Decay of cosmic global string loops

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We numerically study the decay of cosmic global string loops due to radiation of Goldstone bosons and massive scalar (χ) particles. The length of loops we study range from 200 to 1000 times the width of the string core. We find that the lifetime of a loop is $\approx 1.4L$. The energy spectrum of Goldstone boson radiation has a k^{-1} falloff, where k is the wave number, and a sharp peak at $k \approx m_\chi/2$, where m_χ is the mass of χ . The latter is a new feature and implies a peak at high energies (MeV-GeV) in the cosmological distribution of QCD axions.

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I. INTRODUCTION

If high energy particle physics contains a global U(1) symmetry that spontaneously breaks at lower energies, the universe would be left with a network of global strings [1]. Loops of such cosmic global strings would oscillate and decay by the emission of massless and massive radiation that could form part of the dark matter density today [2]. The scenario is relevant to models in which axions are proposed as a means to solve the strong CP problem and is frequently studied in this context [2–14].

The emission of Goldstone radiation from global string loops has been investigated in the Nambu-Goto limit in which the string core has negligible thickness [2]. The calculation is enabled by replacing the scalar field for the Goldstone degree of freedom by a two form gauge field, leading to the Kalb-Ramond description of the string [15]. The radiation has also been studied by numerically evolving the field theory configuration of a string loop [6,10]. Both methods have their limitations. The Kalb-Ramond approach assumes that there is only Goldstone boson radiation and no massive scalar radiation. Also, the backreaction of Goldstone boson radiation on the string dynamics has not yet been taken into account. The numerical field theory method involves a full description of the problem but is limited in dynamic range. Hence, the results must be extrapolated to long loops such as would arise in cosmology.

Here we adapt our numerical field theory study of local cosmic string loops [16] to the case of global strings. The key features of our work that distinguish it from the earlier field theory loop simulations [6,10] are that (i) we consider

loops that are formed from the collision of long straight strings, much as they might form in cosmology, and (ii) we consider relatively long loops, several hundred times the width of the string core. The first feature helps to set up the initial conditions, as the straight string solution is known and we only need to patch together the solutions. The second feature helps with the extrapolation to cosmological scales.

Our results are summarized as follows. Global string loops emit both massless Goldstone radiation and massive particles (denoted by χ) and decay in a time proportional to the size of the loop L . Initially, our loops have length $4L$ and invariant length (energy divided by string tension) of $5L$ due to a Lorentz boost factor of 1.25. We find that the energy distribution of massive particles is peaked at very low wave numbers and they are nonrelativistic at production. Eventually, the massive particles will decay into Goldstone bosons but their decay leaves a sharp feature in the spectrum of Goldstone bosons. The energy spectrum of radiated Goldstone bosons takes the form

$$\frac{d\mathcal{E}_k}{dk} = \eta^2 L \frac{a}{k}, \quad (2L)^{-1} \leq k \lesssim m_\chi, \quad (1)$$

where η is the vacuum expectation value of the scalar field, k is the magnitude of the wave vector, and $a \approx 4.8$ is a coefficient that we determine numerically. Although the spectrum is peaked at the smallest wave vector, the integrated energy at larger momenta is greater for large loops because this contribution grows as $\ln(m_\chi L)$.

This paper is organized as follows. In Sec. II, we describe the field theory, some basic features of cosmic strings, and our scheme for producing a global string loop. We then turn to our numerical implementation on a lattice with periodic boundary conditions in Sec. III, where we also describe the results of our simulations, contrasting global string evolution with that of gauge strings. In Sec. IV, we turn to the radiation produced by the loops and evaluate the fraction of

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energy in massless to massive radiation. The spectrum of radiation is discussed in Sec. V. Here we analyze the spectral features of both massive and massless radiation from global strings and find a good fit to the form of the spectrum in (1) for the Goldstone radiation. We conclude in Sec. VI, where we also place our results in the context of earlier work. The implementation of periodic boundary conditions requires care as the Goldstone boson cloud around the string loop decays relatively slowly and reaches the boundaries of the lattice. We describe our implementation in Appendix A.

