Degenerate Sub-keV fermion dark matter from a solution to the Hubble tension

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We present a dark sector model addressing both the Hubble tension and the core-cusp problem. The model is based on a hidden Abelian gauge symmetry group with some chiral fermions required by the anomaly cancellation conditions, producing a candidate for the decaying fermion dark matter as a solution to the Hubble tension. Moreover, the sub-keV mass regime and the thermal history of the dark sector help the dark matter candidate resolve the core-cusp problem occurring in the standard Λ cold dark matter cosmology.

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I. INTRODUCTION

The presence of dark matter (DM) becomes an unquestionable fact thanks to various observational evidences. Nevertheless, its nature still remains unclear. So far, at least three physical features of DM are known, i.e., its stability, nonzero mass, and very weak interaction with the ordinary matter. Unfortunately, however, as for the three aspects of DM, none was examined clearly thus far. It still remains unanswered whether its stability is permanent or effective only for the time scale of the age of the Universe today. Also, the uncertainty in a possible DM mass has not been narrowed down yet, ranging from $\mathcal{O}(10^{-22})$ eV for the ultralight bosonic DM [1,2] to $\mathcal{O}(10)$ $M_\odot$ for primordial black holes¹ or a macroscopic compact halo object. Above all, lacking knowledge about nongravitational interaction, which DM does, we are still even unaware of if DM is, either of directly or indirectly, coupled to or totally decoupled from the standard model (SM) sector.

In an effort to address the aforementioned questions, we give a special attention to two arguments based on cosmological and astrophysical phenomena. The recently raised Hubble tension is one of them. This regards the discrepancy reaching $\sim 4\sigma$ level [4] between a local measurement of the Hubble expansion rate (H_0) [5–8] and that inferred from the

cosmic microwave background (CMB) observation [9]. While unknown systematics may affect the discrepancy [10–12], it could be a clue for a nature of DM. In particular, a decaying DM (DDM) solution [13]² to the tension amongst several resolutions proposes a possibility that the decay of DM with a lifetime ~ 35 Gyrs can relieve the tension.³ The advanced starting of the dark energy dominated era due to DM decay enables the faster expansion of the Universe at late times [$z \simeq \mathcal{O}(0.1)$] and eventually the Hubble expansion rate at the recombination era determined by CMB data can evolve to a value close to the SHOES measurement of $H_0 \simeq 73\text{--}74$ Km/sec/Mpc [6]. Inspired by this, we take a DM scenario with a finite lifetime greater than the current age of the Universe as far as the question about stability of DM is concerned.

On the other hand, whereas the correlation of structures on the large scale is in excellent accord with predictions from the Λ cold dark matter (Λ CDM) paradigm (cosmological constant + cold dark matter), discrepancies between results from simulations based on Λ CDM (e.g., cuspy halos [24], too many subhalos [25,26], and dense cores [27]) and the experimental observations on galactic scales may be hinting for a nature of DM distinct from CDM (for a review, see [28]). Particularly, from the disagreement between the cuspy DM density profile for galaxies predicted by Λ CDM simulations [29–31] and observations of rotation curves for low mass galaxies [32–35], there arises the so-called cusp-core problem. As one of the resolutions to the problem, the presence of degenerate fermion DM with sub-keV mass

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¹See, e.g., [3] for the possibility of primordial black holes as all dark matter.

²For an exemplary particle physics model for the DDM solution to the Hubble tension, see [14,15].

³Other scenarios proposed to solve the tension include, for example, dark radiation [5,16,17], dark energy at early time [18–21], interacting dark energy [22,23], etc.

has been suggested in [36–41]. The quantum pressure applied by a degenerate fermion gas can counterbalance the gravity to prevent gravitational collapse, yielding cored DM profiles in dwarf spheroidal (dSphs) galaxies. Modeling DM halo as a degenerate fermion gas with the use of the stellar velocity dispersion data of dSphs and the inferred core size of the Fornax dwarf galaxy favor the fermion DM mass in the sub-keV range.

In this paper, we present a sub-keV fermion DDM scenario which addresses both the Hubble tension and the core-cusp problem. The model is based on a hidden Abelian gauge symmetry group and the fermion DDM is one of the chiral fermions required to cancel the chiral anomaly. The dark matter is nonthermally produced and sufficiently cooled down before virialization to follow the degenerate configuration. The Hubble tension is reconciled by the dark matter decaying into a massive dark photon and a light chiral fermion. We also comment about the constraint from the Lyman- α forest [42,43] and show that our model is not inconsistent with its mass bound.

II. A MODEL FOR THE DARK SECTOR

Let us begin with the basic framework of our model for the dark sector. As a minimal extension of the gauge sector in a full theory, we consider a hidden Abelian gauge symmetry $U(1)_X$ under which matters in the dark sector are charged while SM particles are neutral.

