Connecting the electroweak sphaleron with gravitational waves

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We study and reveal the relation between the electroweak sphaleron energy and the gravitational wave signals from a first-order electroweak phase transition. For the first time, we find that a larger sphaleron energy corresponds to a stronger gravitational wave signal for a sufficiently strong phase transition. We also present the correlation between the sphaleron and the properties of the gravitational waves, such as the signal-to-noise ratio, peak frequency, and amplitude, with the intention that traces of the sphaleron can be obtained with detailed measurement and studies of the stochastic gravitational wave at future space-based detectors. This study paves the way for a more practical revelation of the sphaleron which is otherwise improbable at high energy colliders.

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I. INTRODUCTION

The first direct detection of gravitational wave signals from the binary black hole merger by LIGO [1] and the approval of the space-based interferometer LISA [2] have raised growing interest on the study of gravitational waves from a first-order electroweak phase transition (EWPT) in the early universe. It opens an alternative novel approach to study new physics because the nature of the phase transition in the standard model of particle physics is a *crossover* [3–6]. A strongly first-order EWPT provides a nonequilibrium environment for baryon number generation and fulfills one of the three Sakharov conditions [7], with which it is possible to generate the observed baryon asymmetry of the Universe [8] in the framework of the *electroweak baryogenesis* (EWB). On the other hand, the (B + L)-violating sphaleron process associated with the change of Chern-Simons numbers [9,10] should be highly suppressed to avoid the washout of the baryon asymmetry inside the electroweak bubbles where electroweak symmetry is broken [11]. The behavior of the electroweak sphaleron is directly correlated with the possible patterns of EWPT and the generated gravitational waves since the sphaleron energy is closely connected with the phase transition order parameters at finite temperature as will be shown in this paper. Therefore, the electroweak

sphaleron, which is otherwise inaccessible at colliders, correlates with the properties of the stochastic gravitational waves and can be traced through future measurements and studies of the stochastic gravitational waves.

We first summarize the relations among the phase transition strength, gravitational waves from the EWPT, and the baryon number preservation criterion (BNPC) here: (1) a higher detectability of the stochastic background of gravitational waves generally requires a smaller β/H_n (roughly the inverse time duration of the phase transition) and a larger α (the latent heat normalized by the radiation energy) of the EWPT, where the sound waves in the plasma dominates the gravitational wave production, and this corresponds to a higher strength of the EWPT [12,13]. Both parameters are highly related with the finite temperature potential that determines the sphaleron energy ($E_{sph}(T)$) inside the electroweak bubbles. (2) The sphaleron rate in the broken phase is proportional to a Boltzmann factor $\Gamma \propto \exp[-E_{sph}(T)/T]$ [11,14], implying that the estimation of $E_{sph}(T)$ is crucial to determine if the electroweak sphaleron process is sufficiently quenched inside the electroweak bubbles to keep the baryon asymmetry generated during EWPT. Quantitatively, this means the sphaleron rate in the broken phase should be lower than the Hubble expansion rate, serving as the definition of the BNPC [8,15]. (3) The conventionally adopted strongly first-order EWPT condition [16] ($v(T)/T \gtrsim 1$) is usually obtained after taking into account the scaling law [17], which connects the sphaleron energy at the temperature of phase transition ($E_{sph}(T)$) to that at the zero temperature (E_{sph}) as [18,19] $E_{sph}(T) \approx E_{sph} v(T)/v$, where $v(T)$ and v are, respectively, the phase transition order parameter and Higgs vacuum expectation value (vev) at zero temperature. Here $E_{sph} \approx 9$ TeV, and thus leads to highly rare events for

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sphaleron-induced (B + L)-violating process at the high energy colliders in the current and near future [14,20–23] and makes its detection unlikely at colliders [24].