II. MODEL

We consider the global U(1) field theory with a complex scalar field, $\phi = \phi_1 + i\phi_2$, for which the field equations of motion are

$$\partial_t^2 \phi_a = \nabla^2 \phi_a - \lambda(\phi_b \phi_b - \eta^2) \phi_a, \quad (2)$$

where $a = 1, 2$, λ is a coupling constant. By suitable rescalings of the fields and the coordinates, we can set $\lambda = 1/2$ and $\eta = 1$ and then the (classical) model has no free parameters.

The solution for a straight global string along the z axis is

$$\phi = \eta f(r) e^{i\varphi}, \quad (3)$$

where $r = \sqrt{x^2 + y^2}$, $\varphi = \tan^{-1}(y/x)$, and $f(r)$ is a string profile functions that vanishes at the origin and asymptotes to 1 as

$$f(r) \rightarrow 1 - \mathcal{O}\left(\frac{1}{r^2}\right). \quad (4)$$

The energy density in the scalar field is given by

$$\mathcal{E} = \frac{1}{2} |\partial_t \phi|^2 + \frac{1}{2} |\nabla \phi|^2 + \frac{\lambda}{4} (|\phi|^2 - \eta^2)^2, \quad (5)$$

which, if we write $\phi \equiv \rho \exp(i\alpha)$, can also be expressed as

$$\mathcal{E} \equiv \mathcal{E}_\rho + \mathcal{E}_\alpha, \quad (6)$$

where the energy density in massive modes (ρ) is defined as

$$\mathcal{E}_\rho = \frac{1}{2} (\partial_t \rho)^2 + \frac{1}{2} (\nabla \rho)^2 + \frac{\lambda}{4} (\rho^2 - \eta^2)^2 \quad (7)$$

and that in Goldstone modes (α) as

$$\mathcal{E}_\alpha = \frac{\rho^2}{2} [(\partial_t \alpha)^2 + (\nabla \alpha)^2]. \quad (8)$$

The string energy per unit length (also its tension) is found by integrating the energy density of the solution in (3) in the $z = 0$ plane. The integration of $\mathcal{E}_\rho$ is finite but the integral of $\mathcal{E}_\alpha$ diverges logarithmically with distance. With a long range cutoff at $r = \Lambda$, the energy per unit length is

FIG. 1. Four straight strings are set up with velocities as shown. The strings intersect and reconnect to produce a central “inner” loop and also a second “outer” loop because of periodic boundary conditions. These loops then oscillate and decay. By choosing the spacing of the initial straight strings, we can produce loops of different sizes, though they all have the same initial shape.

$$\mu \approx \pi \eta^2 \ln(\Lambda \eta). \quad (9)$$

We now create a loop for our simulations following the scheme in [16] and as illustrated in Fig. 1. Our initial conditions consist of four straight strings boosted with velocities $\pm \mathbf{v}_1$ and $\pm \mathbf{v}_2$ as shown schematically in Fig. 1. The four string solutions are patched together using the “product ansatz.” If Φ_a ($a = 1, \dots, 4$) denotes the solution for the individual strings, the field is taken to be

$$\phi(t=0, \mathbf{x}) = \frac{1}{\eta^3} \prod_{a=1}^4 \Phi_a, \quad (10)$$

and the time derivatives at the initial time are

$$\dot{\phi}(t=0, \mathbf{x}) = \phi(t=0, \mathbf{x}) \sum_{b=1}^4 \frac{\dot{\Phi}_b}{\Phi_b}, \quad (11)$$

with $\dot{\Phi}_b$ obtained from the boosted solution for a single string.

While this scheme can be used to construct a loop in an infinite spatial volume, our simulations are on a finite lattice and employ periodic boundary conditions. These numerical limitations necessitate some modifications of the initial conditions that are described in Appendix A.

