Heavy exotic scalar meson $T_{bb;\bar{u}\bar{s}}^-$

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The spectroscopic parameters and decay channels of the scalar tetraquark $T_{bb;\bar{u}\bar{s}}^-$ (in what follows $T_{b;\bar{s}}^-$) are investigated in the framework of the QCD sum rule method. The mass and coupling of the $T_{b;\bar{s}}^-$ are calculated using the two-point sum rules by taking into account quark, gluon and mixed vacuum condensates up to dimension 10. Our result for its mass $m = (10250 \pm 270)$ MeV demonstrates that $T_{b;\bar{s}}^-$ is stable against the strong and electromagnetic decays. Therefore to find the width and mean lifetime of the $T_{b;\bar{s}}^-$, we explore its dominant weak decays generated by the transition $b \rightarrow W^- c$. These channels embrace the semileptonic decay $T_{b;\bar{s}}^- \rightarrow Z_{bc;\bar{u}\bar{s}}^0 l \bar{\nu}_l$ and nonleptonic modes $T_{b;\bar{s}}^- \rightarrow Z_{bc;\bar{u}\bar{s}}^0 \pi^- (K^-, D^-, D_s^-)$, which at the final state contain the scalar tetraquark $Z_{bc;\bar{u}\bar{s}}^0$. Key quantities to compute partial widths of the weak decays are the form factors $G_1(q^2)$ and $G_2(q^2)$: they determine differential rate $d\Gamma/dq^2$ of the semileptonic and partial widths of the nonleptonic processes, respectively. These form factors are extracted from relevant three-point sum rules at momentum transfers q^2 accessible for such analysis. By means of the fit functions $F_{1(2)}(q^2)$ they are extrapolated to cover the whole integration region $m_l^2 \leq q^2 \leq (m - \bar{m})^2$, where $\bar{m}$ is the mass of $Z_{bc;\bar{u}\bar{s}}^0$. Predictions for the full width $\Gamma_{\text{full}} = (15.21 \pm 2.59) \times 10^{-10}$ MeV and mean lifetime $4.33_{-0.63}^{+0.89} \times 10^{-13}$ s of the $T_{b;\bar{s}}^-$ are useful for experimental and theoretical investigations of this exotic meson.

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I. INTRODUCTION

Investigation of exotic mesons that are composed of four quarks (tetraquarks) is among the interesting topics of the high energy physics. Experimental information collected during last years by various collaborations and theoretical progress achieved in the framework of different methods and models form rapidly growing field of exotic studies [1–5].

The states observed in experiments till now and interpreted as candidates to exotic mesons have different natures. Thus, some of them are neutral charmonium (bottomonium)-like resonances and may be considered as excited states of the charmonium. Others bear an electric charge and are free of these problems, but reside close to two-meson thresholds permitting an interpretation as bound states of conventional mesons or dynamical effects. It is worth noting that all of the discovered tetraquarks have large full widths and decay strongly to two conventional

mesons. Therefore, four-quark compounds stable against strong and electromagnetic interactions, and decaying only through weak transformations can provide valuable information on tetraquarks.

The stability of the tetraquarks $QQ'\bar{q}\bar{q}'$ (in what follows denoted as $T_{QQ';\bar{q}\bar{q}'}$) were studied already in original articles [6–9], in which it was proved that a heavy $Q^{(\prime)}$ and light $q^{(\prime)}$ quarks may form the stable exotic mesons provided the ratio m_Q/m_q is large enough. In fact, the isoscalar axial-vector tetraquark $T_{bb;\bar{u}\bar{d}}^-$ with the mass lower than the $B\bar{B}^*$ threshold is a strong-interaction stable state [9].

These problems were addressed in numerous later publications using for investigations various approaches, including the chiral, the dynamical, and the relativistic quark models. Computational tools employed in these investigations encompassed all diversity of methods available in the high energy physics. Thus, quark models were used in Refs. [10–14] to explore features and calculate parameters of the states T_{QQ} . These tetraquarks were analyzed in the framework of the QCD two-point sum rule method, as well [15,16]. The masses of the axial-vector states $T_{bb;\bar{u}\bar{d}}^-$ and $T_{cc;\bar{u}\bar{d}}^+$ were extracted in Ref. [15]. In accordance with results of this work, the mass of the tetraquark $T_{bb;\bar{u}\bar{d}}^-$ amounts to 10.2 ± 0.3 GeV, which is

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below the open bottom threshold. In other words, this particle is stable against strong decays. Parameters of the states $QQ\bar{q}\bar{q}$ with the spin-parity $J^P = 0^-, 0^+, 1^-$ and 1^+ were found in the framework of the sum rule method in Ref. [17]. There are publications in the literature devoted to investigation of production mechanisms of the tetraquarks T_{cc} in the heavy ion and proton-proton collisions, in electron-positron annihilations, in B_c meson and heavy Ξ_{bc} baryon decays, and to analysis of their possible decay channels [18–22].

The discovery of the doubly charmed baryon $\Xi_{cc}^{++} = ccu$ by the LHCb Collaboration [23] generated new studies of double-heavy tetraquarks [24–31]. Investigations prove that double-charm exotic mesons are unstable against the strong and electromagnetic decays. Thus, in Ref. [24] it was shown that, the mass of the axial-vector tetraquark $T_{cc\bar{u}\bar{d}}^+$ is equal to (3882 ± 12) MeV, which is above thresholds for decays to $D^0 D^{*+}$ and $D^0 D^+ \gamma$ final states. The states $T_{cc;\bar{s}\bar{s}}^{++}$ and $T_{cc;\bar{d}\bar{s}}^{++}$ that belong to the class of doubly charged tetraquarks were investigated in our article [32]. Performed analysis demonstrated that, masses of these four-quark compounds are above the $D_s^+ D_{s0}^{*+}$ (2317) and $D^+ D_{s0}^{*+}$ (2317) thresholds, and they can decay to these conventional mesons. The widths of these strong decays, evaluated also in Ref. [32], allowed us to classify the tetraquarks $T_{cc;\bar{s}\bar{s}}^{++}$ and $T_{cc;\bar{d}\bar{s}}^{++}$ as relatively broad resonances.

The double-beauty tetraquarks $bb\bar{q}\bar{q}'$ are particles of special interest, because in this case the ratio $m_b/m_{q(q')}$ reaches its maximum value, and they may form stable compositions. Indeed, the mass of the axial-vector state $T_{bb;\bar{u}\bar{d}}^-$ was reevaluated in Ref. [24] using a phenomenological model and experimental information of the LHCb collaboration [23]. In accordance with results of this work the mass of the isoscalar axial-vector state $T_{bb;\bar{u}\bar{d}}^-$ equals to $m = (10389 \pm 12)$ MeV which is 215 MeV below the $B^-\bar{B}^{*0}$ threshold and 170 MeV below the threshold for decay $B^-\bar{B}^0\gamma$. This means that the tetraquark $T_{bb;\bar{u}\bar{d}}^-$ is stable against the strong and electromagnetic decays and transforms to ordinary mesons only through weak processes. The conclusion about the strong-interaction stability of the tetraquarks $T_{bb;\bar{u}\bar{d}}^-$, $T_{bb;\bar{u}\bar{s}}^-$, and $T_{bb;\bar{d}\bar{s}}^0$ was made in Ref. [26] on the basis of the relations extracted from heavy-quark symmetry. The mass $m = 10482$ MeV of the axial-vector tetraquark $T_{bb;\bar{u}\bar{d}}^-$ found there is 121 MeV below the open-bottom threshold.

In Ref. [33] we computed the spectroscopic parameters of the axial-vector tetraquark $T_{bb;\bar{u}\bar{d}}^-$ by means of the QCD sum rule method. Our result for the mass of this particle $m = (10035 \pm 260)$ MeV confirmed once more that it is stable against the strong and electromagnetic decays. In this paper, we evaluated also the total width and mean lifetime of $T_{bb;\bar{u}\bar{d}}^-$ using its semileptonic decay channels (see also

Ref. [34]). The predictions $\Gamma = (7.17 \pm 1.23) \times 10^{-8}$ MeV and $\tau = 9.18_{-1.34}^{+1.90}$ fs provide information useful for experimental investigation of the double-beauty exotic mesons.

The axial-vector four-quark systems $qq'\bar{Q}\bar{Q}$, where Q is one of the heavy b or c quarks and q, q' are light quarks were explored in Ref. [31]. In this work, the authors considered the octet-octet $[8_c]_{q\bar{Q}} \otimes [8_c]_{q'\bar{Q}}$ and singlet-singlet $[1_c]_{q\bar{Q}} \otimes [1_c]_{q'\bar{Q}}$ color configurations and calculated masses of these tetraquarks by means of the QCD sum rule method. Obtained predictions for the masses of the octet-octet tetraquarks $ud\bar{b}\bar{b}$ and $us\bar{b}\bar{b}$ are above corresponding two-meson thresholds, and hence these states can decay through strong interactions. The molecular or color singlet-singlet tetraquarks with masses (10360 ± 150) MeV for $ud\bar{b}\bar{b}$ and (10480 ± 150) MeV for $us\bar{b}\bar{b}$ seem are stable particles.