Based on the above relations, we revisit the relation between BNPC and the strongly first-order EWPT criterion. We first calculate the electroweak sphaleron energy $E_{\text{sph}}(T)$ and the phase transition strength $v(T)/T$ at the temperature of the EWPT. After that, we check if the BNPC is met with the following condition being satisfied [17]:

$$PT_{\text{sph}} \equiv \frac{E_{\text{sph}}(T)}{T} - 7 \ln \frac{v(T)}{T} + \ln \frac{T}{100 \text{ GeV}} > (35.9-42.8), \quad (1)$$

The numerical range here originates from the uncertainty calculating the fluctuation determinant $\kappa = (10^{-4}-10^{-1})$ [25], which is comparable to the uncertainty associated with the numerical lattice simulation of the sphalerons at the standard model electroweak crossover [4]. The lattice simulation of the sphaleron rate can settle down the exact value of the right-hand side in Eq. (1) [26,27], which however is currently absent in the literature. Fortunately, our findings in this paper only mildly rely on this uncertainty. We thus investigate the BNPC condition and the scaling law, and its relation with the phase transition strength. Our study shows that the scaling law can be established when $PT_{\text{sph}} \sim \mathcal{O}(10^2)$ for the EWPT that satisfies the BNPC condition. A strongly first-order EWPT that can produce detectable gravitational waves generally is accompanied by a large sphaleron energy as $E_{\text{sph}} \propto v_n$, as will be explored in this work. Based on that, we build the connection between the sphaleron energy and the signal-to-noise ratio of resulting stochastic gravitational waves from the EWPT.

There are mainly two classes of new physics that can enhance the strength of the EWPT [8]: (1) one class couples the standard model Higgs to new scalars and enhances the cubic terms in the thermal potential through loop effects, with the minimal supersymmetry standard model as the celebrated example; and (2) the other couples the SM Higgs to new scalar that develops a vev near the electroweak scale. The first class is less prioritized due to the current severe theoretical or experimental constraints [5,28], and we thus consider the second class where the first-order phase transition is triggered by tree-level cubic terms. We use the extensively studied singlet extended standard model (“xSM”) and the standard model effective field theory (SMEFT) as two representative examples [29], with the former and the latter driven by tree-level renormalizable and nonrenormalizable operators, respectively [28].

II. EWPT MODELS

It is well known that the standard model cannot accommodate a first-order EWPT, and this has motivated a plethora of beyond the standard model scenarios with an

extended Higgs sector. A simplified benchmark model is the gauge singlet extension of the SM, the xSM [30–32], with the finite temperature potential being [13]

$$V[h, s, T] = -\frac{1}{2}[m^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad (2)$$

where $\Pi_h(T)$ and $\Pi_s(T)$ are the thermal masses of the fields,

$$\begin{aligned} \Pi_h(T) &= \left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24} \right) T^2, \\ \Pi_s(T) &= \left(\frac{a_2}{6} + \frac{b_4}{4} \right) T^2. \end{aligned} \quad (3)$$

The scalar cubic terms in Eq. (2) dominate the phase transition dynamics and can trigger a first-order EWPT after theoretical and experimental bounds on model parameters are taken into account [13]. Moreover, in this work, we focus on the one-step EWPT with the electroweak vacuum denoted by $(\equiv(v, v_s))$. While two-step EWPT can also exist, it is of negligible parameter space here. For the phase transition study, we do not take into account the CP violation since it is generally very constrained by electric dipole moment limits [33].

From an effective field theory point of view, a first-order EWPT can be realized by inclusion of higher dimensional operators, irrespective of a specific scenario. Among the dimension-six operators of the SMEFT, the operator $\mathcal{O}_6$ dominates the contribution to the Higgs potential and leads to the following finite temperature effective potential:

$$V_T(h, T) = -\frac{1}{2}(m^2 - c_{hT})h^2 + \frac{\lambda}{4}h^4 + \frac{h^6}{8\Lambda^2}, \quad (4)$$

where $c_{hT} = (4y_t^2 + 3g^2 + g'^2 + 8\lambda)T^2/16$, and it is the presence of the last term that allows the EWPT to be first order through modifying the cubic terms of the potential at tree level in comparison with the SM case [34,35]. The first-order EWPT parameter spaces with deviation of the Higgs trilinear coupling from the SM can be tested by future colliders [36,37]. The requirement of the electroweak minimum being the global one results in the condition $\Lambda \geq v^2/m_h$, and the EWPT can be first order when the potential barrier can be raised with $\Lambda < \sqrt{3}v^2/m_h$ [34,38].