III. SIMULATION AND RESULTS

Once the four strings collide, they reconnect to form two loops—two because of periodic boundary conditions. If the string velocities are small and not oriented suitably, the resulting loops will have insufficient angular momentum and will collapse quickly. We choose velocity magnitudes that are mildly relativistic, $|\mathbf{v}_1| = 0.6 = |\mathbf{v}_2|$, as expected in a cosmological setting. The directions are taken to be

$(\hat{v}_1)_x = 0.4$, $(\hat{v}_1)_y = \sqrt{1 - 0.4^2} \approx 0.92$ for the two strings oriented along the z axis and $(\hat{v}_2)_z = 0.4$, $(\hat{v}_2)_y \approx 0.92$ for the strings along the x axis.

Next, we use the explicit Crank-Nicholson algorithm with two iterations for the numerical evolution [17] with periodic boundary conditions, keeping track of the energy densities in the core of the string and the Goldstone mode [see Eqs. (7) and (8)] and the total energy and angular momentum. The core of the string is defined as the region where $|\phi|/\eta < 0.9$. We take the initial string separation in Fig. 1 to be half the size of our lattice for all our runs. The simulation then produces two loops due to the periodic boundary conditions but both loops are the same size.

We have run our simulations for a few different values of the lattice spacing, Δx , and found that the results are sensitive to the resolution. For example, the total energy in the simulation box over the entire run is conserved only at $\sim 20\%$ level when $\Delta x = 0.50$ for longer runs (required for large lattices). We have set $\eta = 1$, $e = 1$, $\lambda = 1/2$, and so the string width is ~ 1 . Therefore, with $\Delta x = 0.5$, we only have a few lattice points within the width of the string. The run with $\Delta x = 0.25$ gives better conservation to $\sim 5\%$ level over the entire run and agrees quite well with the much more computationally expensive run with $\Delta x = 0.125$.

In Fig. 2, we plot the loop energy versus time for several loop sizes with $\Delta x = 0.25$. We take the string core to be the lattice cells where $|\phi| < 0.9\eta$, with the sum of energies in all such cells giving the energy of the loop. Unlike in the case of gauge strings [16], the decay of global string loops is not episodic and the energy gradually dissipates.

As the loops evolve, they also shed their angular momentum, defined as

$$L_i \equiv -\frac{1}{2} \epsilon_{ijk} \int_{\text{string core}} d^3x x_j (\partial_i \phi \partial_k \phi^* + \partial_i \phi^* \partial_k \phi), \quad (12)$$

FIG. 2. The loop energy as a function of time for the outer loop for different values of loop sizes $L = 50, 100, 150, 200, 250$ (lowest to highest curve). A similar plot is obtained for the inner loops.

FIG. 3. The loop angular momentum as a function of time for the outer loop for different values of loop sizes $L = 50, 100, 150, 200, 250$ (lowest to highest curve).

where x_j is measured from the center of energy of the loop. In Fig. 3, we plot $|\mathbf{L}|$ versus time and also see gradual decay.

IV. MASSIVE VERSUS MASSLESS RADIATION

The global string loop emits massive and massless Goldstone radiation. The massive radiation corresponds to excitations of the field ρ and its energy density is given in (7); the massless radiation corresponds to excitations of α with energy density given in (8). Note that ρ and α interact, which is evident in (8). However, at late times, we can write $\rho = \eta + \chi$, $\theta = \alpha$, where χ is a small excitation above the true vacuum and expand the energy density functions to lowest order in χ ,

$$\mathcal{E}_\rho = \frac{1}{2} [(\partial_i \chi)^2 + (\nabla \chi)^2 + m_\chi^2 \chi^2] + \dots \equiv \mathcal{E}_\chi + \dots, \quad (13)$$

$$\mathcal{E}_\alpha = \frac{\eta^2}{2} [(\partial_i \alpha)^2 + (\nabla \alpha)^2] + \dots \equiv \mathcal{E}_\theta + \dots, \quad (14)$$

where $m_\chi = \sqrt{2\lambda}\eta$. By integrating these expressions, we obtain the total energy in the two components,

$$E_a = \int d^3x \mathcal{E}_a, \quad (15)$$

where $a = \rho, \alpha$. At early times, E_ρ and E_α will differ from E_χ and E_θ , respectively, but they will coincide at late times, when $\rho \approx \eta$.