It turned out that not only exotic mesons containing bb diquarks, but also tetraquarks built of bc may be stable against the strong and electromagnetic decays (see Refs. [24,26,33,35,36]). Thus, analysis of Ref. [33] proved that the scalar tetraquark $Z_{bc;\bar{u}\bar{d}}^0$ has the mass $m_Z = (6660 \pm 150)$ MeV, which is considerably below thresholds for strong and electromagnetic decays. In other words, $Z_{bc;\bar{u}\bar{d}}^0$ transforms due to weak decays that allowed us to estimate in Ref. [36] its full width and mean lifetime. A situation with the axial-vector tetraquark $T_{bc;\bar{u}\bar{d}}^0$ remains unclear: the mass of this state predicted in the range (1705 ± 155) MeV admits twofold explanations [37]. Indeed, using the central value of the mass one see that it lies below thresholds for the strong and electromagnetic decays, whereas the maximum estimate for the mass 7260 MeV is higher than thresholds for strong and electromagnetic decays to $B^{*-} D^+ / \bar{B}^{*0} D^0$ and $D^+ B^- \gamma / D^0 \bar{B}^0 \gamma$, respectively. In the first case the width and lifetime of the tetraquark $T_{bc;\bar{u}\bar{d}}^0$ are determined by its weak decays. In the second scenario the width of $T_{bc;\bar{u}\bar{d}}^0$ is fixed mainly by strong modes, because widths of weak and electromagnetic processes are small and can be ignored [37].

It is worth noting that some of heavy exotic mesons containing diquarks bs may be stable as well. Thus, the scalar tetraquark $T_{bs;\bar{u}\bar{d}}^-$ is strong- and electromagnetic-interaction stable particle: its spectroscopic parameters and semileptonic decays were explored in Ref. [38].

In the present article we study the scalar tetraquark $T_{b;\bar{s}}^-$ with the quark content $bb\bar{u}\bar{s}$ and compute its spectroscopic parameters, full width and mean lifetime. The mass m and coupling f of $T_{b;\bar{s}}^-$ are extracted from the QCD two-point sum rules by taking into account vacuum expectation values of the local quark, gluon, and mixed operators up to dimension ten. The information on the mass of this state is crucial to determine whether $T_{b;\bar{s}}^-$ is strong- and electromagnetic-interaction stable particle or not. It is not difficult to see that dissociation to a pair of conventional pseudoscalar mesons $B^-\bar{B}_s^0$ is the first S -wave strong decay

channel for the unstable $T_{b:\bar{s}}^-$. Therefore if the mass of $T_{b:\bar{s}}^-$ is higher than the $B^-\bar{B}_s^0$ threshold 10646 MeV then one should calculate the width of the process $T_{b:\bar{s}}^- \rightarrow B^-\bar{B}_s^0$. But, our investigations show (see below) that the mass of the tetraquark $T_{b:\bar{s}}^-$ is equal to $m = (10250 \pm 270)$ MeV, and lies below this bound. The $T_{b:\bar{s}}^-$ is stable against the possible electromagnetic transition $T_{b:\bar{s}}^- \rightarrow B^-\bar{B}_{s1}(5830)\gamma$ as well, because for realization of this process the mass of the master particle should exceed 11108 MeV which is not a case. Therefore to evaluate the full width and lifetime of $T_{b:\bar{s}}^-$ one has to explore its weak decays.

The weak transformations of the $T_{b:\bar{s}}^-$ may run due to the subprocesses $b \rightarrow W^-c$, and $b \rightarrow W^-u$ which generate its semileptonic dissociation to scalar four-quark mesons $Z_{bc;\bar{u}\bar{s}}^0$ (hereafter $Z_{b:\bar{s}}^0$) and $Z_{bu;\bar{u}\bar{s}}^0$. The process $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 l\bar{\nu}_l$ is dominant weak channel for $T_{b:\bar{s}}^-$, because the decay $T_{b:\bar{s}}^- \rightarrow Z_{bu;\bar{u}\bar{s}}^0 l\bar{\nu}_l$ is suppressed relative to first one by a factor $|V_{ub}|^2/|V_{cb}|^2 \approx 0.01$ with $|V_{q_1q_2}|$ being the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

But the subprocess $b \rightarrow W^-c$ can also give rise to nonleptonic weak decays of the $T_{b:\bar{s}}^-$. Indeed, the vector boson W^- instead of a lepton pair $l\bar{\nu}_l$ can produce $d\bar{u}$, $s\bar{u}$, $d\bar{c}$, and $s\bar{c}$ quarks as well. These quarks afterwards form one of the conventional mesons $M = \pi^-, K^-, D^-$ and D_s^- leading to the nonleptonic final states $Z_{b:\bar{s}}^0 M$. Depending on the difference $m - \tilde{m}$, where $\tilde{m}$ is the $Z_{b:\bar{s}}^0$ tetraquark's mass, some or all of these nonleptonic weak decays become kinematically allowed.

We calculate the full width of the $T_{b:\bar{s}}^-$ by taking into account its semileptonic and nonleptonic decay modes. To this end, employing the QCD three-point sum rule approach, we determine the weak form factors $G_{1(2)}(q^2)$ necessary to evaluate the differential rates of the semileptonic decays. Partial width of the processes $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 l\bar{\nu}_l$, $l = e^-, \mu^-$ and τ^- can be found by integrating the differential rates over kinematically allowed momentum transfers q^2 , whereas width of the nonleptonic decays $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 M$ are fixed by values of the $G_{1(2)}(q^2)$ at $q^2 = m_M^2$, where m_M is the mass of a produced meson.

This article is structured in the following way: In Section II, we calculate the spectroscopic parameters of the scalar tetraquarks $T_{b:\bar{s}}^-$ and $Z_{b:\bar{s}}^0$. For these purposes, we derive two-point sum rules from analysis of corresponding correlation functions and include into calculations the quark, gluon and mixed condensates up to dimension ten. In Section III, we derive three-point sum rules for the weak form factors $G_{1(2)}(q^2)$ and compute them in regions of the momentum transfer, where the method gives reliable predictions. We extrapolate $G_{1(2)}(q^2)$ to the whole integration region by means of fit functions and find partial widths of the semileptonic decays $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 l\bar{\nu}_l$, where $l = e^-, \mu^-$ and τ^- . In Section IV we analyze the nonleptonic weak decays $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 M$ of the tetraquark $T_{b:\bar{s}}^-$.

Here we also present our final estimate for the full width and mean lifetime of the $T_{b:\bar{s}}^-$. Section V is reserved for discussion and concluding notes. Appendix contains explicit expressions of quark propagators, and the correlation function used to evaluate parameters of the tetraquark $T_{b:\bar{s}}^-$.

II. MASS AND COUPLING OF THE SCALAR TETRAQUARKS $T_{b:\bar{s}}^-$ AND $Z_{b:\bar{s}}^0$

The spectroscopic parameters of the tetraquark $T_{b:\bar{s}}^-$ are necessary to reveal its nature and answer questions about its stability. The mass and coupling of $Z_{b:\bar{s}}^0$ are important to explore the weak decays of the master particle $T_{b:\bar{s}}^-$. It is worth noting that the $T_{b:\bar{s}}^-$ and $Z_{b:\bar{s}}^0$ have the same heavy diquark-light antidiquark organization.

The parameters of these states can be extracted from the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle, \quad (1)$$

where $J(x)$ is the interpolating current for a scalar particle. It is known that interpolating currents for hadrons, including exotic mesons, should be colorless constructions. A diquark QQ' can belong either to color-antitriplet $[\bar{3}_c]_{QQ'}$ or color-sextet $[6_c]_{QQ'}$ representation of the group $SU_c(3)$. Accordingly, an antidiquark $\bar{q}\bar{q}'$ has triplet or antisextet color structures. Colorless currents of exotic mesons should have color organizations $[\bar{3}_c]_{QQ'} \otimes [3_c]_{\bar{q}\bar{q}'}$ or $[6_c]_{QQ'} \otimes [\bar{6}_c]_{\bar{q}\bar{q}'}$. The color and flavor antisymmetric scalar diquarks are most attractive and stable two-quark structures [39]. But, because the heavy diquark bb in $T_{b:\bar{s}}^-$ contains two quarks of the same flavor, relevant diquark field has to be symmetric in color indices, i.e., belong to sextet representation of the color group. The interpolating current for $T_{b:\bar{s}}^-$ built of color-sextet scalar diquark and antidiquark fields has the following form [17]

$$J(x) = [b_a^T(x) C \gamma_5 b_b(x)] [\bar{u}_a(x) \gamma_5 C \bar{s}_b^T(x)], \quad (2)$$

where a , and b are color indices and C is the charge-conjugation operator. The second term $[b_a^T(x) C \gamma_5 b_b(x)] \times [\bar{u}_a(x) \gamma_5 C \bar{s}_b^T(x)]$ in $J(x)$ is equal to the one presented in Eq. (2), therefore we use this compact expression for the interpolating current.