As will be discussed below, the picture for the DDM solution to the Hubble tension in [13] assumes at least three particle candidates for (1) a decaying dark matter, (2) a radiation resulting from the decay, and (3) a daughter dark matter resulting from the decay. Thus, in the case where none of three is a SM particle, the introduction of fields in the dark sector other than the $U(1)_X$ gauge field is necessary. The anomaly-free condition for $U(1)_X$ is a useful guideline for how the introduction is made. As a matter of fact, a simple setup dubbed “Number Theory Dark Matter” for the dark sector along this line of logic was already discussed in [44,45].

As for the total number of Weyl fermions (N) and their $U(1)_X$ charges (Q_i), anomaly-free conditions demand

$$\sum_{i=1}^N Q_i^3 = 0, \quad \sum_{i=1}^N Q_i = 0, \quad (1)$$

for the cancellation of the $U(1)_X^3$ anomaly and gravitational $U(1)_X \times [\text{gravity}]^2$ anomaly. If the theory allows the vectorlike fermions in the dark sector, the values of their mass parameters are naturally close to a cutoff scale.⁴ This point

⁴Of course, the mass of a vectorlike fermion is a free parameter and, in principle, it can be any value from zero to a UV cutoff of the theory. However, the small mass regime is less appealing from the theoretical point of view unless there is any reasonable supporting explanation like the restoration of certain symmetries in the zero limit.

makes the vectorlike fermions become irrelevant to the low-energy physics once integrating out heavy degrees of freedom is done. Thus, we take an option to exclude the presence of the vectorlike fermions in the dark sector. As DM is massive, this option automatically requires the breaking of $U(1)_X$.

In order to have the particle contents required for the DDM scenario, we realize that the minimum number of Weyl fermions in the dark sector satisfying Eq. (1) amounts to five.⁵ Apart from the gauge field and the five Weyl fields, we introduce two complex scalar fields for the spontaneous breaking of $U(1)_X$ and to make fermions massive. Among several possibilities for a set of $U(1)_X$ charges discussed in [44,45], we consider the following charge assignment:

$$\psi_{-9}, \quad \psi_{-5}, \quad \psi_{-1}, \quad \psi_7 \quad \psi_8, \quad \Phi_1, \quad \Phi_6, \quad (2)$$

where the subscripts denote $U(1)_X$ charges of each field. The first five are Weyl fields and the remaining two are complex scalars. One can see that the anomaly-free conditions in Eq. (1) are satisfied indeed by charges of Weyl fields. Writing the complex scalars as $\Phi_1 = \phi_1 e^{iA_1/V_1}/\sqrt{2}$ and $\Phi_6 = \phi_6 e^{iA_6/V_6}/\sqrt{2}$, we assume $V_1 \equiv \langle \phi_1 \rangle$ is greater than $V_6 \equiv \langle \phi_6 \rangle$ so that the breaking of $U(1)_X$ is induced dominantly by the condensation of Φ_1 and hence the gauge boson mass is given by $m'_A \simeq gV_1$, where g is the gauge coupling constant.⁶

The Yukawa coupling of the dark sector reads

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & y_1 \Phi_1 \psi_{-9} \psi_8 + y_2 \Phi_6 \psi_{-5} \psi_{-1} \\ & + y_3 \Phi_6^\dagger \psi_{-1} \psi_7 + \text{H.c.}, \end{aligned} \quad (3)$$

where $y_i (i = 1-3)$ are taken to be real parameters without loss of generality. We diagonalize the mass matrix for ψ_{-5} , ψ_{-1} and ψ_7 to obtain mass eigenstates. Together with ψ_{-1} , the following linear combination χ of ψ_{-5} and ψ_7

$$\chi \equiv \left(\frac{y_2}{\sqrt{y_2^2 + y_3^2}} \right) \psi_{-5} + \left(\frac{y_3}{\sqrt{y_2^2 + y_3^2}} \right) \psi_7, \quad (4)$$

forms a Dirac fermion $\Psi_{\text{DM}} = (\psi_{-1}, \chi^*)^T$ of the mass $m_{\text{DM}} \equiv \sqrt{y_2^2 + y_3^2} V_6$. Also, the other orthogonal combination to χ forms a massless Weyl field ξ

⁵Three Weyl fermions cannot satisfy Eq. (1) due to the Fermat’s theorem. Both cases of two and four Weyl fermions satisfying Eq. (1) give rise to vectorlike fermions, which is excluded in our option.