In Fig. 4, we plot the total energy in each of the components ρ , χ , α , and θ versus time in the run with lattice size 1600^3 . We see that the Goldstone mode has significantly more initial energy compared to the massive mode and the ratio of the energies in ρ and θ remains approximately constant throughout the evolution. The energies in χ and θ agree with those in ρ and α once the loop has decayed as is expected. This pattern is repeated in

FIG. 4. Energy in massive and massless components and total energy (TE) for $L = 200$. Lower curves are for massive radiation calculated as ρ (green) and χ (red). Middle curve is for Goldstone radiation calculated for θ (blue) and α (orange). Top curve (black) is the total energy.

all our simulations with different loop sizes; however, the ratio E_θ/E_χ increases with loop size, as seen in Fig. 5. This shows that massive radiation become less important for larger loops. To obtain the length dependence of the ratio, we need to find a fit to the plot in Fig. 5. Unfortunately, we could not find an unambiguous fit to the data—a linear dependence, power law dependence, and logarithmic dependence, all seem to fit the data equally well. Yet, based on Fig. 4, there is an alternate way to estimate the length dependence of the ratio. This method uses the observation that the energies in χ and θ are approximately conserved during the collapse time. (Eventually, χ particles will decay into Goldstone bosons.) So, to obtain the ratio we simply need to estimate these energies at the initial time, which we can do using the initial conditions described in Sec. II. Since no evolution is necessary to get the initial energies, we can go to much larger lattices. The initial ratios versus L are shown in Fig. 6 on a log-linear plot, showing that the ratio grows as $\ln(m_\chi L)$. The logarithm can also be

FIG. 5. Plot for the ratio of energy in the Goldstone mode to energy in the massive mode at the decay time as a function of loop size.

FIG. 6. Log-linear plot for the ratio of initial energy in the Goldstone mode to initial energy in the massive mode as a function of loop size.

understood by noting that the energy of the Goldstone cloud around a single global string diverges logarithmically with distance from the string [see (9)]. For a loop, the loop size provides a cutoff on the divergence but it means that the Goldstone cloud has energy proportional to $\ln(m_\chi L)$. In our simulations, we have modified the string ansatz slightly to account for the periodic boundary conditions as described in Appendix A, so we have calculated the energy numerically as shown in Fig. 6.

We use Fig. 4 to define the loop lifetime τ : the θ and α curves coincide once the strings have decayed and $\rho \approx \eta$ is a good approximation. The loop lifetime τ versus loop length is shown in Fig. 7 and is well described by a linear relation $\tau \approx 1.4L$.

In Fig. 8, we show a snapshot of the potential energy density at an intermediate time in the evolution. Unlike the gauge string, the global string is fluffy, which may be understood as due to the soft power law profile function in (4) as opposed to the hard exponential profile functions in the gauge case. Deformations of the core correspond to excitations of the massive degree of freedom. In the animations, we see that the kinks get rounded out but they

FIG. 7. The loop decay time as a function of the loop size. The best fit line is $\tau = 1.39L + 4.75$.

FIG. 8. A snapshot of the potential energy density for a loop with $L = 50$ at an intermediate time showing a “fluffy” deformable core and massive radiation. The full animation can be seen at [18].

also produce bulges in the string core as seen in Fig. 8. The transfer of energy from kink collisions to core oscillations is an intermediate step in the process of the eventual decay of the entire loop energy into Goldstone modes that is not accounted for by the Kalb-Ramond approximation. It remains to be determined if the core oscillations play a significant role for cosmological size loops.