The final-state tetraquark $Z_{b:\bar{s}}^0 = bc\bar{u}\bar{s}$ may have color-antisymmetric $[\bar{3}_c]_{bc} \otimes [3_c]_{\bar{u}\bar{s}}$ or symmetric $[6_c]_{bc} \otimes [\bar{6}_c]_{\bar{u}\bar{s}}$ interpolating currents. Our calculations demonstrate that the tetraquark $T_{b:\bar{s}}^-$ decays weakly only to $Z_{b:\bar{s}}^0$ with color-sextet constituents: a matrix element for weak transition to color-antisymmetric state $bc\bar{u}\bar{s}$ vanishes identically. In other words, in weak transitions of $T_{b:\bar{s}}^-$ to $Z_{b:\bar{s}}^0$ color structures of their constituents remain unchanged. This is

true, at least, for tetraquarks under analysis and for currents employed to interpolate them. Therefore, for $Z_{b:\bar{s}}^0$, we choose also $[\mathbf{6}_c]_{bc} \otimes [\bar{\mathbf{6}}_c]_{\bar{u}\bar{s}}$ type current using information from Ref. [40]

$$\tilde{J}(x) = [b_a^T(x)C\gamma_5 c_b(x)][\bar{u}_a(x)\gamma_5 C\bar{s}_b^T(x) + \bar{u}_b(x)\gamma_5 C\bar{s}_a^T(x)]. \quad (3)$$

The exotic mesons with internal organizations (2) and (3) are ground-state particles with color-symmetric diquarks.

Here, we consider in a detailed form computation of the $T_{b:\bar{s}}^-$ tetraquark's mass m and coupling f , and provide final results for $Z_{b:\bar{s}}^0$. To derive the sum rules for m and f , we need first to find the phenomenological expression of the correlation function $\Pi^{\text{Phys}}(p)$, which should be written down in terms of the spectroscopic parameters of $T_{b:\bar{s}}^-$. Since $T_{b:\bar{s}}^-$ is a ground-state particle, we use the “ground-state + continuum” scheme. Then separating contribution of the tetraquark $T_{b:\bar{s}}^-$ from effects of the higher resonances and continuum states, we can write

$$\Pi^{\text{Phys}}(p) = \frac{\langle 0|J|T_{b:\bar{s}}^-(p)\rangle\langle T_{b:\bar{s}}^-(p)|J^\dagger|0\rangle}{m^2 - p^2} + \dots \quad (4)$$

The phenomenological function $\Pi^{\text{Phys}}(p)$ is obtained by inserting into $\Pi(p)$ a full set of scalar four-quark states and performing integration over x .

Calculation of $\Pi^{\text{Phys}}(p)$ can be finished by employing the matrix element

$$\langle 0|J|T_{b:\bar{s}}^-(p)\rangle = fm. \quad (5)$$

After simple manipulations we get

$$\Pi^{\text{Phys}}(p) = \frac{f^2 m^2}{m^2 - p^2} + \dots \quad (6)$$

The correlation function $\Pi^{\text{Phys}}(p)$ has a trivial Lorentz structure which is proportional to $\sim I$. Hence, the only term in Eq. (6) is nothing more than the invariant amplitude $\Pi^{\text{Phys}}(p^2)$ corresponding to this structure.

Now, we have to fix the second component of the sum rule analysis, and express $\Pi(p)$ in terms of the quark propagators. To this end, we utilize the explicit expression of the interpolating current $J(x)$, and contract relevant heavy and light quark fields to get $\Pi^{\text{OPE}}(p)$. After these manipulations, we find

$$\begin{aligned} \Pi^{\text{OPE}}(p) = i \int d^4x e^{ipx} \text{Tr}[\gamma_5 \tilde{S}_s^{b'b}(-x) \gamma_5 S_u^{a'a}(-x)] \\ \times \{ \text{Tr}[\gamma_5 \tilde{S}_b^{aa'}(x) \gamma_5 S_b^{bb'}(x)] \\ + \text{Tr}[\gamma_5 \tilde{S}_b^{ba'}(x) \gamma_5 S_b^{ab'}(x)] \}, \end{aligned} \quad (7)$$

where $S_b(x)$ and $S_{u(s)}(x)$ are the b - and $u(s)$ -quark propagators, respectively. Here we also use the shorthand notation

$$\tilde{S}_{b(u,s)}(x) = C S_{b(u,s)}^T(x) C. \quad (8)$$

The propagators of heavy and light quarks used in the present work are collected in the Appendix. The non-perturbative part of these propagators contains vacuum expectation values of various quark, gluon, and mixed operators which generate a dependence of $\Pi^{\text{OPE}}(p)$ on nonperturbative quantities.

To derive the sum rules, we equate the amplitudes $\Pi^{\text{Phys}}(p^2)$ and $\Pi^{\text{OPE}}(p^2)$, and apply to both sides of the obtained equality the Borel transformation. This operation is necessary to suppress contributions of higher resonances and continuum states. Afterwards, we carry out the continuum subtraction using the assumption on the quark-hadron duality. The expression found by this way, and an equality obtained by applying the operator $d/d(-1/M^2)$ to the first one form a system which is enough to obtain the sum rules for m

$$m^2 = \frac{\Pi(M^2, s_0)}{\Pi'(M^2, s_0)}, \quad (9)$$

and f

$$f^2 = \frac{e^{m^2/M^2}}{m^2} \Pi(M^2, s_0). \quad (10)$$

In Eqs. (9) and (10) $\Pi(M^2, s_0)$ is the Borel-transformed and subtracted invariant amplitude $\Pi^{\text{OPE}}(p^2)$, and $\Pi'(M^2, s_0)$ equals to

$$\Pi'(M^2, s_0) = \frac{d}{d(-1/M^2)} \Pi(M^2, s_0). \quad (11)$$

In the case under discussion $\Pi(M^2, s_0)$ has the following form

$$\Pi(M^2, s_0) = \int_{\mathcal{M}}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2} + \Pi(M^2), \quad (12)$$

where $\mathcal{M} = 2m_b + m_s$. The $\rho^{\text{OPE}}(s)$ is the two-point spectral density, whereas second component of the invariant amplitude $\Pi(M^2)$ includes nonperturbative contributions calculated directly from $\Pi^{\text{OPE}}(p)$. Explicit expression of $\Pi(M^2, s_0)$ is presented in Appendix.

The sum rules for m and f depend on the Borel and threshold parameters M^2 and s_0 , which appear after the Borel transformation and continuum subtraction procedures, respectively. Both of M^2 and s_0 are the auxiliary parameters a proper choice of which depends on the

problem under analysis, and is one of the important problems in the sum rule computations.

Apart from M^2 and s_0 , the sum rules contain also the universal vacuum condensates and the mass of b and s quarks:

$$\begin{aligned}\langle \bar{q}q \rangle &= -(0.24 \pm 0.01)^3 \text{ GeV}^3, & \langle \bar{s}s \rangle &= 0.8 \langle \bar{q}q \rangle, \\ \langle \bar{q}g_s \sigma G q \rangle &= m_0^2 \langle \bar{q}q \rangle, & \langle \bar{s}g_s \sigma G s \rangle &= m_0^2 \langle \bar{s}s \rangle, \\ m_0^2 &= (0.8 \pm 0.1) \text{ GeV}^2, \\ \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle &= (0.012 \pm 0.004) \text{ GeV}^4, \\ \langle g_s^3 G^3 \rangle &= (0.57 \pm 0.29) \text{ GeV}^6, & m_s &= 93_{-5}^{+11} \text{ MeV}, \\ m_c &= 1.27 \pm 0.2 \text{ GeV}, & m_b &= 4.18_{-0.02}^{+0.03} \text{ GeV}.\end{aligned}\quad (13)$$

The working windows for the auxiliary parameters M^2 and s_0 have to satisfy some essential constraints. Thus, at maximum of M^2 the pole contribution (PC) should exceed a fixed value, which for the multiquark systems is chosen in the form

$$\text{PC} = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)} > 0.2. \quad (14)$$

The minimum of M^2 is extracted from analysis of the ratio

$$R(M^2) = \frac{\Pi^{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)} \leq 0.01. \quad (15)$$

Fulfilment of Eq. (15) implies the convergence of the operator product expansion (OPE) and obtained sum rules. Here, $\Pi^{\text{DimN}}(M^2, s_0)$ denotes a contribution to the correlation function coming from the last term (or a sum of last few terms) in the expansion. In the present calculations we

use a sum of last three terms, and hence DimN means $\text{Dim}(8 + 9 + 10)$.

The numerical analysis proves that the working regions for the parameters M^2 and s_0

$$M^2 \in [9, 12] \text{ GeV}^2, \quad s_0 \in [115, 120] \text{ GeV}^2, \quad (16)$$

satisfy all aforementioned constraints on M^2 and s_0 . Namely, at $M^2 = 12 \text{ GeV}^2$ the pole contribution is 0.22, whereas at $M^2 = 9 \text{ GeV}^2$ it amounts to 0.56. These two values of M^2 determine the boundaries of a window within of which the Borel parameter can be varied. At the minimum of $M^2 = 9 \text{ GeV}^2$ we get $R \approx 0.001$. Apart from that, at the minimum of M^2 the perturbative contribution amounts to 85% of the whole result overshooting significantly the nonperturbative terms.