III. ELECTROWEAK SPHALERON

The electroweak sphaleron is a static but unstable solution to the classical equations of motion of the electroweak theory, which corresponds to a saddle-point configuration in the field space and sits at the top of the potential barrier between two topologically distinct vacua

with adjacent values of the Chern-Simons number [9,10]. To calculate the energy of the sphaleron configuration, we adopt the spherically symmetric ansatz and neglect the much smaller $U(1)_Y$ contribution [10,39–41]. In the xSM, it follows that

$$E_{\text{sph}}(T) = \frac{4\pi v[T]}{g} \int_0^\infty d\xi \left[4 \left(\frac{df}{d\xi} \right)^2 + \frac{8}{\xi^2} f^2 (1-f)^2 + \frac{\xi^2}{2} \left(\frac{dh}{d\xi} \right)^2 + (1-f)^2 h^2 + \frac{\xi^2 v_s[T]^2}{2v[T]^2} \left(\frac{dk}{d\xi} \right)^2 + \frac{\xi^2}{g^2 v[T]^4} (V_{\text{eff}}[h, k, T]) \right], \quad (5)$$

with $\xi = gv[T]r$, and $V_{\text{eff}}[h, k, T] = V[h, k, T] - \Delta[T]$, with $\Delta[T]$ being the cosmological constant which can be regarded as the minimal value of the potential at temperature T [42]. Here $4\pi v[T]/g$ has the unit of energy and the integral gives a dimensionless number; $v[T]$ is the vev of h at temperature T and $v[T] = v$ at $T = 0$. When the singlet part is absent, the above $E_{\text{sph}}(T)$ reduces to the form of the SMEFT case, with the potential $V_{\text{eff}}[h, T] = V_T(h, T) - V_T(v(T), T)$. The sphaleron energy at the bubble nucleation temperature $T_n(E_{\text{sph}}(T_n))$ can be obtained through the EWPT analysis. The h, f, k are functions used to define the classical field configurations [43],

$$A_i(\mu, r, \theta, \phi) = -\frac{i}{g} f(r) \partial_i U(\mu, \theta, \phi) U^{-1}(\mu, \theta, \phi), \quad (6)$$

$$H(\mu, r, \theta, \phi) = \frac{v(T)}{\sqrt{2}} \left[(1-h(r)) \begin{pmatrix} 0 \\ e^{-i\mu} \cos \mu \end{pmatrix} + h(r) U(\mu, \theta, \phi) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right], \quad (7)$$

$$S(\mu, r, \theta, \phi) = v_s(T) k(r), \quad (8)$$

where A_i are $SU(2)$ gauge fields, and the matrix U is defined as

$$U(\mu, \theta, \phi) = \begin{pmatrix} e^{i\mu}(c_\mu - i s_\mu c_\theta) & e^{i\phi} s_\mu s_\theta \\ -e^{-i\phi} s_\mu s_\theta & e^{-i\mu}(c_\mu + i s_\mu c_\theta) \end{pmatrix}, \quad (9)$$

with here $s_{\mu(\theta)} = \sin \mu(\theta)$ and $c_{\mu(\theta)} = \cos \mu(\theta)$. The sphaleron configuration is obtained at $\mu = \pi/2$ [10].

IV. GRAVITATIONAL WAVES

With the finite temperature effective potential given above, the Higgs vev at high temperature ($v(T)$) can be obtained. Here and in the following sections we define the temperature of the EWPT as $T_* \approx T_n$ [44], with T_n being the bubble nucleation temperature. The phase transition

order parameter v_n/T_n (the phase transition strength at the bubble nucleation temperature) and the two crucial parameters for the gravitational wave spectrum from the EWPT are calculated using CosmoTransitions [45]. The first parameter crucial for the gravitational wave spectrum is the ratio of released latent heat from the transition to the total radiation energy density [46],

$$\alpha = \frac{1}{\rho_R} \left[-(V_{\text{EW}} - V_f) + T \left(\frac{dV_{\text{EW}}}{dT} - \frac{dV_f}{dT} \right) \right] \Big|_{T=T_*}, \quad (10)$$

where V_f is the value of the potential at the metastable vacuum and V_{EW} is that in the electroweak vacuum. Another parameter β/H_n serves as a time scale for the EWPT,

$$\frac{\beta}{H_n} = \left[T \frac{d}{dT} \left(\frac{S_3(T)}{T} \right) \right] \Big|_{T=T_*}, \quad (11)$$

where H_n is the Hubble rate at T_n and $S_3(T)$ the action for the $O(3)$ symmetric bounce action.