The bulges in Fig. 8 suggest the existence of a bound state on a global string and we can confirm this explicitly. Consider a perturbation of a straight global string oriented along the z axis,

$$\phi(t, r, \theta, z) = (f(r) + e^{-i\omega t}g(r))e^{i\theta}, \quad (16)$$

where we have set $\eta = 1$ for convenience. (To obtain bound states that propagate along the string, we would replace ωt by $\omega t - kz$.) The string profile function satisfies

$$-f'' - \frac{f'}{r} + \left[\frac{1}{r^2} + \frac{1}{2}(f^2 - 1) \right] f = 0, \quad (17)$$

with $f(0) = 0$ and $f(\infty) = 1$. Upon linearization, the perturbation $g(r)$ satisfies the Schrodinger-type equation,

$$-g'' - \frac{g'}{r} + \left[\frac{1}{r^2} + \frac{3}{2}(f^2 - 1) \right] g = \Omega g, \quad (18)$$

where $\Omega \equiv \omega^2 - 1$ and $g(0) = 0 = g(\infty)$. A nontrivial bound state solution, i.e., with $\Omega < 0$, of this equation corresponds to a bound state deformation of the global string profile. That a bound state should exist can be seen by comparing Eqs. (17) and (18). The potential term in square brackets in the Schrodinger equation (18) is deeper than the corresponding term appearing in (17) by the extra term $2(f^2 - 1)/2 < 0$. We know that $f(\infty) = 1$, so the

extra term in the potential in (18) will have the effect of decreasing g as compared to f and can make it vanish asymptotically for the correct eigenvalue Ω . We have confirmed this by solving (17) and (18) and determine the lowest energy eigenvalue to be $\Omega \approx -0.19$, thus explicitly showing the existence of massive bound states on the global string.

V. ENERGY SPECTRUM

We begin by decomposing the fields χ and $\theta = \alpha$ into Fourier modes,

$$\chi = \int \frac{d^3k}{(2\pi)^3} [\chi_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + \chi_{\mathbf{k}}^*(t)e^{i\mathbf{k}\cdot\mathbf{x}}], \quad (19)$$

$$\theta = \int \frac{d^3k}{(2\pi)^3} [\theta_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + \theta_{\mathbf{k}}^*(t)e^{i\mathbf{k}\cdot\mathbf{x}}]. \quad (20)$$

The energy densities in a Fourier mode labeled by $\mathbf{k}$ are given by

$$\mathcal{E}_{\chi\mathbf{k}} = \frac{1}{2} [|\partial_t \chi_{\mathbf{k}}|^2 + (k^2 + m_\chi^2)|\chi_{\mathbf{k}}|^2], \quad (21)$$

$$\mathcal{E}_{\theta\mathbf{k}} = \frac{\eta^2}{2} [(\partial_t \theta_{\mathbf{k}})^2 + k^2|\theta_{\mathbf{k}}|^2]. \quad (22)$$

In general, the spectra will depend on all three components of the wave vector $\mathbf{k}$. However, if we sum over a large number of loops, with different shapes, sizes, and orientations, we can expect an isotropic spectrum. To extract an isotropic spectrum from our simulation, we bin the spectral components according to their $k = |\mathbf{k}|$ value and sum over all modes with $|\mathbf{k}|$ in the interval $\mathcal{R}(k) = (k - \Delta k, k)$, where $\Delta k = 2\pi/(2L)$ ($2L$ is the lattice size in our simulations),

$$\mathcal{E}_{\chi k} = \frac{(\Delta k)^3}{(2\pi)^3} \sum_{|\mathbf{k}| \in \mathcal{R}(k)} \mathcal{E}_{\chi\mathbf{k}}, \quad \mathcal{E}_{\theta k} = \frac{(\Delta k)^3}{(2\pi)^3} \sum_{|\mathbf{k}| \in \mathcal{R}(k)} \mathcal{E}_{\theta\mathbf{k}}. \quad (23)$$