Our results for m and f are

$$\begin{aligned}m &= (10250 \pm 270) \text{ MeV}, \\ f &= (2.69 \pm 0.58) \times 10^{-2} \text{ GeV}^4,\end{aligned}\quad (17)$$

where we indicate also uncertainties of the computations. These theoretical errors stem mainly from variation of the parameters M^2 and s_0 within allowed limits. It is seen, that for the mass these uncertainties equal to $\pm 2.6\%$ of its central value, whereas for the coupling f they are larger and amounts to $\pm 22\%$. In other words, the result for the mass is less sensitive to the choice of the parameters than the coupling f . The reason is that the sum rule for the mass (9) is given as a ratio of two integrals of the function $\rho^{\text{OPE}}(s)$ which stabilizes undesired effects, but even in the situation with the coupling f uncertainties do not exceed limits accepted in sum rule computations. In Fig. 1 we plot the sum rule's prediction for m as a function of the parameters M^2 and s_0 , where one can see its residual dependence on them.

FIG. 1. The mass m of the tetraquark $T_{b;s}^-$ as a function of the Borel M^2 (left panel) and continuum threshold s_0 parameters (right panel).

 FIG. 2. The same as in Fig. 1, but for the mass $\tilde{m}$ of the tetraquark $Z_{b:\bar{s}}^0$.

The mass and coupling of the scalar tetraquark $Z_{b:\bar{s}}^0$ are calculated by the same way. The QCD side of relevant sum rules is given by the following formula

$$\begin{aligned} \tilde{\Pi}^{\text{OPE}}(p) = & i \int d^4x e^{ipx} \text{Tr}[\gamma_5 \tilde{S}_b^{aa'}(x) \gamma_5 S_c^{bb'}(x)] \\ & \times \{ \text{Tr}[\gamma_5 \tilde{S}_s^{b'b}(-x) \gamma_5 S_u^{a'a}(-x)] + \text{Tr}[\gamma_5 \tilde{S}_s^{a'b}(-x) \\ & \times \gamma_5 S_u^{b'a}(-x)] + \text{Tr}[\gamma_5 \tilde{S}_s^{b'a}(-x) \gamma_5 S_u^{a'b}(-x)] \\ & + \text{Tr}[\gamma_5 \tilde{S}_s^{a'a}(-x) \gamma_5 S_u^{b'b}(-x)] \}. \end{aligned} \quad (18)$$

The mass $\tilde{m}$ and coupling $\tilde{f}$ of the tetraquark $Z_{b:\bar{s}}^0$ can be found from Eqs. (9) and (10) after replacing $\rho^{\text{OPE}}(s)$ by a relevant spectral density $\tilde{\rho}^{\text{OPE}}(s)$ and using $\tilde{\mathcal{M}} = m_b + m_c + m_s$ instead of $\mathcal{M}$. Predictions for $\tilde{m}$ and $\tilde{f}$ read

$$\begin{aligned} \tilde{m} &= (6830 \pm 160) \text{ MeV}, \\ \tilde{f} &= (7.1 \pm 1.8) \times 10^{-3} \text{ GeV}^4. \end{aligned} \quad (19)$$

The $\tilde{m}$ and $\tilde{f}$ are extracted using the following regions for the parameters M^2 and s_0

$$M^2 \in [5.5, 6.5] \text{ GeV}^2, \quad s_0 \in [53, 55] \text{ GeV}^2. \quad (20)$$

These working windows meet standard requirements of the sum rule computations which have been discussed above. In fact, since at $M^2 = 5.5 \text{ GeV}^2$ the ratio R is equal to 0.008, the convergence of the obtained sum rules is guaranteed. The pole contribution at maximum of the Borel parameter $M^2 = 6.5 \text{ GeV}^2$ amounts to $\text{PC} = 0.25$, which is in accord with the restriction (14), and reaches $\text{PC} = 0.64$ at $M^2 = 5.5 \text{ GeV}^2$. Theoretical uncertainties of calculations for the mass $\pm 2.3\%$ are considerably smaller than ambiguities of the coupling $\pm 25\%$ due to reasons explained above. In Fig. 2 we depict our prediction for the mass of the tetraquark $Z_{b:\bar{s}}^0$ and show its dependence on M^2 and s_0 .

In the framework of the QCD sum rule method the mass of scalar tetraquark $bb\bar{q}\bar{s}$ was evaluated in Ref. [17]. Computations there were carried out using different interpolating currents and by taking into account non-perturbative terms up to dimension 8. Predictions obtained in Ref. [17] $m = (10.2 \pm 0.3) \text{ GeV}$ and $m = (10.3 \pm 0.3) \text{ GeV}$ confirm a stable nature of this exotic meson, and are very close to our result.

III. SEMILEPTONIC DECAY $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 l \bar{\nu}_l$

Our result $m = (10250 \pm 270) \text{ MeV}$ for the mass of the tetraquark $T_{b:\bar{s}}^-$ demonstrates its stability against the strong and electromagnetic decays to final states $B^- \bar{B}_s^0$ and $B^- \bar{B}_{s1}(5830) \gamma$, respectively. In fact, the central value of the mass $m = 10250 \text{ MeV}$ is 396 MeV lower than the threshold for strong decay to the conventional mesons $B^- \bar{B}_s^0$. Even its maximal value 10520 MeV obtained by taking into account uncertainties of the method is 126 MeV below this limit. Because the threshold 11108 MeV for electromagnetic dissociation of the $T_{b:\bar{s}}^-$ is considerably higher than m the similar arguments hold for the corresponding process as well.

Therefore, the full width and lifetime of $T_{b:\bar{s}}^-$ are determined by its weak transitions. In this section we concentrate on the dominant semileptonic decay $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 l \bar{\nu}_l$, which is depicted in Fig. 3. It is clear, that due to large mass difference $m - \tilde{m} \approx 3420 \text{ MeV}$ decays $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 l \bar{\nu}_l$ are kinematically allowed for all lepton species $l = e, \mu$ and τ . We do not consider processes triggered by $b \rightarrow W^- u$, because they are suppressed relative to dominant ones by a factor $|V_{bu}|^2 / |V_{bc}|^2 \simeq 0.01$.

The transition $b \rightarrow W^- c$ at the tree-level can be described by means of the effective Hamiltonian

$$\mathcal{H}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l, \quad (21)$$

where G_F and V_{bc} are the Fermi coupling constant and the relevant CKM matrix element, respectively:

FIG. 3. The Feynman diagram for the semileptonic decay $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \ell \bar{\nu}_\ell$. The black square denotes the effective weak vertex.

$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2},$$

$$|V_{bc}| = (42.2 \pm 0.08) \times 10^{-3}. \quad (22)$$

After sandwiching $\mathcal{H}^{\text{eff}}$ between the initial and final tetraquark fields, and removing a leptonic part from an obtained expression, we get the matrix element of the current

$$J_\mu^{\text{tr}} = \bar{c} \gamma_\mu (1 - \gamma_5) b. \quad (23)$$

The latter can be written down using the form factors $G_i(q^2)$ ($i = 1, 2$) which parametrize the long-distance dynamics of the weak transition. In terms of $G_{1(2)}(q^2)$ the matrix element of the current J_μ^{tr} has the form

$$\langle Z_{b;\bar{s}}^0(p') | J_\mu^{\text{tr}} | T_{b;\bar{s}}^-(p) \rangle = G_1(q^2) P_\mu + G_2(q^2) q_\mu, \quad (24)$$

where p and p' are the momenta of the initial and final tetraquarks, respectively. Here we also introduce variables $P_\mu = p'_\mu + p_\mu$ and $q_\mu = p_\mu - p'_\mu$. The q_μ is the momentum transferred to the leptons, and q^2 changes within the limits $m_l^2 \leq q^2 \leq (m - \tilde{m})^2$ with m_l being the mass of a lepton l .

To derive the sum rules for the form factors $G_{1(2)}(q^2)$, we begin from analysis of the three-point correlation function

$$\Pi_\mu(p, p') = i^2 \int d^4x d^4y e^{i(p'y - px)} \times \langle 0 | \mathcal{T} \{ \tilde{J}(y) J_\mu^{\text{tr}}(0) J^\dagger(x) \} | 0 \rangle. \quad (25)$$

In accordance with standard prescriptions, we write the correlation function $\Pi_\mu(p, p')$ using the spectroscopic parameters of the tetraquarks, and get the physical side of the sum rule $\Pi_\mu^{\text{Phys}}(p, p')$. The function $\Pi_\mu^{\text{Phys}}(p, p')$ can be presented in the following form

$$\Pi_\mu^{\text{Phys}}(p, p') = \frac{\langle 0 | \tilde{J} | Z_{b;\bar{s}}^0(p') \rangle \langle Z_{b;\bar{s}}^0(p') | J_\mu^{\text{tr}} | T_{b;\bar{s}}^-(p) \rangle}{(p^2 - m^2)(p'^2 - \tilde{m}^2)} \times \langle T_{b;\bar{s}}^-(p) | J^\dagger | 0 \rangle + \dots, \quad (26)$$

where the contribution of the ground-state particles is shown explicitly, whereas effects of excited resonances and continuum states are denoted by dots.