The dominant sources for gravitational wave production during the EWPT are the long-lasting sound waves in the plasma [47,48] and the possible magnetohydrodynamic turbulence [47,48] [49]. A key parameter governing the strength of the dominant sound wave contribution is the fraction of released energy density that is transferred into the kinetic energy of the plasma. We calculate it with given inputs of wall velocity v_w and α from a hydrodynamic analysis [50] of the fluid profiles. We note that a significant gravitational wave production usually needs a very relativistic value of v_w [51], while the baryon asymmetry generation from EWB generally requires a relatively small wall velocity, say ~ 0.05 [33,52]. To deal with this conundrum, we take the plasma hydrodynamics into account and distinguish between v_w and the velocity used in baryogenesis calculations [13,53,54], such that a supersonic v_w can be realized while still maintaining a subsonic plasma velocity outside the bubble wall in the wall frame. To a good approximation, the total energy density of the gravitational waves in unit of the critical energy density of the universe is given by [46]

$$\Omega_{\text{GW}}(f) h^2 \approx \Omega_{\text{sw}}(f) h^2 + \Omega_{\text{turb}}(f) h^2, \quad (12)$$

with detailed formulas for the gravitational wave energy density spectra given in the Supplemental Material [55]. Due to its stochastic nature, this kind of gravitational waves can be searched for by cross-correlating outputs from two or more detectors, with the resulting signal-to-noise ratio (SNR) taking the form [46]

$$\text{SNR} = \sqrt{\mathcal{T} \int df \left[\frac{h^2 \Omega_{\text{GW}}(f)}{h^2 \Omega_{\text{exp}}(f)} \right]^2}, \quad (13)$$

where $\mathcal{T}$ is the duration of the data in years and Ω_{exp} the power spectral density of the detector.

V. NUMERICAL RESULTS

We first revisit and clarify the relation between the BNPC and the phase transition strength. Figure 1 shows that (1) the scaling law is established with $E_{\text{sph}}(T_n)v/E_{\text{sph}}v_n \approx 1$ when the BNPC is satisfied, i.e., when PT_{sph} is above the horizontal shaded region, with the band originating from the uncertainty in Eq. (1); (2) PT_{sph} is proportional to the phase transition strength v_n/T_n with the slope being 34.5 and 34.8 for the xSM and SMEFT when the BNPC is met; and (3) the BNPC yields $v_n/T_n \geq 1(1.5)$ for xSM (SMEFT), showing that the strongly first-order phase transition condition given by BNPC is different for different model classes, though the results suffer from some uncertainty due to the calculation of phase transition and to sphaleron rate simulations [15].

We proceed to explore the correlations of the sphaleron properties with features of the resulting gravitational waves, with the hope of portraying the sphalerons with future gravitational wave detections. These correlations are shown in Fig. 2. The top panel shows that a larger magnitude and a smaller peak frequency of the gravitational waves correspond to a larger sphaleron energy, applicable to both models. The bottom panel tells that the sphaleron energy increases as the phase transition strength grows, accompanied by increasing SNR of the gravitational wave signals. Here we find that $SNR > 10$ corresponds to $v_n/T_n \geq 3.46$ (and $v_n/T_n \geq 3.92$) in the xSM (and SMEFT) along the tendency of the plots [56], and $E_{\text{sph}}(T_n)$ with the largest SNR at LISA is of a typical value $\sim 1.8 \times 4\pi v/g$ which is smaller than the sphaleron energy in the standard model. The observation of the correlation may serve as a feature with which the electroweak sphaleron can be traced by gravitational wave studies for sufficiently strong phase transition [57]. At last, we check the relation between PT_{sph} and SNR in Fig. 3. An increasing tendency of PT_{sph} with growing SNR for both models is shown, which is consistent with Fig. 2 as PT_{sph} is

FIG. 1. PT_{sph} versus v_n/T_n for xSM (left) and SMEFT (right), with color denoting $E_{\text{sph}}(T_n)v/(E_{\text{sph}}v_n)$.