⁶To have values of V_1 and V_6 discussed in Sec. III, we suppress operators composed of both Φ_1 and Φ_6 . To this end, as an exemplary way to do suppression as we desire, in an extra dimension picture we assume that the two complex scalars are localized in the different branes separated by a distance along an extra dimension which is large enough for suppressing effective interactions induced at $3 + 1$ dimensional spacetime.

$$\xi \equiv \left(\frac{y_3}{\sqrt{y_2^2 + y_3^2}} \right) \psi_{-5} - \left(\frac{y_2}{\sqrt{y_2^2 + y_3^2}} \right) \psi_7. \quad (5)$$

From here on, we assume y_2 and y_3 are of a similar order and $y_2 \simeq y_3 \equiv y_*$ for simplicity.

III. A CONCRETE SCENARIO

In this section, we discuss how each resolution to phenomenological problems can fit into our model for the dark sector presented above. To have a simple thermal history of the dark sector, we first assume that the mass ($\simeq y_1 V_1$) of ψ_{-9} and ψ_8 is greater than the inflaton mass m_I so that we can neglect them in the cosmological history of the Universe.⁷

After the inflation ends, both the SM and the dark sector are reheated by the inflaton decay. As a $U(1)_X$ singlet, the inflaton Φ_I couples to both Φ_1 and Φ_6 through the following renormalizable operators⁸:

$$\mathcal{L}_\Phi = b_1 \Phi_I |\Phi_1|^2 + b_6 \Phi_I |\Phi_6|^2, \quad (6)$$

where $b_i (i = 1, 6)$ is a dimensionful coupling coefficient. Assuming quartic couplings $\lambda_1 \simeq \lambda_6 \simeq \mathcal{O}(1)$ for each of complex scalars Φ_1 and Φ_6 , we realize that decay of the inflaton to two ϕ_1 s is kinematically suppressed because of $m_I < m_1 \simeq \sqrt{\lambda_1} V_1$. Hence, for the dark sector, reheating is accomplished via inflaton decay to two ϕ_6 s. We define the decay rate to SM particles and the dark sector species (ϕ_6) to be $\Gamma_{\text{SM}} \equiv \Gamma(\Phi_I \rightarrow \text{SM})$ and $\Gamma_{\text{DS}} \equiv \Gamma(\Phi_I \rightarrow \phi_6 + \phi_6)$, respectively, and make an approximation $\Gamma_{\text{tot}} = \Gamma_{\text{SM}} + \Gamma_{\text{DS}} \simeq \Gamma_{\text{SM}}$. Unless a reheating temperature is so large as to be close to the Planck scale, ϕ_6 's form a dark thermal bath via scattering with one another among them. Thus, below we consider the evolution of the dark thermal bath comprising purely ϕ_6 s.⁹ Notice that the purity of the thermal bath is guaranteed due to the smallness of both y_* and $U(1)_X$ gauge coupling (g), which will be shown later.

The concrete thermal history that we will show below as a viable physical scenario is the following. After the reheating era, the dark sector begins with the temperature

$$T_{\text{DS}}(a_{\text{RH}}) \simeq 0.43 \times \left(\frac{m_{\text{DM}}}{1 \text{ keV}} \right)^{-1/3} \times T_{\text{RH}}, \quad (7)$$

where $T_{\text{DS}}(a)(T_{\text{SM}}(a))$ denotes the temperature of the dark sector (the SM sector) at the time of the scale factor a . a_{RH} is the scale factor at the reheating time, and

⁷We also assume that the reheating temperature T_{RH} is much less than the inflaton mass.

⁸In the extra dimension picture, taking the inflaton field Φ_I as the bulk field, we treat its coupling to Φ_1 and Φ_6 on an equal footing.

⁹In our work, we assume a negligibly small Higgs portal coupling between SM Higgs doublet and Φ_6 .

$T_{\text{RH}} \equiv T_{\text{SM}}(a_{\text{RH}})$ is assumed.¹⁰ From here on, whenever we encounter T_{RH} , we can convert it using $(a_{\text{RH}} T_{\text{RH}}) = (a_{\text{EW}} T_{\text{EW}}) \simeq 10^{-13} \text{ GeV}$ with $a_{\text{EW}} \simeq 10^{-15}$ and $T_{\text{SM}}(a_{\text{EW}}) \simeq 100 \text{ GeV}$. Afterwards, the temperature of the dark thermal bath continues to decrease as

$$T_{\text{DS}}(a) \simeq 0.43 \times \left(\frac{m_{\text{DM}}}{1 \text{ keV}} \right)^{-1/3} \times \frac{10^{-13}}{a} \text{ GeV}, \quad (10)$$

and becomes lower than the mass of ϕ_6 . The comoving number density of ϕ_6 is preserved until the time when the rate of ϕ_6 decay to a DM pair (ψ_{-1} and χ) becomes comparable to the Hubble expansion rate, i.e., $\Gamma(\phi_6 \rightarrow \text{DM} + \text{DM}) \simeq H$. Then, nonrelativistic ϕ_6 's start to decay to produce free-streaming DMs. The free-streaming of DM is ensured owing to the small y_* . From $\Gamma(\phi_6 \rightarrow \text{DM} + \text{DM}) \simeq H$, we infer the temperature of the SM thermal bath at this time