Note that the sum is over vectors $\mathbf{k}$ with the same magnitude. Hence, it includes the $4\pi k^2$ factor that arises from the phase space volume factor and to obtain the total energy one only needs to sum over all the modes,

$$E_\chi = \sum_k \mathcal{E}_{\chi k}, \quad E_\theta = \sum_k \mathcal{E}_{\theta k}. \quad (24)$$

We plot $\mathcal{E}_{\chi k}$ versus k on a log-log scale in Fig. 9. The energy in the higher k modes does not depend on the size of the loop, but the energy in the lowest few modes grows with the size of the loop. It is worth noting that the spectrum gets cut off at $k \approx 0.1 \times 2\pi \approx 0.6$ which corresponds to a momentum less than $m_\chi = 1$. Hence, the massive particles emitted by the string are nonrelativistic especially for long loops.

FIG. 9. A log-log plot of the energy spectrum of massive radiation after the loops in the simulation have collapsed for the runs with initial loop size 50 (blue circles), 100 (red crosses), 150 (green triangles), 200 (orange squares), and 250 (black pluses).

The spectrum for Goldstone radiation is shown in the log-log plot of Fig. 10. We see that the spectrum decays as $15/k$ until a cutoff wave number $k_c \approx 0.5$, after which there is essentially negligible energy contribution. To obtain the continuum version of the energy spectrum as given in (1), we divide both sides of (23) by $\Delta k = \pi/L$. Then,

$$\frac{d\mathcal{E}_{\theta k}}{dk} = \frac{L}{\pi} \frac{15\eta^2}{k} \approx 4.8 \frac{\eta^2 L}{k}, \quad (2L)^{-1} \leq k \lesssim m_\chi. \quad (25)$$

One additional feature we see in the spectrum of Goldstone radiation is the peak at $k \approx 0.08 \times 2\pi$ in Fig. 10 for all the loops we have simulated. The location of the peak, at $k \approx 0.5 = m_\chi/2$, reveals the origin of this feature. It is caused by the massive particles that decay into Goldstone bosons due to the interaction $\chi(\partial_\mu \theta)^2$. Since a

FIG. 10. A log-log plot of the energy spectrum of Goldstone radiation when the initial loop size is 50 (blue circles), 100 (red crosses), 150 (green triangles), 200 (orange squares), and 250 (black pluses). The overlaid black dashed line is given by $15/k$ and is a good fit out to $k \approx 0.08 \times 2\pi \approx 0.5$. The peak at $k \approx 0.5$ corresponds to energy $\approx m_\chi/2$.

radiated χ is nonrelativistic, it would decay into two Goldstone bosons, each with energy of about $m_\chi/2$.

VI. CONCLUSIONS

In this paper, we have focused on cosmic global string loops, their dynamics and decay. We numerically evolved loops of global string with length up to 1000 times the width of the string core. By extrapolating our results, we can meaningfully discuss cosmologically relevant loops whose size can be comparable to the cosmic horizon and many orders of magnitude larger than the core thickness.

Our results show that global string loops decay very quickly with lifetime $\sim L$ by radiating Goldstone bosons (θ) and massive particles (χ). Most of the energy is radiated in Goldstone bosons and the emission of χ is suppressed by $1/\ln(m_\chi L)$, which in a cosmological setting would be ~ 0.01 – 0.1 . The emitted χ particles are nonrelativistic and they eventually decay into Goldstone bosons, producing a sharp peak in the Goldstone boson energy spectrum at $m_\chi/2$. In cosmology, the decay takes place continually during the evolution of the global string network and the sharp peak at $m_\chi/2$ gets smeared out due to cosmological redshifting of the energy. Even if the global string network were to decay at a specific epoch, as in the axion scenario, a sharp feature in the light particle (axion) spectrum that is produced due to heavy particle decay is not expected to survive. Such particles would be relativistic and their energy density would dilute like radiation instead of matter. The energy density of ultrarelativistic axions would be suppressed by the factor $z_{\text{QCD}} \sim 10^{12}$ as compared to nonrelativistic axions.