FIG. 2. Top panel: correlation of the gravitational wave peak frequency and the amplitude, color-coding the sphaleron energy. Bottom panel: correlation of sphaleron energy and SNR of gravitational waves, color-coding v_n/T_n . In both panels, the left (right) is for the xSM (SMEFT).

FIG. 3. PT_{sph} versus the SNR of the gravitational wave spectra for the xSM (left) and SMEFT (right) with the color-coding v_n/T_n .

dominated by the sphaleron energy [see Eq. (1)]. Therefore, the correlation between the sphaleron energy and SNR observed above implies that an EWPT strong enough to produce an observable gravitational wave signal can effectively “switch off” the sphaleron rate as required by the EWB.

VI. CONCLUSIONS

The electroweak sphaleron, which mediates a nonperturbative baryon number violating interactions, remains currently unprobable at colliders (see Refs. [14,20–23] for example). Deducing its existence in alternative methods is therefore of paramount significance. We propose a correlation between the electroweak sphaleron energy and

gravitational waves from a first-order cosmological phase transitions for the first time, which is the first step toward revealing the electroweak sphaleron through gravitational waves, with the anticipation that future detailed studies can provide a more complete picture of the sphaleron. We expect the findings of this paper to be generic for beyond standard model theories accommodating a first-order EWPT in the visible sector. On the experimental side, LIGO, which covers slightly larger frequencies, has performed searches of the stochastic gravitational waves using the O1 and O2 data [58], poised for improved analysis from the ongoing O3 data. LIGO will also search beyond the current simple power law spectra [59] to broken power laws, more appropriate for the signals studied here. Future space-based detectors, such as LISA, are better suited for making the discovery, with gravitational waves from the EWPT being a key target in the stochastic search [60]. Once detected, detailed properties of the typical broken power-law shape spectra, such as the peak position

(frequency) and amplitude can help to portray the sphaleron [36,61]. At last, we note that the gravitational wave from the inflation is flat and very different from the phase transition case, with a scale invariant spectrum produced in the slow roll inflationary scenarios; see Ref. [62].

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- [1] B. P. Abbott *et al.* (LIGO Scientific, Virgo Collaborations), Observation of Gravitational Waves from a Binary Black Hole Merger, *Phys. Rev. Lett.* **116**, 061102 (2016).
 - [2] P. A. Seoane *et al.* (eLISA Collaboration), The gravitational universe, [arXiv:1305.5720](https://arxiv.org/abs/1305.5720).
 - [3] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, Is there a Hot Electroweak Phase Transition at $m(H)$ Larger or Equal to $m(W)$?, *Phys. Rev. Lett.* **77**, 2887 (1996).
 - [4] M. D’Onofrio, K. Rummukainen, and A. Tranberg, Sphaleron Rate in the Minimal Standard Model, *Phys. Rev. Lett.* **113**, 141602 (2014).
 - [5] A. Mazumdar and G. White, Review of cosmic phase transitions: Their significance and experimental signatures, *Rep. Prog. Phys.* **82**, 076901 (2019).
 - [6] N. Christensen, Stochastic gravitational wave backgrounds, *Rep. Prog. Phys.* **82**, 016903 (2019).
 - [7] A. D. Sakharov, Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe, *Pis'ma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967) [*Usp. Fiz. Nauk* **161**, 61 (1991)].
 - [8] D. E. Morrissey and M. J. Ramsey-Musolf, Electroweak baryogenesis, *New J. Phys.* **14**, 125003 (2012).
 - [9] N. S. Manton, Topology in the Weinberg-Salam theory, *Phys. Rev. D* **28**, 2019 (1983).
 - [10] F. R. Klinkhamer and N. S. Manton, A saddle point solution in the Weinberg-Salam theory, *Phys. Rev. D* **30**, 2212 (1984).
 - [11] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, On the anomalous electroweak baryon number nonconservation in the early universe, *Phys. Lett.* **155B**, 36 (1985).
 - [12] L. Bian, H.-K. Guo, Y. Wu, and R. Zhou, Gravitational wave and collider searches for the EWSB patterns, *Phys. Rev. D* **101**, 035011 (2020).
 - [13] A. Alves, T. Ghosh, H.-K. Guo, K. Sinha, and D. Vagie, Collider and gravitational wave complementarity in exploring the singlet extension of the Standard Model, *J. High Energy Phys.* **04** (2019) 052.
 - [14] V. A. Rubakov and M. E. Shaposhnikov, Electroweak baryon number nonconservation in the early universe and in high-energy collisions, *Usp. Fiz. Nauk* **166**, 493 (1996) [*Phys. Usp.* **39**, 461 (1996)].
 - [15] H. H. Patel and M. J. Ramsey-Musolf, Baryon washout, electroweak phase transition, and perturbation theory, *J. High Energy Phys.* **07** (2011) 029.
 - [16] M. Quiros, Finite temperature field theory and phase transitions, in *Proceedings, Summer School in High-Energy Physics and Cosmology: Trieste, Italy, 1998* (1999), pp. 187–259.
 - [17] X. Gan, A. J. Long, and L.-T. Wang, Electroweak sphaleron with dimension-six operators, *Phys. Rev. D* **96**, 115018 (2017).
 - [18] S. Braibant, Y. Brihaye, and J. Kunz, Sphalerons at finite temperature, *Int. J. Mod. Phys. A* **08**, 5563 (1993).
 - [19] Y. Brihaye and J. Kunz, Electroweak bubbles and sphalerons, *Phys. Rev. D* **48**, 3884 (1993).
 - [20] F. L. Bezrukov, D. Levkov, C. Rebbi, V. A. Rubakov, and P. Tinyakov, Semiclassical study of baryon and lepton number violation in high-energy electroweak collisions, *Phys. Rev. D* **68**, 036005 (2003).
 - [21] F. L. Bezrukov, D. Levkov, C. Rebbi, V. A. Rubakov, and P. Tinyakov, Suppression of baryon number violation in electroweak collisions: Numerical results, *Phys. Lett. B* **574**, 75 (2003).
 - [22] A. Ringwald, Electroweak instantons/sphalerons at VLHC?, *Phys. Lett. B* **555**, 227 (2003).