$$T_{\text{SM}}(a_{\text{FS}}) \simeq 537 \times g_{\text{SM}}(a_{\text{FS}})^{-1/4} \times \left(\frac{V_6}{1 \text{ GeV}} \right)^{-1} \left(\frac{m_{\text{DM}}}{1 \text{ keV}} \right) \sqrt{\frac{m_6}{1 \text{ GeV}}} \text{ GeV}, \quad (11)$$

where a_{FS} is the scale factor for the onset of the free-streaming of DM, $g_{\text{SM}}(a)$ is the number of relativistic degrees of freedom in the SM sector evaluated at the scale factor a , and m_6 is the mass of ϕ_6 . Here we used $m_{\text{DM}} \simeq y_* V_6$. Note that the decay of ϕ_6 and the onset of DM's free-streaming take place at the same time since ϕ_6 is nonrelativistic already at this time. For consistency, $T_{\text{DS}}(a_{\text{FS}})$ should be less than m_6 , which constrains $V_6 (\simeq m_6 / \sqrt{\lambda_6} \simeq m_6)$ for each m_{DM} . In Fig. 2, the region above the green dashed line satisfies $T_{\text{DS}}(a_{\text{FS}}) < m_6$. Because of the entropy conservation, on the other hand, we can write down the temperature of the SM thermal bath as

¹⁰To get Eq. (7), the following relation is used:

$$Y_{\text{DM}} \equiv \frac{n_{\text{DM}}}{s_{\text{SM}}} \simeq 4.07 \times 10^{-4} \times \left(\frac{m_{\text{DM}}}{1 \text{ keV}} \right)^{-1}, \quad (8)$$

where n_{DM} is the number density of the dark matter, s_{SM} is the entropy density of the SM sector, and we used the values, $\Omega_{\text{DM},0} = 0.24$, $H_0 = 70 \text{ km/sec/Mpc}$ [13], and $s_{\text{SM},0} \simeq 2.945 \times 10^{-11} \text{ eV}^3$ (entropy density today). Using the approximation $2n_{\phi_6} = n_{\text{DM}}$ at production time, one obtains

$$\text{Br} \frac{T_{\text{RH}}}{m_{\phi_1}} \simeq 2.7 \times 10^{-4} \times \left(\frac{m_{\text{DM}}}{1 \text{ keV}} \right)^{-1}, \quad (9)$$

where Br is defined via $n_6 = \text{Br} \times n_I \simeq \text{Br} \times (\rho_{\text{SM}}/m_I)$ with n_6 and n_I being the number density of ϕ_6 and inflaton (Φ_I), respectively.

$$T_{\text{SM}}(a) = \left(\frac{106.75}{g_{\text{SM}}(a)} \right)^{1/3} \frac{10^{-13}}{a} \text{ GeV}. \quad (12)$$

Equating Eq. (11) with Eq. (12) gives

$$a_{\text{FS}} = 8.8 \times 10^{-16} \times g_{\text{SM}}(a_{\text{FS}})^{-1/12} \times \left(\frac{V_6}{1 \text{ GeV}} \right) \left(\frac{m_{\text{DM}}}{1 \text{ keV}} \right)^{-1} \left(\frac{m_6}{1 \text{ GeV}} \right)^{-1/2}, \quad (13)$$

which will be used later for computing the free-streaming length for DM. After recombination, DM (Ψ_{DM}) gradually decays to a massless radiation ξ and a $U(1)_X$ gauge boson A'_μ serving as a daughter DM. On the other hand, due to a large redshift, DM gets into the motionless stage near the recombination era, completely behaving as a matter. Furthermore, its “would-be” temperature today without taking into account virialization due to gravity in the galaxy formation is sufficiently low to enable DM to be in a degenerate configuration.

In the following two subsections, we probe the parameter space of the model by applying the constraints on the lifetime of DDM and mass obtained in [13,36,37,39]. Eventually we show that there exists a parameter space where the desired physics can be realized in the model.