The phenomenological side of the sum rules can be detailed by expressing the matrix elements in terms of the tetraquarks' mass and coupling, and weak transition form factors. For these purposes, we use Eqs. (5) and (24), and employ the matrix element of the state $Z_{b;\bar{s}}^0$

$$\langle 0 | \tilde{J} | Z_{b;\bar{s}}^0(p') \rangle = \tilde{f} \tilde{m}. \quad (27)$$

Then it is not difficult to find that

$$\Pi_\mu^{\text{Phys}}(p, p') = \frac{f m \tilde{f} \tilde{m}}{(p^2 - m^2)(p'^2 - \tilde{m}^2)} \times [G_1(q^2) P_\mu + G_2(q^2) q_\mu] + \dots \quad (28)$$

We determine $\Pi_\mu(p, p')$ also by utilizing the interpolating currents and quark propagators, which lead to QCD side of the sum rules

$$\begin{aligned} \Pi_\mu^{\text{OPE}}(p, p') &= i^2 \int d^4x d^4y e^{i(p'y - px)} (\text{Tr}[\gamma_5 \tilde{S}_s^{b'b}(x-y) \\ &\times \gamma_5 S_u^{a'a}(x-y)] + \text{Tr}[\gamma_5 \tilde{S}_s^{b'a}(x-y) \gamma_\nu S_u^{a'b}(x-y)]) \\ &\times (\text{Tr}[\gamma_5 \tilde{S}_b^{aa'}(y-x) \gamma_5 S_c^{bi}(y) \gamma_\mu (1 - \gamma_5) S_b^{ib'}(-x)] \\ &+ \text{Tr}[\gamma_5 \tilde{S}_b^{ia'}(-x) (1 - \gamma_5) \gamma_\mu \tilde{S}_c^{bi}(y) \gamma_5 S_b^{ab'}(y-x)]). \end{aligned} \quad (29)$$

One can obtain the sum rules for the form factors $G_{1(2)}(q^2)$ by equating invariant amplitudes corresponding to structures P_μ and q_μ from $\Pi_\mu^{\text{Phys}}(p, p')$ and $\Pi_\mu^{\text{OPE}}(p, p')$. It is known that, these invariant amplitudes depend on p^2 and p'^2 , and therefore in order to suppress contributions of higher resonances and continuum states we have to apply the double Borel transformation over these variables. As a result, the final expressions contain a set of Borel parameters $\mathbf{M}^2 = (M_1^2, M_2^2)$. The continuum subtraction should be carried out in two channels which generates a dependence on the threshold parameters $\mathbf{s}_0 = (s_0, s'_0)$.

These operations lead to the sum rules

$$G_i(\mathbf{M}^2, \mathbf{s}_0, q^2) = \frac{1}{f m \tilde{f} \tilde{m}} \int_{\mathcal{M}^2}^{s_0} ds \times \int_{\mathcal{M}^2}^{s'_0} ds' \rho_i(s, s', q^2) e^{(m^2 - s)/M_1^2} e^{(\tilde{m}^2 - s')/M_2^2}, \quad (30)$$

where $\rho_{1(2)}(s, s', q^2)$ are the spectral densities calculated as the imaginary part of the correlation function $\Pi_\mu^{\text{OPE}}(p, p')$ with dimension-7 accuracy. The first pair of parameters (M_1^2, s_0) in Eq. (30) is related to the initial state $T_{b;\bar{s}}^-$,

FIG. 4. Predictions for the form factors $|G_1(q^2)|$ (the lower red circles) and $G_2(q^2)$ (the upper blue squares). The lines are fit functions $|F_1(q^2)|$ and $F_2(q^2)$, respectively.

whereas the second set (M_2^2, s_0') corresponds to the final particle $Z_{b;\bar{s}}^0$. Explicit expressions of $\rho_{1(2)}(s, s', q^2)$ are rather cumbersome, therefore we do not provide them here.

In numerical computations of $G_{1(2)}(q^2)$ the working regions for the parameters M^2 and s_0 are chosen exactly as in the corresponding mass calculations. Values of the vacuum condensates are collected in Eq. (13), whereas the masses and couplings of the tetraquarks $T_{b;\bar{s}}^-$ and $Z_{b;\bar{s}}^0$ have been calculated in the present work and written down in Eqs. (17) and (20), respectively. Obtained sum rule predictions for the form factors $G_1(q^2)$ and $G_2(q^2)$ are shown in Fig. 4.

The sum rules give reliable results for $G_{1(2)}(q^2)$ in the region $m_l^2 \leq q^2 \leq 9 \text{ GeV}^2$. But this is not enough to calculate the partial width of the decay $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 l \bar{\nu}_l$ under analysis. Indeed, the form factors determine the differential decay rate $d\Gamma/dq^2$ of the process through the following expression

$$\begin{aligned} \frac{d\Gamma}{dq^2} = & \frac{G_F^2 |V_{bc}|^2}{64\pi^3 m^3} \lambda(m^2, \tilde{m}^2, q^2) \left(\frac{q^2 - m_l^2}{q^2} \right)^2 \\ & \times \left\{ (2q^2 + m_l^2) \left[G_1^2(q^2) \left(\frac{q^2}{2} - m^2 - \tilde{m}^2 \right) \right. \right. \\ & \left. \left. - G_2^2(q^2) \frac{q^2}{2} + (\tilde{m}^2 - m^2) G_1(q^2) G_2(q^2) \right] \right. \\ & \left. + \frac{q^2 + m_l^2}{q^2} [G_1(q^2)(m^2 - \tilde{m}^2) + G_2(q^2)q^2]^2 \right\}, \quad (31) \end{aligned}$$

where

$$\lambda(m^2, \tilde{m}^2, q^2) = [m^4 + \tilde{m}^4 + q^4 - 2(m^2 \tilde{m}^2 + m^2 q^2 + \tilde{m}^2 q^2)]^{1/2}. \quad (32)$$

To find the partial width of the semileptonic decay, $d\Gamma/dq^2$ should be integrated over q^2 in the limits $m_l^2 \leq q^2 \leq (m - \tilde{m})^2$. But the region $m_l^2 \leq q^2 \leq 11.7 \text{ GeV}^2$ is wider

than a domain where the sum rules lead to strong predictions. This problem can be solved by introducing model functions $F_i(q^2)$ which at the momentum transfers q^2 accessible for the sum rule computations coincide with $G_i(q^2)$, but can be extrapolated to the whole integration region. These functions should have a simple form and be suitable to perform integrations over q^2 .

To this end, we use the functions of the form

$$F_i(q^2) = F_0^i \exp \left[c_1^i \frac{q^2}{m^2} + c_2^i \left(\frac{q^2}{m^2} \right)^2 \right], \quad (33)$$

where F_0^i , c_1^i , and c_2^i are constants which have to be fixed by comparing $F_i(q^2)$ and $G_i(q^2)$ at common regions of validity. Numerical analysis allows us to fix

$$\begin{aligned} F_0^1 &= -0.30, & c_1^1 &= 9.98, & c_2^1 &= -10.07, \\ F_0^2 &= 0.41, & c_1^2 &= 8.67, & c_2^2 &= -7.15. \end{aligned} \quad (34)$$

The functions $F_i(q^2)$ are plotted in Fig. 4, where one can see their nice agreement with the sum rule predictions.

Other input information to calculate the partial width of the process $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 l \bar{\nu}_l$, namely the masses of the leptons $m_e = 0.511 \text{ MeV}$, $m_\mu = 105.658 \text{ MeV}$, and $m_\tau = (1776.82 \pm 0.16) \text{ MeV}$ are borrowed from Ref. [41].

Our results for the partial widths of the semileptonic decay channels are presented below:

$$\begin{aligned} \Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 e^- \bar{\nu}_e) &= (6.16 \pm 1.74) \times 10^{-10} \text{ MeV}, \\ \Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \mu^- \bar{\nu}_\mu) &= (6.15 \pm 1.74) \times 10^{-10} \text{ MeV}, \\ \Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \tau^- \bar{\nu}_\tau) &= (2.85 \pm 0.81) \times 10^{-10} \text{ MeV}. \end{aligned} \quad (35)$$

As we shall see below, the semileptonic decays $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 l \bar{\nu}_l$ establish an essential part of the full width of $T_{b;\bar{s}}^-$.

IV. NONLEPTONIC DECAYS

$$T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \pi^- (K^-, D^-, D_s^-)$$

In this section, we investigate the nonleptonic weak decays $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \pi^- (K^-, D^-, D_s^-)$ of the tetraquark $T_{b;\bar{s}}^-$ in the framework of the QCD factorization method, which allows us to calculate partial widths of these processes. This approach was applied to investigate nonleptonic decays of the conventional mesons [42,43], and used to study nonleptonic decays of the scalar and axial-vector tetraquarks $Z_{bc;\bar{u}\bar{d}}^0$ and $T_{bc;\bar{u}\bar{d}}^0$ in Refs. [36,37], respectively.

Here, we consider in a detailed form the decay $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \pi^-$ shown in Fig. 5, and write down final predictions for remaining channels. At the quark level, the effective Hamiltonian for this decay is given by the expression

FIG. 5. The same as in Fig. 3, but for the nonleptonic decay $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 \pi^-$

$$\mathcal{H}_{\text{n.-lep}}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} V_{ud}^* [c_1(\mu) \mathcal{Q}_1 + c_2(\mu) \mathcal{Q}_2], \quad (36)$$

where

$$\begin{aligned} \mathcal{Q}_1 &= (\bar{d}_i u_i)_{V-A} (\bar{c}_j b_j)_{V-A}, \\ \mathcal{Q}_2 &= (\bar{d}_i u_j)_{V-A} (\bar{c}_j b_i)_{V-A}, \end{aligned} \quad (37)$$

and i, j are the color indices, and notation $(\bar{q}_1 q_2)_{V-A}$ means

$$(\bar{q}_1 q_2)_{V-A} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2. \quad (38)$$

The short-distance Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$ are given at the factorization scale μ .