- [23] A. Ringwald, An upper bound on the total cross-section for electroweak baryon number violation, *J. High Energy Phys.* **10** (2003) 008.
- [24] We note that very recently the Bloch wave approach was proposed such that there is a chance to observe a $(B + L)$ -violating event at the Large Hadron Collider (LHC) [63–67].
- [25] M. Dine, P. Huet, and R. L. Singleton, Jr., Baryogenesis at the electroweak scale, *Nucl. Phys.* **B375**, 625 (1992).
- [26] G. D. Moore, Measuring the broken phase sphaleron rate nonperturbatively, *Phys. Rev. D* **59**, 014503 (1998).
- [27] Y. Burnier, M. Laine, and M. Shaposhnikov, Baryon and lepton number violation rates across the electroweak crossover, *J. Cosmol. Astropart. Phys.* **02** (2006) 007.
- [28] D. J. H. Chung, A. J. Long, and L.-T. Wang, 125 GeV Higgs boson and electroweak phase transition model classes, *Phys. Rev. D* **87**, 023509 (2013).
- [29] This cannot be matched to SMEFT for the SFOEWPT parameter spaces [12]. See Refs. [38,68] for the mismatch between the SMEFT and xSM model.
- [30] S. Profumo, M. J. Ramsey-Musolf, and G. Shaughnessy, Singlet Higgs phenomenology and the electroweak phase transition, *J. High Energy Phys.* **08** (2007) 010.
- [31] S. Profumo, M. J. Ramsey-Musolf, C. L. Wainwright, and P. Winslow, Singlet-catalyzed electroweak phase transitions and precision Higgs boson studies, *Phys. Rev. D* **91**, 035018 (2015).
- [32] T. Huang, J. M. No, L. Pernié, M. Ramsey-Musolf, A. Safonov, M. Spannowsky, and P. Winslow, Resonant di-Higgs boson production in the $b\bar{b}WW$ channel: Probing the electroweak phase transition at the LHC, *Phys. Rev. D* **96**, 035007 (2017).
- [33] G. C. Dorsch, S. J. Huber, T. Konstandin, and J. M. No, A second Higgs doublet in the early universe: Baryogenesis and gravitational waves, *J. Cosmol. Astropart. Phys.* **05** (2017) 052.
- [34] C. Grojean, G. Servant, and J. D. Wells, First-order electroweak phase transition in the standard model with a low cutoff, *Phys. Rev. D* **71**, 036001 (2005).
- [35] C. Delaunay, C. Grojean, and J. D. Wells, Dynamics of non-renormalizable electroweak symmetry breaking, *J. High Energy Phys.* **04** (2008) 029.
- [36] C. Caprini *et al.*, Detecting gravitational waves from cosmological phase transitions with LISA: An update, *J. Cosmol. Astropart. Phys.* **03** (2020) 024.
- [37] J. Ellis, M. Lewicki, and J. M. No, On the maximal strength of a first-order electroweak phase transition and its gravitational wave signal, *J. Cosmol. Astropart. Phys.* **04** (2019) 003.
- [38] P. Huang, A. Joglekar, B. Li, and C. E. M. Wagner, Probing the electroweak phase transition at the LHC, *Phys. Rev. D* **93**, 055049 (2016).
- [39] F. R. Klinkhamer and R. Laterveer, The Sphaleron at finite mixing angle, *Z. Phys. C* **53**, 247 (1992).
- [40] B. Kleihaus, J. Kunz, and Y. Brihaye, The electroweak sphaleron at physical mixing angle, *Phys. Lett. B* **273**, 100 (1991).
- [41] J. Kunz, B. Kleihaus, and Y. Brihaye, Sphalerons at finite mixing angle, *Phys. Rev. D* **46**, 3587 (1992).