A. Constraints from the Hubble Tension

DDM solution to the Hubble tension is described by four parameters: lifetime of DDM, a fraction of a rest mass of DDM transferred to energy of the resulting radiation (ϵ), DM abundance today $\Omega_{\text{DM},0} \equiv \rho_{\text{DM},0}/\rho_{\text{cr},0}$ and a reduced Hubble parameter $h \equiv H_0/\text{Km/sec/Mpc}$ [13]. Below we refer to constraints on these parameters reported in [13] which were found by carrying out a Monte Carlo Markov Chain against the late universe measurements of the Hubble expansion rate for $0 \lesssim z \lesssim 2.4$. The underlying idea of the solution is to introduce a DDM whose decay leads to the production of a massless and a massive particle with four momenta $p_\mu = (\epsilon m_{\text{DM}}, \vec{p})$ and $p'_\mu = ((1-\epsilon)m_{\text{DM}}, -\vec{p})$, respectively. A decrease in $\rho_{\text{DM}} a^3$ after recombination due to the decay of DDM is compensated by an increase in ρ_Λ , which induces the earlier transition from the matter-dominated era to the dark energy dominated era. In the end, $H(a)$ evolves to a larger H_0 value than that resulting from CMB based on ΛCDM . As phenomenological consequences, enhancement of the late integrated Sachs-Wolfe effect for low ℓ regime ($\ell \sim 10$) can be induced regarding the CMB temperature anisotropy power spectrum [46,47]. Also, the reduction of $f\sigma_8$ in comparison with that from ΛCDM and suppression of the kinetic Sunyaev-Zel'dovich power spectrum was further discussed in [47] as impacts that DDM can make. (Here f is the matter growth rate, defined to be $f \equiv d \ln \delta_m / d \ln a$.)

In our model, we identify the Dirac fermion $\Psi_{\text{DM}} = (\psi_{-1}, \chi^*)^T$ generated from the decay of ϕ_6 with DDM, the

gauge boson of $U(1)_X$ with the resulting massive particle, and the massless Weyl fermion ξ with the resulting massless radiation. The dispersion relation of the resulting massive particle gives

$$\frac{m_{A'}}{m_{\text{DM}}} = \sqrt{1 - 2\epsilon}. \quad (14)$$

From the coupling of the Dirac fermion DM to the gauge boson, the decay rate of DDM can be read as

$$\begin{aligned} \Gamma_{\text{DM} \rightarrow A'\xi} &= \frac{9}{4\pi} \epsilon g^2 m_{A'} \left[\left(\frac{m_{\text{DM}}}{m_{A'}} \right)^3 - 2 \frac{m_{A'}}{m_{\text{DM}}} + \frac{m_{\text{DM}}}{m_{A'}} \right] \\ &= \frac{9g^2 m_{A'}}{2\pi} \times \frac{\epsilon^2 (3 - 4\epsilon)}{(1 - 2\epsilon)^{3/2}}. \end{aligned} \quad (15)$$

Applying 1σ level constraints $1.376 \times 10^{-43} \text{ GeV} \lesssim \Gamma_{\text{DM} \rightarrow A'\xi} \lesssim 1.04 \times 10^{-42} \text{ GeV}^{11}$ and $0.0013 \lesssim \epsilon \lesssim 0.229$ [13] to Eq. (15), we obtain the constraints on (g, V_1) which is shown as the yellow shaded region in Fig. 1. The right panel is the magnification of the left panel for the smaller coupling regime where not only the 1σ but also the 2σ level is shown as a green shaded region (the region above the green solid line).¹²

B. Constraints from the core-cusp problem

Because of DM's fermionic nature, provided that a thermal history of DM can allow for its degenerate configuration near the time of the structure formation, then our DM candidate may form cored density profiles in the Milky Way's dSphs satellites by balancing the attractive gravitational force with the repulsive Fermi quantum pressure. To see whether our DM can go through a sufficient redshift for achieving the degenerate configuration, we need to estimate DM's would-be temperature today, $\tilde{T}_{\text{DM},0}$, in the absence of a structure formation and compare it to a degeneracy temperature, T_{DEG} , for dSphs to check if $\tilde{T}_{\text{DM},0}$ is less than T_{DEG} of dSphs [37]. Starting with $T_{\text{DM}}(a_{\text{FS}}) \simeq m_6/2$, we obtain $T_{\text{DM}}(a_{\text{NR}}) \simeq (m_6 a_{\text{FS}})/(2a_{\text{NR}}) \simeq m_{\text{DM}}$ where a_{NR} is the scale factor when DM becomes nonrelativistic. Combined with Eq. (13), this yields

$$a_{\text{NR}} = 4.4 \times 10^{-10} \times g_{\text{SM}}(a_{\text{FS}})^{-1/12} \times \left(\frac{V_6}{1 \text{ GeV}} \right) \left(\frac{m_{\text{DM}}}{1 \text{ KeV}} \right)^{-2} \left(\frac{m_6}{1 \text{ GeV}} \right)^{1/2}. \quad (16)$$

Since $a = a_{\text{NR}}$, T_{DM} scales as $\sim a^{-2}$, implying that

¹¹The constraint on the DDM lifetime discussed in Refs. [48,49] is not applied here because DDM only decaying into dark radiations is assumed there.

¹²The lifetime of A' can be longer than that of Ψ_{DM} depending on y_2 and y_3 .