We have also extracted the energy spectrum for the Goldstone boson radiation from global string loops in our simulations. We obtain a $1/k$ spectrum, confirming that the results of Refs. [6,10] hold even for loops formed by processes expected in a cosmological setting. The peak in our simulations, at $k \approx m_\chi/2$, is a new feature, though one that should have been expected in hindsight. The suppressed strength of the peak (1%–10%) is due to the logarithmic divergence associated with the Goldstone component of global strings.

To summarize, we have obtained a remarkably simple picture for the decay of cosmic global string loops. Initially, the loop consists of a massive field and a Goldstone field. The loop quickly collapses, releasing radiation of massive field and Goldstone bosons, with the Goldstone radiation retaining its original $1/k$ spectrum. The radiated total energies correspond to the initial energies in these components, except that the massive particles decay into Goldstone bosons. As the massive particles are nonrelativistic, they produce a line signature in the spectrum of Goldstone bosons at $k \approx m_\chi/2$ on top of the $1/k$ continuum. We have tested the sensitivity of our results to the initial velocity and the field profile and find that the results are qualitatively unchanged as seen in Appendix B.

In contrast, studies of loop decay [2,7] using the Kalb-Ramond description assume that there is negligible massive radiation and that radiation backreaction can be ignored. Then it is found that all the massless radiation is in the first few radiation harmonics. This is at odds with our results obtained by evolving the full field theory for loops that have length about 1000 times the width of the string. It is clear that radiation backreaction is very important for us, and we see a conversion of the kink collision energy into core oscillations that enhance massive radiation. Even if backreaction is included in the Kalb-Ramond approach, it does not allow for core oscillations and cannot account for the massive radiation. The caveat in the simple picture we have suggested is if our loops are large enough that the results can be extrapolated to vastly larger loops of cosmological interest.

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APPENDIX A: IMPLEMENTING PERIODIC BOUNDARY CONDITIONS

The ansatz in Eqs. (10) and (11) does not satisfy periodic boundary conditions and requires further adjustments. This is done in several steps that we now outline. The basic idea is to first ensure that a parallel string-antistring pair satisfies periodic boundary conditions and then to patch together the vertical and horizontal pairs by using the product ansatz.

We set the phase of the field and time derivatives of the field to zero at the boundaries of the planes orthogonal to each string-antistring pair. For example, for the pair along z direction, we have the field

$$\begin{aligned}\phi_{s\bar{s},z} &= \frac{\phi_{s,z}\phi_{\bar{s},z}}{\eta} = \frac{|\phi_{s,z}||\phi_{\bar{s},z}|}{\eta} e^{i(\theta_{s,z}-\theta_{\bar{s},z})} \\ &= \phi_{s\bar{s},z1} + i\phi_{s\bar{s},z2}.\end{aligned}\quad (\text{A1})$$

We now multiply the imaginary part of the field above with the function

$$f_z = \left(1 - \frac{x^2}{L^2}\right)\left(1 - \frac{y^2}{L^2}\right), \quad (\text{A2})$$

where $2L$ is the size of the numerical domain. This forces the phase of the field to vanish at the boundaries but also

alters the magnitude of the field. We then normalize the field so that the magnitude is reset to its original value on the boundaries,

$$\phi_{s\bar{s},z1}^{(1)} = \sqrt{\frac{\phi_{s\bar{s},z1}^2 + \phi_{s\bar{s},z2}^2}{\phi_{s\bar{s},z1}^2 + \phi_{s\bar{s},z2}^2 f_z^2}} \phi_{s\bar{s},z1}, \quad (\text{A3})$$

$$\phi_{s\bar{s},z2}^{(1)} = \sqrt{\frac{\phi_{s\bar{s},z1}^2 + \phi_{s\bar{s},z2}^2}{\phi_{s\bar{s},z1}^2 + \phi_{s\bar{s},z2}^2 f_z^2}} \phi_{s\bar{s},z2} f_z. \quad (\text{A4})$$

Note that the locations of the strings do not change by the above manipulations because the zeros of the field are unaffected.