- [42] A. Ahriche, What is the criterion for a strong first order electroweak phase transition in singlet models?, *Phys. Rev. D* **75**, 083522 (2007).
- [43] K. Fuyuto and E. Senaha, Improved sphaleron decoupling condition and the Higgs coupling constants in the real singlet-extended standard model, *Phys. Rev. D* **90**, 015015 (2014).
- [44] The approximation is justified for an EWPT without significant reheating [46].
- [45] C. L. Wainwright, CosmoTransitions: Computing cosmological phase transition temperatures and Bubble profiles with multiple fields, *Comput. Phys. Commun.* **183**, 2006 (2012).
- [46] C. Caprini *et al.*, Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions, *J. Cosmol. Astropart. Phys.* **04** (2016) 001.
- [47] M. Hindmarsh, S. J. Huber, K. Rummukainen, and D. J. Weir, Gravitational Waves from the Sound of a First Order Phase Transition, *Phys. Rev. Lett.* **112**, 041301 (2014).
- [48] M. Hindmarsh, S. J. Huber, K. Rummukainen, and D. J. Weir, Numerical simulations of acoustically generated gravitational waves at a first order phase transition, *Phys. Rev. D* **92**, 123009 (2015).
- [49] We neglect here the contribution from the bubble wall collisions [69–74], as it is now generally believed to be negligible [75].
- [50] J. R. Espinosa, T. Konstandin, J. M. No, and G. Servant, Energy budget of cosmological first-order phase transitions, *J. Cosmol. Astropart. Phys.* **06** (2010) 028.
- [51] D. Bodeker and G. D. Moore, Electroweak Bubble wall speed limit, *J. Cosmol. Astropart. Phys.* **05** (2017) 025.
- [52] L. Bian, T. Liu, and J. Shu, Cancellations between Two-Loop Contributions to the Electron Electric Dipole Moment with a CP -Violating Higgs Sector, *Phys. Rev. Lett.* **115**, 021801 (2015).
- [53] J. M. No, Large Gravitational wave background signals in electroweak Baryogenesis scenarios, *Phys. Rev. D* **84**, 124025 (2011).
- [54] A. Alves, T. Ghosh, H.-K. Guo, and K. Sinha, Resonant Di-Higgs production at gravitational wave benchmarks: A collider study using machine learning, *J. High Energy Phys.* **12** (2018) 070.
- [55] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevD.101.091903> for sphaleron solutions and gravitational waves calculation.
- [56] These points not along the tendency of the left plot are those points with a relatively large v_s in the range 200–800 GeV.
- [57] Note that a recent numerical simulation shows that for a strong EWPT (i.e., those with $\alpha \sim 1$), there appears significant deficit in the gravitational wave production from the sound waves [76]. For the xSM, this leads to a shrinking of the parameter space that gives detectable gravitational waves while the main features of the resulting parameter space remain qualitatively unchanged (see [77] for a detailed study). For the SMEFT, however, it leads to a more severe reduction of the parameter space, making it difficult to generate detectable gravitational waves.
- [58] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaboration), Search for the isotropic stochastic background using data from Advanced LIGO’s second observing run, *Phys. Rev. D* **100**, 061101 (2019).