FIG. 1. Constraints on (g, V_1) space where g is the gauge coupling of $U(1)_X$. The right panel is a magnification of the left panel for the relatively smaller coupling regime. The yellow shaded region between the yellow solid and dashed lines is mapping onto the (g, V_1) space of constraints on the life of DDM and the ϵ parameter in the DDM solution to the Hubble tension [13] at 1σ level. (On the right panel, we showed the same mapping at 2σ level with the green solid line and green shaded region.) The blue shaded region corresponds to (g, V_1) values that produce a sub-keV decaying fermion DM as a solution to not only the Hubble tension, but also the core-cusp problem, satisfying 1σ level constraints.

$$\tilde{T}_{\text{DM},0} \simeq \frac{m_6 a_{\text{FS}}}{2a_{\text{NR}}} \left(\frac{a_{\text{NR}}}{a_0} \right)^2, \quad (17)$$

where $a_0 = 1$ is the scale factor today. Below we will see values of V_1 obtained from the free-streaming length constraint lie in $V_1 \simeq \mathcal{O}(10) - \mathcal{O}(100)$ GeV. Using Eq. (16) and Eq. (17) with $V_6 \simeq \mathcal{O}(10) - \mathcal{O}(100)$ GeV, we estimate would-be temperature today of sub-keV DM to obtain $\mathcal{O}(10^{-14}) - \mathcal{O}(10^{-11})$ eV. This corresponds to $\mathcal{O}(10^{-10})$ K– $\mathcal{O}(10^{-6})$ K, which is lower than the degeneracy temperature for dSphs $T_{\text{DEG}} \simeq \mathcal{O}(10^{-4})$ K– $\mathcal{O}(10^{-3})$ K [37]. Hence, it is confirmed that our DM is capable of achieving the degenerate configuration today when the heat-up due to the virialization is neglected.

As such, the mass of our DM is subject to the Tremaine-Gunn bound [50] which arises due to the presence of the maximum phase space density of the degenerate Fermi gas in a galaxy. When applied to dSphs, for a generic fermion DM, the logic underlying the Tremaine-Gunn bound gives $m_{\text{DM},\text{min}} \sim 70 - 300$ eV as a lower bound on mass [39,51]. Also, recently, a fitting analysis for the stellar kinematics of the relatively smaller dSphs (Leo II, Willman I, Segue I) set a conservative mass lower bound ~ 100 eV [52].¹³ In light of these results, we focus on the fermion DM mass greater than ~ 100 eV in our model. Combined with a constraint on the ϵ parameter in the DDM solution to the Hubble tension, Eq. (14) tells us that $m_{A'}$ is at least 70% of m_{DM} . Thus, we may approximate $m_{\text{DM}} \simeq m_{A'} = gV_1$

and apply $m_{\text{DM}} > 100$ eV to the (g, V_1) plane to improve the constraints obtained in Sec. III A. In Fig. 1, $m_{\text{DM}} = 100$ eV is represented as the blue solid line. Thereby for the right panel, the allowed parameter space at 1σ (2σ) level reduces to the subarea of the yellow (green) colored region which intersects with the region above the blue solid line.

Next, in order for the degenerate fermion DM to serve as a solution to the core-cusp problem, it cannot have too large a mass because the core size of the fermion DM density profile tends to decrease when m_{DM} increases. In this context, there have been efforts to constrain fermion DM mass by using the kinematic of dSphs. In [37], the use of a fermion DM density profile obtained by solving the Lane-Emden equation achieves a good fit to the stellar velocity dispersion of eight classical dwarf galaxies especially for the DM mass regime 100–200 eV.¹⁴ In addition, in [39], the use of the core size of the Fornax dSphs gives $70 \text{ eV} \lesssim m_{\text{DM}} \lesssim 400 \text{ eV}$ under the assumption of quasidegenerate Fermi gas as DM. In contrast, the relatively large fermion DM mass near 1–2 keV was pointed out in [36] by requiring the mass of dSph Willman I be greater than the minimum halo mass made up of the degenerate fermion DM. Given these arguments for m_{DM} , in this work we concentrate on sub-keV fermion DM mass regime, i.e., $m_{\text{DM}} < 1 \text{ keV}$. Similarly to the previous section, we apply $m_{\text{DM}} \lesssim 1 \text{ keV}$ to the (g, V_1) plane to get the blue dashed line and area below it in Fig. 1. As the final intersection of several constraints at 1σ level discussed so far, the blue shaded region in Fig. 1 is obtained, which allows our model to produce a *degenerate decaying*

¹³These small size galaxies are particularly important for studying the possibility of a degenerate fermion DM and its mass since an increase in the DM temperature due to virialization is minimized. For large size galaxies, the fermion DM would be present there in nondegenerate configurations, which makes inferring the fermion DM mass from the stellar kinematics of the large size galaxies more difficult.