The amplitude of the decay $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 \pi^-$ can be presented in the factorized form

$$\begin{aligned} \mathcal{A} &= \frac{G_F}{\sqrt{2}} V_{bc} V_{ud}^* a_1(\mu) \langle \pi^-(q) | (\bar{d}_i u_i)_{V-A} | 0 \rangle \\ &\times \langle Z_{b:\bar{s}}^0(p') | (\bar{c}_j b_j)_{V-A} | T_{b:\bar{s}}^-(p) \rangle, \end{aligned} \quad (39)$$

where

$$a_1(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu), \quad (40)$$

and $N_c = 3$ is the number of quark colors. The amplitude $\mathcal{A}$ describes the process in which the pion π^- is generated directly from the color-singlet current $(\bar{d}_i u_i)_{V-A}$. The matrix element $\langle Z_{b:\bar{s}}^0(p') | (\bar{c}_j b_j)_{V-A} | T_{b:\bar{s}}^-(p) \rangle$ has been introduced in Eq. (24), while the matrix element of the pion is given by the expression

$$\langle \pi^-(q) | (\bar{d}_i u_i)_{V-A} | 0 \rangle = i f_\pi q_\mu, \quad (41)$$

where f_π is the decay constant of π .

Then, it is not difficult to see that $\mathcal{A}$ takes the form

$$\mathcal{A} = i \frac{G_F}{\sqrt{2}} f_\pi V_{bc} V_{ud}^* a_1(\mu) [G_1(q^2) P q + G_2(q^2) q^2]. \quad (42)$$

The partial width of this process is determined by the simple expression

$$\begin{aligned} \Gamma(T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 \pi^-) &= \frac{G_F^2 f_\pi^2 |V_{bc}|^2 |V_{ud}|^2}{32\pi m^3} a_1^2(\mu) \\ &\times \lambda(m^2, \tilde{m}^2, m_\pi^2) [G_1(m^2 - \tilde{m}^2) + G_2 m_\pi^2]^2, \end{aligned} \quad (43)$$

where the weak form factors $G_{1(2)}(q^2)$ are computed at $q^2 = m_\pi^2$. The similar analysis can be performed for the decay modes $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 K^- (D^-, D_s^-)$ as well. The partial width of these channels can be obtained from Eq. (43) by replacing (m_π, f_π) with the spectroscopic parameters of the mesons K , D , and D_s , and implementing substitutions $|V_{ud}| \rightarrow |V_{us}|$, $|V_{cd}|$, and $|V_{cs}|$, respectively.

The masses and decay constants of the final-state pseudoscalar mesons, as well as values of the CKM matrix elements used in computations are collected in Table I. The Wilson coefficients $c_1(m_b)$, and $c_2(m_b)$ with next-to-leading order QCD corrections can be found in Refs. [44–46]

$$c_1(m_b) = 1.117, \quad c_2(m_b) = -0.257. \quad (44)$$

For the decay $T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 \pi^-$ calculations lead to the result

$$\Gamma(T_{b:\bar{s}}^- \rightarrow Z_{b:\bar{s}}^0 \pi^-) = (6.97 \pm 1.99) \times 10^{-13} \text{ MeV}. \quad (45)$$

For the remaining nonleptonic decays of the tetraquark $T_{b:\bar{s}}^-$, we get

TABLE I. Spectroscopic parameters of the final-state pseudoscalar mesons, and the relevant CKM matrix elements.

Quantity	Value
m_π	139.570 MeV
m_K	$(493.677 \pm 0.016) \text{ MeV}$
m_D	$(1869.61 \pm 0.10) \text{ MeV}$
m_{D_s}	$(1968.30 \pm 0.11) \text{ MeV}$
f_π	131 MeV
f_K	$(155.72 \pm 0.51) \text{ MeV}$
f_D	$(203.7 \pm 4.7) \text{ MeV}$
f_{D_s}	$(257.8 \pm 4.1) \text{ MeV}$
$ V_{ud} $	0.97420 ± 0.00021
$ V_{us} $	0.2243 ± 0.0005
$ V_{cd} $	0.218 ± 0.004
$ V_{cs} $	0.997 ± 0.017

$$\begin{aligned}
\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 K^-) &= (5.33 \pm 1.47) \times 10^{-14} \text{ MeV}, \\
\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 D^-) &= (1.13 \pm 0.31) \times 10^{-13} \text{ MeV}, \\
\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 D_s^-) &= (3.88 \pm 1.01) \times 10^{-12} \text{ MeV}. \quad (46)
\end{aligned}$$

It is seen that partial widths of the nonleptonic decays are negligibly smaller than widths of the semileptonic decays. Only widths of the processes $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \pi^-$ and $T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 D_s^-$ affect the final result for Γ_{full} .

Collected information on the partial widths of the weak decays of the tetraquark $T_{b;\bar{s}}^-$ allow us to find its full width and mean lifetime:

$$\begin{aligned}
\Gamma_{\text{full}} &= (15.21 \pm 2.59) \times 10^{-10} \text{ MeV}, \\
\tau &= 4.33_{-0.63}^{+0.89} \times 10^{-13} \text{ s}. \quad (47)
\end{aligned}$$

Predictions for Γ_{full} and τ are the main results of the present work.

V. DISCUSSION AND CONCLUDING NOTES

In this article we have evaluated the mass and coupling of the scalar tetraquark $T_{b;\bar{s}}^-$. Our analysis has proved that the exotic meson $T_{b;\bar{s}}^-$ composed of the heavy diquark bb and light antidiquark $\bar{u}\bar{s}$ is the strong- and electromagnetic-interaction stable state, and dissociates to conventional mesons only through the weak decays. This fact places it to a list of stable axial-vector $T_{bb;\bar{u}\bar{d}}^-$ and scalar $Z_{bc;\bar{u}\bar{d}}^0$ and $T_{bs;\bar{u}\bar{d}}^-$ tetraquarks.

We have investigated also the dominant weak decay modes of the $T_{b;\bar{s}}^-$, and computed their partial widths. These results have allowed us to estimate the full width and mean lifetime of the $T_{b;\bar{s}}^-$. The collected information is enough to find the branching ratios of the various decay modes as well (see Table II). It is worth noting that only the semileptonic decays of the $T_{b;\bar{s}}^-$ play a dominant role in forming of Γ_{full} .

The tetraquark $T_{b;\bar{s}}^-$ can be considered as “s” member of a multiplet of the scalar $bb\bar{q}\bar{q}'$ states with q being one of the light quarks. In Ref. [33], we studied the stable axial-vector particle $T_{bb;\bar{u}\bar{d}}^-$. It will be very interesting to investigate the scalar partner of $T_{bb;\bar{u}\bar{d}}^-$ as well as the axial-vector state $bc\bar{u}\bar{s}$ which may shed light on others members of scalar and axial-vector multiplets $bb\bar{q}\bar{q}'$.

We have computed the mass and coupling of the scalar tetraquark $Z_{b;\bar{s}}^0$: these parameters are required to explore the weak decays of the $T_{b;\bar{s}}^-$. The state $Z_{b;\bar{s}}^0 = bc\bar{u}\bar{s}$ belongs to a famous class of exotic mesons composed of four different

TABLE II. Dominant weak decay channels of the tetraquark $T_{b;\bar{s}}^-$, and corresponding branching ratios.

Channels	$\mathcal{BR}$
$\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 e^- \bar{\nu}_e)$	0.404
$\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \mu^- \bar{\nu}_\mu)$	0.404
$\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \tau^- \bar{\nu}_\tau)$	0.187
$\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 \pi^-)$	4.58×10^{-4}
$\Gamma(T_{b;\bar{s}}^- \rightarrow Z_{b;\bar{s}}^0 D_s^-)$	2.55×10^{-3}

quarks [47]. A simple analysis confirms that it is a strong-interaction stable particle. Indeed, the scalar tetraquark $Z_{b;\bar{s}}^0$ in S -wave may decay to a pair of pseudoscalar mesons $B^- D_s^+$ and $\bar{B}_s^0 D^0$. Thresholds for production of these pairs are 7248 MeV and 7237 MeV, respectively. Because the maximum allowed value of the $Z_{b;\bar{s}}^0$ tetraquarks’s mass is $\tilde{m} = 6990$ MeV, it is stable against these strong decays. The $\bar{u}\bar{d}$ member of the scalar multiplet $bc\bar{q}\bar{q}'$ was investigated in Ref. [36], in which it was found that this particle is a strong- and electromagnetic-interaction stable state. It seems scalar particles with such diquark-antidiquark structures are among real candidates to stable four-quark compounds.

The revealed features of the $Z_{b;\bar{s}}^0$ determine a decay pattern of the master particle $T_{b;\bar{s}}^-$. Indeed, the tetraquark $Z_{b;\bar{s}}^0$ created at the first stage of the decays, at the next step due to subprocesses $b \rightarrow W^- c$ and $c \rightarrow W^+ s$ should have undergone weak transformations. Such cascade picture of decays was encountered in theoretical investigations of other tetraquarks [33,36], and studied in a detailed form in Ref. [38]. Of course, there are nonleptonic decays of $T_{b;\bar{s}}^-$ when it transforms to a pair of ordinary mesons at the first phase of a weak process. A comprehensive analysis of the $T_{b;\bar{s}}^-$ tetraquark’s decays will be finished in our forthcoming publications.

