phenomenon of alignment. But whether or not a hint emerges with run 2 + 3 data, our working assumption that $\rho_{\tau\mu} = \mathcal{O}(\lambda_\tau)$ and $\rho_{tt} = \mathcal{O}(\lambda_t)$ makes $\tau \rightarrow \mu\gamma$ very interesting at Belle II, with a broad parameter range for discovery. If nature provides a finite c_γ that is on the larger side, on one hand, it increases the likelihood that $h \rightarrow \tau\mu$ may emerge; on the other hand, the interference of h with H, A effects in $\tau \rightarrow \mu\gamma$ decay, in principle, probes the phase of ρ_{tt} . We look forward to the unfolding of these two search modes in the coming decade.

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