¹⁴The Lane-Emden equation results from combining the continuity equation and the hydrostatic equilibrium equation with the Fermi pressure.

fermion DM resolving the Hubble tension and the core-cusp problem.¹⁵

Intriguingly, we observe that the blue shaded region (1σ) in Fig. 1 can cover the V_1 value up to 10^{12} GeV for the gauge coupling as small as $g \simeq 10^{-18}$. Besides, when the 2σ level constraints on the (g, V_1) plane are considered, even the V_1 value as large as 10^{13} GeV is allowed. As we mentioned above, V_1 is assumed to be larger than the inflaton mass m_I in the model. This implies that the model can be consistent with even the high scale inflation models such as chaotic inflation [53] or topological inflation [54] where the inflaton mass is $m_I \simeq 10^{13}$ GeV.

C. Constraints from the free-streaming length

Here we discuss constraints on the Yukawa coupling y_* as well as $V_6 (\simeq m_6)$. As a decay product of a nonrelativistic ϕ_6 , which was never in a thermal equilibrium with other species, our Dirac fermion DM (Ψ_{DM}) is expected to follow the delta-function-like distribution in its momentum space, centered on $|\vec{p}| = m_6/2$ at $a = a_{\text{FS}}$. From then on, DM becomes nonrelativistic around the time $a = a_{\text{NR}}$ given in Eq. (16). Bearing in mind $a_{\text{BBN}} \simeq 10^{-10} - 10^{-9}$, we realize that for sub-keV DM, $V_6 (\simeq m_6)$ around the GeV scale or greater than that makes DM still relativistic at the big bang nucleosynthesis (BBN) era. DM may contribute to the energy budget of the Universe at the BBN era as a radiation. Therefore, we apply $\Delta N_{\text{eff}}^{\text{BBN}} \lesssim 1$ [55] to the DM energy density at the BBN era to constrain V_6 . Using Eq. (8) and Eq. (13) with

$$\rho_{\text{DM}}(a_{\text{BBN}}) = \sqrt{m_{\text{DM}}^2 + \left(\frac{m_6 a_{\text{FS}}}{2a_{\text{BBN}}}\right)^2} Y_{\text{DM}}^{\text{SSM}}(a_{\text{BBN}}), \quad (18)$$

yields the parameter space in the (m_{DM}, V_6) plane satisfying the condition $\Delta N_{\text{eff}}^{\text{BBN}} \lesssim 1$. In Fig. 2, this corresponds to the region below the black dashed line.

Given the constraint on a value of V_6 around $\mathcal{O}(10) - \mathcal{O}(100)$ GeV (the region between the black and green dashed lines in Fig. 2), it becomes clear that sub-keV DM is produced from the ϕ_6 decay as a relativistic particle. So the next question that should be further asked is whether DM in the model can be consistent with Lyman- α forest data or not. Should the free-streaming length (λ_{FS}) of DM be too large, the structure formation at the scale below λ_{FS} would have been erased, likely to cause a disagreement with the nonvanishing matter power spectrum at the large scales. On the contrary, if λ_{FS} is too short (i.e., $\lambda_{\text{FS}} < 0.1$ Mpc), the DM model tends to produce too many satellite galaxies of Milky and Andromeda as opposed to the observed number. Therefore, we require $0.3 \text{ Mpc} < \lambda_{\text{FS}} < 0.5 \text{ Mpc}$ to further constrain V_6 [37,56–58].

¹⁵In the right panel, the intersection coming from constraints at the 2σ level is the subspace between the blue solid and dashed lines which overlaps the green shaded region above the green solid line.

FIG. 2. (m_{DM}, V_6) plane which is constrained by (1) $T_{\text{DS}}(a_{\text{FS}}) < m_6$ (region above the green dashed line), (2) $\Delta N_{\text{eff}}^{\text{BBN}} \lesssim 1$ (region below the black dashed line), and (3) $0.3 \text{ Mpc} < \lambda_{\text{FS}} < 0.5 \text{ Mpc}$ (region between the solid and dot-dashed red lines). The eventual intersection of these lies is in the red colored shaded region.