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APPENDIX: THE PROPAGATORS $S_{q(q)}(x)$ AND INVARIANT AMPLITUDE $\Pi(M^2, s_0)$

In the present work we use the light quark propagator $S_q^{ab}(x)$ which is given by the following formula

$$\begin{aligned}
S_q^{ab}(x) &= i\delta_{ab} \frac{\not{x}}{2\pi^2 x^4} - \delta_{ab} \frac{m_q}{4\pi^2 x^2} - \delta_{ab} \frac{\langle \bar{q}q \rangle}{12} + i\delta_{ab} \frac{\not{x} m_q \langle \bar{q}q \rangle}{48} - \delta_{ab} \frac{x^2}{192} \langle \bar{q}g_s \sigma G q \rangle + i\delta_{ab} \frac{x^2 \not{x} m_q}{1152} \langle \bar{q}g_s \sigma G q \rangle \\
&\quad - i \frac{g_s G_{ab}^{\alpha\beta}}{32\pi^2 x^2} [\not{x} \sigma_{\alpha\beta} + \sigma_{\alpha\beta} \not{x}] - i\delta_{ab} \frac{x^2 \not{x} g_s^2 \langle \bar{q}q \rangle^2}{7776} - \delta_{ab} \frac{x^4 \langle \bar{q}q \rangle \langle g_s^2 G^2 \rangle}{27648} + \dots \quad (A1)
\end{aligned}$$

For the heavy quarks Q we utilize the propagator $S_Q^{ab}(x)$

$$S_Q^{ab}(x) = i \int \frac{d^4 k}{(2\pi)^4} e^{-ikx} \left\{ \frac{\delta_{ab}(\not{k} + m_Q)}{k^2 - m_Q^2} - \frac{g_s G_{ab}^{\alpha\beta} \sigma_{\alpha\beta}(\not{k} + m_Q) + (\not{k} + m_Q) \sigma_{\alpha\beta}}{4(k^2 - m_Q^2)^2} \right. \\ \left. + \frac{g_s^2 G^2}{12} \delta_{ab} m_Q \frac{k^2 + m_Q \not{k}}{(k^2 - m_Q^2)^4} + \frac{g_s^3 G^3}{48} \delta_{ab} \frac{(\not{k} + m_Q)}{(k^2 - m_Q^2)^6} [k(k^2 - 3m_Q^2) + 2m_Q(2k^2 - m_Q^2)](\not{k} + m_Q) + \dots \right\}. \quad (\text{A2})$$

Above, we have introduced the notations

$$G_{ab}^{\alpha\beta} \equiv G_A^{\alpha\beta} t_{ab}^A, \quad G^2 = G_{\alpha\beta}^A G_A^{\alpha\beta}, \quad G^3 = f^{ABC} G_{\alpha\beta}^A G^{B\beta\delta} G_\delta^{C\alpha}, \quad (\text{A3})$$

where $G_A^{\alpha\beta}$ is the gluon field strength tensor, $t^A = \lambda^A/2$ with λ^A being the Gell-Mann matrices, f^{ABC} are the structure constants of the color group $SU_c(3)$, and $A, B, C = 1, 2, \dots, 8$.

The invariant amplitude $\Pi^{\text{OPE}}(p^2)$ used for calculation of the mass and coupling of the tetraquark $T_{b:\bar{3}}^-$ after the Borel transformation and subtraction procedures takes the following form

$$\Pi(M^2, s_0) = \int_{\mathcal{M}^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2} + \Pi(M^2), \quad (\text{A4})$$

where

$$\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \sum_{N=3}^8 \rho^{\text{DimN}}(s), \quad \Pi(M^2) = \sum_{N=6}^{10} \Pi^{\text{DimN}}(M^2). \quad (\text{A5})$$

Components of the spectral density are given by the formulas

$$\rho(s) = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \rho(s, \alpha, \beta), \quad \rho(s) = \int_0^1 d\alpha \rho(s, \alpha), \quad (\text{A6})$$

depending on whether $\rho(s, \alpha, \beta)$ is a function of α and β or only α . The same is true also for terms $\Pi(M^2)$, i.e.,

$$\Pi^{\text{DimN}}(M^2) = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \Pi^{\text{DimN}}(M^2, \alpha, \beta), \quad \Pi(M^2) = \int_0^1 d\alpha \Pi^{\text{DimN}}(M^2, \alpha). \quad (\text{A7})$$

In these expressions α and β are Feynman parameters.

The perturbative and nonperturbative contributions of dimensions 3, 4, 5 and 7 are terms of (A6) types. For relevant spectral densities, we get

$$\rho^{\text{pert}}(s, \alpha, \beta) = \frac{\Theta(L_1)}{128\pi^6 L^2 N_1^7} [s\alpha\beta L - m_b^2 N_2]^3 \{3s\alpha\beta L^2 + m_b^2 N_1 [\alpha(\alpha - 1) + \beta(\beta - 1)]\}, \quad (\text{A8})$$

$$\rho^{\text{Dim3}}(s, \alpha, \beta) = \frac{m_s [\langle \bar{s}s \rangle - 2\langle \bar{u}u \rangle]}{8\pi^4 N_1^5} \Theta(L_1) \alpha\beta \{2s^2 \alpha\beta L^3 + m_b^4 (\alpha + \beta) N_1^2 - m_b^2 s [2\beta^5 + 2\alpha^2 (\alpha - 1)^3 \\ + \beta^4 (9\alpha - 6) + \alpha\beta (\alpha - 1)^2 (9\alpha - 4) + 2\beta^3 (3 - 11\alpha + 8\alpha^2) - \beta^2 (2 - 17\alpha + 31\alpha^2 - 16\alpha^3)]\}, \quad (\text{A9})$$

$$\rho^{\text{Dim4}}(s, \alpha, \beta) = \frac{\langle \alpha_s G^2/\pi \rangle}{768\pi^4 (1 - \beta) L^2 N_1^5} \Theta(L_1) \alpha \{6s^2 \alpha\beta^2 (\beta - 1) L^3 [2\beta^2 + 2(\alpha - 1)^2 + \beta(5\alpha - 4)] \\ - m_b^4 N_1^2 (\alpha + \beta) [5\beta^4 + 4\beta^3 (1 - 3\beta) - 8\alpha^3 (\alpha - 1) - 3\beta^2 (5 - 7\alpha + \alpha^2) + \beta(6 - 9\alpha + 3\alpha^2 - 8\alpha^3)] \\ - sm_b^2 \beta L N_1 [4\beta(\beta - 1)^2 (3 - 6\beta + 2\beta^2) + \alpha(\beta - 1)(-12 + 78\beta - 119\beta^2 + 41\beta^3) \\ + \alpha^2 (\beta - 1)(36 - 119\beta + 88\beta^2) + \alpha^3 (\beta - 1)(49\beta - 32) + 16\alpha^4 (\beta - 1) + 8\alpha^5]\}, \quad (\text{A10})$$

$$\rho^{\text{Dim5}}(s, \alpha) = - \frac{m_s[\langle \bar{s}g_s \sigma G s \rangle - 3\langle \bar{u}g_s \sigma G u \rangle]}{48\pi^4} \Theta(L_2)(3m_b^2 + s - 4s\alpha + 3s\alpha^2), \quad (\text{A11})$$

$$\rho^{\text{Dim7}}(s, \alpha, \beta) = - \frac{\langle \alpha_s G^2 / \pi \rangle m_s[\langle \bar{s}s \rangle - 2\langle \bar{u}u \rangle]}{96\pi^2 N_1^3} \alpha \beta L \Theta(L_1). \quad (\text{A12})$$

The dimension 6 and 8 terms have mixed compositions: they contain components expressed through both $\rho^{\text{DimN}}(s)$ and $\Pi^{\text{DimN}}(M^2)$. For dimension 6 term, we find

$$\Pi^{\text{Dim6}}(M^2, s_0) = \int_{\mathcal{M}^2}^{s_0} ds e^{-s/M^2} \int_0^1 d\alpha \rho^{\text{Dim6}}(s, \alpha) + \int_0^1 d\alpha \int_0^{1-a} d\beta \Pi^{\text{Dim6}}(M^2, \alpha, \beta),$$

where

$$\rho^{\text{Dim6}}(s, \alpha) = \frac{\Theta(L_2)}{3\pi^2} \left[\langle \bar{s}s \rangle \langle \bar{u}u \rangle + \frac{g_s^2}{108\pi^2} (\langle \bar{s}s \rangle^2 + \langle \bar{u}u \rangle^2) \right] (3m_b^2 + s - 4s\alpha + 3s\alpha^2), \quad (\text{A13})$$

$$\begin{aligned} \Pi^{\text{Dim6}}(M^2, \alpha, \beta) = & - \frac{\langle g_s^3 G^3 \rangle m_b^4}{3840M^2\pi^6\alpha^2\beta^2L^4N_1^3} \exp \left[- \frac{m_b^2}{M^2} \frac{N_1(\alpha + \beta)}{\alpha\beta L} \right] \{ m_b^2(\alpha + \beta)N_1[5\beta^8 + 2\beta^5\alpha^2(3 - 4\alpha) + 2\beta^3\alpha^4(5 - 4\alpha) \\ & + 3\beta\alpha^6(\alpha - 1) + 5\alpha^6(\alpha - 1)^2 + \beta^7(3\alpha - 10) + \beta^4\alpha^2(-5 + 2(5 - 4\alpha)\alpha) - \beta^2\alpha^4(5 + (\alpha - 1)\alpha) \\ & - \beta^6((3 + \alpha)\alpha - 5)] + M^2\alpha\beta L[14\beta^8 + 14\alpha^6(\alpha - 1) + 2\beta^3\alpha^4(4\alpha - 3) + \beta^5\alpha(\alpha - 1)(8\alpha - 17) \\ & + \beta^3\alpha^4(\alpha - 1)(22\alpha - 3) + \beta^7(23\alpha - 28)\beta\alpha^5(\alpha - 1)(23\alpha - 17) + 3\beta^4\alpha^2(1 + 2\alpha(\alpha - 1)) \\ & + 2\beta^6(7 + \alpha(11\alpha - 20))] \}. \end{aligned} \quad (\text{A14})$$