- [59] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaboration), Upper Limits on the Stochastic Gravitational-Wave Background from Advanced LIGO? First Observing Run, *Phys. Rev. Lett.* **118** (2017) 121101; Erratum, *Phys. Rev. Lett.* **119**, 029901 (2017).
- [60] P. Amaro-Seoane *et al.* (LISA Collaboration), Laser interferometer space antenna, [arXiv:1702.00786](#).
- [61] C. Caprini, D. G. Figueroa, R. Flauger, G. Nardini, M. Peloso, M. Pieroni, A. Ricciardone, and G. Tasinato, Reconstructing the spectral shape of a stochastic gravitational wave background with LISA, *J. Cosmol. Astropart. Phys.* **11** (2019) 017.
- [62] C. Caprini and D. G. Figueroa, Cosmological backgrounds of gravitational waves, *Classical Quantum Gravity* **35**, 163001 (2018).
- [63] J. Ellis and K. Sakurai, Search for sphalerons in proton-proton collisions, *J. High Energy Phys.* **04** (2016) 086.
- [64] Y.-C. Qiu and S. H. H. Tye, Role of Bloch waves in baryon-number violating processes, *Phys. Rev. D* **100**, 033006 (2019).
- [65] S. H. H. Tye and S. S. C. Wong, Baryon number violating scatterings in laboratories, *Phys. Rev. D* **96**, 093004 (2017).
- [66] J. Ellis, K. Sakurai, and M. Spannowsky, Search for sphalerons: IceCube vs. LHC, *J. High Energy Phys.* **05** (2016) 085.
- [67] S. H. H. Tye and S. S. C. Wong, Bloch wave function for the periodic sphaleron potential and unsuppressed baryon and lepton number violating processes, *Phys. Rev. D* **92**, 045005 (2015).
- [68] P. H. Damgaard, A. Haarr, D. O'Connell, and A. Tranberg, Effective field theory and electroweak Baryogenesis in the singlet-extended Standard Model, *J. High Energy Phys.* **02** (2016) 107.
- [69] A. Kosowsky, M. S. Turner, and R. Watkins, Gravitational radiation from colliding vacuum bubbles, *Phys. Rev. D* **45**, 4514 (1992).
- [70] A. Kosowsky, M. S. Turner, and R. Watkins, Gravitational Waves from First Order Cosmological Phase Transitions, *Phys. Rev. Lett.* **69**, 2026 (1992).
- [71] A. Kosowsky and M. S. Turner, Gravitational radiation from colliding vacuum bubbles: Envelope approximation to many bubble collisions, *Phys. Rev. D* **47**, 4372 (1993).
- [72] S. J. Huber and T. Konstandin, Gravitational wave production by collisions: More bubbles, *J. Cosmol. Astropart. Phys.* **09** (2008) 022.
- [73] R. Jinno and M. Takimoto, Gravitational waves from bubble collisions: An analytic derivation, *Phys. Rev. D* **95**, 024009 (2017).
- [74] R. Jinno and M. Takimoto, Gravitational waves from bubble dynamics: Beyond the Envelope, *J. Cosmol. Astropart. Phys.* **01** (2019) 060.
- [75] D. Bodeker and G. D. Moore, Can electroweak bubble walls run away?, *J. Cosmol. Astropart. Phys.* **05** (2009) 009.
- [76] D. Cutting, M. Hindmarsh, and D. J. Weir, Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions, [arXiv:1906.00480](#).
- [77] A. Alves, D. Gonçalves, T. Ghosh, H.-K. Guo, and K. Sinha, Di-Higgs Production in the $4b$ channel and gravitational wave complementarity, *J. High Energy Phys.* **03** (2020) 053.