The free-streaming length is given by

$$\lambda_{\text{FS}} = \int_{t_{\text{FS}}}^{t_0} \frac{v(t)}{a(t)} dt \simeq \int_{a_{\text{FS}}}^1 \frac{(a_{\text{FS}} m_6 / H_0) da}{\sqrt{\Omega_{\text{rad},0} + a \Omega_{\text{m},0}} \sqrt{(2am_{\text{DM}})^2 + (a_{\text{FS}} m_6)^2}}, \quad (19)$$

where $v(t)$ is the DM velocity, and $\Omega_{\text{rad},0}, \Omega_{\text{m},0}$ denote the radiation and matter density parameter, respectively. The dependence of V_6 appears through a_{FS} in Eq. (13). We show the result of mapping $0.3 \text{ Mpc} < \lambda_{\text{FS}} < 0.5 \text{ Mpc}$ to the (m_{DM}, V_6) plane in Fig. 2 as the space between the two red solid and dot-dashed lines. The two red lines correspond to each of $\lambda_{\text{FS}} = 0.5$ and 0.3 Mpc . The eventual intersection in the (m_{DM}, V_6) plane reflecting four constraints discussed so far is shown as the red shaded region in Fig. 2. For DM with mass $100 - 300 \text{ eV}$, we found that there is no parameter space that allows DM to travel the smaller free-streaming length than 0.5 Mpc . In contrast, we found that there exists indeed a consistent parameter space for the mass regime $\sim 300 - 1000 \text{ eV}$. The strength of the Yukawa y_* associated with this V_6 regime reads $10^{-8} - 10^{-7}$. This may appear to be in tension with [37,39]. However, note that to resolve the core-cusp problem with $100 \text{ eV} \lesssim m_{\text{DM}} \lesssim 400 \text{ eV}$, either of a fully or quasi degenerate fermion DM configuration was assumed in [37,39] in the absence of considering a baryonic effect in classical dSphs. As was pointed out in [39], when additional effects such as baryonic feedback or nontrivial DM momentum distribution skewed to lower energies are taken into account together with Fermi repulsion, the upper bound of m_{DM} might be alleviated. Additionally, in our work, we expect the actual momentum distribution of our DM on production to have a broader width due to nonzero velocity dispersion of DM when produced and a center smaller than $m_6/2$, although we approximated

it as a delta-function-like peak with the assumption of instantaneous decay of ϕ_6 . This consideration may help some parts of $100 \text{ eV} \lesssim m_{\text{DM}} \lesssim 300 \text{ eV}$ in Fig. 2 produce the smaller λ_{FS} than we obtained in the present analysis.

IV. DISCUSSION

In this article, we present a particle physics model which describes a completely isolated dark sector lacking any way to communicate with the SM sector. The dark sector enjoys an Abelian gauge symmetry $U(1)_X$ which meets the anomaly-free conditions solely due to fields in the dark sector. We assume $U(1)_X$ without the kinetic mixing with $U(1)_{\text{EM}}$ in the SM. This setup trivially does not affect any known experimental result in the SM. With the purpose to keep the model minimal and consistent with the decaying DM solution to the Hubble tension, we introduced two darkly charged complex scalars and five darkly charged Weyl fermions along with the $U(1)_X$ gauge boson. Starting from an inflation model with inflaton mass smaller than the $U(1)_X$ breaking scale, we showed that one linear combination of two of five Weyl fermions with sub-keV mass can play a role of DM which starts to decay after recombination with the lifetime $\sim 35 \text{ Gyr}$. Within the parameter space of the model we found, the redshift that the DM candidate experiences since its production from the decay of a complex scalar makes the would-be temperature of DM today without virialization low enough to form a cored density profile for the Milky Way's dwarf spheroidal satellites. The main conclusion in this paper is based on the DM production mechanism. Although we restrict our discussion to the specific model, the similar conclusions regarding the free-streaming length of DM will be obtained as long as a DM production mechanism presented in this work is used.

As such, the DM candidate in our model is expected to serve as a solution to both the Hubble tension and the

core-cusp problem. Originating from the dark sector lacking a coupling to the SM sector, our DM candidate interacts with the SM particles only via gravitational interaction. We found that for resolving the Hubble tension, DM needs to decay via the gauge interaction with the coupling as tiny as $\sim 10^{-18}$ – 10^{-15} for having the required long life time. Interestingly, it turns out in the model that the strength of $U(1)_X$ gauge coupling is also linked to the a scale of consistent inflation models. Particularly, inflation models with an inflaton mass as large as 10^{13} GeV require the gauge coupling as small as 10^{-18} . For the sub-keV DM mass regime required to solve the core-cusp problem, this coupling strength is still larger than the strength of the gravitational interaction that DM does, i.e., $\sim m_{\text{DM}}/M_P \simeq 10^{-24}$. As for the sub-keV DM mass regime, we found that the DM mass allowed in the model is in a slight tension with what is required for a degenerate fermion DM solution to the core-cusp problem, which we believe can be alleviated by a more detailed analysis about the phase-space distribution of DM and the use of unknown baryonic effects on dSphs galaxy formation.¹⁶

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¹⁶Another possibility to alleviate the tension is to produce the fermion DM nonthermally by the inflaton decay. In this case, even the dark matter mass as low as $m_{\text{DM}} \simeq 200 \text{ eV}$ can achieve a short enough free-streaming length [37]. We apply a similar nonthermal DM production mechanism to a future study within the present framework [59].

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