The time derivatives can now be set to zero at the boundaries by simply multiplying the derivatives with the function f_z , that is,

$$\dot{\phi}_{s\bar{s},z}^{(1)} = \partial_t \phi_{s\bar{s},z}^{(1)} f_z. \quad (\text{A5})$$

At this stage the field denoted by $\phi_{s\bar{s},z}^{(1)}$ has zero phase on the boundaries, but its magnitude still varies and does not satisfy periodic boundary conditions. In a cosmological setting, the field magnitude will be η ($=1$) far from the strings. Hence, we force the magnitude to be η on the boundaries of the lattice. To achieve this, we successively divide the field $\phi_{s\bar{s},z}^{(1)}$ by functions that interpolate between boundary values along orthogonal directions. The first interpolating function is defined as

$$I_{zx} = \frac{|\phi_{s\bar{s},z}^{(1)}|_{x=L} - |\phi_{s\bar{s},z}^{(1)}|_{x=-L}}{2L} (x+L) + |\phi_{s\bar{s},z}^{(1)}|_{x=-L}, \quad (\text{A6})$$

which gives us a new field periodic in the x direction,

$$\phi_{s\bar{s},z}^{(2)} = \frac{\phi_{s\bar{s},z}^{(1)}}{I_{zx}}. \quad (\text{A7})$$

Then we divide by a corresponding interpolation function in the y direction,

$$I_{zy} = \frac{|\phi_{s\bar{s},z}^{(2)}|_{y=L} - |\phi_{s\bar{s},z}^{(2)}|_{y=-L}}{2L} (y+L) + |\phi_{s\bar{s},z}^{(2)}|_{y=-L} \quad (\text{A8})$$

to get a field that satisfies periodic boundary conditions in both x and y directions,

$$\phi_{s\bar{s},z}^P = \frac{\phi_{s\bar{s},z}^{(2)}}{I_{zy}}. \quad (\text{A9})$$

The steps above ensure that the complex scalar field and its time derivatives are periodic along each direction for both string-antistring pairs. As a final step, we construct the field and its time derivative using the product ansatz,

$$\phi = \frac{\phi_{s\bar{s},z}^P \phi_{s\bar{s},x}^P}{\eta}, \quad (\text{A10})$$

$$\dot{\phi} = \frac{\dot{\phi}_{s\bar{s},z}^P \phi_{s\bar{s},x}^P + \phi_{s\bar{s},z}^P \dot{\phi}_{s\bar{s},x}^P}{\eta}. \quad (\text{A11})$$

Now, ϕ and $\dot{\phi}$ satisfy periodic boundary conditions and can be used as initial conditions in our simulations.

APPENDIX B: TESTS WITH DIFFERENT INITIAL CONDITIONS

To examine the dependence of our results on the way the initial conditions were prepared, we ran two tests. First, we

change the initial velocity of the straight strings from 0.6 to 0.8. The results for the energy decay and the spectrum in the two cases are shown in Fig. 11. As one can see, some of the numerical values change, as expected, but the qualitative features are unchanged.

To test the sensitivity of our results to the specific way in which the initial configuration was prepared, we have run the simulation with the function f_z in Eq. (A2) replaced by f_z^2 , since f_z^2 also has the property that it vanishes on the boundaries and is 1 at the origin. The results for the energy decay and the spectrum are shown in Fig. 12. As in Fig. 11, the qualitative features are same in both cases with slight differences in the numerical values.

FIG. 11. The core energy and the spectra of the decay products for loops of size 100 formed from straight strings moving at velocities $v = 0.6$ (red crosses) and $v = 0.8$ (blue dots).

FIG. 12. The core energy and the spectra of the decay products for loops of size 100 with functions f_z (red crosses) and f_z^2 (blue dots) used to ensure periodic boundary conditions.

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