Dimension 8 contribution is given by expression

$$\begin{aligned} \Pi^{\text{Dim8}}(M^2, s_0) = & \int_{\mathcal{M}^2}^{s_0} ds e^{-s/M^2} \int_0^1 d\alpha \int_0^{1-a} d\beta \rho_1^{\text{Dim8}}(s, \alpha, \beta) + \int_{\mathcal{M}^2}^{s_0} ds e^{-s/M^2} \int_0^1 d\alpha \rho_2^{\text{Dim8}}(s, \alpha) \\ & + \int_0^1 d\alpha \int_0^{1-a} d\beta \Pi^{\text{Dim8}}(M^2, \alpha, \beta). \end{aligned} \quad (\text{A15})$$

Here the relevant functions are equal to:

$$\begin{aligned} \rho_1^{\text{Dim8}}(s, \alpha, \beta) = & \frac{\langle \alpha_s G^2 / \pi \rangle^2}{1536\pi^2 N_1^3} \Theta(L_1) \alpha \beta (\alpha + \beta - 1), \quad \rho_2^{\text{Dim8}}(s, \alpha) = - \frac{\langle \bar{s}g_s \sigma G s \rangle \langle \bar{u}u \rangle}{3\pi^2} \Theta(L_2)(1 - 4\alpha + 3\alpha^2), \\ \Pi^{\text{Dim8}}(M^2, \alpha, \beta) = & \frac{\langle \alpha_s G^2 / \pi \rangle^2 m_b^2}{9216M^4\pi^2\alpha^2\beta^2(1 - \beta)L^4N_1^3} \exp \left[- \frac{m_b^2}{M^2} \frac{N_1(\alpha + \beta)}{\alpha\beta L} \right] \{ 96m_b^4\alpha^2\beta^2(\alpha + \beta)(\beta - 1)N_1^2[2\beta^2 + 2\alpha(\alpha - 1) \\ & + \beta(3\alpha - 2)] - M^4\alpha\beta L^2[4\beta^8 - \beta^7(\alpha + 16) + 4\alpha^4(\alpha - 1)^3(2\alpha - 1) + \beta^6(24 + 4\alpha - 206\alpha^2) \\ & + \beta\alpha^3(\alpha - 1)^2(-1 - 19\alpha + 28\alpha^2) - \beta^5(16 + 6\alpha - 432\alpha^2 + 23\alpha^3) + \beta^4(4 + 4\alpha - 256\alpha^2 \\ & + 67\alpha^3 - 199\alpha^4) + \alpha^2\beta^2(-10 + 23\alpha - 173\alpha^2 + 313\alpha^3 - 153\alpha^4) - \beta^3\alpha(1 - 40\alpha + 66\alpha^2 \\ & - 385\alpha^3 + 358\alpha^4)] + 4m_b^2M^2(\alpha + \beta)LN_1[\beta^4(\beta - 1)^4 + 2\alpha\beta^3(\beta - 1)^3(2\beta - 1) \\ & - \alpha^2\beta^2(\beta - 1)^2(-2 + \beta(9 + 40\beta)) - \alpha^3\beta(\beta - 1)^2(-2 + \beta(9 + 37\beta)) \\ & - \alpha^4(\beta - 1)(1 + \beta(-8 + \beta(-31 + 85\beta))) - \alpha^5(\beta - 1)(4\beta + 1)(22\beta - 3) \\ & + \alpha^6(3 - \beta(7 + 44\beta)) + \alpha^7(\beta - 1)] \}. \end{aligned} \quad (\text{A16})$$

The Dim9 and Dim10 contributions are exclusively of (A7) types. Thus, we have

$$\Pi_1^{\text{Dim}9}(M^2, \alpha, \beta) = -\frac{m_b^2 \langle g_s^3 G^3 \rangle m_s [2\langle \bar{u}u \rangle - \langle \bar{s}s \rangle]}{2880M^6 \pi^4 \alpha^4 \beta^4 (\beta-1)L^4 N_1^2} R_1(M^2, \alpha, \beta), \quad (\text{A17})$$

and

$$\Pi_2^{\text{Dim}9}(M^2, \alpha) = -\frac{\langle \alpha_s G^2 / \pi \rangle m_s [\langle \bar{s}g_s \sigma G s \rangle - 3\langle \bar{u}g_s \sigma G u \rangle]}{3456M^4 \pi^2 \alpha^4 (\alpha-1)^2} R_2(M^2, \alpha). \quad (\text{A18})$$

The dimension 10 term has the following components:

$$\Pi_1^{\text{Dim}10}(M^2, \alpha, \beta) = \frac{\langle \alpha_s G^2 / \pi \rangle \langle g_s^3 G^3 \rangle m_b^2}{46080M^6 \pi^4 \alpha^4 \beta^4 (\beta-1)L^4 N_1^2} R_1(M^2, \alpha, \beta) \quad (\text{A19})$$

and

$$\Pi_2^{\text{Dim}10}(M^2, \alpha) = \frac{\langle \alpha_s G^2 / \pi \rangle}{216M^4 \alpha^4 (\alpha-1)^2} \left[\langle \bar{s}s \rangle \langle \bar{u}u \rangle + \frac{g_s^2}{108\pi^2} (\langle \bar{s}s \rangle^2 + \langle \bar{u}u \rangle^2) \right] R_2(M^2, \alpha) \quad (\text{A20})$$

where functions $R_1(M^2, \alpha, \beta)$ and $R_2(M^2, \alpha)$ are given by formulas:

$$\begin{aligned} R_1(M^2, \alpha, \beta) = \exp \left[-\frac{m_b^2 N_1 (\alpha + \beta)}{M^2 \alpha \beta L} \right] & \{ -6M^4 \alpha^2 \beta^2 L^3 [\alpha^7 + 2\alpha^6 (\beta-1) + \alpha^5 (\beta-1)^2 + \beta^5 (\beta-1)^2] \\ & + m_b^4 (\beta-1) N_1^2 [5\beta^9 + 5\alpha^7 (\alpha-1)^2 + \alpha^5 \beta^2 (\alpha-1)(5+2\alpha) + 2\beta^8 (4\alpha-5) + \beta \alpha^6 (\alpha-1)(8\alpha-5) \\ & + \beta^3 \alpha^4 (-5 + (16-2\alpha)\alpha) + \beta^4 \alpha^3 (-5 + 4(5-4\alpha)\alpha) + \beta^5 \alpha^2 (-5 - 16(\alpha-1)\alpha) + \beta^6 \alpha (5 + 3\alpha - 9\alpha^2) \\ & + \beta^7 (5 + \alpha(-13 + 2\alpha))] + m_b^2 M^2 \alpha \beta N_1^2 [8\beta^8 + \alpha^5 (\alpha-1)^2 (11\alpha-8) + \beta^7 (23\alpha-24) + \beta^3 \alpha^4 (41\alpha-45) \\ & + 3\beta^2 \alpha^4 (15 - 30\alpha + 16\alpha^2) + \beta^6 (24 - 61\alpha + 30\alpha^2) + 15\beta^4 \alpha (-1 + 2\alpha - \alpha^2 + \alpha^3) \\ & + 3\beta \alpha^4 (-5 + 19\alpha - 25\alpha^2 + 11\alpha^3) + \beta^5 (-8 + 53\alpha - 60\alpha^2 + 15\alpha^3)] \}, \end{aligned} \quad (\text{A21})$$

and

$$R_2(M^2, \alpha) = \exp \left[-\frac{m_b^2}{M^2 \alpha (1-\alpha)} \right] [-3M^4 \alpha^3 (1+\alpha)(\alpha-1)^2 + 8m_b^4 (1-3\alpha+3\alpha^2) - m_b^2 M^2 \alpha (16-51\alpha+48\alpha^2+3\alpha^3)]. \quad (\text{A22})$$

In expressions above, $\Theta(z)$ is Unit step function. We have used also the following short hand notations

$$\begin{aligned} N_1 &= \beta^2 + \beta(\alpha-1) + \alpha(\alpha-1), & N_2 &= (\alpha+\beta)N_1, & L &= \alpha + \beta - 1, \\ L_1 &= \frac{(1-\beta)}{N_1^2} [m_b^2 N_2 - s\alpha\beta L], & L_2 &= s\alpha(1-\alpha) - m_b^2. \end{aligned} \quad (\text{A23})$$

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